

The Future for  
Honours Degree Courses  
in Mathematics and  
Statistics



# The Future for Honours Degree Courses in Mathematics and Statistics

Final report of a group working  
under the auspices of the London Mathematical Society

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The London Mathematical Society, Burlington House, Piccadilly, London W1V 0NL  
The Royal Statistical Society, 25 Enford Street, London W1H 2BH  
The Institute of Mathematics and its Applications, 16 Nelson Street, Southend-on-Sea, Essex SS1 1EF

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## Summary

A working group of mathematicians concerned with higher education was established in January 1991 and instructed to review the structure of first degree courses in mathematics. An interim report (June 1991) was widely circulated as a discussion document and this final report takes account of the responses.

We have found widespread agreement with the proposition that there is an urgent need to accommodate

- the impending changes in secondary education and in access to higher education;
- the continuing and rapid growth of mathematics and statistics;
- the national requirement for graduates who are as well-trained as those of other European countries.

We have also found widespread agreement that this can best be achieved by the introduction of four-year degree courses for some students. Accordingly we make proposals for the future of first-degree courses in mathematics and in statistics which may be summarised as follows:

- (1) that UK higher education institutions outside Scotland consider introducing four-year first-degree courses for *some* (notionally and nationally one-third of all) students;
- (2) that all courses begin with a two-year Part I providing a common core and appropriate as a DipHE qualification where needed;
- (3) that the three-year degree be re-designed to contain a little less material than at present so as to encourage interest and understanding;
- (4) that the four-year degree be designed to contain more material than is presently covered in three-year courses, but not so much as to inhibit proper understanding;
- (5) that in future higher education offer a variety of possibilities, including three-year first-degree courses, four-year first degree courses (of various kinds), and specialised MSc courses following three-year or four-year first degrees.

Our recommendations are formulated more precisely in Part V, pp. 13, 14. Discussion of how they could be implemented will be found in Part IV.

## The Future for Honours Degree Courses in Mathematics and Statistics

### PART I: INTRODUCTION

1. This is the final report of a working group which was established by the Council of the London Mathematical Society (LMS) in January 1991. Influenced by the report [3] on first-degree courses in physics, and asked by the North of England Mathematics and Statistics Forum to extend its discussions (see the summary in [2]) to the national scene, Council of the LMS accepted the need to provide for a review of the structure of first-degree courses in mathematics. Pressure arises from changes in school curricula, over-crowding of undergraduate syllabuses, and the need to produce graduates at a level of education comparable with those from universities of other European countries. The group, whose membership is listed in Appendix B, was asked to promote wide discussion of the structure of first-degree courses and to report on what consensus there might be, particularly in relation to the introduction of new four-year courses.

2. The name 'Mathematical Sciences' is nowadays used to indicate a great range of intellectual activity. The scope of this report is a subset of the mathematical sciences, namely the areas covered by degrees in Mathematics, in Statistics or in a mixture of Mathematics and Statistics. In terms of admissions to universities, it may be described as the great majority of the courses classified by UCCA as group G, but excluding G5 (Computer Studies).

3. The pattern of secondary and tertiary education in Scotland with 'Highers' and four-year honours degrees, differs substantially from what is usual elsewhere in the UK. Moreover, secondary education in Scotland is the subject of an independent and wider review by a committee set up by the Secretary of State for Scotland under the chairmanship of Professor J. M. Howie, which is expected to report early in 1992. We have therefore excluded Scottish degrees from our main discussion. Nevertheless, we hope and believe that much of this report may apply with appropriate modifications to the Scottish scene.

4. The working group held seven meetings between March and June 1991. In order to promote discussion. It circulated an interim report [1] to all university and polytechnic departments of mathematics, and to professional bodies. By the middle of November a substantial number of responses had been received (73 written and several oral communications), including comments from over three-quarters of UK universities, from the Conference of Polytechnic Heads of Mathematical Sciences as well as from five individual polytechnics, and from the Conference of Professors of Applied Mathematics, the Edinburgh Mathematical Society, the Mathematical Association, and the Institute of Mathematics and its Applications. There had also been a joint meeting of the LMS and the Royal Statistical Society (RSS), which, being in the standard form of an RSS discussion meeting, naturally focussed attention on issues of particular interest to statisticians. The working group held three further meetings to consider the responses and the best way forward. This report summarises the arguments for change and the reasons for proposing four-year first-degree courses for some students, and makes proposals for course structures to implement the recommended changes. The paper is self-contained, except that some of its arguments are given in more detail in the interim report [1] of June 1991.

5. Since the working group was set up, several major reports have been published which are relevant to its work. The White Paper on Higher Education [5] indicated the Government's plans for a rapid expansion of student numbers over the next ten years, with consequences for entry levels and course provision. The report [8] of the Science and Engineering Research Council (SERC) panel chaired by Sir John Kingman, while concerned mainly with research, drew attention to the needs of the higher-education system for recruitment of academic staff at PhD level, with implications for postgraduate student numbers. In [6], the Advisory Council on Science and Technology (ACOST), a body charged with advising the Government on scientific and technological issues, recommended a substantial re-design of degree courses in physics, chemistry and engineering, including an optional fourth year of study leading to MPhys, MChem and MEng degrees respectively; it suggested that 30% to 40% of students might take either the fourth year or an existing MSc course. The Royal Society report [7] supports the recommendations of the Institute of Physics (IOP) report [3] and commends its reasoning as applicable to all branches of science.

