

# TACKLING THE MATHEMATICS PROBLEM

London Mathematical Society  
Institute of Mathematics and its Applications  
Royal Statistical Society

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© The London Mathematical Society  
Burlington House Piccadilly  
London W1V 0NL  
Telephone +44 171 437 5377  
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## OUTLINE OF CONTENTS

The first two sections summarise why this report was commissioned, and what issues it seeks to address. Sections 3–5 underline the key role of mathematics in the modern world, indicate the profound concerns of those in higher education about the mathematical background of students applying for courses in mathematics, science and engineering, and explain their perception of a marked change in mathematical preparedness even among the very best applicants. The more detailed sections (6–15) address a number of other specific concerns. The main recommendations and issues for further discussion are presented in Section 16.

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# SUMMARY

1. There is unprecedented concern amongst mathematicians, scientists and engineers in higher education about the mathematical preparedness of new undergraduates. There is also a very long-standing worry about the numbers of prospective students in these disciplines. This report, on behalf of the London Mathematical Society, the Institute of Mathematics and its Applications, and the Royal Statistical Society, explains these concerns and suggests some actions.
2. While we believe that our analysis is factually correct, we recognise that ‘facts’ and ‘trends’ in education are open to different interpretations. Any such analysis is based on *judgement*, and is not susceptible of absolute ‘proof’. Some may dispute details of our analysis but they cannot ignore the fact that the underlying concerns shared, with increasing conviction, by almost all those who have experience of teaching mathematically-based courses in higher education.
3. It is economically important to this country that it produces both numerate citizens and top class mathematicians, scientists and engineers. This is almost impossible if the mathematical foundations for all these disciplines are not laid at the appropriate time.
4. Recent changes in school mathematics may well have had advantages for some pupils, but they have not laid the necessary foundations to maintain the quantity and quality of mathematically competent school leavers and have greatly disadvantaged those who need to continue their mathematical training beyond school level.
5. The serious problems perceived by those in higher education are:
  - (i) a serious lack of essential technical facility — the ability to undertake numerical and algebraic calculation with fluency and accuracy;
  - (ii) a marked decline in analytical powers when faced with simple problems requiring more than one step;
  - (iii) a changed perception of what mathematics is — in particular of the essential place within it of precision and proof.
6. International comparisons confirm many of these perceptions and also indicate what other countries have achieved. Insufficient attention has been paid to such findings.
7. A major cause of these problems has been the flawed method of planning change in the past decade. There is no representative, authoritative, continuing forum for mathematics, bringing together mathematicians, scientists, engineers, employers, teachers etc. Rather, there is a one-sided dialogue between SCAA and individual bodies, with agenda-setting and decision-making controlled by a small and necessarily unrepresentative group within SCAA.
8. Most of the suggestions and proposals contained in this document are tentative and need to be tested both by open debate and by discussion within such a representative forum. Hence, our two major proposals are organisational.

9. **We strongly recommend that the DFEE set up a standing committee, including substantial representation from higher education, to provide an overview of education in mathematics from primary school through to university, and to ensure that sound advice and adequate support are provided to those involved in its organisation and delivery.** Such a group should ensure that the many issues raised in this report are debated openly and fully by all parties concerned. The process of identifying suitable representation from higher education should include consultation with professional and learned societies.
10. Given the concern about the current mathematics curriculum in schools, **we recommend that, as a matter of urgency, a Committee of Enquiry be established by the DFEE with the express task of examining the current situation and making proposals in time to allow carefully considered action at the end of the current moratorium on change.**

# 1 WHO COMMISSIONED THIS REPORT? AND WHY?

Mathematicians in higher education have responsibilities of many kinds. They must ensure the welfare of their discipline, in part by preparing new generations of teachers and researchers. They are extensively involved in providing the requisite mathematical knowledge, skills and understanding for students of science, technology, economics and a host of other subjects which are increasingly dependent on quantitative methods. They have been heavily involved during recent years in unprecedented changes in the scope and organisation of higher education. Their specialist knowledge and experience gives them unique authority to comment on broad issues affecting the mathematical life of the nation, and obliges them to take some responsibility for school mathematics in this country<sup>1</sup>.

During the last two to three decades, there have been many radical changes in school mathematics. These include, for example, the rise and fall of ‘modern maths’; the raising of the school leaving age and the growth of comprehensive education; the Cockcroft Report (1982)[3] and the recognition of the need for a curriculum serving ‘the lower half’; the introduction of GCSE in 1986/88; the Education Reform Act (1988); the introduction of national testing at 7, 11, and 14, and of the National Curriculum with several hastily revised versions of the Mathematics Orders; and the vast expansion in the proportions of the age cohort remaining in full-time education beyond the ages of 16 and 18.

Unfortunately, despite the evident interdependence of primary, secondary and higher education, these changes have increasingly proceeded without sufficiently serious, mutual consultation and planning. This lack of genuine consultation has had numerous negative consequences. It has led many mathematicians in higher education to feel released from their responsibilities at school level and, in particular, has led them to give lower priority to their key role in helping to improve the supply of suitably qualified teachers. Moreover, their exclusion from real involvement in decision making has led to school curricula which are, in important ways, inconsistent with mathematics as specialists understand it: in short, which misrepresent what to many is “the essence of the discipline”. This has caused severe problems for those teaching or using mathematics in higher education and has had the regrettable consequence that many in higher education have lost confidence in the manner in which school mathematics is organised and decisions regarding it are taken.

**In the light of the unprecedented level of concern, in January 1995 the Council of the London Mathematical Society established a working group to identify the problems more clearly, and to suggest steps that might be taken to improve matters.**

The Institute of Mathematics and its Applications and the Royal Statistical Society accepted invitations to participate directly in the group’s work. Moreover, in view of the extent of concern among colleagues in other disciplines, the group sought at all stages to consult other interested bodies with a view to presenting a report which might reflect the concerns of as wide a community as possible.

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<sup>1</sup>The issues addressed in this document appear to be most acute in England and Wales, and it is from these two countries that most data in this report are taken.

## 2 A BRIEF OUTLINE OF THE PROBLEM AND THE REQUIRED RESPONSE

This section presents a brief summary of our position; the background discussion and the supporting justification for assertions in this section will be presented in later sections. Our first concern has been to document as clearly as we can the nature of the problems as seen from the viewpoint of those in higher education. Moreover, though the working group included only one practising schoolteacher and one teacher trainer, we have consulted widely with teachers and teacher's organisations to try to ensure that, insofar as possible, our findings are presented in a way which takes account of their concerns. **We stress at the very outset that, in our view, the main responsibility for the weaknesses we identify cannot be laid at the door of classroom teachers.**

The report DfE (1994) [8] highlighted inadequacies in the flow of qualified young people into science and technology. We welcome this official recognition of some of the problems and support much that is to be found therein. However, the picture presented by it, and by the subsequent report OFSTED (1994) [22], is too often obscured by a general air of complacency. For example, both reports take comfort in the fact that graduate numbers in mathematics and computer science combined have expanded steadily, even though the picture for mathematics alone is more worrying. All our evidence suggests (see Sections 5–10) that such statements as “The picture, therefore, can be said to be relatively reassuring” (OFTSED, 1994 [22], p.1) are dangerously misleading.

An equally serious concern of higher education is its observation of a qualitative change in the mathematical preparation of incoming undergraduates (see Section 4). This is in no way restricted to those ‘new undergraduates’ who ten years ago would not have proceeded to higher education. The problem is more serious; it is not just the case that some students are less well-prepared, but that many ‘high-attaining’ students are seriously lacking in fundamental notions of the subject. This trend is new, and is a significant indicator that something has gone wrong.

**The working group’s primary finding, recognised in Dearing (1995) [7] p.39, is that the United Kingdom faces extremely serious problems relating to the supply and the mathematical preparation of entrants to university courses in mathematics, science, engineering and technology.**

We need to improve the mathematical foundations laid at school level for all our students, whether or not they are likely to proceed to higher education. There is no painless or short-term way of achieving this; in particular, it is clear to us that recently implemented and currently envisaged initiatives will not have the required effect. This report puts forward in its conclusions (Section 16) the need for amendments to National Curriculum and A-level mathematics, while stressing that any changes instituted must be extremely carefully thought through. At the same time, it makes proposals concerning assessment, teacher recruitment and training, vocational courses, the effect of ‘market forces’, and other matters — for the impact of mere curriculum change on the problems noted above will be limited if these other aspects

of the educational system are not also subjected to serious scrutiny. On all these matters, higher education has a distinctive perspective and authority — but so too have others. Therefore our conclusions in such matters are presented as a contribution to serious discussion leading to action.

Our two main recommendations concern the manner in which discussion and decision making should be conducted. Discussion and review of the kind we recommend must begin *quickly* and be carried out thoroughly in a way that, much more than at present, commands the confidence of all concerned, including mathematicians throughout higher education. **This national problem can be overcome only by much more carefully considered long-term planning than hitherto, and by determined, co-operative implementation.**

### 3 WHY MATHEMATICS STILL COUNTS

A sound education in mathematics, both for the mass of ordinary pupils and for the mathematically more able, is important for any modern economy. This has been well-argued elsewhere (e.g. Dainton (1968) [5], Cockcroft (1982) [3]). Here we draw attention to the following points.

- *“Mathematics as a means of communicating quantifiable ideas and information”*  
(Dainton (1968) [5] p.89 (i))

In an increasing number of areas of science, technology, management and commerce, mathematics is the only effective language for the analysis of problems and for the communication of results and ideas. If our ordinary school leavers do not achieve a level of fluency in this language comparable to that in other countries, Britain will be at a considerable disadvantage. In addition, if our more able students lag behind those in other countries, British graduates will be unable to keep up with developments in their fields. We will then become ever more dependent on other countries for inventions, specialists and products.

