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DEFINING THE KILOGRAM

With the help of mathematics

mass | kilogram | Planck's constant | watt balance | gravity | force | quantum physics

An African elephant has 8500, the average adult human has about 60, whereas a guinea pig only has one. We're talking kilograms of **mass**, but what does it actually mean to say a guinea pig has a mass equal to a **kilogram**? It means its mass matches that of a lump of metal stored in France. One kilogram is currently defined to be the mass of a particular platinum/iridium block held under lock and key at the International Bureau of Weights and Measures six miles from the centre of Paris.

But what if the block was lost or damaged? Even in its guarded location the mass of the block can subtly change. If the block's mass varies, so does the size of the kilogram. This used to happen with the second, which once was defined as a small portion of the Earth's rotation time. However, the period of the Earth's rotation (1 day) changes slightly due to factors like the Moon's gravitational pull on the oceans. Instead a second is now defined using radiation emitted by caesium atoms, which never changes.

Scientists would like to do the same thing with the kilogram by linking it to a fundamental constant of nature called **Planck's constant** (which, among other things, relates the frequency of light to the energy it carries). This can be done using a **watt balance**. In a simplified version the electrical current flowing in a loudspeaker coil is adjusted until the force produced just supports a small object whose mass is being measured.

The force of **gravity** pulling the mass down must be balanced by the **force** generated within the speaker:

$$mg = ILB$$

where m is the mass, g is the acceleration due to gravity, B is the strength of the speaker's magnetic field, I is the current put through the speaker and L is the length of current-carrying wire inside the speaker.

However, the value of LB is not easy to know because it is a property of the speaker construction and so needs to be replaced by something that is exactly the same, but which can be measured easily. That requires a second measurement:

$$\frac{U}{v} = LB$$

where the wire inside the speaker is made to move at a velocity v, causing the voltage U to appear at the ends of the wire. Combining that with the first equation means that:

$$m = \frac{UI}{vg}$$

Two effects in **quantum physics** are then used to re-write current and voltage in terms of Planck's constant, making the link between mass and a fundamental constant of nature. The Josephson effect is used to calibrate voltmeters; the quantum Hall effect is used to calibrate resistors. Both effects are used in combination to measure current. Since the UK's National Physical Laboratory already uses these effects to calibrate electrical standards, traceability of electrical measurements to the NPL automatically ensures ultimate traceability to these quantum effects for the demonstration model.

Steady progress is being made on refining the watt balance technique to the highest possible accuracy and a decision to do away with the metal block definition of the kilogram is expected to be implemented later this decade.

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