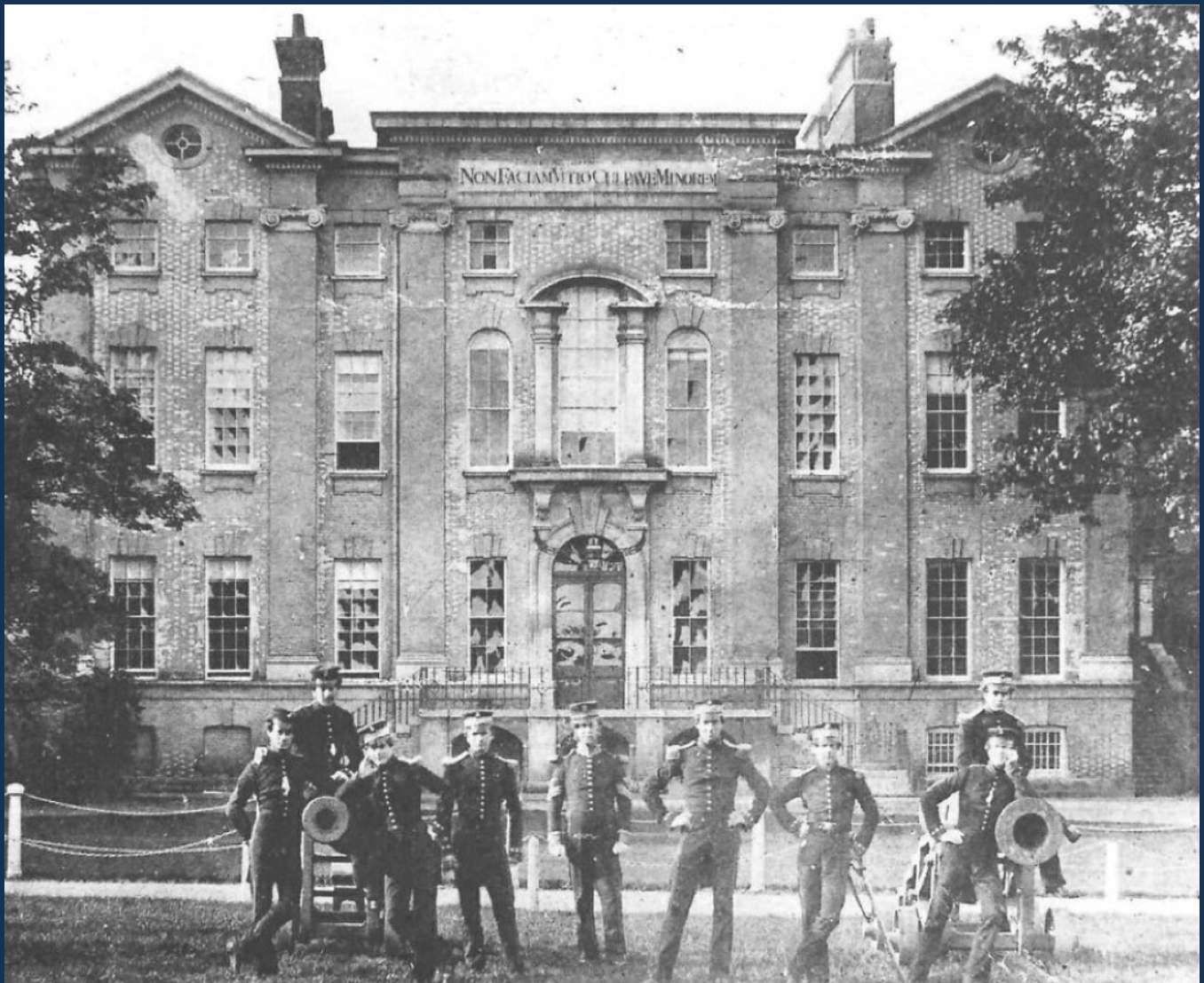




LONDON  
MATHEMATICAL  
SOCIETY  
EST. 1865

## NEWSLETTER

*Issue: 495 - July 2021*



A LIFE  
OF A  
COMPUTER

MATHEMATICAL  
APPROACH TO SERVICE  
APPORTIONMENT

NOTES OF  
A NUMERICAL  
ANALYST

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## COVER IMAGE

Cadets posed in front of Addiscombe Military Academy c. 1859 (from the article 'Allan J. C. Cunningham (1842-1928): A Life Of A Computer' by Tomoko L. Kitagawa; page 22)

Do you have an image of mathematical interest that may be included on the front cover of a future issue? Email [images@lms.ac.uk](mailto:images@lms.ac.uk) for details.