- *“High technology is mathematical technology”* (David (1984) [6])
- *“Mathematics as a tool in activities arising from the developing needs of engineering, technology, science, organisation, economics, sociology, etc.”*  
(Dainton (1968) [5] p.89 (iii))

Mathematics is now important in many areas where it has not previously played much of a role (these include biology and the social sciences). Mathematically based techniques are increasingly used in the workplace, and have transformed the way decisions are made and the way business is done (for example, in stock control, in scheduling, to improve industrial design,

to achieve reduced energy consumption, to increase efficiency, and to reduce manufacturing costs). Before implementing new regimes it is frequently necessary to construct mathematical models of processes or operations (prior to subsequent computer simulation and analysis). If specialists, despite their training, do not fully understand the principles underlying these models, or the conditions limiting their validity, their advice will leave decision makers vulnerable in unintended and unpredictable ways. Some people have mistakenly concluded that the increased availability of computers reduces the need for mathematics: in fact the very opposite is true. Computers make mathematical techniques readily available to many who might previously have never thought of using them. The user does not need to understand all the details of the associated software, but a basic understanding of elementary mathematics is crucial if the output is to be used wisely and critically.

The pace of change in technology is increasing. In order to be able to adapt to new technologies and techniques, people in a variety of fields need to undergo periods of retraining throughout their working lives. Such retraining often presumes a significant element of basic mathematics. Some things have to be learned when young, and basic mathematics would appear to be one example. If the proper mathematical foundation has not been laid during adolescence, it becomes increasingly difficult to address these weaknesses in later life. The attempt to confront these shortcomings during retraining in later life is wasteful, painful, and usually only partly successful.

- *“Mathematics as a study in itself, where development of new techniques and concepts can have economic consequences akin to those flowing from scientific research and development”* (Dainton (1968) [5] p.89 (iv))

The mathematics used today was developed in earlier times — sometimes centuries ago, sometimes relatively recently. Much of the theory required was developed not by users for practical applications, but rather for its own sake by research mathematicians. Modern examples include the application of chaos theory to studies of turbulence, number theory to cryptography, and abstract algebra to error correcting codes. The notion that mathematics is of interest in its own right must be made clear within the curriculum. It follows that there should not be an insistence on all problems being presented ‘in context’ — simple arithmetic questions on fractions should not automatically be translated into problems about dividing pizzas.

- *“Mathematics as a training for discipline of thought and for logical reasoning”* (Dainton (1968) [5] p.89(ii))

The primary concern of scientists, engineers, and other potential users of mathematics is likely to be whether their students or employees can reliably perform certain basic routines. It has therefore not always been easy to convince them that one of the most important things that mathematics has to offer the mass of pupils is that of an intellectual training for the mind. One of the striking consequences of the current widespread concern has been the realisation by many engineers and scientists that this kind of preparation has been perhaps the most significant loss

in recent years. This was part of what was meant when the working group was told by a leading figure from the Engineering Council: “What is missing is the idea of mathematics as a precise analytic tool”.

The fundamental importance of mathematics is well summed up in the assertion by a Fellow of the Royal Academy of Engineering that “The one thing that would most help recruitment to engineering is better mathematics teaching in schools”. Students appreciate the key role of mathematics in such subjects as physics and engineering and are often deterred from following them because of inadequate mathematical preparation.

## 4 CONCERNS ABOUT QUALITY

### 4.1 The view of higher education

There is unprecedented concern among academics about the decline in the mathematical preparedness of those entering undergraduate courses in science and engineering. This can be seen, for example, in the reports of the Engineering Council (1995) [9], the Institute of Physics (1994) [17], and the Institute of Mathematics and its Applications (1995) [16]. (There are some signs that students *with A-level mathematics* who enter courses such as biology, geography, or business studies, cope relatively well; but many such courses have serious problems with the bulk of their students who gave up the study of mathematics at 16.) While we acknowledge that such a change is, in part, due to the greater numbers now entering higher education, and to the increasing number of options available to students with good grades in mathematics, this does not begin to explain the deficiencies that are being observed. As we have already remarked, the same shortcomings are being seen in those universities which select the best of our young mathematicians, scientists and engineers.

**Mathematics, science, and engineering departments appear unanimous in their perception of a *qualitative* change in the mathematical preparedness of incoming students.**

Their criticisms of student preparedness concentrate on three main areas.

- 4A Students enrolling on courses making heavy mathematical demands are hampered by a serious lack of *essential technical facility* — in particular, a lack of fluency and reliability in numerical and algebraic manipulation and simplification.
- 4B Compared with students in the early 1980s, there is a marked decline in students’ analytical powers when faced with simple two-step or multi-step problems.
- 4C Most students entering higher education no longer understand<sup>2</sup> that mathematics is a

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<sup>2</sup>Things may never have been good, but we are bound to report (and are reluctantly convinced by) consistent

precise discipline in which exact, reliable calculation, logical exposition and proof play essential roles; yet it is these features which make mathematics important.

These criticisms affect all those undergraduate courses which presuppose a basis of mathematical technique and are well-illustrated by the report of the Engineering Council (1995) [9], and the problems listed in Appendix A. Faced with modest weaknesses of the kind indicated in 4A, mathematics departments can try to adjust the level of their courses to fit the entry standards of their own specialist mathematics students — though there is a limit to how many basic weaknesses can be (or should need to be) overcome by undergraduates. Any such adjustment is made much more difficult by the diversity of syllabuses indicated in Appendix B.

It is much less clear how higher education can respond effectively to the more disturbing weaknesses of types 4B and 4C. Science and engineering departments are in a more difficult position than mathematics departments, since they can allocate only a limited proportion of their time to mathematics teaching, and frequently need to make use of mathematical knowledge from the very outset in teaching their own disciplines. If standards of preparedness continue to decline, then so too will the standard of degrees. This will place in jeopardy the value of UK degrees, diminishing their status in relation to qualifications from Europe and elsewhere, and making them less attractive to overseas students.

## 4.2 Some possible causes

In recent years English school mathematics has seen a marked shift of emphasis, introducing a number of time-consuming activities (investigations, problem-solving, data surveys, etc.) at the expense of ‘core’ technique. In practice, many of these activities are poorly focused; moreover, inappropriate insistence on working within a context uses precious time and can often obscure the underlying mathematics. Such approaches, if well-directed, have value, but priorities must be agreed. At no stage has there been a serious debate as to which topics or skills are of primary, and which are of secondary, importance for students’ subsequent progress. The position was made worse by the recent administrative squeeze which effectively reduced the amount of time available for mathematics by around 20%. **This reduction should be urgently reviewed.**

During the same period we have also seen implicit ‘advice’ (from HMI (1985) [14], from OFSTED(1994) [22], in the wording of the National Curriculum [20], and from elsewhere) that teachers should reduce their emphasis on, and expectations concerning, technical fluency. This trend has often been explicitly linked to the assertion that “*process* is at least as important as technique”. Such advice has too often failed to recognise that to gain a *genuine* understanding of any process it is necessary first to achieve a *robust technical fluency* with the relevant

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evidence from many quarters of a genuine qualitative change in the last decade. The extent of the phenomenon makes it difficult to summarise this evidence. Something of the spirit was conveyed by one lecturer in engineering mathematics who observed: “Students have always made mistakes such as replacing  $1/a + 1/b$  by  $1/(a + b)$ . The difference is that now, when I try to correct them, they do not believe me and there seems to be no way of convincing them. Some even insist that I must be wrong and they are right.”

content. Progress in mastering mathematics depends on reducing familiar laborious processes to automatic mental routines, which no longer require conscious thought; this then creates mental space to allow the learner to concentrate on new, unfamiliar ideas (as one sees, for example, in the progression from arithmetic, through fractions and algebra, to calculus).

In parallel with these changes in emphasis, evidence that many English pupils were unable to solve standard problems involving, for example, decimals, fractions, ratio, proportion and algebra (Hart (1981) [12], APU (1980-85) [1, 2] etc.) was interpreted by many curriculum developers and those responsible for defining national curricula, as meaning that such topics were ‘too hard’ for most English pupils in the lower secondary years. This interpretation ignored two key facts.

- (i) Many of the topics that were postponed, or eliminated, are fundamental to subsequent progress in mathematics, and cannot be neglected without serious consequences.
- (ii) Many countries (where pupils often start school at a later age than in England) managed to teach these topics effectively to a far larger proportion of pupils.

Many ‘hard’ topics must be introduced early if students’ subsequent progress is not to be blocked. And it is often the case that, the longer one postpones a first treatment, the more inaccessible a topic will become. Despite recent advances in cognitive psychology, few quotes express the general point more cogently than the following :

*“It is not true that the easier subjects should precede the harder. On the contrary, some of the hardest must come first because nature so dictates, and because they are essential to life. The first intellectual task which confronts an infant is the acquirement of spoken language. What an appalling task: the correlation of meanings with sounds! [...] All I ask is that, with this example staring us in the face we should cease talking nonsense about postponing harder subjects.”* (Whitehead (1922) [27]).

### 4.3 Evidence

Some data relating to the first two criticisms 4A and 4B in Section 4.1 have been publicised in the press during the last year. While symptomatic of our concerns, none of this information is as robust as one would like it to be, either in the range of mathematics tested or in its use of strictly comparable groups of students<sup>3</sup>.

Various universities have offered us data supporting the case we are putting forward. We have chosen not to reproduce, or even to try to summarise, all these, largely because of difficulty in

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<sup>3</sup>It is therefore regrettable that, despite these obvious weaknesses, two such findings have been used repeatedly and selectively by officials to argue that “there is no evidence of any decline”. We shall return to this in Section 8.

ensuring that the groups of students involved are strictly comparable. However, in Appendix A, we give some results (from a test given in October 1994 to students entering the honours school of mathematics at one university) which illustrate the problems now faced by universities. We also present, in the same spirit, some evidence from large-scale national mathematics competitions aimed at the top third of the relevant age groups in secondary schools. We suggest that these should be studied bearing in mind the question ‘Are these standards appropriate?’ rather than ‘Were students better in the past?’

