Deloitte.

EPSRC
Engineering and Physical Sciences Research Council

The Council for the Mathematical Sciences

Measuring the Economic Benefits of Mathematical Science Research in the UK

June 2013

Contents

Important notice
Deloitte has been commissioned by the Engineering and Physical Sciences Research Council (EPSRC) to undertake an independent study that assesses the economic benefits of mathematical science research in the UK.

This is the first time such an analysis has been attempted in the UK and, as such, the estimates represent the first step in a longer process of impact evaluation of MSR in the UK.

The research was originally carried out between April and November 2012.

Glossary of key terms

Broad impacts	Wider impacts caused by the activities of organisations and employees which either entail mathematical science research or which directly require the usage of mathematical science research-dependent tools and techniques. These impacts can sometimes be quantified and assigned a monetary value in GVA and job terms.
Direct impact of mathematical science research	Those initial and immediate economic activities (jobs and GVA) attributable to the activities of organisations and employees which either entail mathematical science research or which directly require the usage of mathematical science research-dependent tools and techniques. These effects are often referred to as first-round impacts as they coincide with the first round of spending in the economy.
Gross Value Added (GVA)	A measure of the value of goods and services produced by a business, industry, sector or region of the economy. The OECD defines Gross Value Added as the value of output less the value of intermediate consumption. It is analogous to Gross Domestic Product.
Indirect impacts of mathematical science research	Changes in the number of jobs and GVA in associated industries that supply inputs to organisations generating and applying mathematical science research-dependent tools and techniques (sometimes referred to as 'supply-chain' or impacts).
Induced impacts of mathematical science research	The spending by households that result in changes to the number of jobs and GVA due to direct and indirect impacts.
Mathematical science research (MSR)	For the purposes of this report, this term is taken to refer to high-end mathematics research, as carried out in academic institutions, research centres, the private sector, private individuals and Government that adds to the store of accumulated mathematical knowledge. Those involved in research are not always involved in its application.
Mathematical science occupations	These are occupations which either entail mathematical science research, or which use mathematical science research-derived tools and techniques. This includes the generation of new mathematical science research. The direct usage of mathematical science research-derived tools and techniques may, in some circumstances, require a high degree of understanding of the underpinning mathematical concepts, but this is not always essential.
Narrow impacts	The 'traditional' economic impacts caused by the activities of organisations and employees which either entail mathematical science research or which directly require the usage of mathematical science research-derived tools and techniques. These typically cover jobs and GVA generated. These are the sum of direct, indirect and induced effects. This report focuses on such 'narrow impacts'.
Standard Industrial Classification (SIC)	First introduced in the UK in 1948, this is a framework for classifying business establishments and other statistical units by the type of economic activity in which they are engaged. There are a number of levels of the classification, with subsequent levels becoming more detailed.
Standard Occupational Classification (SOC)	A common classification framework of occupational information for the UK on the basis of skill level and skill content.

Foreword

The mathematical sciences play a vital part in all aspects of modern society. Without research and training in mathematics, there would be no engineering, economics or computer science; no smart phones, MRI scanners, bank accounts or PIN numbers.

Through a combination of quantitative analysis and qualitative assessment, this ground-breaking independent report by Deloitte provides a snapshot of how maths broadly impacts the UK economy. It is the first study of its kind to provide quantitative insights into the value across all sectors of such a pervasive subject as mathematics research in terms of the employment it supports, and is also the first to indicate the gross added value of mathematics to the UK economy.

It is the acknowledged excellence of the UK mathematics research base that has led to such impressive and far-reaching impacts. This reputation for excellence has ensured that the UK remains the partner of choice for international collaboration and continues to attract high levels of foreign investment.

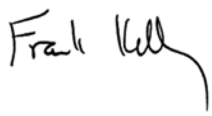
Mathematics is playing a key role in tackling the modern-day challenge of cybersecurity: ensuring that the UK is a safe place to do business and that we all benefit from a secure and resilient cyberspace.

And in UK manufacturing sectors such as aerospace, the second largest in the world, the industry benefits from a highly-skilled home-grown workforce, superior manufacturing processes and sophisticated quality management systems – all made possible by superior research and training in mathematics.

Likewise the UK life sciences sector, with significant potential for economic growth, wouldn't be in such a strong position without mathematics research and training, providing the expertise integral to the development of areas such as personalised healthcare and pharmaceuticals, and underpinning the development of many medical technologies.
The emergence of truly massive datasets across most fields of science and engineering, and in business, government and national security, increases the need for new tools from the mathematical sciences.

Mathematical sciences are vital for the future prosperity of the UK and its position in a world economy; this report tells us why.

Professor David Delpy
EPSRC Chief Executive



Professor Frank Kelly
Chair of Council for the Mathematical Sciences

Executive summary

Introduction

Deloitte has been commissioned by the Engineering and Physical Sciences Research Council (EPSRC) to undertake a study that assesses the economic benefits of mathematical science research in the UK.

Mathematical science research (MSR) refers to the high-end mathematics research, as carried out by academic institutions, research centres, businesses, individuals and Government, that adds to the store of accumulated mathematical knowledge. Individuals in **mathematical science occupations** are in occupations which either entail mathematical science research, or which use mathematical science research-derived tools and techniques.

The fruits of MSR affect the daily lives of everyone in the UK and the application of contemporary MSR can be seen in[1]:

- Smart phones which use mathematical techniques such as **linear algebra** to maximise the amount of information that can be transmitted across a limited spectrum;

- **mathematical models** predicting the movement of weather systems to allow airplanes to quickly and safely return to the skies after major meteorological events such as the 2010 Icelandic volcanic ash cloud;

- healthcare that applies the insights from **fluid mechanics** to better understand blood-related diseases in order to save lives;

- the latest Hollywood blockbusters that take advantage of the mathematics behind **software for 3D modelling** to showcase cutting-edge special effects; and

- the performance of elite athletes at the 2012 Olympic Games who have maximised their performance using tools that harness mathematical tools and techniques such as **inverse dynamics**.

It is not just contemporary MSR that can have an impact – research conducted during the last century and beyond has paved the way for the technology that has come to facilitate a range of activities, goods and services, for example, mobile telecommunications and medical devices. Carl Friedrich Gauss' work on number theory from the 18th and 19th centuries, previously thought to have little practical use, now underpins much modern work in cryptology, data management and the encoding of digital data.

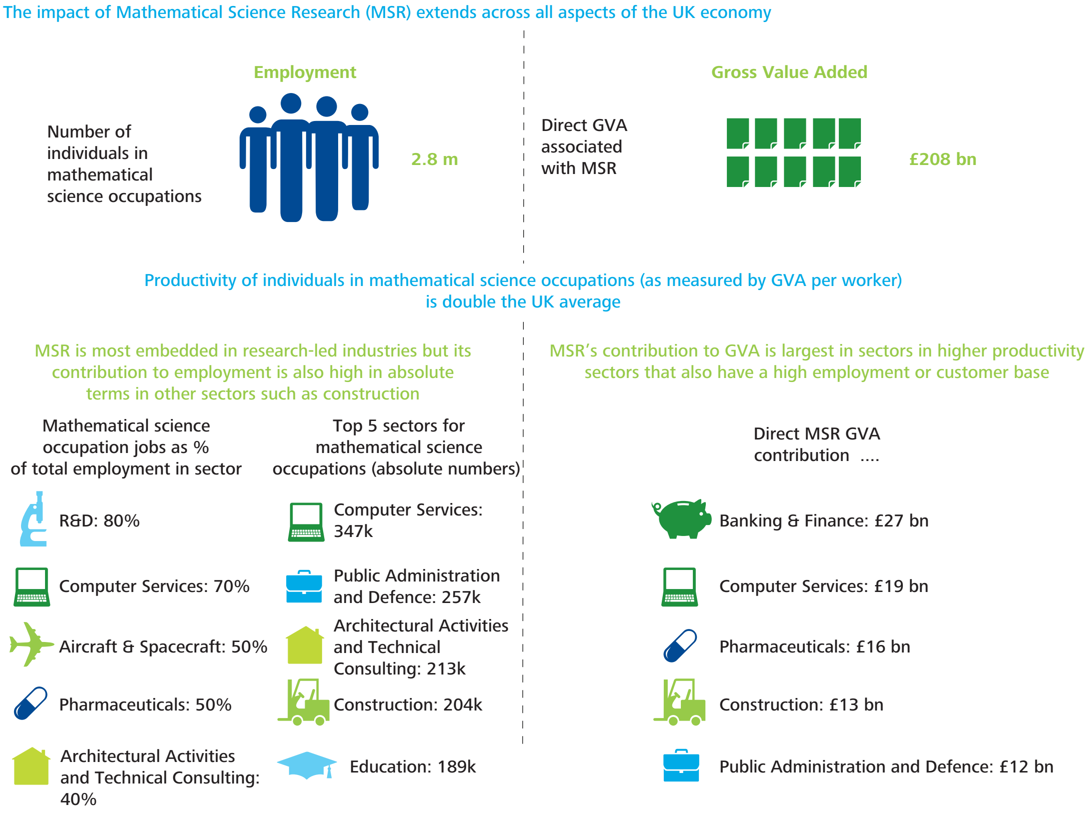
However, as highlighted by the Institute of Mathematics and its Applications, many people remain unaware of the importance of MSR despite the importance placed on mathematics, and other so-called 'STEM' subjects, in the Government's *Plan for Growth*.

This study by Deloitte considers, qualitatively, the ways in which MSR influences economic performance in the UK and then quantifies the economic value of MSR in terms of **direct employment supported** and **Gross Value Added** (GVA) generated in 2010. Our approach has been to first identify those jobs that can be classified as mathematical science occupations and then to use publicly available data to examine how these occupations are distributed across different sectors of the UK economy. These insights on direct employment are then inputted into a bespoke Deloitte model to calculate GVA. Where appropriate, this quantitative analysis has been supplemented by qualitative analyses.

This is the first time such an analysis has been attempted in the UK and, as such, the estimates represent the first step in a longer process of impact evaluation of MSR in the UK[2].

Deloitte thanks the EPSRC, learned societies and other stakeholders in the mathematics community for their valuable inputs and comments on the methodological approach throughout the study.

The Quantifiable Impacts of MSR in 2010 – a 60-second summary



Source: Deloitte

The quantified contribution of mathematical science research to the UK economy in 2010 is estimated to be approximately 2.8 million in employment terms (around 10 per cent of all jobs in the UK) and £208 billion in terms of GVA contribution (around 16 per cent of total UK GVA)[3].

As a proportion of all jobs in a sector, mathematical science occupations are the majority of jobs in the R&D[4] and computer science sectors[5], with the latter accounting for the largest absolute number of jobs in mathematical science occupations (nearly 350,000). However, high absolute numbers of mathematical science occupations can also be found in aerospace, pharmaceuticals, public administration and defence, architectural activities and technical consulting, construction[6] and education. Individuals in MSR occupations include professional mathematicians and statisticians, engineers, physical scientists, IT professionals, social scientists, finance professionals, selected medical practitioners, administrators and senior managers. A full list of MSR occupations and the rationale for their inclusion is given in the appendix.

Unsurprisingly, the banking and finance sector accounts for the largest direct GVA attributable to MSR, but there are also significant contributions from the computer services, pharmaceuticals, construction and public administration and defence sectors. Together, these five sectors account for over 45 per cent of all direct GVA attributable to MSR.

Productivity (as measured by direct GVA per worker) is significantly higher in mathematics science occupations compared to the UK average (approximately £74,000 versus £36,000), and as such the direct GVA impact of mathematical science research in 2010 is proportionately higher than the share of direct employment (16 per cent versus 10 per cent).

A qualitative take on MSR's economic contribution

As well as the narrow economic impact on jobs and GVA, MSR has **broader impacts** on society and the economy. There are many specific routes through which MSR can impact the UK economy and which do not come to light through an empirical analysis of MSR.

The main body of this report considers a number of case studies showing the contribution of MSR across specific sectors of the economy. Very broadly these contributions can be thought of as using MSR to:

- **make sense of data** and **better understand the world**;
- **safeguard society**; and
- **forecast, address uncertainty,** and **optimise processes**.

Qualitative analysis has shown that the UK economy benefits from mathematical science through a number of routes, including:

- building the **'information infrastructure'** upon which myriad businesses and individuals rely;
- supplying the **tools and techniques** to analyse and interpret large datasets;
- providing a **public good** such as modelling the impacts of natural disasters and testing drugs;
- contributing to **national security** and other necessary 'public goods' through advanced data security tools and infrastructure provision;
- creating robust **forecasts** to address uncertainty and allow for better planning; and
- optimising processes to increase **efficiency**.

Issues in considering MSR's economic contribution

Definition

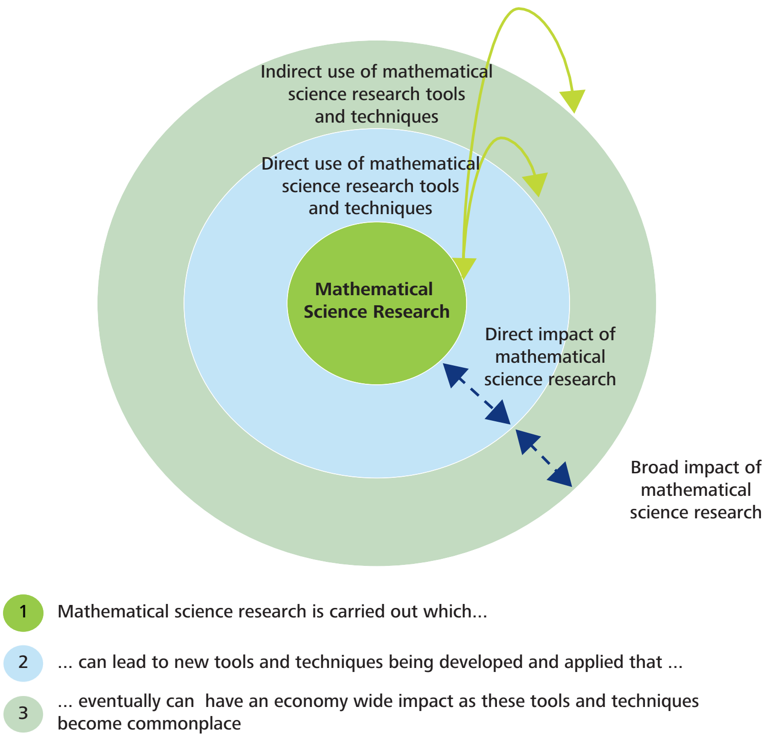
Given the diversity of the subject, it is perhaps unsurprising that there is no single agreed definition of MSR or indeed mathematics.

Mathematical science includes such diverse areas as algebra and analysis, dynamical systems, mathematical physics, operational research, probability and statistics: areas which touch on all aspects of everyday life.

For the purposes of this study we use the working definition of MSR as being high-end research in mathematical sciences carried out in academic institutions, research centres, businesses, individuals and Government that adds to the store of accumulated mathematical knowledge.

Mathematical Science occupations are therefore those occupations which either entail MSR[7], or which directly require the usage of MSR-derived tools and techniques. The direct usage of MSR-derived tools and techniques may, in some circumstances, require a high degree of understanding of the underpinning mathematical concepts, but this is not usual.