6. The working group was originally asked to discover what consensus there may be about the structure of honours degree courses in mathematics in universities. As is indicated in §5, the calendar year 1991 saw the publication of a spate of plans for the development of Higher Education and research. This led to pressure for the group to consider statistics as well as mathematics, and polytechnics as well as universities. We have discovered that, even in this wider context, there is a substantial amount of agreement. Since our brief was to report on what consensus there might be we have restricted our report to those areas where we have found almost universal agreement. These include

- what we had reported about developments in secondary education and their consequences for higher education;
- appreciation of the continued (and accelerating) growth of mathematics;
- appreciation that UK degrees should have the capability of producing graduates competitive with those of continental Europe;
- a commitment to introduce four-year honours degrees for some students.

**Our conclusions and recommendations are formulated explicitly in Part V.**

These have been modified from those of the interim report [1] to take account of responses and to concentrate on matters on which a very large majority of respondents could agree.

## PART II: THE CASE FOR CHANGE

7. The pattern of three-year honours degree courses (four-year honours courses in Scotland) in the British higher education system has served well over a long period of time. Nevertheless, the scene of which it is a part has changed greatly over the years and is expected to change rapidly in the coming decade. Three main influences have been identified:

- impending changes in secondary education and in access to higher education;
- the continuing growth of mathematics both in depth and in breadth, leading to the danger of overloading of undergraduate courses in mathematics;
- the need for comparability with degree qualifications from the universities of continental Europe.

All these provide reasons why the structure of degree courses in mathematics and statistics should be reviewed as a matter of urgency.

### CHANGES IN SECONDARY EDUCATION AND IN ACCESS

8. The syllabuses of GCE A-levels and AS-levels are likely to undergo substantial change as a consequence of the introduction of GCSE and the National Curriculum. The GCSE examination in mathematics was designed to test not only the mastery of a body of knowledge, but also problem-solving and practical work. Its expected positive effects include the acquisition of new skills of questioning and investigation by the students. Many teachers remark, however, on the problems faced by students embarking on sixth-form mathematics studies straight after GCSE. At present A-level courses do not build on the strengths of GCSE, and there is a gap in content between the National Curriculum and the beginning of A-level. Possible changes in A-level are likely to involve a reduction of the common core, increased use of programmable calculators and computers, and also a reduction of expertise in algebraic manipulation, proof, calculus, geometry, etc. Alongside changes in syllabus, different styles of teaching which are currently being introduced in schools will have a considerable impact. Thus the changes in the background of students entering our departments will be in knowledge, skills and attitude. Matching adjustments of course content and teaching style in higher education are essential.

9. Until quite recently the majority of students entering honours degree courses in mathematics had studied two separate mathematical subjects at GCE A-level, but there has been a trend away from 'double-subject' mathematics in the sixth form [see 1, Appendix A]. Although Pure Mathematics, Statistics and Applied Mathematics offer just as wide a range of intellectual experience as combinations such as Economics and Business Studies or French and German, hard-pressed teachers working in sixth forms and FE colleges are increasingly starved of the time and resources needed to teach double-subject mathematics. It is likely therefore that before long the great majority of mathematics students will arrive with only one A-level in the subject.

10. There will also be changes in access to higher education, and the broadening of its role. In 1960 only about 8% of each age-group was offered an education to honours degree level; in 1988 it was about 14%; government plans are to increase the age participation rate for higher education beyond 30% before the end of the present decade [see 5, §13]. Courses can and must be designed for a wide ability-range. What

is needed is the flexibility to offer courses of different kinds and levels to meet the needs both of the students and of their future employers.

11. Fewer than 5% of the students who pass an A-level examination in mathematics each year enter higher education to study for an honours degree in the subject. A-level mathematics is a general qualification sought by a large number of students, and cannot be designed primarily for those heading for degrees in mathematics. The future of post-16 education is far from settled. Nevertheless, the pressures for substantial change, and most of the directions of change, are already clearly visible. The future single A-level may contain little more than the currently agreed common core, amounting to approximately 60% of the present syllabus. The aim must be to design degree course structures in mathematics and statistics that will be capable of accommodating these imminent changes.

#### THE GROWTH OF MATHEMATICS

12. Mathematics, in company with other subjects, has grown enormously in the twentieth century and continues to grow in a rapid and healthy way. Most of our graduates are appreciated by employers simply for the fact that an academic education in the subject produces a clear-thinking, numerate mind and an ability to cope with a wide range of problems. Many, on the other hand, are valued in industry, commerce and higher education for the specific knowledge and skills that they have acquired in their studies. Because of advances in mathematics and increasing technical demands on many graduates it is already doubtful whether the level of training required by this latter group can be packed successfully into the traditional three-year course.

13. It is also doubtful whether such a level of training *ought* to be packed into a three-year course. In its report [6] ACOST advises that courses in physics, chemistry and engineering give:

... insufficient emphasis on the development of critical skills such as originality, conceptual thinking, written communication, presentational skills and self-confidence. ...

A number of respondents to the discussion paper [1] indicated a belief that the same comment applies to courses in mathematics.

