Pólya Prize: citation for Ehud Hrushovski

Short citation:
Professor Ehud Hrushovski FRS, of the University of Oxford, is awarded a Pólya Prize for his profound insights that transformed very abstract model-theoretic ideas into powerful methods in well-established classical areas of geometry and algebra.

Long citation:
Professor Ehud Hrushovski FRS, of the University of Oxford, is awarded a Pólya Prize for his profound insights that transformed very abstract model-theoretic ideas around definability into powerful methods that have had spectacular success in diverse areas of geometry and algebra, such as diophantine geometry, several varieties of motivic mathematics, $p$-adic integration, rigid geometry, permutation groups and algebraic groups, and approximate groups. This creates an amazing complex of deep connections.

Hrushovski obtained his PhD from Berkeley in 1986. Model theory had, in the previous twenty years, developed rapidly, with two outstanding achievements in the mid 1960s being Morley's work on omega-stable theories (with a prominent role for indiscernibles and emerging geometric notions, for example around dimension.), and the Ax–Kochen–Ershov work on Henselian fields (followed by Ax's work on the model theory of finite fields).

Already in the 1980's Shelah and Zilber were inspirational figures in model theory. Shelah had his eyes on general classification of models of theories, while Zilber was intent on a geometrical approach to theories, based on definability considerations such as interpretability. An idea that links both approaches is that of stable theories. Stability generalises Morley's omega-stability, linked to algebraically closed fields, thus connected to geometric notions. In particular it links to the notion of irreducible curve, with the general notion being called strong minimality. In stable theories in general there are various notions of a geometric and combinatorial nature, often forbidding to a novice. Hrushovski, by exceptional efforts both mathematical and methodological, has revealed the fertility of the geometric theory for various sophisticated classical mathematical subjects.

His thesis concerned groups interpretable in stable theories, and can be seen as a vast generalisation of the fundamental coordinatisation theorem of projective geometries. Zilber had made a bold and beautiful conjecture about strongly minimal sets. Hrushovski gave an intriguing family of counterexamples. Hrushovski and Zilber reflected deeply, repaired, and then proved the conjecture, by imposing extra conditions. This allowed Hrushovski to make a startling contribution to diophantine geometry, with a proof of the characteristic $p$ version of a conjecture of Mordell–Lang.

He wrote a rich and beautiful book with Cherlin, connecting permutation group theory and associated geometry to basic problems in the model theory of finite structures. Hrushovski has had a major influence on profound recent work on approximate groups by Breuillard, Green and Tao, via stability-theoretic insights.

Much of his recent work is connected to valued fields and pseudofinite structures. With Haskell and MacPherson he gave a comprehensive study of definable equivalence relations in $p$-adic fields (such fields are not stable). This deep analysis is of basic importance in
extending greatly Denef's work which uses model theory to analyse the structure of $p$-adic Poincaré series. Issues of uniformity in $p$ are crucial for the work of Denef and Loeser on a powerful theory of motivic integration. Hrushovski and Kazhdan, in turn, developed a different motivic integration using ideas of geometric model theory, and obtained new results in algebraic geometry, as well as a quite new perspective on model theory of henselian fields.

Hrushovski and Loeser gave a model-theoretic version of Berkovich spaces, using the refined model theory to make the topology closer to classical topologies. O-minimality (which is nowadays very important for diophantine geometry) is deeply involved, as it is in the Hrushovski–Kazhdan work. More recent novel model theoretic work, with Ducros, connects complex and nonarchimedean integrals.