Our report has focused on quantifying the impact arising from MSR and the direct use by organisations and employees of MSR tools and techniques.



Source: Deloitte

The enabling effects of this research and tools and techniques are felt by a wide range of organisations and individuals who use goods and services that have relied, in some form, on MSR, but who are not directly involved in MSR. These broader effects are not readily quantifiable and we have discussed them qualitatively.

The issue of time

The temporal contribution of MSR should also be acknowledged and understood. This is why our occupation-based definition used acknowledges those occupations that are likely to rely on MSR which was commercialised some time before 2010.

A contemporary measure of MSR might include those individuals directly engaged in the generation and application of MSR at present and the associated economic output on a variety of measures such as employment supported, GVA contribution, or patents. However, the full economic impact of a given piece of research may not be felt immediately due to time lags; indeed it may be many years until there is an impact on the economy.

As the International Review of Mathematical Sciences 2010 (IRM) notes, a mathematical idea initially viewed as purely abstract can later turn out to have important real world applications.

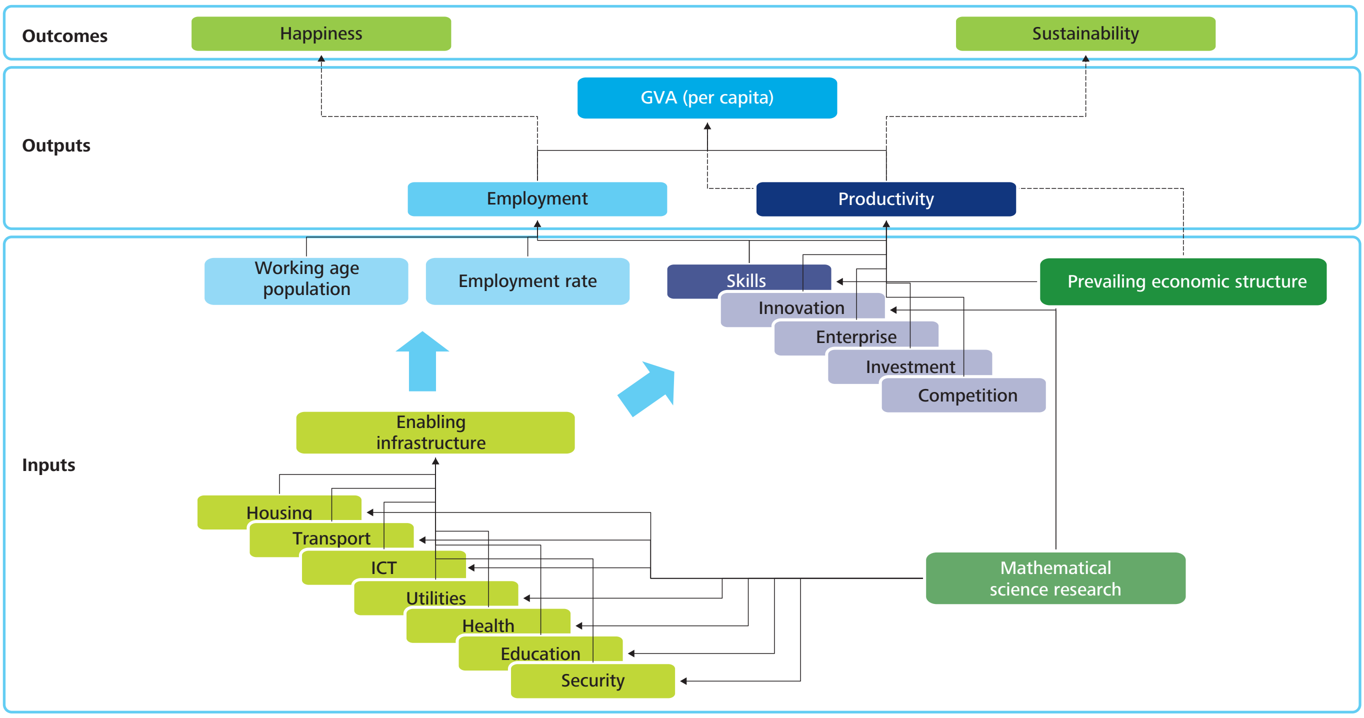
Equally, the Radon Transformation in topography, first introduced by Johann Radon in 1917 is now widely used: it was not until much after the initial research that the transformation formed the mathematical basis for the technology behind a non-invasive imaging technique that can recover the internal structure of an object from external measurements, as used in CAT scans and barcode scanners.

Clearly, research performed nearly a century ago still has routine work-task applications and continues to benefit the economy and society. The same 'everyday' notion can also been seen in a piece of software used routinely by a shop assistant processing a payment with no perceptible link to mathematical science research that helped to create it.

MSR and interdisciplinary-relatedness
MSR is intertwined to a great extent with other disciplines. This interaction of mathematical science with other disciplines makes it difficult to accurately segment what is truly MSR and what is not. In this study we do not seek to make such a distinction. Rather, by using an occupation-led approach to identifying 'MSR occupations' that directly entail MSR and/or apply MSR tools and techniques, the analysis captures both 'pure MSR' and that research where mathematical science has played a key role.

MSR as a generic driver of long-term economic growth
MSR generates an increase in understanding, new tools and techniques and refinements to existing tools and techniques. These then give way to outcomes and applications, such as an increase in the skills-base of the UK; new patents and commercial applications; product and process innovation; improved decision-making and more efficient management practices.

The generation and application of MSR can have a positive impact on enabling infrastructure, drivers of productivity, employment and in doing so overall levels of economic output. Indeed, MSR cuts across the long-term economic growth drivers as shown below. MSR can be leveraged by other disciplines to develop new products and services, promoting skills, innovation and increasing efficiency, which can result in economic growth, or hinder it in the long-term if MSR skills and knowledge are not in sufficient supply.



Source: Deloitte

1. Introduction

1.1 Background to this study

"What is mathematics helpful for? Mathematics is helpful for physics. Physics helps us make fridges. Fridges are made to contain spiny lobsters, and spiny lobsters help mathematicians who eat them and have hence better abilities to do mathematics, which are helpful for physics, which helps us make fridges which...".

Laurent Schwartz

Mathematical science research (MSR) affects the daily lives of everyone in the UK. The application of contemporary MSR can be seen everywhere, ranging from facilitating the development of the latest electronic devices, predicting the consequences of climate change, ensuring the successful organisation of the London 2012 Olympic Games and maximising the performance of elite athletes[8]. It is not just contemporary MSR that can have an impact – research conducted almost a century ago has paved the way for the technology that has come to facilitate a range of activities, goods and services, for example, mobile telecommunications.

However, as highlighted by the Institute of Mathematics and its Applications, many people remain unaware of the importance of MSR[9]. This despite the importance placed on mathematics, and other so-called 'STEM' subjects[10], in the Government's *Plan for Growth*[11].

Deloitte has been commissioned by the Engineering and Physical Sciences Research Council (EPSRC), working with the Council for Mathematical Sciences (CMS) to undertake a study that assesses the economic benefits of mathematical science research in the UK.

In particular, this study considers the ways in which MSR influences economic performance in the UK and quantifies the economic value of mathematical science in terms of employment supported and Gross Value Added (GVA) generated.

This research has been carried out between May and September 2012 and has used a combination of bespoke modelling and qualitative research to identify the economic contribution of mathematical science to the UK economy. Due to data limitations, the focus of this study is on 2010 only.

Deloitte would like to place on record its thanks to the EPSRC, learned societies and other stakeholders for their inputs throughout the study[12].

This introductory chapter sets out our broad approach to this study and the definitions used. This is the first time such an analysis has been attempted in the UK and, as such, the methodology used and estimates calculated represent the first step in a longer process of impact evaluation of MSR in the UK with subsequent attempts refining the methodological process as better data become available. The estimates should be read as indicative and be read in conjunction with the stated limitations and assumptions. Full methodological details can be found in the appendix.

1.2 Definitions

Before we can answer the question of how MSR makes a contribution to the UK economy, it is important to answer the question:

- **What is MSR?**

Following on from this key question, are a series of related questions:

- **How does MSR differ from the everyday use of mathematics across the economy?**
- **How does one capture the impact of MSR over time?**
- **How does one distinguish the contribution of MSR from that of other disciplines?**
- **How does one measure the impact of MSR and which metrics should be used?**

This section answers each of these questions in turn.

1.2.1 Definition of mathematical science research

"Mathematical reasoning may be regarded rather schematically as the exercise of a combination of two facilities, which we may call intuition and ingenuity."

Alan Turing

Given the diversity of the subject, it is perhaps unsurprising that there is no single agreed definition of MSR or indeed mathematics. For the purposes of this study we use the working definition of MSR as being high-end research in mathematical sciences carried out in academic institutions, research centres, the private sector and Government that adds to the store of accumulated mathematical knowledge[13].

As recognised in the 2010 International Review of Mathematical Sciences[14], there is a diverse array of study areas within the subject. Sub-fields of mathematical science include, in alphabetical order[15]:

- **algebra:** the study of operations and relations and the concepts arising from them such as polynomials and equations;
- **analysis:** a branch of pure mathematics that involves, among other things calculus and series;
- **combinatorics and discrete mathematics:** a branch of mathematics that explores finite or countable structures;
- **differential equations:** an area of mathematics which focuses on mathematical equations that relate a function to one or more of its derivatives, commonly used in engineering, economics and physics;
- **dynamical systems and complexity:** the study of relationships and linkages within a given system;
- **fluid mechanics:** the study of liquids, gases and plasmas and the forces on them;
- **geometry and topology:** the study of shape, size and relative position and properties of objects;
- **logic and foundations:** the study of logic (modes of reasoning) and its applications to other areas of mathematics;
- **mathematical physics:** the application of mathematical methods to problems in physics;
- **number theory:** a branch of pure mathematics that is devoted to the study of numbers;
- **numerical analysis/scientific computing:** the development and analysis of algorithms for approximation, and their applications.
- **operational research:** the application of advanced analytical methods and models to improve decision-making;
- **probability:** the study of uncertainty; and
- **statistics:** the study, collection and analysis of data.

In addition, there are a number of other fields in which mathematical science is closely connected to other scientific disciplines such as computer science, engineering, physics, biology, materials science and medicine. Further, social science disciplines such as economics and psychology heavily draw upon tools and techniques developed by MSR. Increasingly mathematics is also being used in the arts[16].

1.2.2 Mathematical research and the everyday use of mathematics

"All science requires mathematics. The knowledge of mathematical things is almost innate in us. This is the easiest of sciences, a fact which is obvious in that no one's brain rejects it; for laymen and people who are utterly illiterate know how to count and reckon."

Roger Bacon

While it is a relatively straightforward exercise to define the term MSR, moving from its definition to its application is less clear-cut. This is especially so when one considers that mathematics is used daily by individuals from all walks of life to carry out basic calculations such as counting the change received back when shopping. Mathematics is also used in a number of work-tasks ranging from applying a spreadsheet that was created using algorithms developed by mathematicians and computer scientists; developing a business plan using insights from probability theory to help mitigate risks and make better decisions; to relying on weather forecasts, that are built using complex statistical models, to plan major events. The following quote from Camilla Cavendish highlights the importance of mathematics in professional and personal life:

"As a management consultant I used statistics and built computer models to analyse companies. When I ran a small company I had to learn about accrual and cashflow accounting to keep track of the money. As a journalist, I have found it most helpful to be able to read a balance sheet.[17]"

How is one to distinguish between the economic impact of specialised MSR that requires detailed subject knowledge to apply and generate it, compared to the economic impact of using more generic tools and techniques created by such MSR and the application of more fundamental mathematics skills such as arithmetic that are used by everyone?

Figure 1.2.1 summarises our approach to distinguishing the economic impact of MSR from mathematics more generally using occupation classifications[18] created by the Office of National Statistics.

As Figure 1.2.1 shows, our approach has been to identify those occupations which either entail mathematical science research, or which directly require the usage of mathematical science research-derived tools and techniques as 'mathematical science occupations'. This includes the generation of new MSR. The direct use of MSR-derived tools and techniques may, in some circumstances, require a high degree of understanding of the underpinning mathematical concepts (at least to degree level), but this is not often the case. This list of occupations (which is placed in the appendix) has been identified through stakeholder discussions, a literature review and an exploration of mathematics (and related sciences) graduates' employment destinations and employment profiles of the membership of mathematics learned societies.

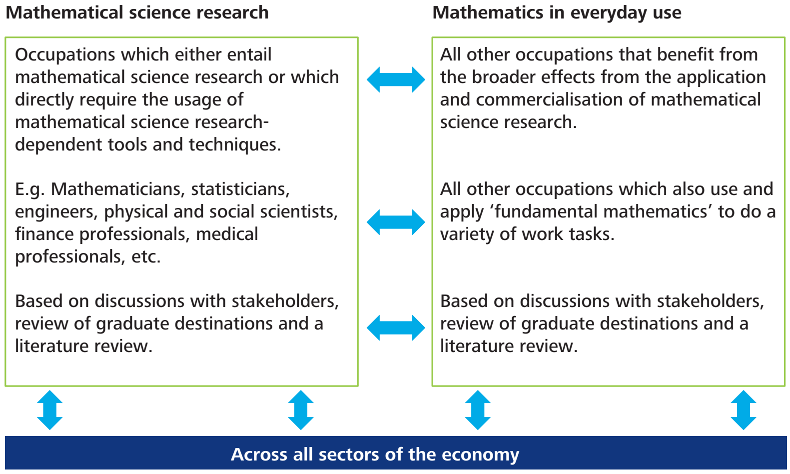
Critically, these occupations are not limited to particular industries or sectors of the economy, meaning occupations that generate and apply MSR can be found across the economy. For example, the agricultural sector, which is perhaps not an obvious employer of staff with mathematics backgrounds, will employ chemical engineers who will apply MSR when considering crop fertilisation.

These so-called 'mathematical science occupations' will generate insights from MSR that will have a much wider (generic) application across the economy and impact upon other occupations[19] that are not directly involved in MSR generation and application.

For example, in the case of the agricultural sector, as well as directly employing staff in mathematical science occupations, the sector will simultaneously use insights from MSR developed elsewhere, for instance, using insights from mathematical models forecasting weather patterns. Equally in other sectors, insights from the analysis of 'Big Data' by statisticians can help design more effective retailing strategies, even if the final beneficiaries do not directly apply or generate mathematical science research.

In this way, it is possible to delineate between the economic impact of MSR and mathematics more generally.

Figure 1.2.1. Mathematical science research and mathematics in everyday use



Source: Deloitte

1.2.3 Mathematical science research over time

"A mathematical theorem remains true and a counterexample remains false regardless of subsequent advances in computing and technology – the passage of time does not make mathematical research obsolete."

International Review of Mathematical Sciences, 2010

There is another complicating factor in attempting to quantify the economic contribution of mathematical science research to the UK economy: time.

A contemporary measure of MSR might include those individuals engaged in the generation and application of MSR at present and the associated economic output on a variety of measures such as employment supported, GVA contribution, or patents. However, the full economic impact of a given piece of research may not be felt immediately due to time lags; indeed it may be many years until there is an impact on the economy.