14. As mathematics and its applications have grown both in breadth and depth, so new topics have been introduced into degree courses. For example during the last forty years, areas such as functional analysis, mathematical logic, topology, mathematical biology, non-linear dynamical systems, numerical analysis, modern applied mathematics and mathematical physics, mathematical statistics, combinatorics, optimisation theory, theoretical computation have been introduced either as compulsory elements or as options. Some of these are precisely the topics that are motivated by applications in economics, computation, business administration and other areas of importance in commerce; others are the key to much industrial technology and biotechnology. Mathematicians are required wherever it is necessary to model physical phenomena on large computers. Although many older topics have been pruned from the curriculum to make way for new, the serial nature of the subject imposes limitations on the amount that can be cut. Compared with fifty years ago, students have more work to do to master the ideas and the structure of modern mathematics which enable them to see the potential for further advances and new applications. We are well aware that the number of students who proceed to doctorates and careers in research in industry or higher education is relatively small,

and that it is quite unreasonable to allow the needs of this group to distort the general pattern of higher education. Nevertheless, a first degree course should expose our students to some of the exciting new ideas and concepts of recent mathematics and statistics.

#### COMPARISON WITH CONTINENTAL EUROPE

15. The need for comparability of UK degrees with those awarded on the Continent and elsewhere has become urgent. It is clearly in the national interest that British universities should produce mathematicians competitive in quality and training with those produced by our partners in the Community. There are already indications that first degrees in England, Wales and Northern Ireland are not of as high a standard as those in many other European countries where, in most cases, a first degree course lasts at least four years (see [1, Appendix E]). We note that in Germany it is usual to take six years or more to reach the *Diplom*. A university course in France is divided into three 'cycles', the first two of which are undergraduate. The first cycle can be completed in two years and leads to the *Diplôme d'Études Universitaires Générales*. The second cycle consists of the *Licence* (year 3) and the *Maîtrise* (year 4). It has become increasingly difficult to maintain comparability of standards within the three years of a degree course while the corresponding courses in Europe are of much longer duration and are no less well taught. We must ensure that our graduates are not disadvantaged in the Community of which we are a part.

16. Student mobility is another important consideration. Some institutions in the UK are already offering the option of four-year courses in connection with ERASMUS exchange schemes, with the third year of the course spent in another European country. There is also growing pressure for students who have attended school in one European country to attend a university in another. Such facilities, which are in accord with Government policy and must be expected to grow in the foreseeable future, increase the need for comparability between UK degrees and equivalent qualifications from higher education institutions elsewhere in Europe.

#### THE PRESENT POSITION

17. Students entering degree courses have been successful in the mathematics that they studied at school. They have experienced satisfaction and pleasure in their ability to solve problems and to some extent they appreciate the beauty, power and utility of mathematics. When these students enter the first year of an honours degree course two problems emerge.

(a) The background of students is very varied. Different schools may have taught the core syllabus in different styles and with differing levels of thoroughness, and students entering higher education with a qualification in mathematics may have studied different branches of the subject. They may, for example, have chosen to specialize in mechanics or in statistics. In these topics first-year higher education courses therefore need to cope with students having very different levels of knowledge.

(b) Few students begin with any idea of the nature of the mathematics they will meet in an honours level course. Although the subject has the same name as the one they studied at school, its approach is significantly different. In pure mathematics students are expected to deal with a high level of abstraction and to understand the important role played by precise logical argument and proof. In applied mathematics and theoretical physics the relationship between the

perceived physical world and a mathematical model is much more subtle and complicated than might have been anticipated. Statistical inference is an unexpectedly new and sophisticated mode of reasoning. Naturally, some students are more successful than others in coming to terms with the nature of degree-level mathematics. There is a reasonable correlation between success and a student's school performance, but some well-qualified students experience considerable difficulties, and some apparently weak students are very successful in adjusting to the new mathematics.

18. In their second year many students develop a preference for one or other of the different broad areas of mathematics, with their very different styles, but the majority should still be sampling rather widely. What is offered to final-year students is choice from a wide range of options, many of which are in-depth presentations of special topics. In the past it had been hoped that some of these might bring students within range of the frontiers of research but this is increasingly unrealistic. Indeed, it has become clear that a three-year course can no longer be adequate, without further advanced training, as a preparation for research in mathematics. There are some students for whom mathematics has begun to be a real struggle in the second year, and for whom the third year is merely something to be survived for the sake of the degree qualification. But for the majority, even for many of those who have found second-year mathematics difficult, the chance to concentrate on chosen specialisms in the third year provides real pleasure and excitement.

19. We firmly believe that there is a need to increase the number of students who choose to study mathematics and statistics and that our subject can be made attractive enough for that to happen. However, there is a danger that, unless our courses are changed, too high a proportion of these extra students may fall into the category of 'strugglers'. There is already a tension between the wish to provide experience of the best of modern mathematics and the clear need to design our syllabuses to be within the range of students' background and abilities; between the ambition to take students to the point where they may be shown the frontiers of research and the requirement that our teaching be appropriate to the needs of the majority. This is another reason for re-thinking the structure of our degree courses.

20. The development of mathematical education must reflect changing perceptions of training within the wider European framework. Creativity, analytical thinking, ability to work in a team, independence and communication skills are cited as particular qualities needed to cope with rapid change. Above all, initial education must be a basis for continuing education through a career which may change several times. The balance between specialization and the breadth needed for this adaptability is critical in mathematics degree courses.