The third area of criticism, 4C, depends crucially upon what experienced university lecturers *infer* about the perceptions of their students from their spoken or written work. It is therefore more difficult to substantiate this criticism in quantitative terms. However, the criticism as stated would appear to reflect the considered judgement of almost everyone we have consulted (including many scientists and engineers (see Engineering Council (1995) [9])). There is some research evidence which supports these widely held concerns: for example, the study of Coe and Ruthven (1994) [4] showed that, of sixty mathematics students engaged in a piece of coursework to complete an A-level module “directly concerned with proof”, only two students appeared to understand what was required in order to prove a hypothesis. Evidence over the last 5–6 years from national mathematics ‘olympiads’, each aimed at one thousand or so of the very best students in School Years 7–8, 10–11, and 12–13 reinforces this concern.

No evidence was presented to the working group which would contradict any of the criticisms 4A, 4B, 4C.

We were unanimous in our judgement that:

- (a) **such criticisms are justified;**
- (b) **most of the observed weaknesses stem from current content, emphases, aims and assessment procedures; they will not be removed as a result of recent changes to the National Curriculum; and they will only be remedied by a far-reaching review and by much improved guidance for teachers;**
- (c) **urgent attention must be paid to ensure that students bound for higher education in future leave school better prepared to continue their studies;**
- (d) **while higher education has adjusted, and will continue to adjust, to changes in the preparation of its entrants, the nature and scope of the current shortfall cannot be addressed satisfactorily at 18+ or, indeed, at 16+.**

## 5 CONCERNS ABOUT NUMBERS

Funding mechanisms now encourage HE institutions at 18+, and schools and colleges at 16–19, to ‘fill places’ in numerate disciplines as best they can. Though numbers have increased, there are nevertheless clear grounds for concern at the number of adequately prepared students

taking courses in mathematics at 16-19, and the number of suitably qualified applicants wishing to specialise in mathematics, science or engineering at 18+.

We recognise that such concerns are not new: the “relative decline in [the number of students undertaking] the study of science and technology” was explicitly addressed in Dainton (1968) [5]. When considering the nature of our present difficulties at 16–19 and at 18+, it is important to understand some of the changes that have occurred within the English and Welsh educational systems since the Dainton committee began its work in 1965. This section highlights some of the more dramatic changes.

In the thirty years since 1965 the country’s educational systems have been extensively reorganised, and the school-leaving age has been raised to 16. In 1995 most 11-16 year old students attend comprehensive schools, and a much larger proportion of the age cohort (72% according to some estimates) now continues in full-time education beyond the age of 16.

Since 1965 the total number of A-levels taken each year has doubled, but this increase is far from uniform. For example, the number of students entering for physics A-level *fell* by almost a fifth between 1965 and 1995. Part of this drop in numbers may be attributable to demographic trends. However, it is easy to misinterpret this crude statistic. For example, the demographic change among the social groups providing the bulk of undergraduates has been relatively modest. Moreover, as the number of students staying on after the age of 16 has expanded, some subjects have apparently flourished. For example, over the period of demographic decline since 1965, numbers taking English A-level have more than doubled.

The changes over the same period in numbers taking mathematics A-level have been both more modest and more complicated. Appendix C shows an increase of one third in single mathematics entries (with a corresponding two thirds increase in single mathematics *passes*), and a two thirds decline in double mathematics entries. Overall there has been a significant increase in participation since 1965; but there has also been a substantial reduction in the number who study the subject in greater depth. More serious is the fact that, in the last decade, there has been a dramatic decline on both fronts (see Appendix D). The sequence of expansion followed by decline means that, when compared with 1965, the number of *male* A-level mathematics entries is now no different from what it was; any growth can be attributed to the welcome increase in the number of female candidates from 6,400 in 1965 to 20,400 in 1995 (see also DfE (1994) [8]).

One change which deserves more serious attention is the dramatic trend away from students combining mathematics and science A-levels as constituents of a coherent course of study leading to higher level courses in engineering, science and technology. In 1965 38% of A-level students studied only science and mathematics; in 1993 the percentage had dropped to 16%. (Indeed, DFEE data suggest that in 1994 less than 9% of 17 year-old A-level students studied only science and mathematics.) This change has profound implications.

Increasing numbers of students choose, and are often encouraged to choose, a ‘more balanced’ A-level course, including subjects drawn from both sides of the Arts-Science divide: 14% in 1965, 40% in 1991. This desire to seek a broadly based education creates severe problems

when most students take only two or three main subjects. For example, the mutual support which mathematics and physics traditionally provided for each other can no longer be assumed. Moreover, where able students taking a fourth A-level used to take further mathematics, they are now often required to take general studies. The position has recently been further exacerbated in sixth form colleges by the funding policies of the FEFC. As a result of these various pressures, many students are effectively dissuaded from taking a second mathematics A-level (the number doing so has decreased by over 60% since 1965).

One significant change to patterns of university entrance since 1965 is the marked increase in the percentage of entrants who do not come straight from school, or who have qualifications other than A-levels. The number of such students entering degrees in mathematics and physics may still be small; but the number of ‘non A-level’ students entering engineering courses has increased substantially. Teaching mathematics to such students has created additional serious problems, which have mostly been tackled *ad hoc*.

We shall refer to these changes again in Section 12. Meanwhile, we shall concentrate our attention on concerns affecting A-level, starting with the mathematical preparation underpinning A-level mathematics courses.

## 6 STANDARDS AT GCSE

In this section we look briefly at the interface between GCSE and A-level. The most immediate question is:

- Are current standards in mathematics at GCSE (Grades A–C) in any way comparable to those associated with GCE O-level and CSE (Grade 1)?

The percentages of students obtaining Grades A to C have risen sharply in the last decade, and there would seem to be clear evidence of grade dilution. This and other structural changes have had a marked effect on the ‘gap’ between GCSE and A-level. Since 1994 students can routinely obtain a Grade B taking only the intermediate tier GCSE papers, which assess students on a reduced syllabus (requiring, for example, very little algebra). Moreover, the mathematical knowledge now required to obtain an A\* on GCSE papers in no way corresponds to that needed to obtain a good grade on the GCE additional mathematics formerly taken by many mathematically high-attaining students.

The difficulties such changes can cause are illustrated in the Engineering Council report (1994) [9] which describes the serious problems faced in higher education by students who obtain a Grade C in GCSE mathematics. Now, as a result of the change in grading procedures on the intermediate tier papers, such problems will be exacerbated. That change, like so many others, was made without a proper consideration of the consequences. Some attempts to avoid the more extreme abuses may have been introduced this year, but underlying issues, including comparability with other subjects, have still to be addressed, and the matter needs urgent review.

It is important to try to achieve some sort of consensus among professionals — academics in mathematics, science and engineering, teachers in schools and colleges, educationalists and administrators — as to the way forward. We cannot expect complete agreement on the extent to which standards have changed. However, few would dispute the observations that:

- (a) in recent years less emphasis has been placed on the acquisition of skills involving arithmetic, fractions, ratios, algebraic technique, and the basic geometry of triangles, lines and circles;
- (b) all of these neglected topics are vital for further study in mathematics, science and engineering.

These two observations suggest strongly that we have paid insufficient attention to the effectiveness of the current curriculum for the mass of students with the potential for further study.

At the same time we have seen greatly increased dependence upon calculators and computers. These are invaluable tools which have a place in the mathematics classroom; but their advent does not greatly change the mathematics which beginners need to master — a fact which is reflected in the relatively conservative mathematics curricula of those countries with the strongest commitment to modern technology. Nor does it mean that pupils need no longer achieve fluency in traditional written methods. This applies especially when laying key numerical or algebraic foundations with beginners. Therefore, while we welcome the review of calculator usage to be carried out by SCAA, we urge that it should not be restricted to short-term effects on arithmetical competence, but that the experiences of those in higher education should be sought to try to assess how *typical* uses of calculators in schools have affected students' long-term mathematical thinking and behaviour.

**We need an urgent and serious examination of what levels of 'traditional' numerical and algebraic fluency are needed as a foundation for students' subsequent mathematical progress, and how such levels of fluency can be reliably attained.**

## 7 RECRUITMENT TO A-LEVEL

As noted in Section 5, the annual number of A-level entries, over all subjects, has doubled since 1965. However, despite numerous reports and well-intentioned reforms, the number of A-level entries in mathematics, single and double combined, has not increased significantly — see Appendix C. (In France, over the same period, the number of candidates for the Mathematics and Physical Sciences Baccalauréat has tripled — see Section 10). In addition to such disturbing statistics, one needs to ask:

- Are we recruiting A-level mathematics students from the same 'attainment stratum' as previously?

Certainly, we are now recruiting A-level mathematics students from 'further down' the cohort than in earlier times. In 1979, well over 80% of them had obtained A or B at GCE O-level mathematics and so came from the top 11% of the age cohort at 16; the vast majority of the rest came from the top 20%. In 1994, the percentage of A-level students who had obtained an A or B Grade at GCSE showed little change, but now the comparable figures within the age cohort were the top 17% and 40% , respectively. We have mentioned in Section 6 the large numbers of students obtaining GCSE mathematics via intermediate papers, and so having been assessed on a reduced syllabus. This means that A-level teachers must now deal with students drawn from a much wider range of mathematical ability and attainment than formerly.

A related question is:

- Are we still attracting the same percentage of the very high-attaining 16+ students to A-level mathematics?

Further attention must be given to this issue. Such students are important for all mathematically-based disciplines, yet there are signs that some may be repelled from mathematics by an unchallenging diet. Data on the attitudes of students towards mathematics and their wish to proceed further with its study should soon become available from the Exeter-Kassel Project and from the responses of 13 year-olds involved in the Third International Science and Mathematics Study. These should be studied with care and, if necessary, a further, more focused, study mounted.