As the International Review of Mathematical Sciences 2010 (IRM) notes, a mathematical idea initially viewed as purely abstract can later turn out to have important real world applications. A classic example quoted is the Radon Transformation in topography, first introduced by Johann Radon in 1917. It was not until much after the initial research that the transformation formed the mathematical basis for the technology behind a non-invasive imaging technique that can recover the internal structure of an object from external measurements, as used in CAT scans and barcode scanners.

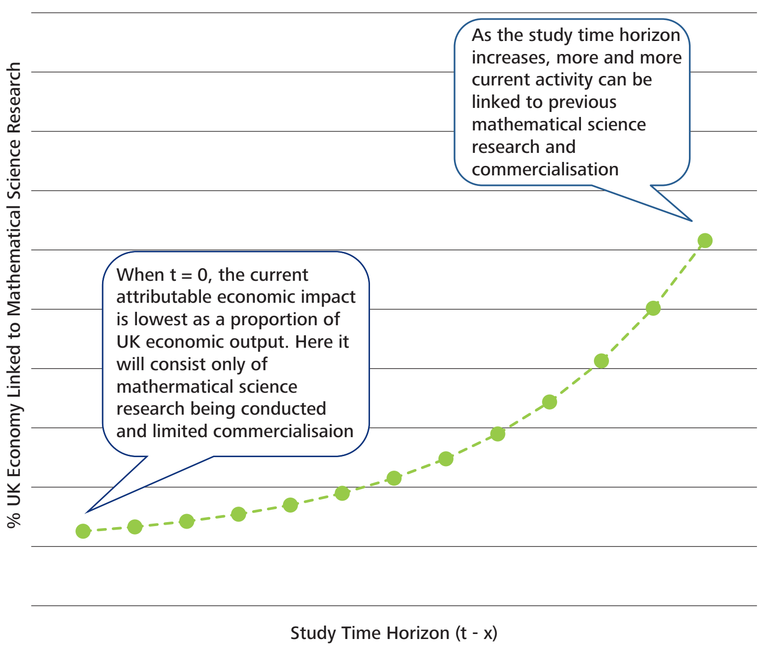
Equally, Carl Friedrich Gauss' work on number theory from the 18th and 19th centuries previously thought to have little practical use now underpins much work in cryptology, data management and the encoding of digital data.

Clearly, research performed nearly a century or more ago still has routine work-task applications and continues to benefit the economy and society. For example, Pythagoras' Theorem formulated over 2,000 years ago and taught to school children everyone is regularly used in engineering, architecture and meteorology, among other sectors. The same 'everyday' notion can also been seen in a piece of software used routinely by a shop assistant processing a payment with no perceptible link to the MSR that helped to create it.

Forecasting the value of commercialised revenues from research just completed or in progress is not feasible[20] and beyond the scope of this study. However, the extent to which MSR conducted many years ago is included in the definition will influence the results of any quantification exercise greatly.

Things which are ubiquitous and taken for granted now can in almost all cases be traced back to MSR conducted in the UK and around the world. Given that research tends to build in most cases on previous research, it is not hard to posit that the further back in time MSR is considered, the greater the permeation of MSR across the UK economy.

Figure 1.2.2. Mathematical science research over an extended time horizon



Source: Deloitte

As the hypothetical relationship shown in Figure 1.2.2 shows, the further the definition is extended into the past the greater the proportion of total economic activity that might be attributed to it.

Related to this, when one considers where mathematical science occupations are to be found, they are likely to be concentrated in those sectors of the economy that are highly technical and rely on mathematics to innovate and develop new products and services. This might include sectors such as aeronautics, computer science, engineering and research and development. The impact of MSR research is likely to be felt immediately in these sectors.

The extent to which other sectors are able to apply these mathematical insights immediately will depend on their absorptive capacity and the wider applicability of MSR. There may be a time lag between the insight being generated in 'research' sectors and its wider application in 'non research' sectors.

As agreed with the EPSRC, our study focuses on the impact of MSR in 2010. This means the economic impact measured will be restricted to the activity associated with mathematical science occupations themselves and research deployed for commercial gain in that given year as well as the legacy impact of MSR carried out before 2010 (though these impacts are not separated out).

1.2.4 Separating the impact of mathematical science research from other disciplines

"Physics is mathematical not because we know so much about the physical world, but because we know so little; it is only its mathematical properties that we can discover."

Bertrand Russell

As has been recognised elsewhere (e.g. the International Review of Mathematical Sciences 2010 (IRM)), MSR and its application does not take place in a disciplinary vacuum. For example, significant academic MSR not only takes place in mathematics and statistics departments, but also in engineering, physics and medical departments. It is not just the EPSRC that gives grants for work that generates and applies MSR, but also the other research councils – indeed, the shift towards 'Big Science' and 'grand design' projects is likely to increase the role of mathematical science in multi-disciplinary projects.

As an example of multi-disciplinary projects, the IRM notes how the Scottish Financial Risk Academy (which was founded by a consortium of members led by the Maxwell Institute for Mathematical Sciences) works to share knowledge between MSR and the financial services industry.

Outside of academia, staff in mathematical science occupations increasingly work with other occupations to bring a multi-disciplinary approach to problems. This can be seen by the role played by statisticians in clinical trials for new drugs.

This interaction of mathematical science with other disciplines makes it difficult to accurately segment what is truly pure MSR and what is not. In this study we do not seek to make such a distinction. Rather, by using our occupation-led approach to identifying the impact of MSR we will capture both 'pure mathematical science research' and that research where mathematical science has played a key role.

1.2.5 Capturing the metrics of impact

As agreed with the EPSRC, our study considers the economic impact of MSR in 2010 across two key measures:

- **jobs** attributable to mathematical science research; and
- **Gross Value Added[21] (GVA)** attributable to mathematical science research.

In addition to this quantitative analysis, this study also carries out qualitative analysis to capture how mathematical science broadly impacts the economy. The following section contains a broad outline of our methodological approach.

1.3 Our approach

Broadly speaking our approach has had four stages:

- **Stage 1:** confirming definitions and collecting data;
- **Stage 2:** testing assumptions and definitions;
- **Stage 3:** estimating jobs and GVA attributable to MSR; and
- **Stage 4:** qualitative analysis of the role of mathematical science in the UK economy and case study analyses.

Data has been collected from a number of public sources, including the Office of National Statistics and the Department of Business, Innovation and Skills. We have presented our assumptions and approach to the EPSRC, a committee of specialists and the wider mathematics community through a workshop[22]. Our approach has subsequently been agreed with the EPSRC.

Within Stage 3, a further three-part bespoke approach was adopted to quantify the economic impact of MSR.

- **Identifying occupations directly involved in the generation and application of MSR ('MSR occupations'):** as discussed above, using the 2000 Standard Occupational Classification (SOC) codes, a list of occupations that could be classed as 'mathematical science occupations' was identified – this list, which can be found in the appendix, was based on our literature review, a review of graduate destinations and stakeholder comments. 70 occupations were identified as mathematical science occupations.

- **Allocating these occupations across the different sectors of the UK economy:** using the SIC-SOC matrix provided by the ONS, we identified how these mathematical science occupations were distributed across the 600+ sectors of the economy. Some adjustments were made to each sector in order not to over-state mathematical science occupation employment numbers, i.e. where only a proportion of those in mathematical science occupations are likely to be directly involved in the application of MSR. Details of such adjustments can be found in the appendix.

  Following these adjustments, we were able to reach our estimates of the direct number of individuals in mathematical science occupations in the in the UK in 2010.

- **Applying a UK Input-Output[23] model to calculate GVA attributable to MSR.**

The Input-Output model* approximates supply chain linkages and consumer spending effects to quantify three different categories of effect:

- **direct GVA impact:** those initial and immediate economic activities caused by the activities of organisations and employees which either entail mathematical science research or which directly require the usage of mathematical science research-derived tools and techniques. These effects are often referred to as first-round impacts as they coincide with the first round of spending in the economy;
- **indirect GVA and employment impact:** changes in the number of jobs and GVA in associated industries that supply inputs to organisations generating and applying mathematical science research-dependent tools and techniques (sometimes referred to as 'supply-chain' impacts); and
- **induced GVA and employment impact:** the spending by households that result in changes to the number of jobs and GVA due to direct and indirect impacts.

The focus of this study is direct employment and GVA impacts of MSR, but for completeness we have also calculated indirect and induced impacts. These results are placed in the Appendix.

In this study, a detailed counterfactual case was not necessary, given mathematics is so heavily engrained in economic activity.

1.4 Document structure

This report is structured as follows:

- **Chapter 2:** outlines a framework for thinking about MSR;
- **Chapter 3:** contains our qualitative analysis, exploring how MSR can impact the UK economy;
- **Chapter 4:** presents out quantitative analysis of the direct employment levels and GVA attributable to mathematical science in 2010;
- **Chapter 5:** contains some closing thoughts;
- **Appendix 1:** provides further estimates of indirect and induced effects; and
- **Appendix 2:** contains further details of our methodological approach.

* The process behind constructing an Input-Output model is complex and involves considering the employment supported and gross revenue earned in each sector and applying adjustments so that it only represents additional revenue as compared to the counterfactual case of there being no MSR. In this study, it was the case that a detailed counterfactual case was not necessary given mathematics is so heavily engrained in economic activity.

2. A framework for considering mathematical science research and the UK economy

When calculating the quantitative impact of MSR in the UK, it is important to place the results in context and consider the impact within the context of the current MSR landscape and wider economic trends and circumstances. This chapter provides a short overview of the MSR landscape in the UK. Drawing on this, we outline a framework for tracing through the economic (and broader) impacts of MSR.

2.1 The mathematical sciences landscape in the UK[24]

"...in the fields of mathematics and sciences...Britain Has Talent."

Professor Adrian Smith Director General, Knowledge and Innovation, Dept for Business Innovation & Skills (2010-2012)

In its *Plan for Growth*[25], the Government noted that "access to a skilled workforce, particularly science, technology, engineering and mathematics (STEM) skills is vital" for a number of sectors identified as being key drivers of growth such as advanced manufacturing, life sciences, financial services, green energy and non-financial business services.

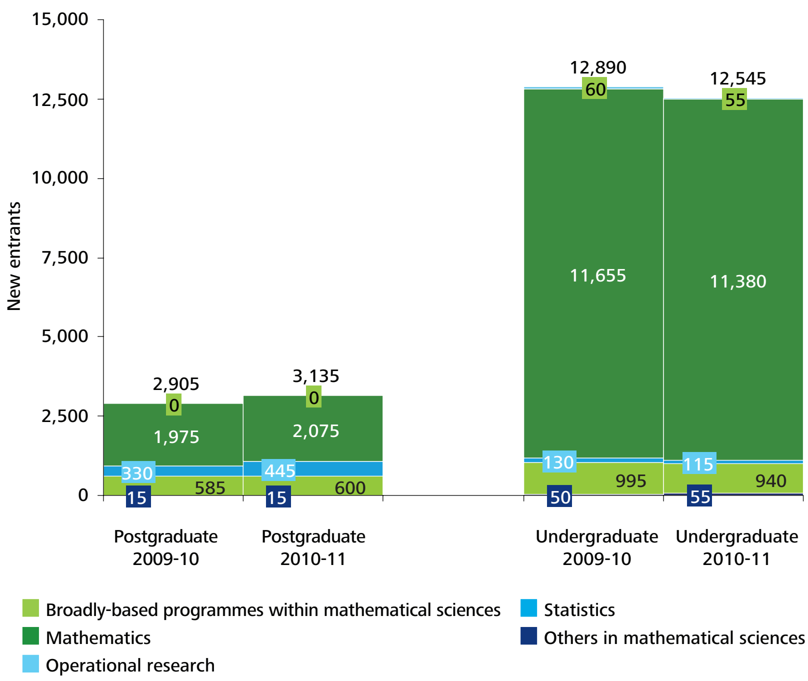
It is therefore unsurprising that the study of mathematics and related disciplines remains popular in the UK. Writing in *Science in Parliament*, Professor Adrian Smith quotes evidence showing a steady uptake in the numbers of students taking mathematics A-Level and improving performance[26]. Data on new entrants to undergraduate and postgraduate courses also remains strong, though the numbers for undergraduates fell slightly between 2009-10 and 2010-11[27].

The UK is widely recognised as having a world-class MSR base. This is evidenced by figures quoted by Professor Smith on academic research outputs:

- with 3.9 per cent of world researchers and 3 per cent of world gross expenditure on R&D, the UK produces 6.4 per cent of all mathematical science articles;
- these articles have 9.4 per cent of article usage; and
- achieve 10.9 per cent of citations and comprise 14 per cent of the top 1 per cent highly-cited articles.

This can be summarized by saying that the UK mathematical science researchers produce more articles per researcher and these articles are more widely read than articles from other countries.

Figure 2.1.1. New entrants to undergraduate and postgraduate mathematical science programmes[28], 2009-10 to 2010-11



Source: Deloitte using HESA data

While this may be helped due to English being the international language of academic research, the UK is said to vie with the USA as a world leader in MSR, and well ahead of other European countries.

A similar set of findings was found by the EPSRC's International Review of Mathematical Sciences (IRM) in 2010, which examined the quality and impact of the mathematical science base in the UK compared to the rest of the world. The IRM, which comprised of an international panel of 16 experts reviewing the entire mathematical sciences portfolio in the UK in academia and collaborations and users outside academia found UK MSR to be world-leading in some fields: outstanding in many others and strong overall.

The key contributors to the current excellence of UK MSR were identified as its diversity – in area, group size and size of institution – and its geographically distributed nature. Mathematical science researchers in academia were also found to participate actively in multidisciplinary collaborations involving important and complex problems and serve as valuable partners for industry in addressing its long-term challenges.

Professor Smith notes that one way of ensuring the UK remains 'best in class' is through greater international co-operation. In 2010, nearly half of all UK papers in mathematics were internationally co-authored, a figure much higher than any other G8 or BRIC country except France.