### PART III: THE CASE FOR THREE-YEAR AND FOUR-YEAR DEGREE COURSES

#### POSSIBILITIES

21. One possibility would be to make no planned changes but to let existing three-year courses evolve as the need arises. In view of the considerable problems described in Part II above, this could only lead to a substantial decline in the standard of the first degree. It is clear that what is needed is greater flexibility than honours degree courses have offered hitherto – flexibility to cope with a differently trained intake, and flexibility to offer a larger menu of degree options appropriate to a broader ability range and the widening needs of employers.

22. The four-year degree course provides that flexibility. It would enable higher education departments to adapt to changes in the mathematical background of students at entry, to lessen the pace of the course as a whole so that students might achieve a broader and deeper understanding, and to include in the final year at least some introduction to current developments. In Scotland the four-year course for honours is normal. In the rest of the UK four-year courses are normal in some subjects (modern languages because of the year abroad, subjects allied to medicine, engineering, education, social work), and are possible in many other subjects at a number of universities. Even in mathematics the concept is not new. The majority of polytechnic mathematics degrees are four-year sandwich courses, the normal pattern being 2 + 1 + 1, with the third year spent undertaking professional training. Similar structures are available also at a few universities, and, as has been noted in §16, there are some four-year university courses connected with ERASMUS schemes.

23. We do not see four-year degrees as appropriate for all honours students, however. With widening access it is by no means clear that a four-year degree course would be attractive to, or educationally desirable for, everyone studying mathematics. What is required is a new course structure that will satisfy the needs both of those students who seek a good intellectual education through mathematics but who do not need specialised mathematical knowledge, and of those who have the ambition to pursue careers as professional mathematicians. We must emphasize that this means a system whereby some students graduate after three years, others after four. For present purposes we suggest that a notional figure of perhaps one-third of students nationally choosing the four-year option, with substantial local variation from place to place, would not be unrealistic.

24. We have examined two natural ways to design courses that offer the choice between three-year and four-year degrees:

- the 3 + 1 structure;
- the 3 *or* 4 structure.

By 3 + 1 we mean a scheme whereby all students share the first three years, and some stay for a fourth year. This is the structure proposed by the Institute of Physics, which has attracted substantial, though by no means universal, approbation from Physics departments nationally. By 3 *or* 4 we mean a scheme in which the three-year and four-year courses diverge at an earlier stage – in practice this is likely to mean that for many students the choice will be made at the end of the second year of study. Thus 2 + (1 *or* 2) would be a more appropriate, if less concise, mnemonic for the second structure.

25. Discussion has included the case of a 1 + 3 structure, meaning a three-year honours course preceded by a one-year ‘access’ course. Such schemes already exist. The purpose of the preliminary year is to prepare students who have inappropriate or unusual qualifications for entry to degree-level courses, and so to widen access. Insofar as the access year may be more a matter for the secondary and further education sectors than for higher education, and as the primary concern of the working group has been the structure of first-degree courses given some appropriate entry qualification, we have not pursued this matter. Nevertheless, we warmly welcome arrangements which lead to more well-motivated students entering mathematics degree courses.

#### THE MATHEMATICS GRADUATE

26. The requirements of industry and commerce for graduate mathematicians have not been assessed directly, but demand appears to be increasing. Mathematics and statistics graduates have the numeracy and the computer literacy that provide a sound basis to which vocational skills can be added. The need for mathematicians to be able to communicate and work with engineers and applied scientists in high technology industry is certain to increase. The report [4], for example, puts forward a number of ways of reducing student overload in first-degree courses in engineering, including a suggestion to “Teach only the mathematics ... applicable to their chosen kind of engineering degree” and a proposal to “Reduce ‘analytical theory’”. It is accepted in manufacturing and business that training is required throughout a professional’s career. Where mathematics is not covered in engineering degrees it will need to be supplied by mathematicians – but by mathematicians who know how to communicate with technologists. In fact, there is already a need in industry for well-trained mathematicians with a sufficiently deep and wide range of knowledge to enable them to develop the applications of mathematics to major technological problems. In mathematics the required training may well be identical to that provided in final-year options in first-degree courses, or in MSc courses. It should be noted that the Kingman report [8, (paras 5.23–5.25)] concluded that there was a pressing need to increase the number of PhDs in mathematics, both to maintain staff levels in higher education, and to supply research agencies, industry and commerce. Appendix A (§A4) gives outline information about careers taken up by mathematics graduates in recent years.

27. In proposing a revised structure for honours degrees in Mathematics, we need to understand the aims of the three-year and four-year courses. Like many other subjects, mathematics can be studied both for its value as a general training of the mind, and for the specific skills and knowledge which it provides.

(a) Many graduates in mathematics seek careers in areas such as accountancy, management, administration, sales, where they need analytical and problem-solving skills and a high level of numeracy and conceptual thinking. The three-year degree should be designed to develop these attributes. Since it should not be distorted by the need also to train future research workers and specialists, it ought to provide an even better basis for careers in these areas than the mathematics degree does at present, and quite as good as degrees in other subjects.

(b) Other graduates will choose to work as practising mathematicians in industrial research groups or higher education, or as professional statisticians, in fields where specific and broadly based advanced mathematical knowledge and

techniques are essential. The four-year degree should be designed with these in mind.

Of course students do not have their career plans fully worked out at the start of the degree course, and it is important to provide enough flexibility to enable them to choose an appropriate programme as their interests develop. Generally speaking, however, students aiming at careers which do not depend on specific knowledge of advanced mathematical technique would be likely to choose the three-year degree, while those aiming to work in specialized mathematical fields would normally choose the four-year course. We must emphasise, however, that this really is a generalization, and that exceptions should not be rare.