**We must ensure that our very best students are being provided with a mathematical diet which not only provides a foundation for further studies but is presented in a way which will encourage them to continue their mathematical studies.**

Of course, such needs have not passed unnoticed by governments. The 'Great Debate' of the 1970s and the steps taken by Conservative governments since 1979 attest to this. The introduction of a National Curriculum presented the country with a unique opportunity to set agreed goals for students and teachers and to bring more coherence to a sadly inhomogeneous system. Unfortunately, the imposition of a National Curriculum in mathematics was mishandled, and after two attempts to rewrite the Orders for Mathematics we still have National Criteria for GCSE mathematics which are less specific and represent little, if any, advance on those in operation before the 1988 Education Act. The current five-year moratorium offers us an opportunity to set out a more coherent curriculum. But that opportunity will be missed if the lessons to be learned from the past seven years are ignored.

**One of these lessons is that professional mathematicians and bodies should have a much bigger role to play alongside teachers, educators and employers in determining aims and means.**

## 8 A-LEVEL STANDARDS

Similar questions concerning standards must be asked about A-level mathematics. The statistics we present in this section constitute only part of the picture, and we welcome the inquiry recently initiated by the government into mathematics and science qualifications. Simply inspecting examination papers tells one little. The preparedness of incoming university students depends not only on grades, but on a combination of syllabuses, style of examination questions, mark schemes and their implementation — above all on the way they have been taught and on the mathematical experience they bring with them<sup>4</sup>.

Until 1986, A-level grading in mathematics was roughly ‘normed’, i.e. a Grade A was awarded to about 10% of candidates, Grade B to 15% , Grade C to 10% , and so on, with some 30% being condemned to fail. Since then the percentage of higher grades awarded has increased significantly. The comparable percentages<sup>5</sup> for 1994 were: A 25%, B 18%, C 16.5%, ‘fail’ 15%. The evidence presented to this working group is clearly inconsistent with any suggestion that the much larger proportion of today’s students who achieve a Grade A at A-level mathematics can in any sense be said to perform at a level comparable to the smaller proportion of the cohort pre-1986. There is no doubt that there has been, in an obvious sense, a devaluation of grades<sup>6</sup>. This observation is supported by evidence presented by Professor C. Fitz-Gibbon to the 1995 meeting of the British Association for the Advancement of Science in Newcastle in which she reported that A-level mathematics candidates in 1994 with a given score in the International Test of Developed Abilities achieved roughly two grades higher than candidates with comparable scores in the same test in 1988.

This devaluation, by itself, is not what concerns us. In those cases where previous exchange rates have ceased to reflect underlying conditions and needs, devaluation of a currency can sometimes bring economic benefits. It can be argued that there were strong reasons for reducing grade standards in mathematics A-level. For example, it has repeatedly been shown to be more difficult to obtain good grades in mathematics, physics and chemistry than in other subjects — see, for example, Nuttall (1974) [21] and most recently Fitz-Gibbon and Vincent (1994) [10]. Insofar as this may have been a major deterrent to recruitment, one can understand the pressures to award higher grades. This in itself would have no harmful consequences, provided the mathematical integrity of the courses had been maintained (for example, by deliberately reducing difficulty levels while ensuring that fluency was achieved in a large coherent core of common material), and provided the reliability of the assessment procedures made it possible to interpret the new grades (e.g. if a 1994 Grade A were reliably equivalent to a 1979 Grade A or B). **The neglect of these provisos has devalued our academic currency in a way which threatens to undermine the credibility of all 18+ assessment.**<sup>7</sup>

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<sup>4</sup>For this, and other reasons, we should treat with great caution certain recently publicised claims that A-level standards have been maintained, or have risen, in recent years.

<sup>5</sup>Strict comparability is impossible because of the recent introduction of modular A-level schemes. These, because of their structure, effectively discount many of those who would previously be deemed to have failed the course.

<sup>6</sup>One is thus less surprised than one should be to read (IMA (1995) [16]) of Hong Kong universities which now adopt the exchange rate: 1 Hong Kong A-level point is equivalent to 1.67 UK A-level points.

<sup>7</sup>Despite this perception from higher education, officials have repeatedly claimed that “what little evidence

## 9 CORE MATERIAL AT A-LEVEL

As mentioned in Section 8, it had become increasingly clear over the past 20 years that mathematics A-levels were comparatively too hard. Some relaxation was needed — both in the way grades were awarded, and in the demands made on candidates. A number of projects and examination boards therefore sought to devise new A-level courses which were more accessible to students at the outset, more successful in retaining those who started the course, and more effective in rewarding students for what they achieved. (For example, the Mathematics in Education and Industry project, the Nuffield project, and the School Mathematics Project 16–19 group have all developed distinctive courses of this kind.) Regrettably, in the absence of any centrally coordinated response to the problem, each of these groups acted unilaterally.

The situation was exacerbated by the revised compulsory A-level core produced by SEAC/SCAA which appears to attempt more to accommodate diverse types of courses than to provide the predictable base needed by end-users. As will be seen from Appendix B, the amount of material that is now common to all the boards has been reduced to the point where those in higher education can infer relatively little from the fact that a student has a ‘mathematics’ A-level. The kind of differences revealed in that table not only make life very difficult for those in higher education, but (as far as one can tell) bring no obvious benefits to schools and colleges. We recognise the need for a mechanism which allows significant curriculum development to take place, but see no value in diversity for its own sake.

Mathematics A-level has a broader function in 1995 than it did in, say, 1980. Rather than hark back to the past, one has to ask whether the current A-level structure is meeting the challenges of today. As is clear from the thrust of this report, changes are certainly needed. But before launching into further ‘reforms’, we must answer some serious questions.

- To what extent are mathematics A-level courses hamstrung by weaknesses and omissions in the mathematical attainment students bring with them at 16+?
- How can mathematics A-level be modified (to make it more accessible to students at 16+) without making the transition at 18+ more traumatic than it is at present?<sup>8</sup>
- Is the current A-level content sufficiently focused to provide a useful foundation for further studies?<sup>9</sup>
- Are our current expectations and standards at GCSE and at A-level appropriate for a modern economy?

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there is” indicates that standards may even have improved. We have examined the evidence on which this claim is based (and the more significant evidence which it ignores) and we cannot attach any weight to the claim.

<sup>8</sup>Students who in 1992 had obtained a B Grade in GCSE mathematics, had a median grade of D in the 1994 A-level examinations; for those with a C Grade the median grade was E (see Appendix E — and note the large numbers who failed to pass).

<sup>9</sup>This is an important question especially since over 90% of those with two A-levels now proceed to higher education (Smithers and Robinson (1995) [26] p7).

Any informed response to such questions must draw on the knowledge and experience of those in higher education, and on their understanding of what mathematics is fundamental for subsequent progress.

**There is a clear need for greater awareness on the part of the Schools Curriculum and Assessment Authority of the mathematics which will be used in employment and in higher education. It is vital that the Authority involves bodies representative of higher education, and of employers of mathematicians, scientists and engineers, in decision-making processes in a direct and consistent fashion. Concerted action is needed to implement an effective A-level structure and usable cores appropriate to the particular needs of A-level mathematics students.**

## 10 INTERNATIONAL COMPARISONS

One way of highlighting possible weaknesses in our current systems is to ask how our students, our curricula, our assessment procedures, and the support we give to our teachers compare with those in other countries — especially other European ones. It was disappointing, therefore, that the DfE (1994) [8] report included no comparative data. Again, comparisons will centre on two issues: *quantity* (the percentage of students opting for mathematical studies once it is no longer compulsory) and *quality*.

Considerable attention has been paid to quality in numerous international studies (e.g. Robitaille and Garden (1989) [24], IAEP (1989) [15], Prais (1994) [23]) and the messages are consistently discouraging: pre-16 we lag behind many European and Asian countries. One apparent success is reported in Fitz-Gibbon and Vincent (1994) [10]: “in international comparisons ... [England’s] mathematics achievement at A-level was ahead of most countries”. However, this was based on a misconception, since it failed to note that what were compared were the average marks of all 17 year-olds specialising in mathematics — and that, among the countries surveyed, England and Hong Kong had the smallest percentages of the age cohort doing this. When the marks of the top 1% or the top 5% were compared, England sank to its usual middle-of-the-table position (Robitaille and Garden (1989) [24] p.151, McKnight *et al* (1987) [19] p.26). Regrettably, England is not taking part in that section of the current Third International Study which is comparing the attainments of 17 year-olds specialising in mathematics and which is also studying general numeracy amongst all 17 year-olds.

The failure over the last decade and more to expand significantly the percentage of 16–19 year-olds opting to study mathematics is a serious one. To argue, as in DfE (1994) [8], that A-level mathematics is, by its nature, only suitable for a select few ignores the fact that in England we are now recruiting A-level mathematics students from a broader intellectual range than formerly (and that A-level mathematics entries almost trebled in the decade 1955-1965). Such a view also ignores experiences elsewhere. In Metropolitan France, for example, the number of students obtaining the Series C Mathematics and Physical Sciences Baccalauréat has mushroomed from 21,443 in 1970 to 66,438 in 1993. (In the same period, the numbers admitted to all sections of

the Arts Baccalauréat increased only from 64,502 to 74,431 - a striking contrast with changes in numbers at A-level). The data for the 1994 Baccalauréat<sup>10</sup> were:

<u>General</u>	<u>Stream</u>	<u>Entrants</u>	<u>Passes</u>	<u>Hours of study per week</u>		
				<u>Y11</u>	<u>Y12</u>	<u>Y13</u>
A	Arts–Maths	45,516	32,762	4	5	5
B	Economics–Social	96,900	64,481	4	5	5
C	Maths–Physics	75,291	64,572	4	6	9
D	Maths–Biology	87,202	66,264	4	6	6
<u>Technical</u>						
E	Science–Technology	14,000	10,577	3	6	9

(Other streams lay less emphasis on mathematics)

(Regarding hours per week, it should be noted (see, e.g. (Dearing (1995) [7]) that French students have considerably more time-tabled hours per week than do English and Welsh sixth-formers.)