2.2 A framework for analysing the impact of mathematical science research

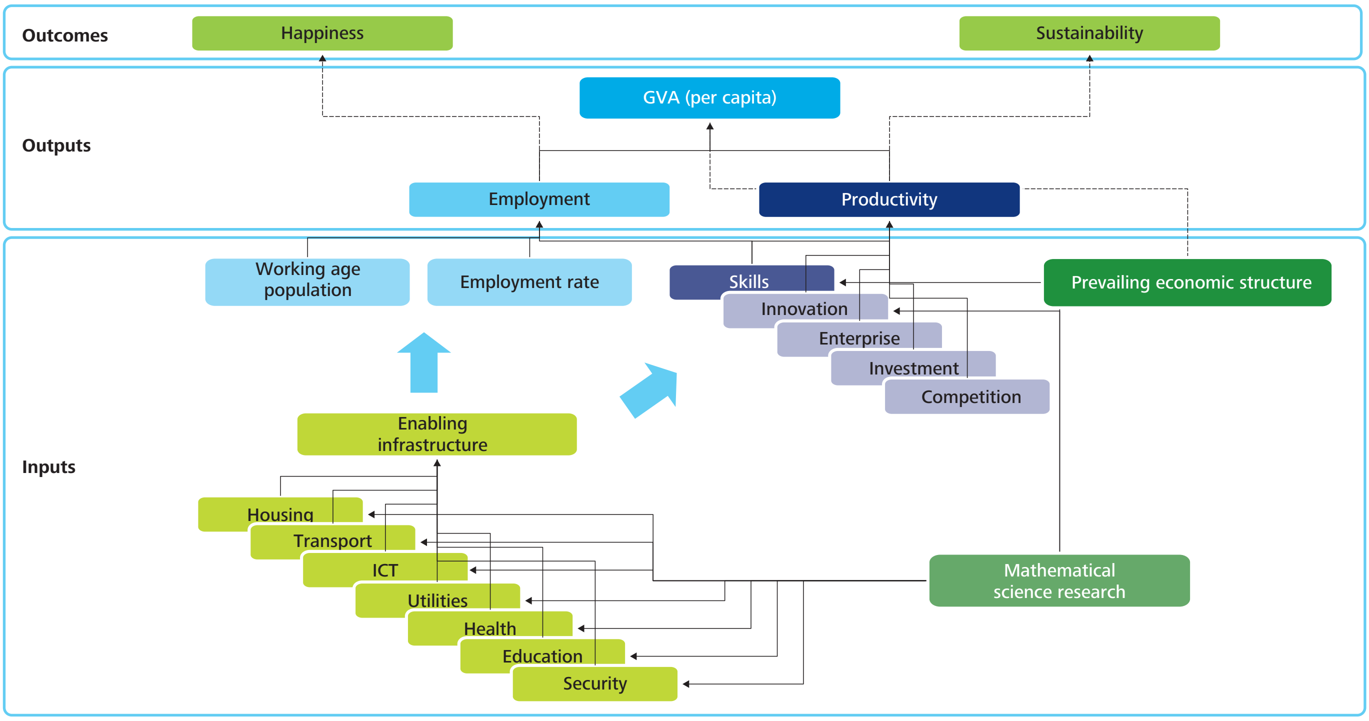
"Mathematical science research underpins a wide range of activities that benefit society, including engineering, economics and computer science. Just as technology plays an increasing vital part in virtually every aspect of modern society, mathematical sciences research continues to be indispensable."

Professor David Delpy, CEO EPSRC and Professor Tim Pedley, International Review of Mathematical Sciences Steering Committee Chair

As discussed in the preceding section, what we might call the UK's 'Mathematical Infrastructure' – a combination of human capital, research institutions and facilities, and the legacy of previous research is formidable in an international context and has the potential to continue to yield and stimulate economic growth for the UK into the future. There are a number of routes through which MSR can have an impact on the economy and its generation and application of can have a positive impact on enabling infrastructure and driving productivity, which in turn can generate employment and wealth. Indeed, MSR cuts across the long-term economic growth drivers as shown below in Figure 2.2.1.

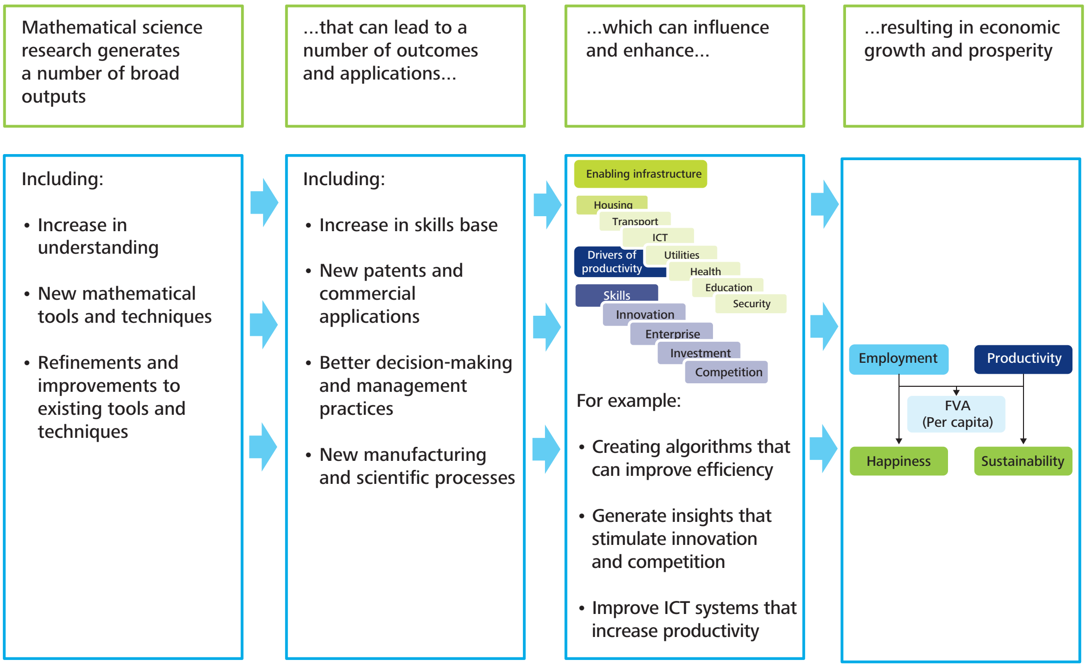
The simplified framework in Figure 2.2.1 illustrates the long-term drivers of economic growth. Measurable outputs (such as GVA, employment and productivity) as well as less-easily measurable outcomes (such as happiness and sustainability) are determined by a number of inputs to the economy. In the long-term and considering the supply-side of the economy only, the economic output of the UK is a function of only two things: the number of people engaged in gainful employment and the amount each person in employment is capable of producing.

Figure 2.2.1. Long-term UK economic growth framework



Source: Deloitte (drawing on HMT drivers of growth)

Figure 2.2.2. Economic impact chain for UK mathematical science research



Source: Deloitte

Drawing on the HMT's research into the *Five Drivers of Productivity*[29], the amount each worker produces is determined by skill levels; the extent of innovation in products and processes; the degree of investment in capital; entrepreneurial activity; and, lastly, levels of competition.

Underpinning employment and productivity are seven necessary enablers. These are related to the infrastructure required to facilitate long-term economic growth. A deficit in these enablers will equate to a supply-side constraint on economic growth in the long-run. This could be caused either by limiting the growth in working population (through insufficient housing capacity or supporting utilities) or by acting as a drag on productivity growth (through below-par ICT connectivity or a sclerotic transport system). Expanding and enhancing these enablers can therefore positively impact employment and productivity, which in turn generate economic growth and greater prosperity.

From this framework, it is possible to identify the routes through which MSR can positively influence economic growth and prosperity.

As Figure 2.2.2 depicts, the generation and application of MSR can have a positive impact on enablers of infrastructure and drivers of productivity. For example, mathematical algorithms can improve online security infrastructure which in turn can increase the number of online retailers as customers' trust in shopping on the internet increases – this can then lead to the recruitment of more employees[30] causing an increase in GVA and wellbeing. Equally, mathematical research can be leveraged by other disciplines to develop new products and services, promoting skills, innovation and increasing efficiency, which can result in economic growth.

The following chapters consider in more detail how mathematics can have an economic impact[31] and a broader societal impact.

3. Ways in which mathematical science research can benefit the UK economy

This chapter contains the qualitative analysis conducted as part of the study. This is based on a literature review and case study research, examining ways in which MSR broadly impacts the UK economy in 2010. The three ways identified include:

- making sense of data and better understanding the world;
- safeguarding society; and
- forecasting, address uncertainty, and optimise processes.

It is important to note that MSR does not occur in a vacuum and, as has been noted by a number of stakeholders, is part of British culture and history. Individuals in MSR occupations work closely with professionals from other disciplines and widely disseminate their findings. The ways in which MSR has an impact on society and the economy are manifold and constantly changing – meaning any attempt to (qualitatively or quantitatively) capture its broad impact can never be truly exhaustive. Hence our analysis is very much a snapshot of MSR's economic impact in the UK.

3.1 Using mathematics to make sense of data and better understand the world

"Film is one of the three universal languages, the other two: mathematics and music."

Frank Capra

One of the classic ways in which MSR plays a role in the economy is through the processing and understanding of raw data. At the most fundamental level this can involve the collection of data by the Office of National Statistics, such as Census data that is then made available to a wider audience in Government and the private sector to manipulate and derive their own insights. This data, which forms part of the so-called 'information infrastructure' is a fundamental ingredient for a range of business sectors ranging from management consultancy to education.

Moving the next stage along in the data value-chain from collecting data towards understanding it, by using tools and techniques developed by MSR, individuals and businesses can discover new relationships and opportunities that would have otherwise remained obscure and hidden. In this way, mathematics can help positively influence the Five Drivers of Productivity by providing insights that stimulate competition and new markets, contribute to innovation and lead to new investments.

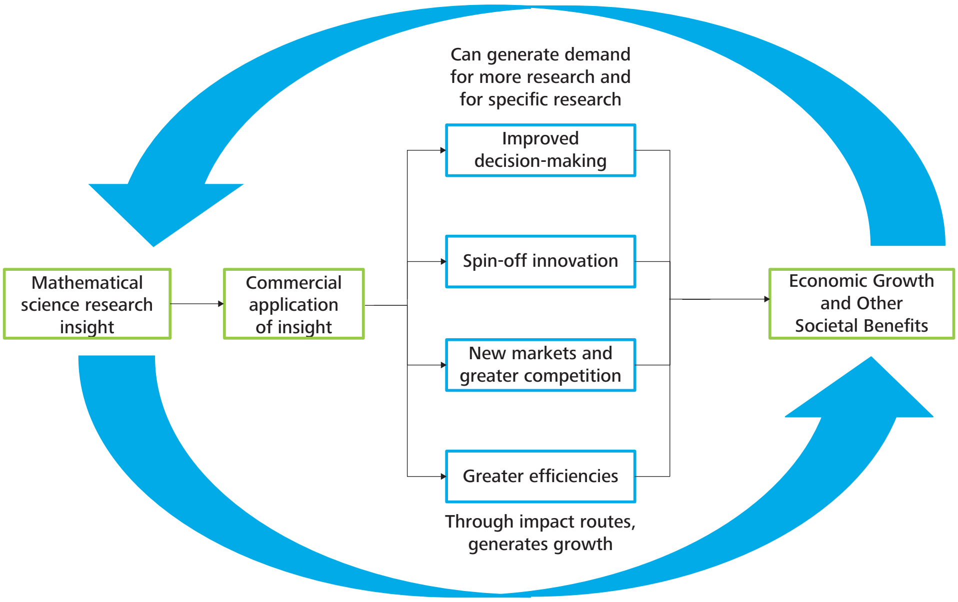
As the Minister of State for Universities and Science, David Willetts, recently pointed out, 2012[32] is the centenary of the birth of Alan Turing, the English mathematician who was highly influential in formalising the concepts of algorithms and computation which underpin much of the computational analysis of data seen today. With the size of datasets increasing significantly, these techniques, alongside those developed by UK researchers such as Bayes, Pearson and Fisher, are being increasingly used to make sense of the data in uncovering patterns, charting relationships and forecasting the future.

Indeed, as Jared Tanner writing in *Science in Parliament*[33] notes, although collecting data and analyzing it has been a cornerstone of scientific research for centuries, the introduction of computers has allowed much larger datasets to be examined, increasing the demand for mathematical science. This has led to the emergence of the 'data analyst' – an individual highly skilled at applying mathematical science insights to identify patterns and trends within large datasets and who works as part of multi-disciplinary teams to answer complex questions.

Analytical driven insights drawing on MSR have become a key basis of competition, underpinning new waves of productivity growth and innovation in every business sector. A virtuous circle can be created between MSR and economic growth and prosperity as shown below in Figure 3.1.1.

Through commercialization, algorithms developed by researchers can help businesses gain a competitive edge through improving efficiencies, developing a better understanding of customers, improving processes and decision-making and reducing market-entry barriers. This in turn can drive economic growth and prosperity. There is positive feedback loop in that greater use of algorithms will drive up demand for more research, which can generate new and more targeted insights, again driving growth.

Figure 3.1.1. Mathematical science research virtuous circle[34]



Source: Deloitte drawing on insights by Rowan Douglas

These algorithms can be found in a variety of industries and economic sectors.

Indeed, there are businesses that have been established, such as the Numerical Algorithms Group (NAG)[35], which specialize in developing and supplying algorithms to a range of clients to help solve their problems. NAG works closely with the academic community to develop commercial and non-commercial applications for algorithms. The company employs a number of mathematics and applied science PhDs and its algorithms have benefit a number of industries including[36]:

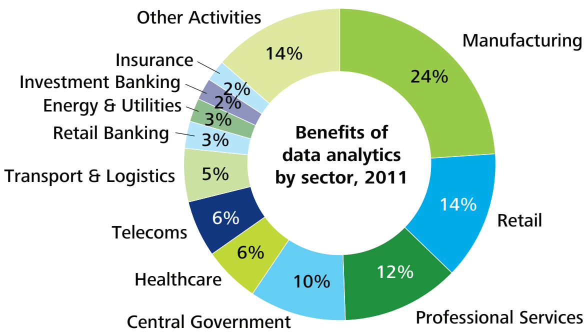
- **Finance:** to assist in portfolio construction and optimization;
- **Business intelligence and business analytics:** to assist in making forecasts, identifying area of opportunity and guiding investment decisions; and
- **Aeronautics, utilities and manufacturing:** using algorithms to model real world events and optimise processes.

As an example, algorithms are now informing mathematical models of real world processes, reducing the need to build expensive test facilities such as wind tunnels and reducing the risk test pilots are exposed to.

Algorithms are used across a range of industries, not just those in the technical sectors. According to research carried out by Vanson Bourne, the majority – 60 per cent – of private sector organisations believe the use of data analytics is the most important factor in increasing growth in UK businesses.

SAS[37] estimated that data analytics of 'big data', which heavily uses algorithms, contributed £25.1 billion to the UK economy in 2011. This is expected to reach £40.7 billion p.a. by 2017 as more companies adopt big data technologies (not just cutting edge companies in technical sectors). This is estimated to lead to the creation of an additional 58,000 new jobs over the next 6 years.

Figure 3.1.2. Estimated spread of benefits from data analytics in the UK



Source: SAS

As David Willetts points out, companies such as WPP increasingly use advanced sentiment analyses to improve their campaigns. Famously, the retail sector uses algorithms to achieve greater customer segmentation and targeted offers. Following the introduction its Clubcard, Tesco's in-house insight analytics business (Dunnhumby) has become a leading data analytics business in the grocery market. Similarly, social media networks make extensive use of algorithms to create targeted advertisements and offers for their users. This can itself open up new market opportunities for third parties – as an example, Deloitte recently estimated the value of broad effects from the Facebook social media platform to be over €2 billion in GVA terms in the UK in 2010[38].

Another area where mathematics, in particular statistics, plays a key role in making sense of large amounts of data in order to better understand the world occurs in retail financial services. One of the key characteristics of this sector is the availability of highly detailed large-scale datasets. Box 3.1 explains how statistics is being used in this sector. As this example shows, it is hard to imagine modern financial services being able to function efficiently without mathematical and statistics at the heart of operations.