#### PART IV: DEGREE COURSE STRUCTURES

##### COURSE STRUCTURE

28. As indicated in §24, the working group considered two possible course structures for first degrees in mathematics. For brevity these are referred to as the 3 + 1 and the 3 *or* 4 structure. Both should start at a point appropriate to the average school-leaving background as measured by GCE A-level achievement. In outline, the 3 + 1 structure would consist of a three-year degree course for all students, aiming to provide a good understanding of rather less material than existing courses, followed by a fourth year on advanced topics for those who wished to undertake more specialized study of mathematics. The 3 *or* 4 structure would begin with a two-year core course for all students and would then divide into two alternative paths: one would be a third-year programme in mathematics and related subjects leading to an Honours degree, for the majority of students; the other would be an integrated programme over two years, developing more specialized topics for those with the desire to pursue the subject further. The skills and knowledge required for different careers vary widely and the 3 *or* 4 scheme is designed to provide the flexibility to offer an appropriate mathematical education, with emphasis on the most appropriate training, to a wide range of student interests, abilities and ambitions. Our interim report recommended the 3 *or* 4 year structure on academic and educational grounds, because courses would be planned in a coherent way to suit the interests and aptitudes of the students.

29. As was to be expected, considerable differences in the circumstances of mathematics departments across the country, both in the courses they offer and in the resources they have available, were clearly reflected in their responses to the discussion paper and their perceptions of our recommendations. Nevertheless, there was near unanimity on the need to provide four-year degrees for some students and to maintain three-year degrees for others. We therefore recommend that UK Higher Education institutions outside Scotland consider introducing four-year courses for first degrees in mathematics and in statistics as soon as possible.

30. A number of responses to the interim report asked about the relationship of our proposals to existing four-year degrees. Successful four-year programmes, such as sandwich courses and degrees that include a year of study in some other European country, should be unaffected. There is no suggestion that they might need to be expanded to five years. We must make it clear that what is proposed is simply that further four-year options be introduced; that students who at present are catered for by

conventional three-year honours degree courses should in future have the possibility of a four-year course.

31. The comments received on our interim report were generally in favour of the 3 or 4 year structure, for the reasons that we had given. A small but significant minority of respondents foresaw difficulties with it, however, and expressed a preference for the 3 + 1 scheme. Most of the problems which they envisaged were in three areas. First, the 3 or 4 scheme would require students to choose between the three-year and four-year programmes at the end of their second year, which would reduce flexibility for those whose interests changed later and would make modular course structures more difficult to operate. Secondly, the resources needed to provide two alternative third-year programmes might not be easily found, particularly in small departments. Thirdly, the four-year structure was seen as conflicting with the provision of MSc courses, such as the well-established courses in statistics. Although we maintain our belief that, where it can be provided, the educational advantages of the 3 or 4 structure weigh heavily in its favour, we see no need to be prescriptive and it therefore seems appropriate to recommend varied provision of undergraduate mathematics courses. The flexibility that is required will be achieved if the majority of departments follow the 3 or 4 model, some introduce a 3 + 1 scheme, some continue with specialized MSc programmes in place of a fourth undergraduate year, and some continue with their well-established sandwich or year-abroad courses in which the year spent learning mathematics in a different environment enhances a student's programme in a different way. In all courses we believe that there should be greater emphasis on critical skills, written and oral communication, and ability to work independently, in line with the ideas of the ACOST report [6, Chap. 4].

#### THE FIRST TWO YEARS

32. There is a clear consensus for our recommendation that the first two years should constitute a Part I with common standards maintained as far as possible across all higher education institutions. It should provide a common currency so that students could transfer between institutions at the end of the second year if their interests developed in a direction which was not covered in their first institution. It could also provide a suitable basis for the award of a DipHE, so that students whose interests moved away from mathematics, or even from academic work entirely, would be able to complete their education early with a recognised qualification, and proceed to employment or to some other kind of training.

33. Many responses expressed anxiety about standards and content of future GCE A-levels. It seems to be accepted that the time is coming, if it has not already arrived, when higher education will need to take responsibility for introducing students to the idea of proof and to a number of techniques of algebra, calculus, geometry and applied mathematics that used to be taught and learned in schools. Furthermore, the expansion of higher education which is envisaged by the White Paper [5] of May 1991 will lead to a greater diversity of entrance qualifications. The White Paper proposes that the age participation rate for higher education will rise from about 20% in 1990 to 30% in 2000, fuelled partly by significant increases in the numbers of mature students and of students with a background of vocational courses instead of GCE A-levels. If mathematics and statistics are to participate in, and contribute to, this expansion, departments will need to review the content and structure of their courses, and to introduce more flexible patterns of teaching [5, §16]. In the light of these predictions, we find that it is also accepted that the aim of Part I should be to

bring students to the point where they have a sound understanding of the amount of material currently reached by about the middle of the second year of present-day degree courses.