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## LMS NEWS

## New Newsletter Editor



I would like to welcome Dr Alina Vdovina, Senior Lecturer in Pure Mathematics at Newcastle University, as Editor-in-Chief of the Newsletter. Alina is already an enthusiastic

contributor to the Society's work. She is currently a member of the Society's Research Grants Committee and the Society Lectures and Meetings Committee, and she was a Member of Council from 2015-19. Alina's research interests include geometric group theory, geometry, knot theory, non-commutative geometry, algebraic geometry, theory of buildings and K-theory of  $C^*$ -algebras. Her editorial experience includes editing *Beauville Surfaces and Groups* published by Springer in 2015. Alina's connections in the mathematical community and her knowledge of the Society's activities will be invaluable to the Newsletter.

The Society warmly thanks immediate past Editor-in-Chief Dr Eleanor Lingham for the significant amount of time she has given to the Newsletter and the production of exceptional issues of the Newsletter during her tenure. Eleanor is only the second Editor of the new format Newsletter, and it has thrived under her leadership. We wish her all the best for the future.

Jon Keating  
LMS President

## Growth in Open Access in the Society's Journals

The Society publishes 12 peer-reviewed journals, five of which it owns and seven of which it publishes in collaboration with other learned societies and institutions. In 2020 these journals published 864 scholarly articles across mathematics.

Seven of the journals operate under a hybrid model offering a gold open access option to authors on acceptance while the *Transactions* is a completely

open access journal. The remaining four journals, those translated from Russian, are subscription only.



Within the seven hybrid journals 131 articles (19%) were published online with gold open access in 2020 compared to 35 (6%) in 2019. This growth continues, with a further 60 articles published openly in 2021, to the end of April. The *Transactions*, with its new scope and separate Editorial Board, has seen seven papers published in the first four months of 2021, all, of course, open access. The decision to publish an article is made independently and in advance of discussions with authors regarding open access arrangements.

Gold open access has the benefit of making the final version of record free to read by any researcher irrespective of location or institution. Authors of these papers benefit from increased circulation and higher download figures. The application of a Creative Commons licence allows liberal reuse in future works subject to proper attribution. This is also the route to compliance with a growing number of funder mandates around open access to research. For more details on Creative Commons see [creativecommons.org/about/cclicenses](https://creativecommons.org/about/cclicenses). The LMS journals offer authors the choice of CC BY, CC BY-NC and CC BY-NC-ND licences

Authors publishing open access in LMS journals have typically been able to do so because of a 'Read and Publish' agreement struck between their institution and one of the Society's publishing partners. These deals have come to replace traditional bundled subscription arrangements and



combine both reading and publishing charges into a single payment, providing access to pay-walled content and open access publishing without individual article publishing (APCs) charges to authors.

Amongst the open access articles published in the *Bulletin* are five survey articles. The Society has agreed to make future surveys open access, subject to the authors' agreement. Where institutional funding is not available the Society will cover APCs supported by legacy by Frank Gerrish in the publication of expository articles and surveys. Open access surveys published to date are:

- **Braids groups and mapping class groups: The Birman-Hilden theory**  
Dan Margalit and Rebecca R. Winarski  
[doi.org/10.1112/blms.12456](https://doi.org/10.1112/blms.12456)
- **New progress on Grothendieck duality, explained to those familiar with category theory and with algebraic geometry**  
Amnon Neeman  
[doi.org/10.1112/blms.12429](https://doi.org/10.1112/blms.12429)
- **Peak sets and boundary interpolation sets for the unit disc: a survey**  
Alan Noell  
[doi.org/10.1112/blms.12414](https://doi.org/10.1112/blms.12414)
- **Haar null and Haar meager sets: a survey and new results**  
Márton Elekes and Donát Nagy  
[doi.org/10.1112/blms.12340](https://doi.org/10.1112/blms.12340)
- **F-Manifolds and geometry of information**  
Noémie Combe and Yuri I. Manin  
[doi.org/10.1112/blms.12411](https://doi.org/10.1112/blms.12411)

The Managing Editors of the *Bulletin* welcome suggestions for further Survey articles for consideration and can be contacted via [bulletinlms@gmail.com](mailto:bulletinlms@gmail.com).