Box 3.1. Mathematics and retail banking[39]

Retail banking includes a range of financial transactions including personal current accounts (and debit cards), mortgages and other loans, credit cards and saving products.

Providers of retail banking services and products regularly collect information on the behavior of customers. This data is used to build models to answer a range of fundamental questions such as:

- What is the likelihood a loan applicant will not repay their loan or be late in their repayments?
- What is the likelihood a customer will enter into financial difficulty and require support?
- Is an anomalous credit card transaction evidence of fraud or simply a one-off purpose?

These questions go well beyond simple credit-scoring exercises.

The models constructed draw heavily on mathematical science, in particular statistics. These models are entirely empirical, meaning they do not make any a priori judgments on customers' behavior, but rather rely on the data. A new characteristic can be added into the model (subject to any legal restrictions) if the analysis shows it to improve the model's predictive capability.

The predictive power of these models has improved significantly since their introduction in the 1960s. Statistical algorithms are now recognized as be as accurate, if not more so, than human decision making.

These models continue to increase in sophistication. One area of research concerns the evaluation of 'score cards'. Populations of customers will change due to changing economic conditions (e.g. a recession reducing the demand for loans or making some people more likely to repay outstanding debts), increased competition (consumers having a wider range of alternative products to choose from), changed regulations (restrictions on loan-to-value ratios) and technological developments (mobile banking). This means the available data for models will be historic and not reflective of the new state of the world – the population for which the model will be applied will not necessarily have the same characteristics as the one upon which it is based. New models have attempted to address the issue of 'distorted population' and 'selection bias' – in 2000 the NobelPrize for economics was awarded to James Heckman for his work in this area.

3.2 Using mathematics to safeguard society

"UK mathematics, especially pure mathematics, makes a major contribution to UK security and intelligence."

Malcolm MacCallum, former Director Heilbronn Institute for Mathematical Research

MSR also has an economic impact through its contribution to security. Through its application, MSR can positively influence infrastructure enablers such as health, ICT and security, which in turn can promote economic and social well being.

In this section we consider the ways in which MSR has an economic impact on security – both in terms of the well-being of individuals and preventing criminal activities online.

3.2.1 Mathematical science research and public health[40]

The insights and applications of MSR have been used in the pharmaceutical sector and more widely in public health laboratories considering contagion management and mitigation. In this way, the insights are used by both the private and the public sectors.

Beginning with the pharmaceutical sector, statisticians are typically involved in the design of clinical trials of new drugs – often more so than doctors and other researchers.

Indeed, statisticians are employed by pharmaceutical companies to work across all areas of R&D, from the initial identification of medicines and the design of clinical trials through to the manufacturing of pharmaceutical products. Box 3.2 summarises the role of statistics in the pharmaceutical sector – and highlights how a piece of research concluded many years ago continues to have commercial resonance and can contribute to economic growth in perpetuity.

New research is examining issues around 'flexible trial design' which allows researchers to modify trials that are already in progress; to still maintain robustness; and to meet regulatory requirements. The techniques developed in a pharmaceutical context are applicable to other sectors.

Mathematical models are also deployed in the area of public health to safeguard the public. These models are typically collaborations between mathematicians, biologists, clinicians and policymakers and use a variety of techniques to outline 'what if' scenarios.

As Deirdre Hollingsworth writing in *Science in Parliament*[41] notes: mathematical models are used to answer a number of policy questions, for example how to design the optimal vaccination strategy. Models can help answer what the critical vaccination level needed is to prevent an outbreak of a disease.

Box 3.2. Mathematics and the pharmaceutical industry

Many of the tools and techniques applied by statistics can be traced back to Sir David Cox's *Planning of Experiments* (1958) and subsequent research into the role of statistics in areas of medical research such as life expectancy and the treatment of disease in the *Analysis of Survival Data study* (1984).

Statisticians are now integral to the development of areas such as pharmacology, biological and process modelling, health economics, personalised healthcare, real world evidence and cost-effectiveness modelling. They make a significant contribution to UK research in pharmaceuticals. In 2010, R&D expenditure in the pharmaceutical sector amounted to £4.6billion – 29 per cent of all UK R&D spend and the greatest in Europe. The Government has identified the pharmaceutical sector to be one of the key industries to pull the UK out of its current recession.

With a high concentration of research-based pharmaceutical and biotechnology companies, leading centres of academic medicine, and a history of pioneering research, Britain is a leading location for running the complex and often multinational studies needed to develop new medicines.

The industry makes a substantial contribution to the British economy in terms of both income and employment and is said to employ over 72,000 highly skilled professionals (including statisticians). The industry has generated a trade surplus throughout the past 13 years: earnings from exports exceeded those from imports by over £5 billion in 2011.

A key driver of its contribution is the productivity of its workforce. GVA per employee within the pharmaceutical industry was over £200,000 in 2010. This is £160,000 more than the average employee in the UK and greater than those employed in the construction and financial services sectors.

A number of complexities can be introduced to take account of local ecological conditions, human contact networks and so forth. In doing so, a range of branches of mathematics interact with biology and ecology including probability, statistics, network theory, mechanics, computation and analysis and pure mathematics.

Similarly, a recent edition of Mathematics Matters[42] describes the role of mathematical science in controlling the swine flu outbreak in the spring of 2009. A team of epidemiologists and mathematicians implemented a three phase to contain the outbreak that consisted of:

- **Phase 1:** using data from Mexico (the source of the outbreak) to build an initial mathematical model of the disease.
- **Phase 2:** when the virus hit the UK, research was carried out to understand how social contact affected transmission using an online flu surveillance system (www.flusurvey.org.uk).
- **Phase 3:** when the number of individual infections in the UK became too great to track, data was aggregated from a number of sources to estimate the number of expected future infections, which in turn informed different vaccination strategies according to different scenarios.

Hollingsworth goes on to point out that the UK is widely seen as being best in class in the field of infectious disease modeling and in the interaction between modeling and public health policy. She cites the example of the Medical Research Council Centre for Outbreak Analysis and Modelling being designated the first World Health Organization Collaborating Centre for infectious disease modeling as an example of this wider recognition.

3.2.2 Mathematical science research and cyber-security

According to the 2012 Information Security Breaches Survey conducted on behalf of the Department for Business Innovation and Skills (BIS), in 2012 93 per cent of large businesses reported having had an information security incident – an increase of 21 per cent since 2008. The cyber security market comprises of companies that provide products and /or services for defensive and offensive applications across IT, telecoms and industrial equipment – all areas where mathematical science research has a key role.

As has been pointed out by Meza, Campbell and Bailey (2009)[43], the field of cyber-security poses a rich set of new research and analytical opportunities for MSR. They highlight developments in automated attack response software and network intrusion detection software that has been informed by MSR (probability theory, algorithms, complexity theory and number and group theory). They conclude by highlighting how non-traditional mathematical and statistical techniques have the potential to significantly improve online security, which in turn can act as an infrastructure enabler.

3.3 Using mathematics to forecast, address uncertainty and optimise processes

"[Statistics] is a discipline in which everyone dabbles. But many outsiders are more than just dabblers: they have profoundly benefited from the science of statistics."

Professor Stephen Senn, Head of Competence Centre for Methodology and Statistics, Centre de Recherche Public de la Santé

Mathematical models can be used to address uncertainty and allow businesses and policymakers to plan ahead through the use of forecasts. David Spiegelhalter in *Science in Parliament*[44] points out that mathematical risk models are very widely used in a number of sectors of the economy including: insurance, pensions, finance, individualised risk assessments for heart attacks, health policy by NICE and for epidemics, weather and climate and associated hazards of flooding and so on.

The National Risk Register[45] has become increasingly sophisticated to the extent that it publicly communicates the assessed numerical chances (except for security events) of various extreme scenarios over future years, such as severe space weather and Icelandic volcanic eruptions. Spiegelhalter notes how these assessments draw on mathematical research tools and techniques such as Monte Carlo analyses in which large numbers of possible future 'worlds' are simulated under slightly different conditions, and the proportion in which a particular extreme event happens reflects the chances of the event occurring.

One clear example of mathematical models being used to forecast and deal with uncertainty occurs in weather forecasting[46].

Weather forecasts are relied upon by businesses (such as insurance companies, commercial shipping, tourism operators and so on) and individuals alike. Weather forecasts also play a key role for the armed forces. While predicting natural disasters is impossible, technological advancements in weather forecasting have allowed governments to better measure and manage the scale of disasters and in doing so, minimise the loss of human life. This is an instance of mathematics providing a 'public good' that has both commercial and wider societal benefits which are far greater than the direct economic impact of the GVA generated by the mathematical science occupations employed in this activity.

The UK is recognized as a global centre of excellence for weather forecasting. The Met Office's Hadley Centre has been described as "probably the world's leading place for combining weather and climate forecasting" (Willetts, 2012).

Box 3.3 outlines how MSR contributes to weather forecasting and the UK economy.

Box 3.3. Mathematics and weather forecasting

As has been highlighted, the cost of not understanding and predicting changes in the physical world can be immense. Natural disasters were estimated to have cost the global economy over £100 billion worth of damage in 2011 – the costliest year in over 300 years of the insurance industry. With the effects of climate change being ever clearer, through extreme weather events, the demand for robust weather forecasts becomes even greater.

Mathematical research has, and is continuing, to play a pivotal role in the field of weather forecasting and modelling. Early mathematical research undertaken by Richardson in 1922 *(Weather prediction by numerical process)* and by Charney in 1948 *(On the scales of atmospheric motions)* are regarded as two influential pieces of research in the field of weather forecasting and climate modeling that have been subsequently used to inform modern-day techniques.

Indeed, a range of mathematical techniques are utilised in the modelling of weather forecasts. These include:

- Deterministic mathematics: this is employed to process the large amount of data that is collected from a series of different sources (e.g. satellites).
- Algorithmic modelling: this is used to incorporate data into forecasting models.
- Complex equations: which are used to estimate and measure uncertainties.

The models also draw on insights from fluid mechanics and statistics.

Within the sector, mathematicians are employed to work alongside computer programmers and software scientists to interpret and evaluate the vast array of data. Around 2,000 mathematicians are employed by the UK Met Office (source: ONS) to analyse and evaluate vast amounts of atmospheric trends and information. Once this information has been processed, it can be utilised by public and private corporations that are vulnerable to certain weather conditions – atmospheric data has significant commercial value once it has been processed, analysed and cleansed. Mathematics has been fundamental in creating international databases that are widely exploited so people don't have to create own models.

The UK is regarded in the meteorological industry as a talent hub with many institutions choosing to locate research facilities in the UK to take advantage of the high quality workforce.

As David Willetts points out, that a record year of natural disasters has not had the corollary of a crisis in the London insurance market is a testament to the continuous engagement between the scientific community and the commercial world.

As well as addressing uncertainty, mathematical science research is also deployed to optimise processes. Operational research techniques are frequently used across a number of industries to optimise processes and maximize value-for-money. Mathematical science research is used not just in the design of processes and projects, but across a project's lifecycle and supply chain to ensure it consistently meets its objectives.

As UK manufacturing is increasingly focused on high-end, high-value products, the role of mathematical science in optimizing processes and designing quality assurance processes is vital. As an example, the aerospace industry is one of the UK's highest value adding manufacturing sectors with the UK home to the second largest aerospace industry in the world. The companies that operate in this sector have been able to tap into the high quality workforce that the UK has to offer as well its superior manufacturing processes and sophisticated quality management systems that both rely on the application of mathematical science research.

It is estimated that the improvement in manufacturing processes leads to productivity gains of 3 per cent per year for UK manufacturers. Research commissioned by the Chartered Quality Institute (CQI) and the Chartered Management Institute (CMI) from the Centre for Economic and Business Research (cebr), calculated that in 2011, quality management practices contributed £90 billion to Gross Domestic Product (GDP), accounting for 6.0% of UK GDP[47].

3.4 Conclusion

"The study of mathematics, like the Nile, begins in minuteness but ends in magnificence."

Charles Caleb Colton

This chapter has considered the many routes through which MSR can impact the UK economy in 2010. Our qualitative analysis has shown that the UK economy benefits from mathematical science through:

- building the 'information infrastructure' upon which a myriad of businesses and individuals rely;
- supplying the tools and techniques to analyse and interpret large datasets;
- providing a public good in modelling the impacts of natural disasters and testing drugs;
- contributing to national security and other necessary 'public goods' through advanced data security tools and infrastructure provision;
- creating robust forecasts to address uncertainty and allow for better planning; and
- optimising processes to increase efficiency.

Thus, the generation and application of mathematical science research can be an engine of growth. Together the above benefits can positively impact upon infrastructure enablers and the Five Drivers of Productivity, which in turn will lead to economic growth and greater prosperity.

4. Mathematical science research – direct economic contribution

"Mathematics is as much an aspect of culture as it is a collection of algorithms."

Carl Boyer

This chapter contains our quantitative analysis of the direct economic contribution of MSR in the UK in 2010. This analysis draws on a UK Input-Output model, which is used to estimate GVA attributable to MSR. Further details of our methodological approach are placed in the appendix. Estimates of downstream indirect and induced impacts of MSR are placed in an appendix.

4.1 Employment

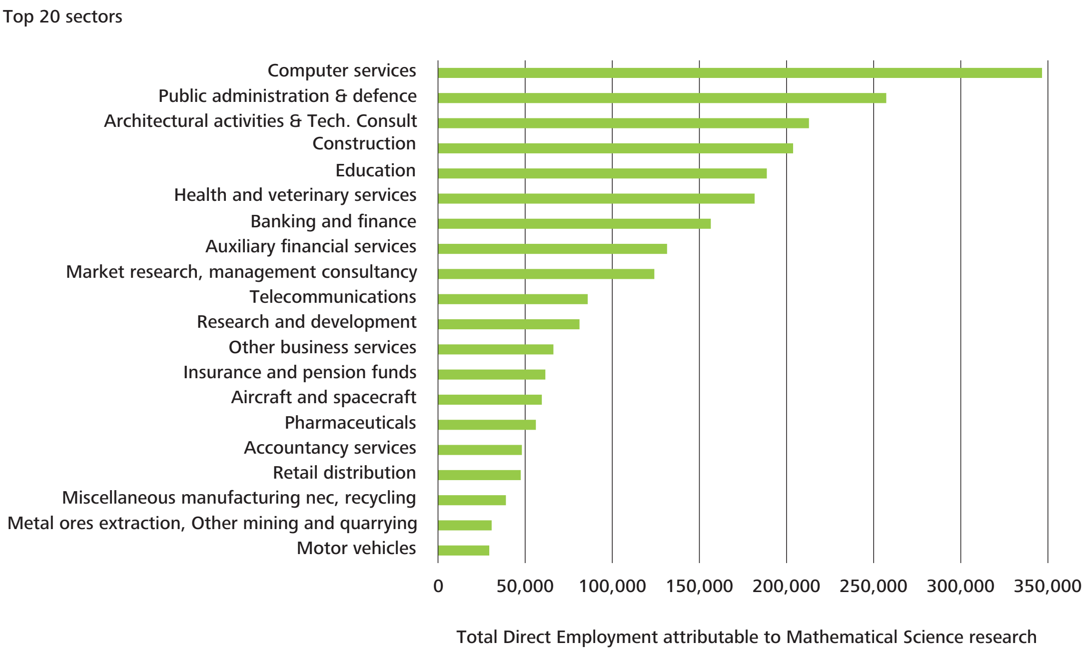
In 2010, there were over 2.8 million individuals in direct employment[48] due to mathematical science research in the UK.