If we compare these figures with those for A-level mathematics in 1994 — roughly, 57,000 attempted ‘single-subject’ A-level (4–5 hours per week in Years 12 and 13) of whom 5,400 took ‘double subject’ (about 9 hours per week) — the contrast in the figures, as well as in the changes since 1970, is quite stark.

The working group was also able to study examination papers from elsewhere in the EU. These indicated, for example, that in one country 20% of the 18 year-old age cohort were studying integral calculus at a standard comparable with that of A-level as opposed to about 7% in England and Wales (Dearing (1995) [7]).

Many countries are concerned about recruitment to science and to mathematics undergraduate courses. However, it would appear that the important aim of sending mathematically competent students to university to study quantitative subjects is being more satisfactorily met elsewhere than it is in the UK.

**It is essential that our national aims and objectives for education in mathematics should take full account of what is being achieved in other countries.**

## 11 THE A-LEVEL STRUCTURE

We referred in Section 8 to a ‘relaxation’ of standards and to attempts to make mathematics more ‘accessible’. We also pointed out in Sections 6 and 7 how mathematics at A-level is no

<sup>10</sup>Important structural changes to the Baccalauréat, affecting the teaching of mathematics, were implemented in 1995.

longer attracting the same percentage of those with high grades in mathematics at GCSE. Are these two facts connected? Are the needs of the mathematical high-flyers being met as well as they were in the past? If not, how can the situation be improved?

Though the problems have their roots in pupils' experience of mathematics at age 5–16, many argue that the inflexibility of the A-level structure presents additional problems at the 16-19 stage. If we adopt the assumptions of the 1988 TGAT 'levels' model, then an A\* student at GCSE would be four or five years in advance of a Grade C student in terms of mathematical knowledge and attainment. Yet both are expected to be examined on the same A-level papers only two years later. (We have remarked in Section 9 on the likely consequences for the student with a Grade C.) Hence, in addition to the need to revise the 5–16 mathematics structure, there is also a need for more flexible 16–19 courses with time allocations and subsequent qualifications geared more to specific needs, and ensuring adequate differentiation.

Traditionally the double subject qualification (in mathematics and further mathematics, or in pure mathematics and applied mathematics) provided a challenging goal for the mathematical high-attainer. As noted in Section 5, the number of candidates has dropped considerably in recent years; in addition, the current funding policies of government and its agencies now make this qualification a far from attractive option for schools and sixth form colleges. In 1965 over one third of all mathematics A-level candidates attempted double subject mathematics; by 1994 the corresponding figure was about 10% . There are signs that the flexibility of modular schemes may bring with them some improvement in the position (mainly because students can now take 'one-and-a-half' mathematics A-levels, leading to both an A and an AS qualification — a welcome, but unplanned spin-off of the introduction of AS-levels<sup>11</sup>

We recall that there have been many previous proposals for changes at this level. Dainton (1968) [5] recommended a revised sixth form curriculum which would provide students with a more broadly-based education that could still contain, as a significant part, a coherent mathematics and science component; Higginson (1988) [13] called for slightly different changes; more recently, the Royal Society (e.g. 1991) [25] has echoed the demands of Dainton. These proposals deserve renewed consideration. The position relating to science and technology has, by most measures, deteriorated since Dainton's day. As was indicated in Section 10, we can no longer claim that our A-level mathematics students begin university better prepared than their peers in countries in which students follow more broadly-based courses.

**We welcome the commitment in §14.5 of the Interim Report of the current review of the 16–19 qualifications (Dearing (1995) [7]) that specific attention will be paid to the particular problems encountered in mathematics (see also Section 13). However, consultations will need to be carried out in an open and collaborative fashion. Moreover, while we acknowledge the need for a review of provision at the 16–19 level, such changes**

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<sup>11</sup>It is unclear from published data how many of the entries for AS mathematics:

- a) took it as a sole mathematics qualification,
- b) gained it *en route* to A-level by studying a subset of the A-level course, or
- c) took it *in addition* to an A-level course (a growing use of AS not mentioned in Dearing (1995) [7]).

**must not preempt a revision of the mathematics curriculum for ages 5–16. We need to work towards a position in which the foundation laid at 5–16 allows for a range of provision at 16–19 comparable with that in other EU countries.**

## **12 NON A-LEVEL ROUTES TO HIGHER EDUCATION**

Smithers and Robinson (1995) [26] point out that, unless the percentage of the cohort taking A-levels is increased (and Dearing (1995) [7] offers little hope of this) there is very limited scope for increasing the number of higher education entrants with an A-level background: already almost all of those who obtain two A-levels proceed to higher education. As mentioned in Section 5, much of the recent growth in undergraduate numbers, over all subjects, is due to non A-level and mature students.

This is less true in mathematics and science. In particular, relatively few mature students opt to study mathematics and science. The Open University, by far the greatest provider for mature students, reported that mathematics attracts fewer students with ‘low’ previous qualifications than any other foundation course and that these few have a poor success rate.

If, as is indicated by Dearing(1995) [7], future growth in science and engineering numbers is likely to come through the GNVQ route, then GNVQs will be of increasing significance for universities. However, GNVQs were established as a more vocationally-inclined qualification to *supplement* A-levels. Few entrants to mathematics degrees are likely to have followed the GNVQ path; but there will be many students of science, engineering and other mathematics-based courses who have done so. For that reason we particularly welcome the establishment by the National Council for Vocational Qualifications of a Mathematics Advisory Group. We hope that it receives the appropriate commitment from the NCVQ and that it will involve mathematicians from a varied selection of universities. We also welcome its funding of research such as is described in Lord *et al* (1995) [18] which proposes, on the basis of evidence from universities, a rationalised set of mathematics units designed to prepare students for a wide range of undergraduate studies.

**However, much remains to be done to ensure that, and then to convince higher education that, mathematics courses within the vocational sector are being described, delivered, and assessed in satisfactory ways. It is important that higher education is fully involved in this process. Until this has been achieved, we should be very hesitant about any attempt to combine A-levels and GNVQs into a uniform framework.**

## **13 TEACHER SUPPLY**

Writing in 1912, the Chief Inspector for Secondary Schools, W.C. Fletcher [11], explained that the efficiency of *individual* teachers cannot be measured by their academic qualifications, since

lack of formal qualifications can sometimes be compensated by personal qualities; nevertheless, “when the question is not of an individual or of a small group, but of a large number, it remains true that the lack of good qualifications must seriously limit the efficiency of teachers”. The lack of sufficient well-qualified mathematics teachers has been a major problem facing English education for over 30 years. Moreover, changes to the educational system have served to exacerbate existing problems. The Dainton report (1968) [5] drew particular attention to a shortage of graduate teachers of mathematics, and the fact that these were concentrated in grammar schools and tended to teach only the older pupils. It recommended that younger and uncommitted pupils should receive higher quality teaching and that positive incentives should be offered to attract more mathematics and science graduates into teaching. The coming of comprehensive schools tended to disperse well-qualified<sup>12</sup> teachers more widely, but did so without increasing their numbers. Subsequent recruitment to newly created sixth form colleges tended once more to concentrate well-qualified teachers in a smaller number of institutions, so that a student in an 11–16 school might never meet a well-qualified mathematics teacher. There was also (and still is) a concentration of well-qualified mathematics teachers outside the maintained sector. Figures drawn from a survey in 1977 and quoted in Cockcroft (1982) [3] show that overall, the percentage of mathematics teaching, in all maintained secondary schools, done by well-qualified staff was about 33%. The corresponding figures for sixth form colleges was 61%, for grammar schools was 59%, for comprehensives with sixth forms was 40%, and for 11–16 schools was 23%.

What has happened since then? Certainly teaching is becoming a ‘graduate profession’ and the percentages of teachers, in all subjects, having degrees of some kind has increased substantially. (The figures for 1970, 1980 and 1990 respectively were 37%, 51% and 66%.) Nevertheless, in the mid-1980s, the number of students recruited to initial teacher training in mathematics (PGCE and concurrent courses combined) regularly failed to meet official targets. The latest published figures (DfE (1993) [8]), based on a 10% sample of schools in England in 1992, suggest that 30% of those teaching mathematics in maintained secondary schools have a degree with mathematics as a main subject, and that 39% of mathematics lessons are taught by such teachers. Despite the peculiarly technical nature of mathematics, these figures are markedly worse than the corresponding figures for most main-stream subjects — see Appendix F. Thus, despite the apparent improvements there is still cause for concern. In particular, the concentration of well-qualified teachers in 16–19 colleges means that younger secondary pupils are less likely to be taught their mathematics by teachers who are both enthusiastic about the subject and have a clear grasp of its internal structure.

At primary level the position is especially serious. Mathematics is a core subject in the National Curriculum yet there must still be many thousands of teachers who never achieved a satisfactory qualification in the subject at age 16. Only 10% or so of those with B.Ed degrees have studied mathematics as a main subject or as a major component of their course, and very few have a degree in mathematics. It is also clear (see Section 6) that the current demand for would-be teachers to have at least a pass at Grade C in GCSE mathematics is no longer comparable with the corresponding requirement made in the late 1970s.

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<sup>12</sup> ‘Well-qualified’ is not easy to define, in terms of official statistics. In the figures from 1977, we are using what Cockcroft (1982) calls a ‘good’ qualification: this includes all trained teachers having a degree with mathematics as main subject, and some with subsidiary mathematics in their degrees.

