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promotion
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The Maths Inside project

How often do we read that 'maths is in everything in the world around us' or 'maths affects everything that we do'?

And yet, it is often extremely difficult to find a good example of mathematics to communicate with a general audience. A key problem is the perception of mathematics. Many people think maths is just too difficult and are put off before even starting to try to understand. Secondly, much cutting edge mathematics research *is* too difficult to understand, even for those with a quite advanced level of mathematics.

A further issue when trying to communicate mathematics is that there is no 'end product' as with much science research. The mathematics is the process by which a hypothesis is proven or disproven and unless an audience can follow this process, there really isn't much to see.

The Mathematics Promotion Unit, which is directed by a joint committee of members of the London Mathematical Society and the Institute for Mathematics and its Applications, discussed how we could get more mathematics 'out there'.

We realised that mathematics is very often hidden. We wanted to come up with a way of drawing attention to the mathematics inside other research, predominantly in science but without forgetting that mathematics can be the basis for research in many other subjects.

We came up with the 'Maths Inside' project, something like the 'Intel inside' stickers that draw ones attention to the chips that power many of our computers yet are hidden away deep inside the box.

For the pilot of this project, the Mathematics Promotion Unit worked with one of the most prestigious annual science events. The Royal Society allowed the MPU to contact exhibitors at its Summer Exhibition. Three research teams enthusiastically agreed to join in and all were very helpful in putting forwards their mathematics.

The CLEVER project (University of Bath) had created a space-saving three wheeled car designed to use less fuel and to be suitable for city use. To do this, the car had to be able to go round corners safely, and the Maths Inside looked at how this had been done.

The other projects were the Stereo Mission (Imperial College London, University College London, Rutherford Appleton Laboratory et al), which had sent satellites to produce pictures of the sun's activity from two angles to create three dimensional images and another research team (Queen Mary, University of London) which was using the Menger Sponge as a demonstration of their three dimensional printers.

With the help of members of both the IMA and the LMS, the MPU produced the three leaflets, printed below, which sat on the exhibits and were given to visitors by the exhibitors.

Feedback has been excellent and next year we are hoping that the Maths Inside will be working with a greater number of exhibitors and incorporated into the Royal Society's overall fact sheet for the event next year.

The MPU is hoping that the Maths Inside project will be working with a variety of other events. It has advised the British Association on content for a leaflet highlighting mathematics themed events at this year's Festival of Science at York and is currently looking for other opportunities.

Narrow cars may take up less space but they have a problem - they tend to roll as they go around corners, and can even roll over if they go round the corner too fast. When a bicycle or a Pendolino train goes round a corner, it tilts into the corner. So the CLEVER team developed a tilting mechanism to make cornering possible using the branch of mathematics called **dynamics**.

When the car goes round a corner at a speed *V* it has to accelerate towards the centre of corner – this happens by the car's tyres creating a sideways force into the corner. The acceleration is given by

$$a_{corner} = \frac{V^2}{R}$$

where *R* is the radius of the corner. The tighter the corner is, the greater the acceleration is. But if this acceleration is too great, the weight lifts off the inside wheel and the car will roll over.

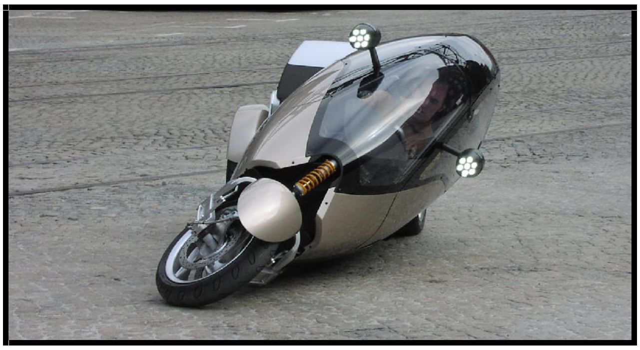
The researchers had to find the greatest acceleration that the car could cope with without rolling over. This is given by the equation

$$a_{\max} = \frac{gt}{2h}$$

where *t* is the width of the car, *h* is the height of the centre of gravity of the car and *g* is the acceleration due to gravity.

The relationship between the width of the car and the height of its centre of gravity is crucial to determining the maximum acceleration round a corner. Since CLEVER is only 1m wide, the team needed to ensure that the height of the centre of gravity *h* is also small, otherwise the car would always have to go very slowly to get around corners safely.

They discovered that *h* needed to be maximum 0.5m. But they did not want the occupants to be sitting so low, as this would be problematic for getting in and out of the car and also make visibility difficult.



So they developed a tilting mechanism which leans the body of the vehicle like a motorbike at angles upto 45°. This shifts the centre of gravity sideways and pushes the weight on to the inside wheel thus avoiding roll-over. In fact, CLEVER is very clever, and uses a computer control system to work out the best tilt angle, which depends on the speed *V* of the car and also on the angle at which the driver steers into the corner as this changes the corner radius, *R*.