Based on our identification of 'mathematical science occupations'[49] as discussed in Chapter 2, our analysis suggests that, in total, over 2.8 million individuals were directly involved in the generation and application of mathematical science research in the UK in 2010. As discussed in Chapter 2, these individuals are found across all sectors of the UK economy, though as one would expect there are certain sectors with higher concentrations of mathematical science occupations than others. The SOC categories that make up mathematical science occupations (and any adjustments made) are given in the appendix.

The top five sectors, in terms of direct MSR employment, are computer services, public administration and defence, architectural activities and technical consulting, construction and education. This list is perhaps not surprising given the role mathematics plays in computer services, technical consulting and construction and that Government (including national, local and quasi-Governmental organizations) is one of the largest, if not largest, employer of mathematicians (e.g. through the Government Statistical Service). Figure 4.1.1 charts the twenty sectors of the UK economy with the highest levels of direct mathematical science employment using this definition.

In terms of the share of total employment in each sector, the analysis suggests that in three sectors (research and development, computer services and aircraft and spacecraft) well over half of all employment is directly involved in the generation and application of mathematical science research. Again, this is not surprising given that these sectors are highly technical and require significant, cutting-edge mathematical science research inputs. These are sectors where the gap between the generation of mathematical science research and its application is very narrow (if there is any gap at all). The full breakdown of jobs by sectors is given in the appendix.

Figure 4.1.1. Top 20 sectors for direct employment in the UK, 2010



Source: Deloitte using ONS data

Figure 4.1.2 summarises the different shares of direct mathematical science employment across 20 sectors.

Even in the case of sectors such as electricity production and distribution, medical and precision instruments and insurance and pension funds, mathematical science occupations account for over a quarter of all employment in 2010.

4.2 Gross value added

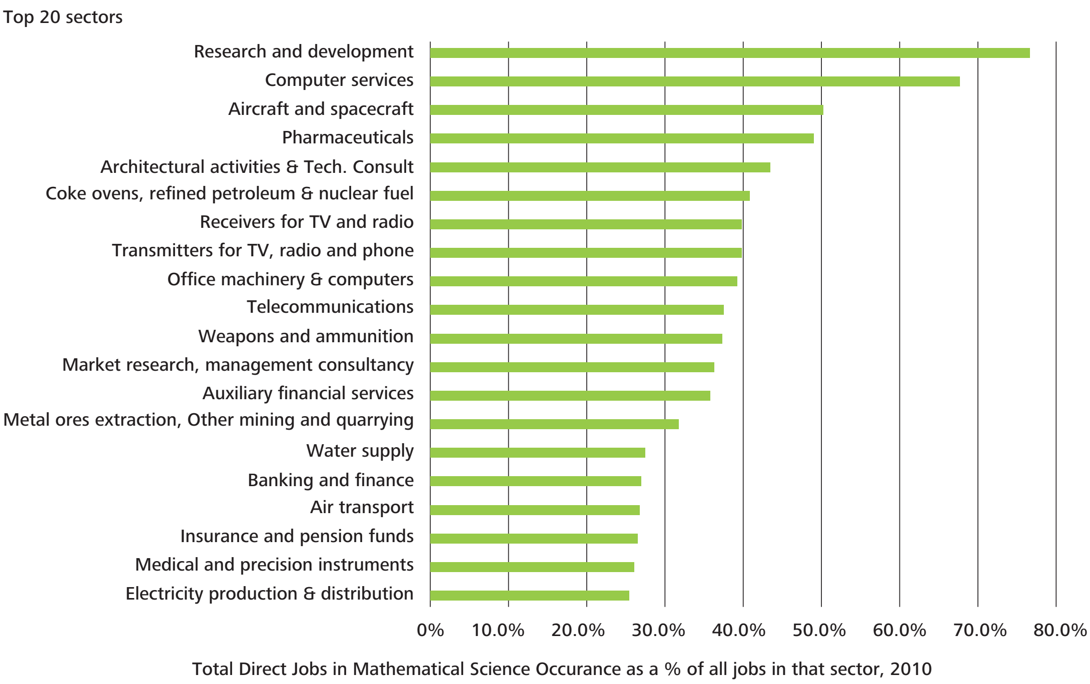
In 2010, mathematical science research in the UK generated direct GVA in excess of £200 billion.

The measure used to evaluate the economic contribution of mathematical science research is gross-value added (GVA). The OECD defines Gross Value Added as the value of output less the value of intermediate consumption. It is analogous to Gross Domestic Product. The GVA analysis presented in this section is based on domestic use Input/output tables from the ONS.

Our analysis suggests that the GVA attributable to the direct application and generation of mathematical science research in the UK in 2010 was approximately £208 billion, or around 16 per cent of total UK GVA.

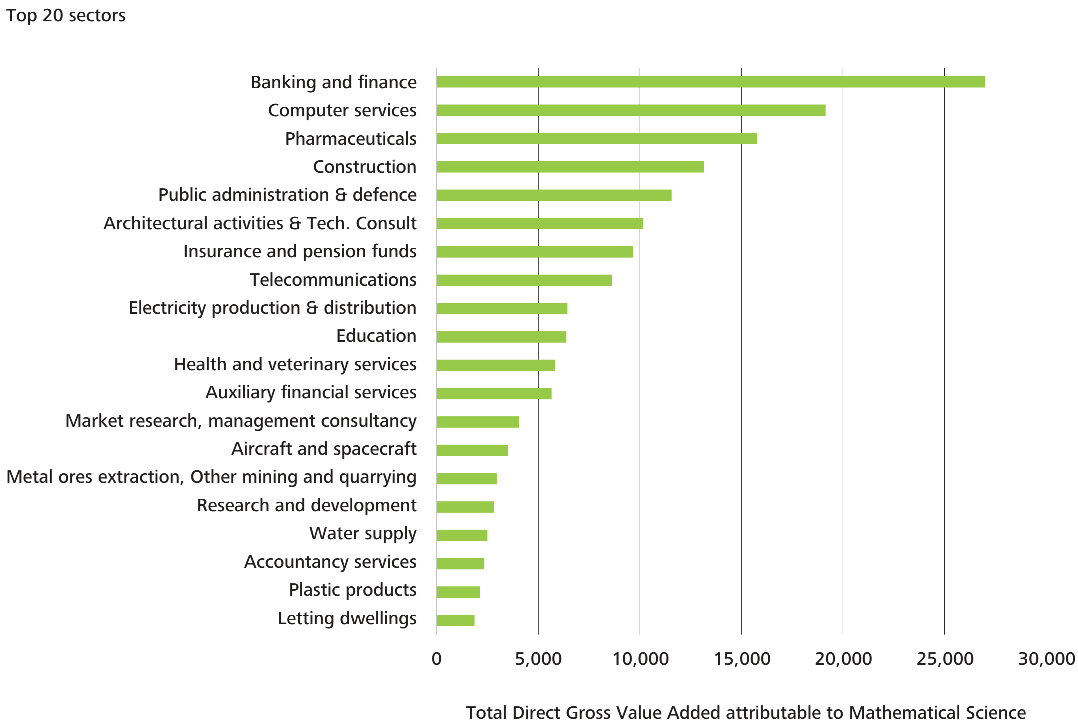
The largest direct contribution of GVA can be found in the banking and finance sector – around £27 billion in 2010. The next largest sectors by direct GVA contribution of mathematical science research are computer services, pharmaceuticals, construction and public administration. Figure 4.2.1 summarises.

Figure 4.1.2. Direct jobs in mathematical science occupation employment as a percentage of all jobs in that sector, 2010



Source: Deloitte using ONS data

Figure 4.2.1. Top 20 sectors for direct mathematical science GVA in the UK, 2010, £m



Source: Deloitte using ONS data

As can be seen, the list of top sectors in terms of GVA is not dissimilar to that of top sectors in terms of employment, although banking and finance rises from sixth place in the rankings to first place on account of the relatively higher levels of per worker productivity.

4.3 Productivity per worker and wages

Productivity, which is measured by direct GVA per worker, in mathematical science occupations in 2010 is calculated as approximately **£74,000 per worker** – based on a mapping of occupations to industry sectors. This compares favourably with the UK productivity average in 2010 which is estimated to be approximately £36,000 (Deloitte calculation).

Using the 2011 ONS Annual Survey of Hours and Earnings (SOC 2000) database and our identified mathematical science occupations, we estimate that the mean wage for those jobs[50] in mathematical science research in 2011 was approximately £44,000.

4.4 Conclusion

"Without maths we can't count on our jobs."

Headline from *The Times*, 26th July 2012

This chapter has considered the direct contribution mathematical science research made to the UK economy in 2010. The analysis suggests:

- **Direct employment in the UK attributable to mathematical science research is over 2.8 million**;
- The total direct **GVA attributable to mathematical science research is over £200 billion**; and
- **Productivity** (as measured by GVA per worker) is significantly higher in mathematics science occupations compared to the UK average – £74,000 compared to £36,000.

Appendix 1: Indirect and induced economic impacts

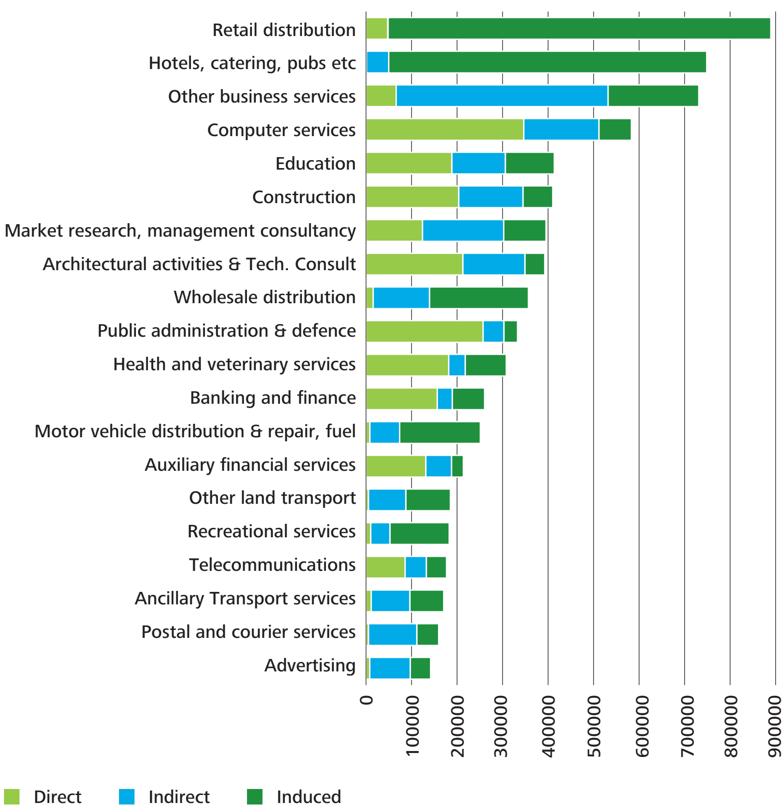
This appendix provides our estimates for the indirect and induced economic impacts of MSR in terms of jobs and GVA: those due to business-to-business purchasing and induced consumer spending. This gives an estimate of the interrelatedness of MSR with the rest of the economy.

5.1 Incremental employment

In 2010, there were over 6.9 million individuals in employment due to the wider ripple effects of mathematical science research in the UK.

In addition to the direct impacts in terms of employment described in Chapter 5, there are number of other ways in which MSR contribute to the UK economy. For example, a 'ripple' or 'multiplier' effect can be created by an MSR-related business and individuals directly generating and applying MSR through their sourcing of supplies from the wider economy or through an employee spending their wages in other sectors.

Figure 5.1.1. Total employment attributable to mathematical science research in the UK, by sector, 2010



Source: Deloitte using ONS data

Accordingly, we consider:

- **indirect employment impact**: changes in employment numbers in associated industries that supply inputs to businesses generating and applying mathematical science research (sometimes referred to as 'supply chain' impacts); and

- **induced employment impact**: the spending by households that result in changes to employment numbers to direct and indirect impacts.

Together, indirect and induced impacts are known as incremental or upstream impacts. These impacts constitute the overall multiplier effect of MSR on the UK economy. This takes account of the proportion of activity in other sectors of the economy that are supported by the intermediate demand of direct MSR, as well as the spending of those involved in MSR and its supply chain.

Figure 5.1.1 shows the total employment (direct plus incremental) attributable to MSR in 2010.

As can be seen from Figure 5.1.1, when considering the wider ripple effects of MSR, a number of observations can be made:

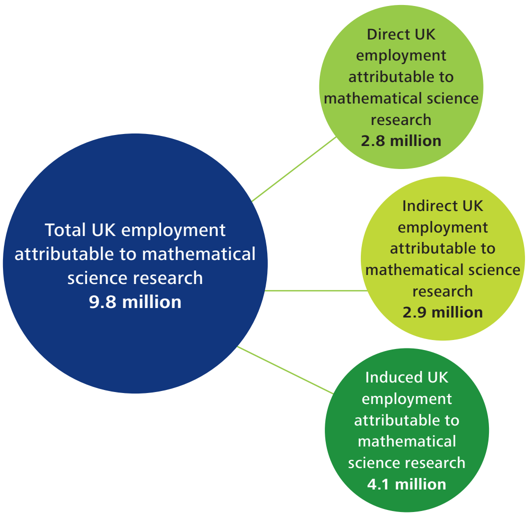
- Sectors such as advertising, postal and courier services and other business services which have low levels of direct employment, have significant indirect employment attributable to mathematical science research due to their role in the supply chain.

- Sectors such as retail, hotels, catering and pubs, motor vehicle distribution and repair and fuel and have high levels of induced employment attributable to MSR. This is wholly expected given these are sectors of the economy where individuals tend to spend large segments of their disposable income, and where a given level of expenditure/output supports more jobs than in direct mathematics-related jobs.

Summing together direct and incremental employment gives **the overall employment number attributable to mathematical science research as 9.6 million in 2010** – around a third of all UK employment.

As Figure 5.1.2 shows, of the total employment figure attributable to mathematical science research, 39 per cent comes from direct employment, 29 per cent from indirect employment and 42 per cent from induced employment.

Figure 5.1.2. Breakdown of mathematical science research in the UK, 2010



Source: Deloitte using ONS data (numbers rounded to 1 decimal point)

Of course, as discussed in previous chapters, all employment in the UK will, at some basic, level be connected to mathematics, but the focus of our analysis has been that connected to mathematical science research in 2010.