There is still a desperate shortage of properly qualified mathematics graduates at all levels of the teaching profession. In the light of this, it is hard to see how the proposal (Dearing (1995) [7]) to introduce compulsory core skills for all students at 16–19, including ‘number’, could possibly be staffed without spreading an already over-stretched resource even more thinly. Although recruitment for PGCE courses in mathematics improved during the recent recession, reports from education departments suggest that many students “appear not to have a very positive sense of mathematics or of their own mathematical ability”. (Despite this, within twelve months, the statistics will count them as ‘well-qualified’ teachers.) It is important for the long-term health of the profession that a substantial number of our better mathematics students consider a teaching career; at present the contrast between working conditions in teaching and in other professions is such that good graduates are required to pay too high a price. University mathematics departments have an important role to play in reversing the trend, but they will need substantial government support.

**In the long term, to improve what is taught and how it is taught, we must raise the competence and confidence of those who choose to become mathematics teachers, and support those who are already teaching in schools and colleges. This will require cooperation between government, university mathematics departments, and those involved with pre-service and in-service training. (See Conclusions 7 and 8, Section 16.)**

## 14 MARKET FORCES

Some of the strongest criticisms brought to the attention of the working group hinged upon (presumably unintended) consequences of the simplistic application of ‘market forces’ to education. We would all like to see standards raised, to drive out bad practices and to foster good ones. However, the use of unnecessarily crude measures of success often militates against what is ostensibly desired. For example, league tables encourage schools to move their examination entries to what are seen as ‘less demanding’ examination boards — and the working group was supplied with compelling evidence that this is happening. Moreover, demands on students and overall standards are slipping as the boards themselves compete to retain market shares. Modular A-levels may have much to offer, but schools should not feel compelled to switch to them simply because they allow a near 100% pass rate, (since the students who fail to achieve that goal can be discounted).

Similar unfortunate effects of market forces are seen in higher education. In the first place, because of prospective science and engineering students’ worries about mathematics, there is pressure on those departments to reduce the mathematical content of their courses. There are also pressures to retain students for the funds they generate, even if they fail mathematics courses which form part of their degree. Finally, there is pressure to award more first and upper second class honours degrees, since this could be taken as a measure of the effectiveness of an institution rather than of its generosity. All these pressures drive down standards.

The losers include the more selective departments whose traditions of excellence are

undermined, and the high-attainers (on whose future so much of the country's prosperity depends) who fail to be challenged and whose achievements are not adequately recognised.

**We urge the government to take serious note of these comments and, after full and open discussion, to act before irreparable damage is done.**

## 15 CONSULTATION

Higher education wishes to play its part in, and bring its knowledge and expertise to bear on, the development of mathematics in the schools of this country. It seeks to do so in co-operation with schools and further education colleges, with government and with other concerned bodies. This, however, is rendered extremely difficult by the techniques adopted by SCAA. It is necessary, therefore, to comment explicitly on SCAA's currently preferred mechanisms — primarily large-scale 'consultation' exercises, and *ad hoc* committees. At best these mechanisms allow for outside bodies to comment, or to provide input. There is no genuine dialogue, or discussion — the system is too one-sided, with the agenda and all decisions about which questions to raise and which issues to pursue, being controlled in-house. The *ad hoc* committees selected by SCAA's secretariat provide those bodies with whom they should be routinely cooperating with no systematic representation, no continuity, and no stake in agenda setting. Thus power is concentrated within SCAA itself. This effectively frustrates all unwelcome attempts at cooperation on the part of bodies representing teachers, academics, or employers.

**We are facing serious challenges. Only if all parts of the education system work together will true progress be made.**

## 16 CONCLUSIONS

This report demonstrates that there are many problems concerning education in mathematics in England and Wales to which solutions must be urgently sought. Below we put forward some proposals. Apart from the first two recommendations, these are intended to fuel **discussion** — for there are many parties not involved in the writing of this report who must be involved in discussions of such proposals. But the two main recommendations, which would provide appropriate settings for such discussions and decision-taking, are for **immediate and urgent action**.

## MAIN RECOMMENDATIONS FOR IMMEDIATE ACTION

- 1 We strongly recommend that the DFEE set up a standing committee, including substantial representation from higher education, to provide an overview of education in mathematics from primary school through to university, and to ensure that sound advice and adequate support are provided to those involved in its organisation and delivery.**

The questions of what mathematics should be taught, how, when and to whom, are not easy to answer. There are strong, but sometimes subtle, connections within the subject; there are also important links with other disciplines. Some of the connections stretch forward several years in the student's experience and can rely heavily on facility in specific techniques or upon understanding of earlier ideas. Therefore any decisions about change need to be made within an informed context, with a complete overview of the subject and in the light of informed international comparisons. The current system, relying merely upon a small secretariat within SCAA, is not adequate for the task. It cannot achieve the level of trust required nor, indeed, can it be relied upon always to provide sound advice.

It is clear, from the earlier sections of this report and from other recommendations below, that there are many issues requiring serious and sensible debate and mediation. These provide both a justification and a remit for such a committee, which would be expected to establish *ad hoc* working groups on specific issues, and to receive and act on their reports. In order to develop the required long term overview and to monitor the effects of the changes made, this needs to be a standing committee.

Its membership should include representation from two distinct groups within higher education — from mathematicians and from other major users of mathematics. Higher education not only contains specialists with the necessary insight in to the internal structure of the discipline, but is now, as we have explained elsewhere, a major 'end-user' of the school system. In particular, almost all students achieving A-level in mathematics now go on to higher education and most will make use, in that context, of their previous training.

- 2 The government should establish, as a matter of urgency, a Committee of Enquiry, on which higher education is strongly represented, with the express task of examining the current curricula in mathematics, both age 5–16 and age 16–19, and making proposals in time to allow carefully considered action at the end of the current moratorium on change in the National Curriculum.**

The committee's brief should include the aims and content of the curriculum, the manner in which it should be specified, principles underlying the ways in which the needs of all types of students can be met, and means of assessment. The committee's concerns should also include mathematics and numeracy provision overseen by NCVQ. Data from examination boards and the DFEE, together with the evidence provided from higher education — both from mathematicians and from engineers and scientists — make it abundantly clear that a rethink is needed, and that the thinking should begin immediately if we are not to repeat the old cycle of over-hasty and ill-considered change.

## PROPOSALS FOR DISCUSSION

### 3 *The National Curriculum 5–16*

- (a) The current system, which aims to cater for different abilities by varying the speed of progress through the same material, must be reconsidered. It is inappropriate for progression to A-level and higher education. (Its suitability for low-attainers must also be questioned.)
- (b) There needs to be more emphasis in national curriculum mathematics on important basic topics and on the acquisition of those techniques which will form a firm foundation for further study. Nowadays, a third of all students progress to higher education and most of these use some mathematics in their degree courses. It is also essential that the exactness of mathematics and its notion of proof should not be distorted and that close attention should be paid to accuracy and clarity of oral and written mathematical communication, including the setting out of logical arguments in good English.
- (c) Teachers would be greatly assisted if the National Curriculum were more explicit about the basic facts, methods and ideas which are fundamental to subsequent mathematical progress.
- (d) Methods of assessment must be reviewed. Until major changes can be made, the decision to allow Grade B to be obtained on GCSE intermediate mathematics papers, other than in exceptional circumstances, should be reconsidered.
- (e) The recent substantial cut in time spent on mathematics, especially at age 11–16, must be urgently reviewed.

### 4 *On alternatives to A-level*

The present A-level system would no longer seem appropriate for many students and militates against their opting for a course in which mathematical and scientific studies form a coherent component. This seriously damages the nation's scientific and technological well-being. The Government should reconsider its support for the *status quo* and immediately begin to investigate the feasibility, on educational and logistic grounds, of alternative systems which could preserve — or increase — academic values while offering students more broadly-based and coherent educational options. However, this should not prejudice the review of the 5–16 curriculum.

### 5 *On changes to the existing A-level in mathematics*

- (a) Methods of dealing with A- and AS-level students with different needs, ability and attainment must be given serious attention. In particular, the desirability of papers differentiated by need (as in the French Baccalauréat), the position of further mathematics, and the need for a Special Paper demand urgent review.
- (b) The size and detail of the 'core' should be reconsidered. A larger, more detailed core would ease the transfer from school to university greatly, with negligible loss to teachers and students.
- (c) Modular A-levels have some advantages. However, a single summative examination on core topics (along the lines of that recommended in IMA (1995) [16]) could have considerable merits — especially for more ambitious students, and for more selective HE institutions.

- (d) We recommend that the government should establish a bursary scheme for students who go to university to study mathematics, physics, physical chemistry or engineering, based on the results of some such examination as that in (c). (To encourage the study of mathematics in maintained schools, we would propose that the proportion of such awards available to students from non-maintained schools should be strictly circumscribed.)
- (e) The comparative difficulty of mathematics A-level should be kept under review. Moreover, simplistic measures of success based on league tables which, for example, fail to distinguish between modular and non-modular syllabuses or mathematics and further mathematics, should be re-examined as a matter of urgency.
- (f) Consideration should be given to the obvious merits of cutting the number of different A-level syllabuses. This would simplify the transition from school to university and would make it possible to insist on improvements in question-setting and in moderation.

## 6 *On GNVQ*

We have recommended (Recommendation 2) that the form GNVQ should take in the medium-term should be considered along with the future of academic 16–19 qualifications. Since GNVQ can lead to higher education, it is important that mathematicians in higher education at all types of university should be included in discussions at NCVQ.

## 7 *On teacher training*

The government and universities must co-operate to improve the supply of highly qualified mathematics teachers at all levels. The government should provide extra support, in the way of subject-based mathematics in-service training, to existing teachers of the subject. There needs to be a greater effort to increase the fund of mathematical expertise in primary, as well as in secondary schools. The minimum mathematical preparation required by those wishing to become teachers of mathematics at different levels should be reviewed (see Section 13). In view of the complexity and the extreme importance of this matter, it may well be appropriate to establish a committee to investigate and report on these issues.