#### THE THREE-YEAR DEGREE

34. The content of the final year of the *three-year* degree should be chosen to give students general mathematical experience, together with an overview of mathematics. We envisage that the three-year course will focus on the potential of diverse branches of mathematics. Emphasis will be less on the rigorous and linear nature of the more technical parts of the subject than on perspective and problem solving. There might be a substantial element of project work, with the possibility of developing students' communication skills. The course should be enjoyable and should promote a positive attitude to mathematics. In many departments, these possibilities could be made available through a modular structure giving great flexibility, subject perhaps to a rule that two-thirds of the courses should be 'mathematical'. As discussed in §27 the course should be designed to qualify students for a wide range of employment. It would also prepare students for further study in courses (such as the PGCE and certain MSc programmes in Applied Statistics, Mathematical Biology, Operations Research, Computation, etc) which make use of mathematics in other areas. The level of mathematics should not be far beyond that currently reached at the end of the second year, that is to say, the quantity of material should be reduced by the exclusion of the specialist courses currently available in the third year at some universities. On the other hand, it could be more adventurous than many present second-year courses, and the level of understanding aimed for and achieved should be significantly higher than at present. The three-year degree should be the equivalent of Honours degrees in any other subject, and it should be classified and externally moderated as at present.

#### THE FOUR-YEAR DEGREE

35. We see the *four-year* degree as laying the foundation of basic advanced mathematics which is necessary for applications of, and research in, a rapidly growing subject. In the 3 or 4 structure the content of the last two years of the four-year option should be planned as a whole, separately from the three-year degree, although for reasons of economy there might be some elements available to both. There would be project work, specialist options, and core courses in areas considered central for those who expect to use their mathematical knowledge in industry, business, or in research. The level of mathematics taught in the four-year degree should be beyond that currently reached at the end of the third year, and the level of understanding achieved by students should be higher than at present. The degree should be classified and examination standards should be externally moderated as at present. All students who aim to become professional mathematicians, as distinct from professionals who use a good background of mathematics in other areas, would be expected to choose the four-year course.

#### JOINT HONOURS DEGREES

36. First degree courses involving mathematics are at present of four main types:
- (a) three-year Honours courses, with most of the time spent on mathematics;
  - (b) sandwich courses, involving three years of mathematical study with a break of one year's work in a mathematically-based job;

(c) three-year Joint Honours courses with half the time spent on mathematics, usually on courses taken from the full Honours programme but covering a restricted range of topics;

(d) courses combining mathematics with other subjects in varying proportions (usually between one-third and two-thirds), for example mathematics and business studies.

The majority of undergraduates take courses of type (a); a small percentage (a little below 10%) take courses of type (b); the numbers choosing types (c) and (d) are increasing, although it is difficult to obtain precise figures because of problems of definition. The expansion of higher education mentioned in §33 is likely to lead to increased demand for more varied provision. The problem of designing mathematics courses for a diverse population of students is particularly acute because, being of a 'serial' nature, the subject needs a continuous development, with careful planning of pre-requisites and of the sequence of courses. However, most departments already have experience of meeting the demand for flexibility by providing Joint Honours courses in which certain areas of mathematics can be studied to a high level without requiring the student to take the full degree programme. Thus it is possible to envisage the degree course as a number of streams in different areas of mathematics, from which several degree programmes can be built up without duplication of teaching. (This structure would fit well with a modular degree scheme.) In their Part I courses, departments would provide full coverage of basic material in the first two years, from which certain courses could be selected for Joint Honours students as at present.

37. The mathematical programme for Joint Honours students would combine a number of courses selected from the full degree programme, giving a continuous development of a limited range of topics to final-year standard. It would be possible to arrange four-year programmes if the other subject of the Joint Honours degree was organized appropriately, for example, if it had a suitable modular structure.

#### STUDENT AND DEPARTMENTAL RESOURCES

38. There are well established procedures for funding students on existing four-year undergraduate programmes. The nearest equivalent to our proposals for mathematics are MEng degrees. Here students register initially for a course that is specified to be of three or four years' duration and must decide by the end of their second year of study which option they will follow.

39. To understand the resource implications of the proposed degree course structure we need to estimate the number of students who would want the four-year option. It is difficult to predict this with any confidence, particularly when the expansion of higher education is likely to result in a student body with a wider range of interests and abilities than at present. As we have indicated earlier, probably the best estimate is about one-third, which is in the range proposed by the ACOST report [6] for students taking four-year courses in science and engineering. Setting aside increases in the age participation rate for higher education, this would indicate an increase of little more than 10% in the overall number of students on course at anyone time. The extra resources needed to provide a four-year programme will depend on the number of options offered, the extent to which course units could be planned for more than one audience, whether the fourth year was intended to supersede existing MSc programmes, and by how much student numbers are planned to increase. All these factors vary from institution to institution. However, we believe that the arguments of

Parts II and III are strong enough to justify additional support for the introduction of four-year programmes for a limited number of students.

#### PART V: CONCLUSIONS

40. Very few responses to the interim report were hostile to, or even neutral about, the suggestion that four-year degrees in mathematics and in statistics were now needed. Accordingly we can report that there is widespread support for

**RECOMMENDATION 1: UK Higher Education institutions outside Scotland should consider introducing four-year courses, alongside their established three-year courses, as an option for some students reading for a first degree in Mathematics or Statistics.**

There was considerable but by no means universal support for the view expressed in the interim report, that the educational advantages of the 3 *or* 4 structure made it preferable to the 3 + 1 structure (see §24 for definitions). A significant minority of respondents saw practical advantages to the latter scheme, especially in institutions where modular course structures are operational or planned. Although national uniformity might be thought desirable, there is also a strong case to be made for variety, and variety is inevitable within so large a system as Higher Education in Britain. Some respondents enquired about the position of established four-year sandwich courses and year-abroad courses. We recommend that these should continue as they are.