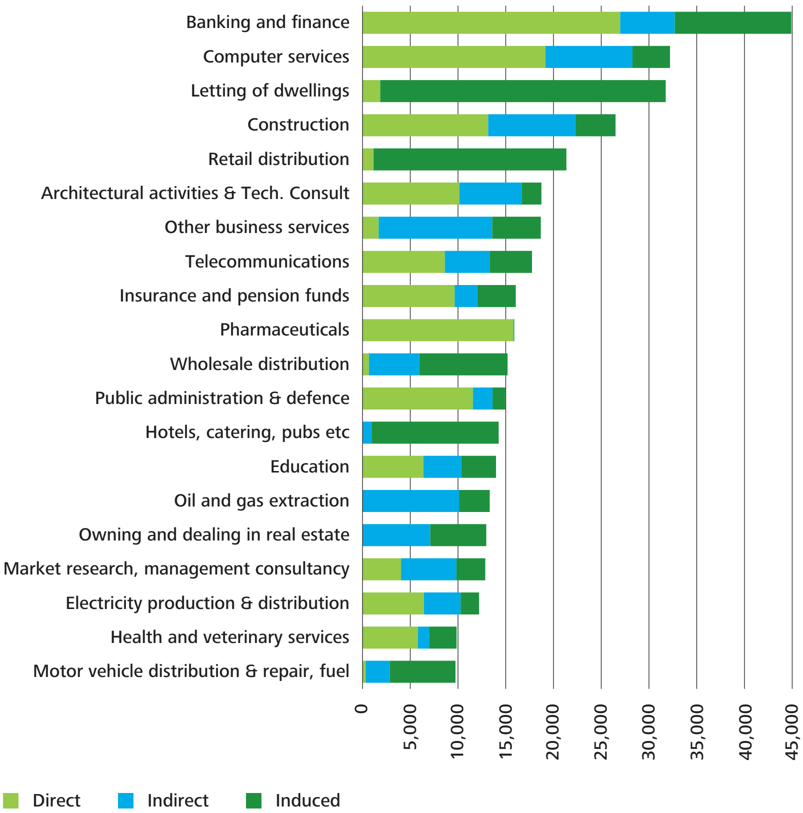
5.2 Incremental GVA

In 2010, mathematical science research in the UK generated incremental GVA in excess of £345 billion.

Our analysis suggests that the incremental GVA (i.e. the indirect and induced GVA) attributable to the application and generation of mathematical science research in the UK in 2010 was approximately £348 billion.

Below we chart the top 20 sectors for total GVA (direct + indirect + induced).

Figure 5.2.1. Top 20 sectors for total GVA in the UK, 2010, £m



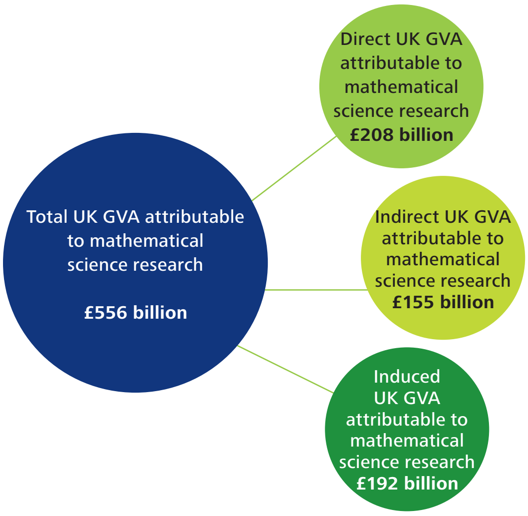
Source: Deloitte using ONS data

Thus, the **total GVA contribution in the UK in 2010 of mathematical science research is £556 billion or over 40 per cent of total GVA**[51]. Certain sectors report much higher levels of indirect and induced GVA compared to direct GVA from MSR. This reflects how these sectors are much more involved in supply-chain activities, selling into MSR sectors and also where individuals in mathematical science occupations spend their wages.

As Figure 5.2.2 shows, of the total GVA figure attributable to mathematical science research, 37 per cent comes from direct GVA, 28 per cent from indirect GVA and 35 per cent from induced GVA.

This represents a more even split between, direct, indirect and induced impacts than seen in employment effects due to relative levels of productivity per worker across each type of impact.

Figure 5.2.2. Breakdown of mathematical science research in the UK, 2010



Source: Deloitte using ONS data (numbers rounded to 1 decimal point)

Appendix 2: Quantification methodology

Broadly speaking, the approach to quantifying the economic impact of mathematical science has involved:

- identifying those occupations that are directly involved in the generation and application of mathematical science research ('**mathematical science occupations**');
- allocating these occupations across the **different sectors** of the UK economy; and
- applying a **UK Input-Output model** to calculate employment and GVA attributable to mathematical science research.

This annex contains further details of each stage.

6.1 Identifying mathematical science research occupations and the number of individuals employed in them

The Office of National Statistics (ONS) and other statistical agencies do not collect information by academic discipline. There is no readily available data on employment in mathematical science research, or for that matter other disciplines. Hence, as part of this research, we have had to create a bespoke classification of what constitutes a mathematical science occupation.

To do this, we have used the 2000 Standard Occupational Classification (SOC) produced by the ONS to list all the occupation categories in the UK. The SOC splits all occupations into groups according to their skill level and skill content. The most detailed level (4-digit SOC) contains over 350 occupations and this is the level we have used.

Based on our understanding of mathematical science research and its applications, mathematical science and other science graduate destinations and our experience of different industries and sectors, we identified a number of occupations as being 'mathematical science occupations'. This list was then presented to a sub-committee of mathematics stakeholders. The list of occupations was subsequently revised to take account of their views.

From this process, 70 4-digit SOC categories were identified as being mathematical science occupations. These are listed in Figure 6.1.1.

Figure 6.1.1. Identified mathematical science occupations

Mathematical science occupations from SOC (2000)	
1111 Senior officials in national government	2211 Medical practitioners
1112 Directors & chief execs of major organisations	2212 Psychologists
1113 Senior officials in local government	2213 Pharmacists & pharmacologists
1121 Prod. works & maintenance managers	2215 Dental practitioners
1123 Managers in mining and energy	2311 Higher education teaching professionals
1131 Financial managers & chartered secretaries	2317 Registrars & senior administrators of educational establishments
1132 Marketing and sales managers	2321 Scientific researchers
1133 Purchasing managers	2322 Social science researchers
1136 Information & communication technology managers	2329 Researchers not elsewhere classified
1137 Research and development managers	2421 Chartered and certified accountants
1141 Quality assurance managers	2422 Management accountants
1151 Financial institution managers	2423 Management consultants, actuaries, economists & statisticians
1181 Hospital and health service managers	2431 Architects
2111 Chemists	2432 Town planners
2112 Bio scientists and biochemists	2433 Quantity surveyors
2113 Physicists, geologists & meteorologists	2434 Chartered surveyors (not quantity surveyors)
2121 Civil engineers	2441 Public service administrative profs
2122 Mechanical engineers	3111 Laboratory technicians
2123 Electrical engineers	3112 Electrical & electronic technicians
2124 Electronics engineers	3113 Engineering technicians
2125 Chemical engineers	3114 Build & civil eng technicians
2126 Design and development engineers	3115 Quality assurance technicians
2127 Production and process engineers	3119 Science & eng technicians not elsewhere classified
2128 Planning and quality control engineers	3131 IT operations technicians
2129 Engineering professionals not elsewhere classified	3132 IT user support technicians
2131 IT strategy and planning professionals	3211 Nurses
2132 Software professionals	3212 Midwives
3533 Insurance underwriters	3213 Paramedics
3534 Financial & investment analyst & advisers	3511 Air traffic controllers
3535 Taxation experts	3512 Aircraft pilots and flight engineers
3537 Financial and accounting techs	3513 Ship and hovercraft officers
3568 Environmental health officers	3531 Estimators, valuers and assessors
4121 Credit controllers	3532 Brokers
4122 Accounts wages clerk, bookkeeper	
5242 Telecommunications engineers	
5245 Comp engineer, installation & maintenance	
5249 Electrical & electronic engineer not elsewhere classified	

Source: Deloitte using ONS data

Having identified 70 mathematical science occupations, the next stage was to make adjustments to each occupation in order to ensure that we were not over-estimating employment levels. For example, potentially not all managers and senior officials should be designated as being in mathematical science occupations as only a portion (or less) of this occupation is likely to directly involved in activities that entail MSR or directly apply MSR tools and techniques. This adjustment was based on our understanding of the sector and mathematical science graduates employed in different occupations (see below), a literature review and discussions with a sub-committee of stakeholders and stakeholders at a workshop.

Figure 6.1.2. Occupation of full-time first mathematics and computer science degree leavers entering employment in the UK[52]

Standard Occupational Classification	% Mathematics and Computer Science graduate destination
Managers and senior officials	5%
Professional occupations	10%
Associate professional and technical occupations	4%
Administrative and secretarial occupations	4%
Skilled trades occupations	6%
Personal service occupations	1%
Sales and customer service occupations	5%
Process, plant and machine operatives	6%
Elementary occupations	4%
Not known	3%

Source: HESA

Additional data on mathematics graduates in different occupations was also received from the learned societies and this also informed our analysis.

Other adjustments included disregarding any unreliable data points or where the sample data was too small.

We acknowledge there is an element of subjectivity in these adjustments as data does not readily exist on mathematical science occupations. As this exercise is repeated in the future, we would recommend the collection of further data that will help better inform the number of individuals in mathematical science occupations.

Figure 6.1.3 presents our rounded estimates of direct employment numbers across each mathematical science occupation, taking into account any adjustments. These percentage adjustments are based on a variety of factors including destinations of mathematical science, our understanding of the stock of mathematics graduates in different occupations, a literature review and discussions with a sub-committee of stakeholders and stakeholders at a workshop.

In some occupations, following the above adjustments the number of individuals in mathematics science occupations was too small to be included for modelling purposes and hence was not included. The level of adjustment ranged from no adjustment to removing the entire occupation category – 17 categories were adjusted.

Figure 6.1.3. Employment numbers in each mathematical science occupation

Level 4 SOC code	Include entire occupation as mathematics occupation?	Apportion needed?	% of category included	Final number of mathematical science occupations	Total number of jobs in SOC category
1111 Senior officials in national government	N	Y	*	400	*
1112 Directors & chief execs of major organisations	N	Y	5%	2,800	60,200
1113 Senior officials in local government	N	Y	5%	1,600	35,100
1121 Prod. works & maintenance managers	N	Y	5%	19,300	413,500
1123 Managers in mining and energy	N	Y	*	12,600	*
1131 Financial managers & chartered secretaries	N	Y	5%	12,100	259,200
1132 Marketing and sales managers	N	Y	5%	25,600	549,400
1133 Purchasing managers	N	Y	5%	2,300	50,100
1136 Information & communication technology managers	Y	N	na	309,900	309,900
1137 Research and development managers	Y	N	na	51,500	51,500
1141 Quality assurance managers	Y	N	na	46,600	46,600
1151 Financial institution managers	N	Y	5%	7,200	154,100
1181 Hospital and health service managers	Y	N	na	77,000	77,000
2111 Chemists	Y	N	na	27,600	27,600
2112 Bio scientists and biochemists	Y	N	na	94,200	94,200
2113 Physicists, geologists & meteorologists	Y	N	na	26,100	26,100
2121 Civil engineers	Y	N	na	78,700	78,700
2122 Mechanical engineers	Y	N	na	78,300	78,300
2123 Electrical engineers	Y	N	na	59,800	59,800
2124 Electronics engineers	Y	N	na	35,900	35,900
2125 Chemical engineers	Y	N	na	9,500	9,500
2126 Design and development engineers	Y	N	na	63,300	63,300
2127 Production and process engineers	Y	N	na	31,200	31,200
2128 Planning and quality control engineers	Y	N	na	29,200	29,200
2129 Engineering professionals not elsewhere classified	Y	N	na	97,100	97,100
2131 IT strategy and planning professionals	Y	N	na	48,900	148,900
2132 Software professionals	Y	N	na	327,500	327,500
2211 Medical practitioners	N	Y	10%	24,200	242,900
2212 Psychologists	N	Y	10%	3,000	29,900
2213 Pharmacists & pharmacologists	Y	N	na	41,800	41,800
2215 Dental practitioners	N	N	na	0	35,700
2311 Higher education teaching professionals	N	Y	10%	13,200	132,600
2317 Registrars & senior administrators of educational establishments	N	Y	10%	4,700	47,200
2321 Scientific researchers	Y	N	na	17,100	17,100
2322 Social science researchers	Y	N	na	17,100	17,100
2329 Researchers not elsewhere classified	Y	N	na	49,100	49,100
2421 Chartered and certified accountants	N	Y	10%	15,900	160,000
2422 Management accountants	Y	N	na	85,700	85,700
2423 Management consultants, actuaries, economists & statisticians	Y	N	na	180,400	180,400
2431 Architects	N	N	na	0	55,400
2432 Town planners	N	Y	10%	2,000	20,700
2433 Quantity surveyors	N	N	0%	0	39,700

Level 4 SOC code	Include entire occupation as mathematics occupation?	Apportion needed?	% of category included	Final number of mathematical science occupations	Total number of jobs in SOC category
2434 Chartered surveyors (not quantity surveyors)	N	N	na	0	63,500
2441 Public service administrative professionals	Y	N	na	34,900	34,900
3111 Laboratory technicians	Y	N	na	64,900	64,900
3112 Electrical & electronic technicians	Y	N	na	26,500	26,500
3113 Engineering technicians	Y	N	na	73,500	73,500
3114 Build & civil eng technicians	Y	N	na	25,800	25,800
3115 Quality assurance technicians	Y	N	na	15,700	15,700
3119 Science & engineering technicians not elsewhere classified	Y	N	na	38,200	38,200
3131 IT operations technicians	N	N	na	0	117,000
3132 IT user support technicians	N	N	na	0	65,400
3211 Nurses	N	N	na	0	509,300
3212 Midwives	N	N	na	0	35,100
3213 Paramedics	N	N	na	0	20,800
3511 Air traffic controllers	Y	N	*	8,900	*
3512 Aircraft pilots and flight engineers	Y	N	na	22,600	22,600
3513 Ship and hovercraft officers	N	N	na	0	15,200
3531 Estimators, valuers and assessors	N	N	na	0	55,300
3532 Brokers	N	Y	0%	0	50,900
3533 Insurance underwriters	Y	N	na	30,100	30,100
3534 Fin. & invest. analyst & advisers	Y	N	na	173,900	173,900
3535 Taxation experts	N	Y	4%	900	21,500
3537 Financial and accounting technicians	N	N	na	0	30,800
3568 Environmental health officers	Y	N	4%	11,900	11,900
4121 Credit controllers	Y	N	4%	45,600	45,600
4122 Accounts wages clerk, bookkeeper	N	N	na	0	512,700
5242 Telecommunications engineers	Y	N	na	42,600	42,600
5245 Comp engineer, installation & maintenance	N	N	na	0	43,500
5249 Electrical & electronics engineer not elsewhere classified	Y	N	na	83,500	83,500

Source: Deloitte using ONS data. Numbers are rounded. * refer to where the underlying data is not available publicly and hence the adjustment percentage cannot be disclosed. Note, numbers do not sum to 2.8 million due to rounding.

6.2 Allocating occupations across different sectors of the UK economy in order to calculate GVA

Having identified 70 mathematical science occupations and the total number of individuals employed in them, the next stage was to identify employment numbers in each occupation by the different sectors of the economy. This is necessary in order to calculate GVA using an Input-Output model which is based on economic sectors rather than employment categories. To make this conversion, we used a bespoke SIC-SOC matrix provided to us by the ONS.