## 8 *On universities*

- (a) All mathematics departments in universities should provide, or collaborate in providing, appropriate courses or modules to attract and stimulate potential school teachers of mathematics.
- (b) All those involved in higher education need to consider further their provision for incoming students — particularly in the short term, before changes can be made at secondary level. The effect of wider access, and the resulting widening in the ability range, must be considered further. The moves to four-year courses leading to MPhys and MMath are one response, but there remains a need to consider whether the current honours degree structure is suited to all, or even the majority, of incoming students.

## 9 *On market forces*

Steps must be taken to counter the unfortunate effects of ‘market forces’. In particular, we note the competition between examination boards which has driven down standards, the incentives to schools to shop around for short-term gains, and the pressures on those in higher education both to reduce the mathematical content of their courses in order to attract applicants and also to lower standards in order to retain and reward students.

10 *On the provision of data*

It is clear from earlier sections that there is a general problem regarding the availability of sensible and reliable statistics about mathematics students, teachers, etc. In particular, at university level, statistics for mathematics and for computing should no longer be aggregated as ‘Mathematical Sciences’. It is important to know the separate figures and trends for these disciplines which are quite distinct, although with an important interface.

11 *On attracting students to mathematics, science and engineering*

Government, learned societies, professional bodies, university mathematics departments, etc. should collaborate to increase publicity for mathematics and for mathematics related careers with a view to increasing the supply of young mathematicians. In particular, government attempts to publicise Science and Technology should be renamed — e.g. from SET to STEM (Science, Technology, Engineering and Mathematics) — making explicit the underlying importance of mathematics.

# APPENDIX A

We present here two illustrations of the problems described in Section 4. In each case, these are presented with the question “Are these standards appropriate?” in mind rather than “Are they worse than 30 years ago?”

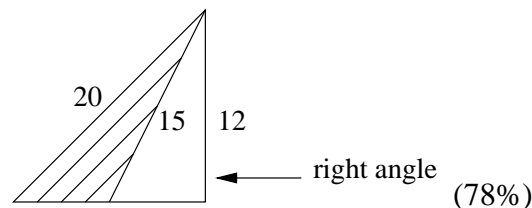
## A1 New undergraduates

The first example concerns a test given to 55 incoming honours mathematics students at one English university in October 1994. The students’ mathematics A-level grades were: A 12, B 13, C 16, D 6, other qualification 8. We choose some of the questions to indicate particular concerns, but other examples could also have been used. After the statement of each question appears the percentage of students who answered the question correctly.

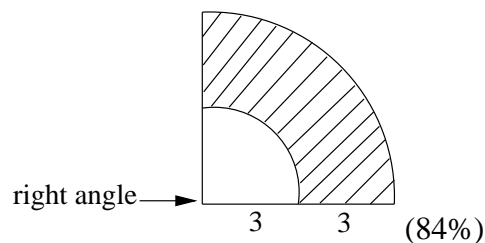
1. Solve the equation  $x^2 - 4x = 0$ . (75%)
2. Factorise the following expressions as far as possible:
  - (a)  $2x^2 - x - 3$  (78%)
  - (b)  $x^2 - 9y^2$  (73%)

*[Note: Of the 6 students who managed to factorise neither of these quadratics, 3 had achieved a Grade B at A-level.]*

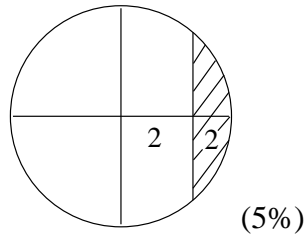
3. Calculate the areas shaded in the diagrams, leaving your answer in terms of  $\pi$  where appropriate.
  - (a)



- (b)



(c)

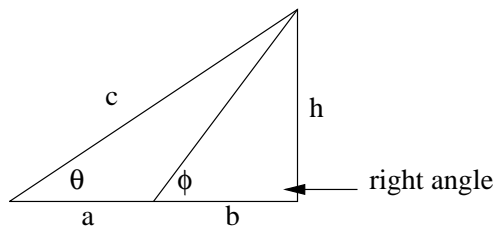


[Note: the lack of success on the third part of this question underlines the dangers of a system which

- (a) sets questions which lead the candidate, step by step and
- (b) rewards superficial knowledge of those items listed in the 'levels' of the National Curriculum.

This question needs no mathematics beyond 'level 7'; but it asks students to use and apply this knowledge in a mathematical way. For they must draw together knowledge from different areas and undertake, without instruction, the extra steps required.]

4. In the diagram below:



- (a) Express  $h$  in terms of  $b$  and  $\phi$ . (91%)
- (b) Express  $h$  in terms of  $c$  and  $\theta$ . (91%)
- (c) Express  $h$  in terms of  $a$ ,  $\theta$  and  $\phi$ . (18%)

[Note: Again, there is a substantial drop in success when the question requires additional, unsignalled steps.]

## A2 Evidence from national mathematics competitions:

The questions below come from two national events taken in 1995. Each is aimed at the top 35% of the relevant age group. Thus the population is reasonably representative of those who might expect at a later date to go on to higher education. Our information suggests that schools do indeed restrict their entries to something like the top 35%, and this should be borne in mind when assessing the data. All indications point to the conclusion that the schools that take part tend to be above average in their aspirations and staffing.

The first group of questions comes from the *UK Intermediate Mathematical Challenge* for pupils in English School Years 9–11 (10–12 in Northern Ireland, S3 or S4 in Scotland), which had an entry of 115,000 from 1,500 schools. The second group of questions comes from the *UK Junior Mathematical Challenge* for pupils in English School Years 7 and 8 (8 and 9 in Northern Ireland, S1 and S2 in Scotland), which had an entry of 105,000 from 1,400 schools.

Both events are one hour papers with twenty five multiple-choice questions of graded difficulty. Candidates are advised to concentrate their efforts, in the first instance, on Questions 1–15, and to attack later questions selectively. While the problems are technically elementary, they all require pupils to select and coordinate two or more simple steps. We have excluded only those problems which are sufficiently unusual, off-beat, or hard to make the interpretation of statistics problematic. The questions remaining are those which test standard curriculum material in a relatively straightforward way. (To allow for differing backgrounds, in the case of the UK JMC we have further restricted the selection to concentrate on problems on ‘number’.) After each option we give the percentage of candidates choosing that answer. The balance consists of those who failed to answer the question. The percentage corresponding to the correct answer is underlined.

Our central professional concern is with the quantity and quality of those proceeding to university. This has led us to consider such students’ preparation at 16–19, and hence to comment on weaknesses in the curriculum at 5–16. Of the two sets of problems given here, those from the Intermediate Challenge are of most immediate concern; the Junior Challenge questions have been included mainly to complete the overall picture.

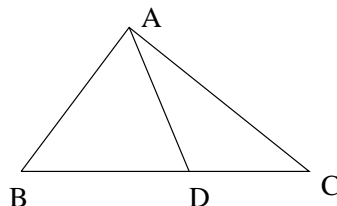
Among pupils’ responses to the Intermediate Challenge questions we draw particular attention to the following. The success rate on Q1 is very low (given that candidates come roughly from the top third of the ability range), and the numbers choosing option C suggests that many think division by single digit numbers depends only on the last two digits. The response to Q7 reflects our concern 4C (Section 4): instead of using the given information to analyse the diagram *mathematically*, pupils have simply assumed that a triangle in the position shown necessarily has a right angle at A. The response to Q12 is most striking: where previous generations would be expected to *calculate* the percentage increase, we only ask how the percentage increase could be calculated. Finally, the responses to Q15 and Q17 illustrate basic weaknesses which hinder the subsequent development of algebraic technique.

## UK Intermediate Mathematical Challenge

1. Which of these divisions has a whole number answer?

**A**  $1234 \div 5$    **B**  $12345 \div 6$    **C**  $123456 \div 7$    **D**  $1234567 \div 8$    **E**  $12345678 \div 9$   
 4%                      8%                      21%                      7%                      58%

7. Find the value of the angle  $BAD$ , where  $AB = AC$ ,  $AD = BD$ , and angle  $DAC = 39^\circ$ .



**A**  $39^\circ$    **B**  $45^\circ$    **C**  $47^\circ$    **D**  $51^\circ$    **E**  $60^\circ$   
 12%      8%      21%      41%      9%

9. The diagonal of a square has length 4 cm. What is its area in  $\text{cm}^2$ ?

**A** 2   **B** 4   **C** 4.2   **D** 8   **E** 16  
 3%   13%   14%   32%   33%

10. If  $x = 3$ , which expression has a different value from the other four?

**A**  $2x^2$    **B**  $x^2 + 9x$    **C**  $12x$    **D**  $x^2(x - 1)^2$    **E**  $2x^2(x - 1)$   
56%                      8%                      6%                      11%                      17%

12. The number of students who sat the 1992 UK SMC was 80,000. The number who sat the 1993 UK SMC was 105,000. Which calculation gives the percentage increase?

**A**  $\frac{80,000}{105,000} \times 100$    **B**  $\frac{80,000}{100} \times 105,000$    **C**  $\frac{105,000}{80,000} \times 100$   
 32%                      10%                      25%

**D**  $\frac{105,000 - 80,000}{80,000} \times 100$    **E**  $\frac{105,000 - 80,000}{105,000} \times 100$   
20%                      11%

15. What is the value of the fraction shown here when written as a decimal?  $1 + \frac{2}{1 + \frac{3}{1+4}}$

**A** 1.5   **B** 2.25   **C** 2.5   **D** 2.6   **E** 3.5  
 17%      23%      11%      17%      19%

17. Augustus Gloop eats  $x$  bars of chocolate every  $y$  days. How many bars does he get through each week?

**A**  $7x/y$    **B**  $7y/x$    **C**  $7xy$    **D**  $1/7xy$    **E**  $x/7y$   
30%      11%      31%      5%      13%

## UK Junior Mathematical Challenge

1. Gill was surprised this year to discover that a thousand million, (the equivalent of a US billion) is sometimes called a *gillion*! Which of the following represents a gillion in figures:

A 1 000 000 000    B 10 000 000    C 1 000 000    D 100 000 000    E 10 000 000 000  
82%                      2%                      3%                      5%                      7%

3. In Birmingham, on the day of the last UK JMC (26 April, 1994) the sun rose at 0550 and set at 2041. For how long, in hours and minutes, was it above the horizon?