All LMS open access content can be found at:

- *Bulletin, Journal, Proceedings, Transactions, Journal of Topology and Matematika*: [bit.ly/LMS\\_OA](https://bit.ly/LMS_OA)
- *Compositio Mathematica*: [bit.ly/OA\\_CMAT](https://bit.ly/OA_CMAT)
- *Nonlinearity*: [bit.ly/OA\\_NON](https://bit.ly/OA_NON)

Suzanne Abbott  
Publications Development Manager, LMS

## New Reciprocity Agreements

The London Mathematical Society is delighted to announce that in May 2021 it signed reciprocity agreements with the Allahabad Mathematical Society and with the Indonesian Mathematical Society.

Founded in 1958 by Professor B.N. Prasad, the Allahabad Mathematical Society is a non-profit scientific organisation dedicated to promoting the cause of advanced study and research in various branches of mathematics including theoretical physics and mathematical statistics. More details about the Allahabad Mathematical Society are available on their website at [amsallahabad.org](https://amsallahabad.org).



The Indonesian Mathematical Society was founded in July 1976 and its aim is to foster and develop mathematics and mathematics education, and enhance the role of mathematics and mathematics education in Indonesia. Further details about the Indonesian Mathematical Society are available on its website at [indoms.id/en/home](https://indoms.id/en/home).

Through these Reciprocity Agreements, members of the Allahabad Mathematical Society and/or the Indonesian Mathematical Society, who are not normally resident in the UK may join or become reciprocity members of the London Mathematical Society. In return LMS members who are not normally resident in India may join the Allahabad Mathematical Society at the AIMS reciprocal membership rate and LMS members who are not normally resident in Indonesia may join the Indonesian Mathematical Society at the IndoMS reciprocal rate.

The LMS now has reciprocity agreements with 23 mathematical societies, which are listed on the LMS website at [tinyurl.com/27ycvnb7](https://tinyurl.com/27ycvnb7).

Elizabeth Fisher  
Membership & Grants Manager, LMS

## Forthcoming LMS Events

The following events will take place in forthcoming months:

**Summer General Society Meeting:** 2 July, online ([tinyurl.com/rt8xwn8](https://tinyurl.com/rt8xwn8))

**Northern Regional Meeting:** 1–10 September, University of Manchester ([tinyurl.com/yamy8uvq](https://tinyurl.com/yamy8uvq))

**South Wales & South West Regional Meeting:** 4–6 January 2022 ([tinyurl.com/y3k9e4wh](https://tinyurl.com/y3k9e4wh))

A full listing of upcoming LMS events can be found on page 46.

## OTHER NEWS

### HoDoMS Annual Conference 2021

The Annual Conference of HoDoMS (Heads of Department of Mathematical Sciences in the UK) gathers a range of speakers with the aim of supporting heads of department and informing them of current issues and good practice. This year, the conference was held on 22 and 23 April and, for the second time, it was held online, with support being provided by ICMS. It is hoped that next year we will be able to resume the usual practice of a face to face conference.

The conference began with a presentation and question session led by Lynn Gladden, the Executive Chair of EPSRC, and Mark Smith, senior independent member of the Council of EPSRC. They came to inform us about the current situation with respect to the additional £300 million EPSRC funding for mathematical sciences. Part of this money is currently held up by the Government spending review. Lynn and Mark reassured us that they were optimistic that this commitment would eventually be honoured. We heard more from EPSRC on the second day with a presentation by Katie Blaney on their funding strategy and the opportunities in mathematical sciences.

The HoDoMS conference always includes presentations on the learned societies. This year we heard about both the IMA and the Edinburgh Mathematical Society, with talks from Nira Chamberlain and Gavin Gibson respectively. Also, David Abrahams gave an update on the progress towards a National Academy for Mathematical Sciences. David also spoke, along with Paul Glendinning, on new funding for mathematics that is available through the Isaac Newton Institute (INI) and the ICMS. In addition, the incoming director

of the INI (and now President-Designate of the LMS), Ulrike Tillmann, gave a personal introduction to herself and her vision for the INI. We had a presentation from Tony Hill and Lindsay Walsh about the Levelling Up project that Tony is sponsoring and the LMS is facilitating. This is an inclusive, collaborative and evidence-based project working with sixth formers from schools in disadvantaged and low participation areas in order to encourage them to study mathematics at university.

