

Berwick Prize: citation for Ailsa Keating

Short citation:

Dr Ailsa Keating of the University of Cambridge is awarded a Berwick Prize for the paper ‘Dehn twists and free subgroups of symplectic mapping class groups’ (Journal of Topology, 7: 436–474). Keating's work sheds light on the global symmetries of symplectic manifolds, by showing that arbitrary products of Dehn twists along two Lagrangian spheres that intersect at least twice never simplify to the identity map.

Long version:

Dr Ailsa Keating of the University of Cambridge is awarded a Berwick Prize for the paper ‘Dehn twists and free subgroups of symplectic mapping class groups’ (Journal of Topology, 7: 436–474). Keating's work addresses a central question in modern symplectic topology: the structure of groups of global symmetries of symplectic manifolds. Seidel's seminal work in the late 1990s made progress on this problem by studying Dehn twists about Lagrangian spheres (generalising classical Dehn twists of surfaces about simple closed curves), using Floer homology to show that products of Dehn twists can be smoothly isotopic without being symplectically isotopic. Meanwhile, in the classical case of surfaces, Ishida showed in 1996 that, if two curves intersect at least twice, then the corresponding Dehn twists do not satisfy any algebraic relation whatsoever: no non-trivial sequence of these Dehn twists ever simplifies to the identity map.

Keating proves a similar result in the much harder higher-dimensional case, by cleverly exploiting the algebraic structure of Lagrangian Floer homology to show again that there are no algebraic relations between two Dehn twists about Lagrangian spheres whose Floer homology detects the presence of at least two intersection points. This shows that many symplectic manifolds have fairly complicated groups of symplectomorphisms, containing a free subgroup; moreover, Keating also shows that even the smaller subset of those symplectomorphisms which are topologically equivalent to the identity map often contains a similarly rich structure.

Besides geometric considerations, Keating's argument makes a significant use of homological algebra to represent the image of a Lagrangian under a power of a Dehn twist by an explicit twisted complex in the Fukaya category, and obtains a lower bound on the rank of its homology. While similar techniques had already been used to study particular examples, Keating's result is remarkable by its level of generality. It also stands as a beautiful instance of the algebraic machinery of Fukaya categories being used to prove genuinely new results about classical objects of symplectic topology.