41. Some common ground in the first two years of first-degree courses will nevertheless be necessary if qualifications are to be comparable and student mobility is not to be inhibited. The interim report recommended a two-year 'Part I' that would provide a good foundation in degree-level mathematics, upon which both the three-year and the four-year courses would be based, and could, perhaps, also provide a satisfactory completion point offering a qualification at the DipHE level. This 'Part I' was to be designed to bring all students to a point where they would have a sound understanding of the amount of material currently reached by about the middle of the second year of an honours degree course. There was little dissent from this and we can report widespread support for

**RECOMMENDATION 2: The first two years of a first-degree course should lead to a 'Part I' examination, with common standards maintained as far as possible across all Higher Education institutions.**

42. What is needed throughout is to concentrate on the quality of learning as well as the quality of teaching. A common criticism of present-day Honours courses is that too much material is packed into the syllabuses, and students have too little time to think about it. There is widespread support for

**RECOMMENDATION 3: The three-year degree course should be newly designed to contain somewhat more material than is presently covered in the first two years of three-year Honours courses.**

This degree should be an excellent mathematically based intellectual education, of the same standard as the Honours BSc or BA in any other subject.

43. To provide some consistency with subjects like Engineering and Physics there is a case for giving the four-year degree a distinctive name like MMath. In order that it might provide a satisfactory foundation for postgraduate work and for research (in HE

or in industry), and in order that it might be comparable with degrees offered in other European countries the four-year degree should provide students of mathematics and statistics with broader and deeper knowledge, and a fuller understanding of their subject than can be achieved at present. There was widespread support for

**RECOMMENDATION 4: The four-year degree course should be designed to contain more material than is presently covered in three-year courses. The increase in material should not be so great as to overcrowd the syllabus.**

The MMath should be rather more broadly based than what is presently represented by BA/BSc + MSc.

44. A number of responses to the interim report raised the question of the future of well-established MSc courses. It is clear that these courses provide a more highly focussed and specialised training than is envisaged in the recommended four-year MMath degree. It is also clear that the need for these courses, particularly in areas such as Applied Statistics, Applied Analysis, Mathematical Biology, Numerical Analysis, etc, will not disappear. Therefore we believe that there is general support for the following recommendation, which was not formulated in the interim report.

**RECOMMENDATION 5: Established MSc courses, especially those with a vocational function, should be maintained. They should be accessible to graduates from both the three-year and the four-year degree programmes.**

45. Our interim report contained the tentative suggestion that departments plan to make any necessary administrative changes in time for 1994 entry to the new courses. We conclude by expressing the hope that the major bodies interested in higher education in mathematics and statistics will encourage departments to take heart from the amount of agreement that has emerged, and to take initiatives to introduce desirable new course structures at the earliest possible opportunity.

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## APPENDIX A: ENTRY LEVELS, STUDENT NUMBERS AND DESTINATIONS

### DOUBLE AND SINGLE A-LEVEL

A1. The number of students taking A-levels in all subjects has been steadily increasing in recent years, but the number of those taking mathematics, after increasing considerably, has begun to decline. The proportion of mathematics students taking the double subject at A-level has been declining steadily, and all degree courses in mathematics have given up the requirement of double-subject mathematics for entry.

The following illustrative figures are taken from the statistics of the Joint Matriculation Board, and show the trend in recent years.

	Single Mathematics	Single and Further Maths	Further Maths only	% of total with Double Maths
1975	10 542	1 547	89	13%
1980	14 267	1 929	106	12%
1985	16 089	1 650	134	10%
1990	14 917	1 383	110	9%

**Table 1:** JMB figures for students obtaining one or two passes in A-level mathematics.

### A-LEVEL SCORES

A2. The A-level grades of students accepted for university degree courses in mathematics in recent years are shown in Table 2. The grades are converted to numerical scores by counting 10 points for grade A, 8 for grade B, etc. Note that mathematics is taken to be the UCCA group G omitting G5 (Computer Studies).

	Total accepted	Total with A-levels	Average score	Average for all subjects
1986	3 096	2 731	23.6	21.8
1987	3 346	2 958	23.3	21.8
1988	3 623	3 229	23.2	21.8
1989	4 033	3 564	23.0	21.9
1990	3 805	3 383	22.9	21.2

**Table 2:** A-level scores of home students accepted for mathematics courses by UK universities (excluding The Open University, but including Buckingham and 13 additional colleges from 1990). Source: UCCA.

## NUMBERS OF STUDENTS

A3. Table 3 shows the recent figures for full-time university undergraduates taking subjects in group G omitting G5 (Computer Studies). Also shown is the total of these students as a proportion of all full-time university students. It can be seen that this figure has remained fairly steady over the five years 1985–90.

	Mathematics	Statistics	Combinations in group G	Total G excl. G5	Proportion of all students
1985–86	7 970	591	1 498	10 059	4.1%
1986–87	7 656	597	1 580	9 833	4.0%
1987–88	7 599	639	1 593	9 831	3.9%
1988–89	7 939	673	1 609	10 221	3.9%
1989–90	8 699	671	1 640	11 010	4.0%

**Table 3:** Numbers of full-time university students studying mathematics (excluding Open University and Buckingham). Source: USR.

