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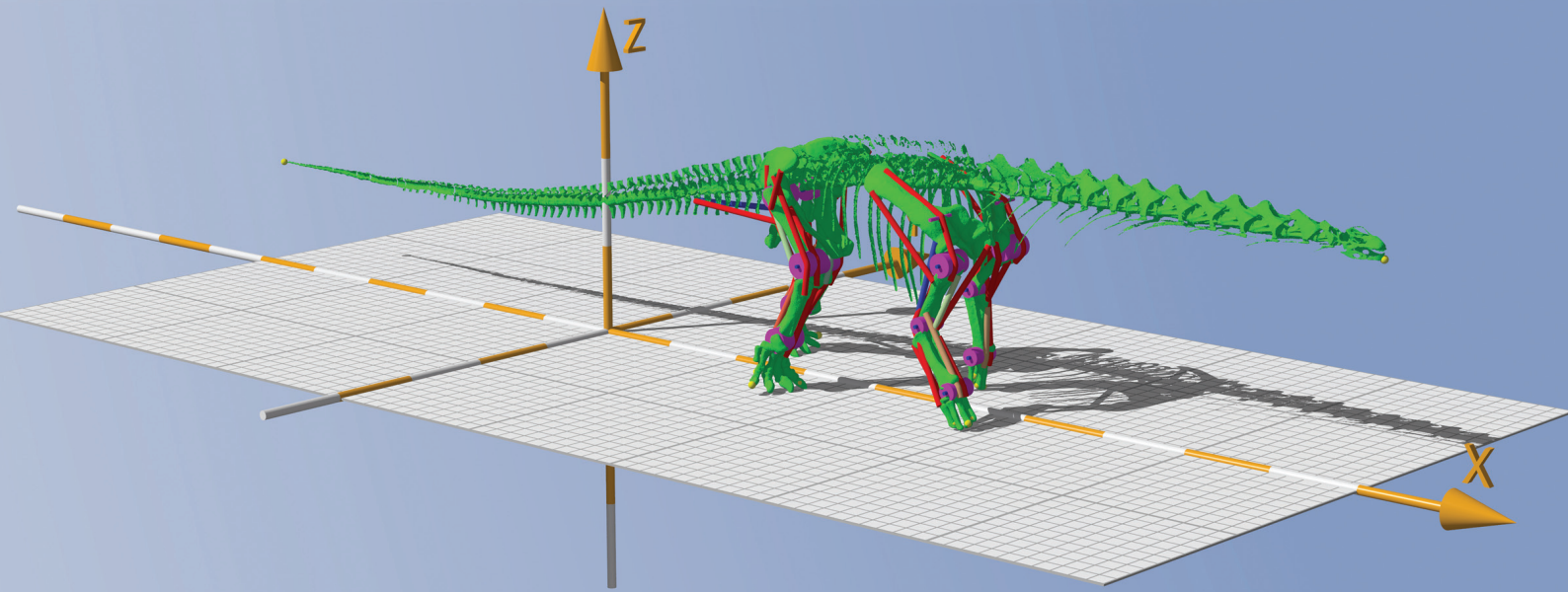
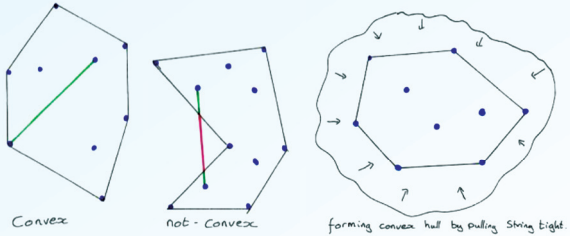


CONVEX HULLS

Using mathematics to recreate the past



Researchers at Manchester are exploring how dinosaurs would have walked and moved. As part of this research they build 3D computer-models of the dinosaurs to understand how the distribution of weight affects their likely movement. They begin by scanning dinosaur skeletons using a technology called Lidar, which uses lasers to scan the surfaces of the fossil bones.

The laser scans produce a large number of points in space that represent the bones surface. This is called a point cloud. Once they have the point cloud the researchers cover the virtual bones with flesh to model the dinosaur and its weight distribution. They do this by finding a 3D shape that encloses the points. Finding any shape that encloses them turns out to be easy — you could take a giant cube for example — finding a biologically reasonable one turns out to be the hard part. They use a construction from mathematics called the convex hull.

You can use the geometry of major load-bearing bones, such as the thigh bone (femur) to estimate the mass of an organism. This can be useful when only single bones or partial skeletons exist. The laser scanning qhull technique is very reliable and has been tested using the known body mass of extant (living) animals. This helps us ground truth our technique when applied to long extinct animals, such as dinosaurs. The laser scanning approach is rapid, accurate and repeatable.

To map a dinosaur using standard surveying techniques yields low resolution models of the animal and can take many hours or days.

However, a laser scanner can provide sub-millimetre accurate models of the whole skeleton in a matter of minutes that are already in a digital format. The data can be easily interrogated using software to 'wrap' the point cloud to yield a volume of the animal. This can then be used, with an estimated density, to calculate the body mass of the dinosaur or other extinct animal... all from a mounted skeleton in a museum!

The convex hull is the smallest shape enclosing all of the points in the point cloud that also satisfies the convexity property; this means that a straight line joining any two points in the shape stays entirely within the shape. To picture constructing the convex hull in two dimensions is relatively easy: imagine sticking some pins into a board and then wrapping some thread, or a rubber band, tightly around the outside of all of them. The shape that is formed is the convex hull of those points.

In three dimensions it's somewhat harder to imagine forming the convex hull of some points, but we could picture wrapping all the points in cellophane or a tight sheet of rubber. The rubber will conform to the shape of the points that it contains. The researchers take separate limbs or sections of the skeleton, form the convex hull of these components, and then combine them to form a realistic dinosaur model.

Convex hulls, and convex shapes in general, see a lot of use in mathematics and applications. In particular they are very useful in understanding and describing multi-dimensional spaces, computer pattern recognition, and in computer games.

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