Unsurprisingly, there were several talks that addressed different aspects of how we have adapted to the pandemic. Katherine Seaton of La Trobe University spoke on incorporating academic integrity considerations in online mathematics assessments. She used her experience to discuss how exams can be set so as to make it harder for students to cheat and signs that markers can look for that may indicate issues that should be investigated. We had a presentation by Matthew Henley, who is an undergraduate in Birmingham. Matthew presented the results a survey he has conducted as part of his undergraduate project on how mathematics departments have adapted their teaching and learning in response to covid-19. Rebecca Hoyle (University of Southampton) spoke about how the Virtual Forum for Knowledge Exchange in the Mathematical Sciences (V-KEMS) had been set up rapidly at the start of the pandemic to give a forum for knowledge exchange while everyone is having to work remotely and she described some of the resulting projects. Finally, there was a discussion session in breakout rooms, led by Jim Anderson (University of Southampton), on how we create the new normal as we move out of the pandemic.

John R. Parker  
LMS Representative

## Double Gold for UK at European Mathematics Championship



Yuhka Machino (left) and Jenni Voon

For the first time the team representing the UK at the European Girls' Mathematical Olympiad (EGMO) has brought home two gold medals. The team also won further silver and bronze medals, with each team member receiving a medal.

The gold medal winners were Yuhka Machino, a student at Millfield School, and Jenni Voon, a student at Landau Forte College. Yuhka and Jenni, both 17, came 6th and 7th respectively at a competition of over 140 students representing 37 European countries. The team's all-round performance meant the UK finished 3rd in Europe, their second ever podium finish.

The team is entered into the competition by the United Kingdom Mathematics Trust (UKMT) and supported by XTX Markets and was identified through a rigorous selection process, which included two British Mathematical Olympiad competitions held over two rounds in UK schools and organised by the UKMT.

More information is available at [ukmt.org.uk](http://ukmt.org.uk).

## The Case of Laila Soueif

The Egyptian mathematician and activist Laila Soueif is a Professor of Mathematics at Cairo University. She was a key founder of the March 9 Movement for University Autonomy in Egypt and has been fighting authoritarianism throughout her life<sup>1</sup>. Members of her family have been repeatedly arrested, over several years, because of their political activity<sup>2</sup>.

<sup>1</sup>[bit.ly/2TImMYA](https://bit.ly/2TImMYA)

<sup>2</sup>[bit.ly/3yIEw0m](https://bit.ly/3yIEw0m); [bit.ly/3uirjxX](https://bit.ly/3uirjxX); [bit.ly/3wNcq2f](https://bit.ly/3wNcq2f); [nyti.ms/34gUD7I](https://nyti.ms/34gUD7I); [bit.ly/3ukZVsL](https://bit.ly/3ukZVsL); [bit.ly/3bUJPA1](https://bit.ly/3bUJPA1); [bit.ly/3oPeDHB](https://bit.ly/3oPeDHB); [bit.ly/3vAsVhy](https://bit.ly/3vAsVhy)

<sup>3</sup>[bit.ly/3fmPlxd](https://bit.ly/3fmPlxd)

Her lifelong work has been recognised by the 2020 MESA Academic Freedom Award<sup>3</sup>. More about Laila Soueif and her family's current plight can be found at [lailasoueif.org/take-action](https://lailasoueif.org/take-action), calling on mathematicians and mathematical societies to act in support.

Iain Gordon  
LMS Vice-President

## Celebration of the International Day of Mathematics 2021



Celebrations took place to mark the annual International Day of Mathematics on Sunday 14 March 2021 with a mixture of virtual and onsite events, as well as celebrations in schools.

The theme for 2021 was "Mathematics for a Better World". The global online celebration started on the IDM website at 00:00 in Oceania and lasted 48 hours, including live-blogged pictures and videos from over 697 IDM events from at least 95 different countries. Altogether more than 17,000 people attended the Global Celebration via the website. Many more joined by posting on Facebook, Twitter, and Instagram using the hashtag #idm314 or tagging the official IDM account. The Mathematics for a Better World Poster Challenge generated 2100 posters.