## DESTINATIONS OF GRADUATES

A4. The number of students who use advanced mathematics in their subsequent careers cannot be determined precisely, but the following figures showing the first jobs taken up by graduates give some indication. Again they refer to UCCA group G omitting G5. Note that Management Services includes operational research, computer programming, data processing and statistics.

	Universities		Polytechnics		Distribution 1989 + 1990
	1989	1990	1989	1990	
Total graduates	2877	2874	471	456	100.0%
Known destination	2633	2648	389	392	90.8%
Permanent home job	1554	1368	231	192	50.1%
Administration	55	72	13	5	2.2%
Science/Eng. and support	125	107	25	24	4.2%
Management Services	453	312	138	102	15.0%
Financial	762	728	33	42	23.4%
Other	159	149	22	19	5.2%
Short-term home job	74	80	24	23	3.0%
Overseas employment	127	160	29	19	5.0%
PG courses and research	352	402	27	46	12.4%
Teacher training	157	167	23	27	5.6%
Other training	138	131	2	2	4.1%
Not available or unemployed	231	340	53	83	10.6%

**Table 4:** Destinations of graduates in mathematics (excluding Open University and Buckingham). Source: USR and CDP.

## APPENDIX B: MEMBERSHIP OF THE WORKING GROUP

The original remit of the working group was limited to a study of the structure of honours degree courses in mathematics and statistics in British universities outside Scotland: it was

- (1) To review the structure of first degree courses in mathematics in the light of
  - (a) changing patterns of secondary education, and
  - (b) the national need to produce graduate mathematicians comparable with those of other countries in Europe, and those of our trade competitors throughout the world,and having regard to national graduate employment needs;
- (2) to extend the NEMAS Forum consensus to discussions in universities nationwide, to evaluate resource implications and to seek mechanisms for the implementation of any national consensus;
- (3) if possible, to seek such a national consensus by the end of the current year.

The chairman was nominated by LMS Council. Membership was constructed with a view to providing input from different kinds of universities and different interest groups, subject to the constraint that the group had to be small enough to be effective. No members were representing any particular body, although they were asked to provide lines of communication with such bodies as LMS Council, IMA Council, RSS Council, EMS Council and various interest groups within higher education. It soon became clear that the interests of universities could not easily be distinguished from those of polytechnics, and this was confirmed early in May 1991, when the Government White Paper [5] announced the impending abolition of the 'binary divide'. Glyn James joined the committee in March 1991 to provide a polytechnic input. Because of his impending year overseas Garth Dales resigned in July 1991 and was replaced for the second stage of the group's work by John McDonnell of British Aerospace. Membership details are as follows.

Stephen Barnett	Professor of Applied Mathematics and Head of Department of Mathematics, University of Essex; Member, IMA Council; Chairman, Conference of Professors of Applied Mathematics.
Alan F. Beardon	University Lecturer in Pure Mathematics, Cambridge; Member, Royal Society Mathematics Instruction Subcommittee.
Alan R. Camina	Reader in Pure Mathematics, University of East Anglia; Member, LMS Education Committee.
Fred H.J. Cornish	Professor of Mathematics, University of York; Chairman, NEMAS Forum.

H. Garth Dales (until June 1991)	Professor of Pure Mathematics, University of Leeds; Secretary, NEMAS Forum.
D.J.G. James	Professor and Head of Mathematics Department, Coventry Polytechnic; Member and past Vice-President, IMA Council; Honorary Secretary, IMA; Chairman, IMA Education Committee.
John H. McDonnell (from November 1991)	British Aerospace; Member, IMA Council; President-elect IMA.
Peter M. Neumann (Chairman)	Fellow and Tutor in Pure Mathematics, Queen's College and Lecturer (CUF), Oxford University; Member and Vice-President, LMS Council.
R.L.E. Schwarzenberger	Professor of Mathematics, Science Education Department, University of Warwick; Member, LMS Council; Chairman, LMS Education Committee.
Brian D. Sleeman	Professor of Applied Analysis, University of Dundee; Member and Ex-President, EMS.
T .M. Fred Smith	Professor of Statistics, University of Southampton; President-elect, then President, RSS; Member, Conference of Professors of Statistics.
Joan Walsh	Professor of Numerical Analysis, University of Manchester; Member of IMA Council.
Henry P. Wynn	Professor of Statistics and Dean of the School of Mathematics, Actuarial Science and Statistics, City University; Member, Committee of Professors of Statistics.

## APPENDIX C: GLOSSARY OF ABBREVIATIONS

3 + 1	Degree course structure where all students follow a three-year course, some stay on for an optional fourth year (see §24)
3 or 4	Degree course structure where students decide after two years whether they will continue for one or two more years (see §24)
GCSE	General Certificate of Secondary Education
GCE	General Certificate of Education
A-level	Advanced level GCE examination or qualification
FE	Further Education
HE	Higher Education
DipHE	Diploma of Higher Education
PG	Postgraduate
PGCE	Postgraduate Certificate of Education
LMS	London Mathematical Society
EMS	Edinburgh Mathematical Society
IMA	Institute of Mathematics and its Applications
RSS	Royal Statistical Society
RS	Royal Society
COPS	Committee of Professors of Statistics
NEMAS	North of England Mathematics and Statistics
IOP	Institute of Physics
ACOST	Advisory Council on Science and Technology
SERC	Science and Engineering Research Council
UCCA	Universities' Central Council on Admissions
USR	Universities' Statistical Record
CDP	Committee of Directors of Polytechnics