This document was produced by the Mathematics Promotion Unit
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July 2007

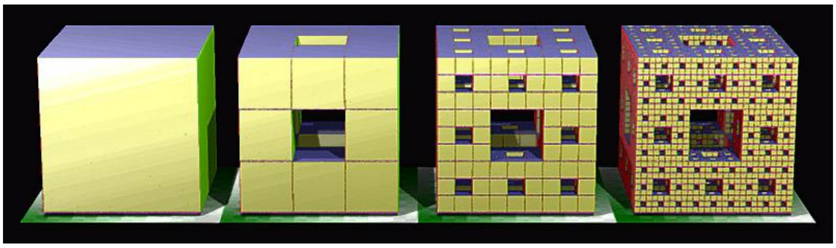
The Maths inside...

THE MENGER SPONGE

Austrian mathematician Karl Menger first described what became known as the Menger Sponge in 1926. He was working in the field of **topology** and was trying to develop a definition of dimension. But the Menger Sponge is in fact an instance of what is nowadays termed a **fractal**.

The Sponge is constructed by dividing a unit cube into an array of 27 smaller cubes of side one third, then removing the central cube and six cubes at the centre of each face. This procedure is repeated with each of the 20 remaining smaller cubes.

At any point in the process, there will be 20^n cubes remaining where n is the number of iterations which have been carried out. The Sponge is the limit of this construction process as n tends to infinity.



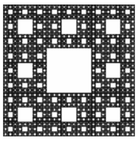
The first four iterations in creating a Menger Sponge

The Sponge is a key and somewhat surprising mathematical object. Menger was able to show that it had topological dimension only 1, which means that in many ways its behaviour is much closer to that of curves and line drawings than to surfaces or solid volumes.

At the same time, the Menger Sponge in fact contains a distorted copy of every other topologically 1-dimensional set. This means that any curve, line, diagram or graph can be distorted to fit in the sponge without self intersection and mathematicians describe the Sponge as a 'universal curve'.

Georg Cantor described a linear version of the Sponge in 1883. Known as the 'Cantor set', this might be considered to be the prototype fractal. The planar analogue, first described by Wacław Sierpiński in 1916, is known as the 'Sierpiński carpet'.

Construction of the Cantor set by repeatedly removing the middle third of intervals



Sierpiński carpet

Very roughly, fractals are geometric shapes that have irregularities at all scales, however small. For instance the Menger sponge has holes of arbitraily small sizes. Moreover, the Menger Sponge is a 'self-similar' fractal in that it is made up of 20 scaled copies of itself, each one scaled by a factor of one third.

Using traditional machining methods, it is very difficult to make a Menger Sponge. The research team show a menger sponge model fabricated by rapid prototyping method.

This document was produced by the Mathematics Promotion Unit
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The Maths inside...
THE STEREO MISSION

The STEREO mission is flying two spacecraft, one that leads Earth in its orbit round the Sun and one that lags behind. These spacecraft produce simultaneous views of the Sun and inner solar system using identical instruments.

The effect is like a pair of human eyes. Each eye sends an image simultaneously to the brain which automatically puts them together to produce a single, 3-D picture.

For the images produced to be viewed by people in stereo the spacecraft must not be too far apart; when they are, it is as if they have gone 'cross-eyed'.

So the researchers had to calculate for what period of the mission the technique would work. A bit of simple **geometry** and **trigonometry** gives the answer.

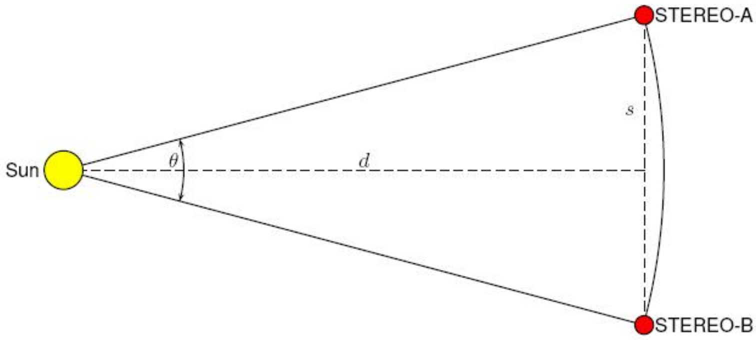

Figure 1: Geometry of the STEREO mission

In Figure 1, the relationship between the distance between the two spacecraft, *s*, and the distance from the spacecraft pair to the Sun, *d*, is given by the formula:

$$\frac{s}{2d} = \tan \theta/2$$

During the mission the two spacecraft gradually move further apart with the angle θ increasing by 45° each year, distance *s* increasing and distance *d* decreasing. This makes the view of the Sun increasingly cross-eyed. To determine the maximum value for θ that makes viewing possible, the research team considered how human eyes work.

Human eyes are about 8cm apart and are unable to focus on objects closer than 10cm. This angle will be exactly the same for the real mission since the same ratios apply. So entering these values into our formula we have:

$$\frac{8}{20} = \tan \theta/2$$

which in turn gives us a maximum value for θ of about 44°.

And this told the researchers that they need to concentrate on data from the first year of the mission because after that the angle of separation would be greater than 45°.

This document was produced by the Mathematics Promotion Unit
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