A 14h 51m    B 14h 91m    C 15h 09m    D 20h 41m    E 25h 91m  
52%                      23%                      17%                      3%                      3%

5. You have to make up a sum using two different numbers chosen from 6, 8 and 72, and one operation chosen from +, −, × and ÷. For example, choosing 8, 6 and + gives you 8 + 6, a sum whose ‘answer’ is 14. Which of the options below is not a possible ‘answer’?

A −2    B 0.75    C 1.3    D 9    E 432  
8%    28%    32%    14%    12%

8. What is the units digit of the product of 12345679 and 63?

A 2    B 3    C 7    D 9    E can't be sure without a calculator  
18%    13%    22%    14%                      26%

9. Peter is working out a sequence of numbers. To get the next number in the sequence he trebles the previous number and adds one. For example if his first number was 10, his second number would be 31, his third would be 94, and so on. In fact, the fifth number in his sequence was 445. What was his first number?

A 1    B 5    C 10    D 16    E can't be sure  
8%    45%    16%    12%                      15%

10. £50,000 in £50 notes weighs 1.3kg. About how much does one £50 note weigh?

A 0.0013g    B 0.13g    C 0.65g    D 1.3g    E 65g  
44%                      17%                      5%                      26%                      2%

19. If five sixths of a number is sixty, what is three-quarters of it?

A  $37\frac{1}{2}$     B 40    C 45    D 54    E 60  
6%                      6%                      45%                      37%                      4%

## APPENDIX B

We present a summary of the extent to which new A-level syllabuses for 1996 cover those topics in the previous A-level core which are not in the new core. The analysis is presented in the form of a table. This may conceal some detailed distinctions, but is done for ease of reference which a more discursive treatment would not provide. Each topic is indicated by a simple Y or -, this showing whether or not that topic is part of the compulsory element of a board's syllabus, taken by all candidates irrespective of what award they are entered for, or which combination of modules they take. In some cases a topic may be in an optional part of the syllabus, but it cannot be assumed to have been covered by all candidates.

Topic	A	B	C	D	E	F	G	H	I	J	K
Rational functions	Y	-	Y	-	-	Y	-	Y	-	Y	-
Partial fractions	Y	Y	Y	-	-	Y	Y	Y	-	Y	Y
Binomial for $ x  < 1$	-	Y	-	-	-	-	Y	Y	Y	-	-
Six trigonometric functions	Y	-	Y	-	-	Y	Y	Y	Y	Y	Y
Sine and cosine rules	Y	Y	Y	Y	-	Y	Y	Y	Y	Y	Y
$a \cos \theta + b \sin \theta = r \cos(\theta + \alpha)$	-	Y	-	-	-	Y	Y	Y	Y	-	-
General soln of trig equations	-	-	-	-	Y	Y	-	-	-	-	-
Small trig approximation	-	Y	-	-	-	Y	-	-	-	-	-
Inverse trig functions	-	Y	Y	-	Y	Y	Y	Y	Y	-	Y
Implicit differentiation	-	-	Y	Y	Y	Y	Y	Y	-	Y	Y
Parametric differentiation	Y	-	Y	Y	Y	Y	Y	Y	Y	Y	Y
Normals	Y	Y	Y	-	Y	-	Y	Y	Y	Y	-
Small increments	-	-	-	-	-	-	-	-	-	-	-
$\int 1/(1+x^2), \int 1/\sqrt{1-x^2}$	-	-	Y	-	-	-	-	-	-	-	Y
Volumes of revolution	-	-	-	Y	-	Y	Y	Y	Y	Y	-
Vectors	-	Y	-	Y	Y	Y	Y	Y	-	-	-
Scalar products	-	Y	-	Y	Y	Y	Y	Y	-	-	-
Vector equation of a line	-	Y	-	Y	Y	Y	Y	-	-	-	-

A:AEB

B:MEI (O&C)

C:NEAB

D:NUFFIELD (OXF)

E:OXFORD

F:O&C

G:SMP 16–19 (NEAB)

H:UCLES (Linear)

I:UCLES (Modular)

J:ULEAC

K:WJEC

## APPENDIX C

The data below refer to the Summer examination taken in England and Wales

	Single Maths A-level entries		Double Maths entries	Single passes (Total)	All Maths entries as % of all A-level entries
	Male	Female			
1965	36,800	6,400	15,600	29,900	15.9%
1994	36,500	20,400	5,400	48,000	8.7%

‘Single maths’ entries are for A-levels such as Pure Mathematics and Mathematics, and represent potential undergraduates.

‘Double maths’ entries are for A-levels such as Applied Mathematics and Further Mathematics (normally taken in conjunction with a single subject).

All numbers have been rounded to the nearest hundred.

In 1994

- Independent schools accounted for 12,700 of the total number of 63,600 entries (England Wales and Northern Ireland) for Mathematics A-levels, i.e. 20%
  - 42.6% obtained A against a national average of 25.5%
  - 63.1% obtained A or B against a national average of 43.6%
- Approximately one third of all ‘Double Maths’ candidates came from independent schools.

(It is extremely difficult to find data on examinations, entries, pass rates, etc. which are both accurate and comparable. We have tried to present information which is both fair and accurate and apologise for any inaccuracies which may remain.)

## APPENDIX D

The table below shows the changes in numbers taking A-level Mathematics and Further Mathematics. It shows, by way of contrast, figures for the numbers in France passing the Series C Baccalauréat. (See Section 10 for more details about the Baccalauréat.)

	O-level A-C, CSE1 or GCSE A-C two years previously	Single A-level entries	Single A-level passes	Double A-level entries	Size of English 18 year cohort	Baccalauréat Series C passes (Metropolitan France)
1960	81,500 <sub>E&amp;W</sub>	29,000 <sub>E&amp;W</sub>	19,200 <sub>E&amp;W</sub>	10,500 <sub>E&amp;W</sub>	550,000	17,100
1970	150,400 <sub>†E&amp;W</sub>	50,200 <sub>E&amp;W</sub>	33,800 <sub>E&amp;W</sub>	14,200 <sub>E&amp;W</sub>	628,000	21,400
1980	224,000 <sub>†E</sub>	67,500 <sub>E</sub>	45,400 <sub>E</sub>	13,400 <sub>E</sub>	761,000	32,700
1985	259,400 <sub>†E</sub>	75,800 <sub>E</sub>	52,100 <sub>E</sub>	11,900 <sub>E</sub>	764,000	33,500
1990	243,500 <sub>E&amp;W</sub>	69,500 <sub>E&amp;W</sub>	50,300 <sub>E&amp;W</sub>	6,900 <sub>E&amp;W</sub>	679,000	56,800
1994	260,600 <sub>E</sub>	56,900 <sub>E&amp;W</sub>	48,000 <sub>E&amp;W</sub>	5,400 <sub>E&amp;W</sub>	557,000	64,600*

- E = England
- E&W = England and Wales
- † = these numbers are inflated since some students entered for and obtained both an O-level A-C and a CSE 1.
- \* = includes Antilles, Guyana & Reunion (very small numbers).

## APPENDIX E

The table below shows the relative performances at A-level of candidates having different GCSE results.

**Table: GCSE Maths Grade of 17 year-olds taking single A-level Maths in 1994**

Base: 15 year-olds in 1992 who attempted at least one A-level in 1994. England

Single A-level maths grade	GCSE Mathematics Grade					
	A	B	C	D-G	No pass	Total
A	6002	531	78	22	98	6,731
B	5315	1300	161	40	81	6,897
C	4139	2066	345	60	54	6,664
D	2884	2285	508	57	41	5,775
E	1862	2034	600	39	19	4,554
All passes	20202	8216	1692	218	293	30,621
Did not pass (N,U,X)	2491	3793	1618	96	35	8,033
Total	22693	12009	3310	314	328	38,654

## APPENDIX F

The following statistics come from [DfE Statistical Bulletin 24/93] and are based on a survey of 10% of maintained secondary schools in England in January, 1992.

Table F1 shows the percentage of mathematics teachers in maintained secondary schools in England having specified levels of qualification.

**TABLE F1**

Degree with maths as a main subject	B.Ed with maths as a main subject	Some other math. qual.	Others
30%	9%	29%	32%

For comparison with the 30% figure above, we note the corresponding percentages of teachers in other subjects who have a degree with that subject as ‘main subject’:

Biology, English, Geography (33%); Art (34%); History (38%); French, Music (41%); German (45%); Economics (49%); Physics (41%); Chemistry (62%).

Table F2 shows the percentage of mathematics lessons taught by teachers in maintained secondary schools in England having specified levels of qualification.

**TABLE F2**

	Degree with maths as a main subject	B.Ed with maths as a main subject	Some other math. qual.	Others
Years 7–9	33%	13%	36%	18%
Years 10–11	39%	12%	35%	12%
Years 12–13	63%	8%	21%	8%
Overall	39%	12%	33%	16%

For comparison with the ‘overall’ figure of 39% above, we note the corresponding percentages of lessons in other subjects taught by a teacher having a degree with that subject as ‘main subject’:

Biology (36%); English (43%); Art (46%); French (47%); Geography (49%); Music (51%); History (53%); German (57%); Physics (59%); Economics (60%); Chemistry (70%).

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A. G. Howson (Chairman), University of Southampton.  
A. D. Barnard, King's College, London, representing LMS.  
D. G. Crighton, University of Cambridge, representing LMS.  
N. Davies, Nottingham Trent University, representing RSS.  
A. D. Gardiner, University of Birmingham, representing LMS.  
J. M. Jagger, Trinity and All Saints College, Leeds, representing LMS.  
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