The SIC-SOC matrix shows the number of each SOC occupation in each sector of the economy. In this case, the 2007 Standard Industrial Classification (SIC) divides up the economy into statistical units by the type of economic activity in which they are engaged. We have used the most detailed sectoral breakdown (4-digit level SIC) which splits the economy into over 600 sectors[53].

Figure 6.2.1. Example SIC-SOC matrix

	Different Occupations by SOC code				
Different SIC sectors	…	…	…	…	…
	…	…	…	…	…
	…	…	…	…	…
	…	…	…	…	…
	…	…	…	…	…

Thus, the full SIC-SOC matrix is approximately 600x350. Having defined the 70 mathematical science occupations it was then possible to identify levels of employment in each sector of the UK economy. This was done as follows:

- refer to the original SIC-SOC matrix to understand how each SOC job is distributed across the 600+ SIC sectors and calculate the distribution ratios; and then
- take the mathematical science occupation SOC codes and the number of jobs in each and distribute them across the 600+ SIC sectors using the distribution ratios calculated above.

This gave us the numbers of mathematical science occupations in each SIC category. However, the UK Domestic Use Matrix (DUM), which forms the basis of our Input-Output model (see below) uses broader categories than the SIC sectors. Rather than 600+ economic sectors, it uses 100+ sectors. Thus, it was necessary to convert the SIC categories into DUM categories. Having made these adjustments, we reached a figure for direct employment attributable to mathematical science research.

Figure 6.2.1. Mathematical science occupations by sector[54]

Sector in model	Number of mathematical science occupation jobs
Agriculture	1,000
Forestry	0
Fishing	0
Coal extraction	1,000
Oil and gas extraction	6,100
Metal ores extraction, Other mining and quarrying	30,000
Meat processing	2,100
Fish and fruit processing	100
Oils and fats processing	0
Dairy products	2,000
Grain milling and starch	0
Animal feed	100
Bread, biscuits, etc	100
Sugar	600
Confectionery	600
Other food products	600
Alcoholic beverages	700
Soft drinks & mineral waters	700
Tobacco products	0
Textile fibres, Textile weaving, Textile finishing	300
Made-up textiles, Carpets and rugs, Other textiles, Knitted goods	0
Wearing apparel & fur products	200
Leather goods, Footwear	0
Wood and wood products	400
Pulp, paper and paperboard	200
Paper and paperboard products	100
Printing and publishing	14,900

Sector in model	Number of mathematical science occupation jobs
Coke ovens, refined petroleum & nuclear fuel	13,500
Industrial gases and dyes	0
Inorganic chemicals, Organic chemicals	4,600
Fertilisers, Plastics & Synthetic resins etc, Pesticides	0
Paints, varnishes, printing ink etc	1,800
Pharmaceuticals	56,200
Soap and toilet preparations	1,200
Other Chemical products, Man-made fibres	1,400
Rubber products	100
Plastic products	8,200
Glass and glass products	100
Ceramic goods	0
Structural clay products, Cement, lime and plaster	0
Articles of concrete, stone etc	100
Iron and steel, Non-ferrous metals, Metal castings	6,700
Structural metal products	300
Metal boilers & radiators	0
Metal forging, pressing, etc	16,200
Cutlery, tools etc	100
Other Metal products	200
Mechanical power equipment	0
General purpose machinery	6,800
Agricultural machinery	1,000
Machine tools	12,900
Special purpose machinery	1,400
Weapons and ammunition	3,900
Domestic appliances not elsewhere classified	1,200
Office machinery & computers	20,900
Electric motors and generators etc, Insulated wire and cable	2,100
Electrical equipment not elsewhere classified	2,600
Electronic components	10,100
Transmitters for TV, radio and phone	2,500
Receivers for TV and radio	7,400
Medical and precision instruments	28,600
Motor vehicles	29,400
Shipbuilding and repair	4,500

Sector in model	Number of mathematical science occupation jobs
Other transport equipment	3,400
Aircraft and spacecraft	59,600
Furniture	2,700
Jewellery & related products	0
Sports goods and toys	200
Miscellaneous manufacturing not elsewhere classified, recycling	39,000
Electricity production & distribution	30,500
Gas distribution	8,700
Water supply	16,000
Construction	203,800
Motor vehicle distribution & repair, fuel	8,500
Wholesale distribution	15,700
Retail distribution	47,500
Hotels, catering, pubs etc	900
Railway transport	3,000
Other land transport	5,500
Water transport	1,200
Air Transport	17,600
Ancillary Transport services	11,400
Postal and courier services	5,500
Telecommunications	85,900
Banking and finance	156,500
Insurance and pension funds	61,600
Auxiliary financial services	131,400
Owning and dealing in real estate	0
Letting of dwellings	4,700
Estate agent activities	1,500
Renting of machinery etc	3,100
Computer services	346,600
Research and development	81,200
Legal activities	3,700
Accountancy services	48,100
Market research, management consultancy	124,100
Architectural activities & Tech. Consult	214,700
Advertising	8,200
Other business services	66,200
Public administration & defence	257,200
Education	188,700
Health and veterinary services	181,700

Sector in model	Number of mathematical science occupation jobs
Social work activities	20,800
Sewage and Sanitary services	2,000
Membership organisations not elsewhere classified	17,500
Recreational services	10,300
Other service activities	22,900
Total	**2,827,100**

Source: Deloitte using ONS data. Note: variance in numbers is due to rounding.

Having made this conversion, we ran our Input-Output model to calculate direct GVA attributable to MSR as well as supply chain linkages and induced impacts.

- **indirect impact**: changes in the number of jobs and GVA in associated industries that supply inputs to businesses generating and applying mathematical science research (sometimes referred to as 'supply-chain' impacts); and

- **induced impact**: the spending by households that result in changes to the number of jobs and GVA due to direct and indirect impacts.

The process behind constructing an Input-Output model is complex[55] and involves considering the gross revenue earned in each sector and applying adjustments so that it only represents additional revenue as compared to the counterfactual case of there being no mathematical science research. In this instance, it was the case that a detailed counterfactual case was not necessary given that mathematics is so heavily engrained in economic activity. We have then applied multiplier effects in the model to trace the impact across all sectors of the economy.

References

1 Many of these examples are taken from *Mathematics Matters*, www.ima.org.uk/i_love_maths/mathematics_matters.cfm

2 The estimates should be read as indicative snapshot and read in conjunction with the stated limitations and assumptions. Full methodological details can be found in the appendix.

3 In addition to these direct impacts, mathematical science research activities by organisations and employees will generate downstream impacts across the supply chain (indirect effects) and changes in household spending (induced effects). An estimate of these impacts is provided in the appendix. There will also be upstream effects of mathematical science research (other organisations using the research) – these have not been quantified due to data limitations, but are discussed qualitatively.

4 This includes research in both physical and social sciences.

5 This includes software publishing, programming, IT consultancy, data processing, web activities etc.

6 This includes civil engineering and survey work.

7 This includes the generation of new mathematical science research.

8 See for example *Ground-breaking UK research helps Team GB become Olympic golden great*, www.rcuk.ac.uk/media/news/2012news/Pages/120808.aspx

9 See *introduction to Mathematics Matters*,www.ima.org.uk/i lobe maths/mathematics matters, cfm

10 As noted by the House of Lords Select Committee on Science and Technology, the acronym 'STEM' encompasses a group of disciplines that teach the skills required for a high-tech economy. For its inquiry, the Select Committee adopts a definition used by the Department for Business, Innovation and Skills (BIS) and the Higher Education Statistics Agency (HESA) which classifies STEM subjects as including : medicine and dentistry; subjects allied to medicine; biological sciences; veterinary science, agriculture and related subjects; physical sciences; mathematical sciences; computer science; engineering; technologies; and architecture, building and planning. See www.publications.parliament.uk/pa/ld201213/ldselect/ldsctech/37/37.pdf for more details.

11 See *The Plan for Growth*, March 2011, http://cdn.hm-treasury.gov.uk/2011budget_growth.pdf

12 While we have sought, and received, thoughts and comments from stakeholders across a range of issues as part of this study, this should not be regarded as an endorsement by stakeholders as to our chosen methodology and subsequent results.

13 This study does not consider issues around mathemeatical literacy and basic skills.

14 Available at: www.epsrc.ac.uk/newsevents/pubs/corporate/intrevs/2010maths/Pages/default.aspx

15 This list of sub-fields is consistent with the JACS3 classification produced by HESA (www.hesa.ac.uk/content/view/1787/281/).

16 See for example *Advancing the digital arts*, (www.ima.org.uk/viewItem.cfm?cit_id=383289) which highlights how the computer animation industry relies on a steady stream of mathematics to produce images found on our cinema and television screens.

17 In *Without maths* we can't count on our jobs, The Times, 26th July 2012.

18 These occupation classifications are based on the Office of National Statistics' Standard Occupational Classification (SOC) system which is common classification framework of occupational information for the UK on the basis of skill level and skill content. At the most detailed level (4 digit) which we have used this contains 70 different occupational types.

19 Of course, there will be mathematics graduates in these other occupations but they will not be engaged in MSR directly. They may still, nonetheless, still apply the skills learnt during their studies in their occupation. The careers profile section of the Maths Careers website describes how mathematics graduates apply their training across a range of diverse occupations (see www.mathscareers.org.uk/16-19/career_profiles.cfm).

20 Forecasting the future economic impacts of R&D is notoriously difficult. While it may be possible to identify the characteristics of research that may make it more or less likely to have a wider economic impact, attempting to quantify the size of this impact (and what proportion of this is 'additionality') is fraught with high levels of uncertainty making any estimates highly speculative. This was also recognised by the Royal Society in its report *Hidden wealth: the contribution of science to service sector innovation,* (page 12).

21 This analogous to GDP, except that it only includes relevant value added at each stage of production.

23 This model, which is tailored for each project, has been used to quantify the economic impact of a range of activities across different sectors in the UK and abroad including transport, retail, telecommunications, defence and tourism.

24 This section draws heavily on the Mathematics Matters seminar facilitated by the Parliamentary and Scientific Committee and the Council for Mathematical Sciences on 15th March 2012 and the subsequent write up in *Science in Parliament*, Vol. 69, No.2, Whitsun 2012.

25 T*he Plan for Growth*, HMT and BIS, March 2011, http://cdn.hm-treasury.gov.uk/2011budget_growth.pdf

26 This trend has continued into 2012 – see for *example A-level results show popularity of maths*, Financial Times, 16 August 2012.

27 If one were to include other physical sciences in these figures, the fall would have been reversed.

28 As defined by HESA as including mathematics, statistics, operational research, 'broadly based programmes within mathematical sciences' and 'others in mathematical sciences'.

29 See www.bis.gov.uk/analysis/economics/productivity-and-competitiveness for further details.

30 There would be a net increase in employment if the number of new jobs created and filled exceeded the number of people simply moving from one job to another.

31 The following sections provide a broad outline of the different ways mathematical science research has an economic impact. It does not go into detail on specific mathematical techniques used. For more detailed examples of how mathematical science can be applied across different industries see the *Mathematics Matters* series.

34 This virtuous circle applies to all facets of mathematical science research, not just algorithms.

35 This section has benefit greatly from discussions with Dr Mike Dewar of NAG. All errors are the authors' own.

36 Detailed case studies can be found on the NAG website: www.nag.co.uk/Market/casestudies.asp.

37 Source: SAS, *Data equity – Unlocking the value of big data*: www.sas.com/offices/europe/uk/downloads/data

32 In a speech to the Royal Institution, London on 8th March 2012.

33 See *Science in Parliament*, Vol. 69, No.2, Whitsun 2012.

34 This virtuous circle applies to all facets of mathematical science research, not just algorithms.

35 This section has benefit greatly from discussions with Dr Mike Dewar of NAG. All errors are the authors' own.

36 Detailed case studies can be found on the NAG website: www.nag.co.uk/Market/casestudies.asp

37 Source: SAS, *Data equity – Unlocking the value of big data*: www.sas.com/offices/europe/uk/downloads/data

38 See www.deloitte.com/view/en_GB/uk/industries/tmt/media-industry/df1889a865f 05310VgnVCM 2000001b 56f00aRCRD.htm

39 This section has benefit greatly from discussions with Professor David Hand of Imperial College, London and material provided by him. All errors are the authors' own.

40 This section has benefit greatly from discussions with Professor Stephen Senn of the Centre de Recherche Public de la Santé and material provided by him. All errors are the authors' own.

41 See *Science in Parliament*, Vol. 69, No.2, Whitsun 2012.

43 *Mathematical and Statistical Opportunities in Cyber Security*, March 2009 (published by Lawrence Berkeley National Laboratory in the USA), available at: http://crd-legacy.lbl.gov/~dhbailey/dhbpapers/CyberMath.pdf

44 See *Science in Parliament*, Vol. 69, No.2, Whitsun 2012.

45 This can be found on the Cabinet Office website, www.cabinetoffice.gov.uk/resource-library/national-risk-register

46 This section has benefit greatly from discussions with Professor Mike Cullen of the Met Office. All errors are the authors' own.

47 Source: Chartered Management Institute, www.managers.org.uk/news/new-research-reveals-quality-management-delivers-%C2%A390-billion-uk-economy

48 This figure includes individuals employed by businesses and other organisations and those that are self-employed.

49 I.e. occupations that require a high-degree of mathematical knowledge (at to least to degree level) and regularly contribute to the development of new mathematical research and apply it in innovative ways.

50 This figure does not include the income of the self-employed.

51 As a point of comparison, total GVA in the UK in 2010 was £1,301 billion (preliminary ONS figures).

52 The HESA data does not disaggregate between mathematics and computer science graduates. While we acknowledge this is not a perfect measure (and does not include postgraduates) and only captures the flow of new graduates into occupations (rather than the stock of mathematics graduates), it is a reasonable first proxy. Subject research could focus on revising these estimates.

53 This matrix is based on the Annual Population Survey, an annual survey of households undertaken by ONS. As this matrix is extremely detailed (615 rows by 354 columns) compared to the relatively small sample size of the Annual Population Survey, in some instances ONS does not disclose employment information which it deems as unreliable.

54 Due to licence restrictions we are unable to show the total number of jobs in each DUMs sector.

55 Broadly speaking, the Domestic Use Matrix (differentiating between domestic purchases and imports) is used to give a matrix of coefficients, detailing the proportion of inputs sourced by an industry from all other industries and labour. The matrix of coefficients is then subtracted from the identity matrix before being inverted to give the *Leontief Inverse*. This matrix then details Type II multipliers for each country, such that a multiplier of, for example, 1.8 in 'mathematics critical sector' means that for a direct impact of £1 million in Gross Revenue terms, a further £0.8 million would be generated by business-to-business purchases in the supply chain and induced consumer spending for a total expenditure (or Gross Output) impact of £1.8 million.

Registered office: Hill House, 1 Little New Street, London EC4A 3TR, United Kingdom. Registered in England No 3311052.

Designed and produced by The Creative Studio at Deloitte, London. 26755A