The main event of the online celebration was a series of short talks in different languages streamed through the IDM website, targeting a general audience, and featuring mathematics and how it can make the world better.

In a project funded by the Simons Foundation, IDM joined hands with partner organisations in three African countries, Algeria, Republic of Congo and Senegal, to plan and organise an African celebration consisting of a series of online events with panel discussions, talks, interactive workshops, classroom activities, competitions and teacher training. The

events took place from 10 to 26 March 2021 and were held in French, English, and Arabic.

Videos from the online celebration and the poster gallery are available on the IDM website: [idm314.org](http://idm314.org).

Christiane Rousseau  
IDM Instigator and Coordinator

## Mathematicians Elected Royal Society Fellows

The Royal Society has released details of its newly elected Fellows, among whom were LMS members Professor Karen Vogtmann (University of Warwick), Professor Endre Süli (University of Oxford) and Professor Jeremy Quastel (University of Toronto). LMS Honorary Member Professor Claire Voisin (CNRS, France) was elected a Foreign Member. Other notable Fellows included Professor Richard Samworth (University of Cambridge).

Professor Vogtmann was awarded the LMS Pólya Prize in 2018 and also took part in the film *Thinking Space*, which was commissioned for the LMS 150th Anniversary. Extracts from the film, *Frames of Mind*, are available on

the LMS website at [tinyurl.com/4s4ykdpx](http://tinyurl.com/4s4ykdpx). Professor Süli was the LMS Forder Lecturer in 2015.

The Fellowship of the Royal Society is made up of the most eminent scientists, engineers and technologists from or living and working in the UK and the Commonwealth. The 52 new Fellows, 10 Foreign Members and one Honorary Fellow have been selected for their 'outstanding contributions to scientific understanding'. The full list of new Royal Society Fellows is available at [tinyurl.com/7td87k7e](http://tinyurl.com/7td87k7e). The Fellows Directory, which has extended biographies of all Fellows, is available at [tinyurl.com/scrps7b2](http://tinyurl.com/scrps7b2).

## Anne-Marie Astad

The Society was sad to learn of the death last summer of Anne-Marie Astad, Senior Communications Adviser of the Norwegian Academy of Science and Letters. She was very actively involved in the publicity for the Abel Prize, and engaged with the LMS in a series of recent UK-based Abel events such as the occasions in 2018 and 2019 when the Society hosted the Abel Prize Committee and attended the reception at the ambassador's residence, and also in 2019 when the Abel Prize announcement was live streamed from the Science Gallery at King's College, London.

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## MATHEMATICS POLICY DIGEST

### Ethnicity in STEM Academic Communities

People from Black backgrounds in science, technology, engineering or mathematics (STEM) higher education in the UK have poorer degree outcomes and lower rates of academic career progression than other ethnic groups, research shows.

The Royal Society has published two reports using Higher Education Statistics Authority (HESA) data that lay out the unacceptable inequalities in UK STEM higher education over the past 10 years, and in the pool of UK-based researchers eligible for the Society's own early career fellowship grants.

The proportion of Black students entering undergraduate and postgraduate education has increased over the past decade, as it has for other minority ethnic groups, but they are leaving STEM in greater numbers at all stages of the career pipeline. The full reports are available at [tinyurl.com/8cuz5paf](http://tinyurl.com/8cuz5paf).

Digest prepared by Dr John Johnston  
Society Communications Officer

*Note: items included in the Mathematics Policy Digest are not necessarily endorsed by the Editorial Board or the LMS.*



## EUROPEAN MATHEMATICAL SOCIETY NEWS

## New EMS website

We remind readers that the European Mathematical Society now has a new internet platform, as well as a new logo and complete rebranding. Visit [euromathsoc.org](http://euromathsoc.org) for information and news updates as well as details of a range of funding opportunities which include EMS Weekends and Summer Schools and (with the support of the Simons Foundation) a programme of research visits to foster research opportunities for young and established researchers from Africa. It is also really worth investigating the

action-packed *Pop Math* online calendar [popmath.eu](http://popmath.eu) giving links to mathematics outreach activities in many countries including the UK.

EMS News prepared by David Chillingworth  
LMS/EMS Correspondent

*Note: items included in the European Mathematical Society News represent news from the EMS are not necessarily endorsed by the Editorial Board or the LMS.*



## ADVERTISE IN THE LMS NEWSLETTER

The *LMS Newsletter* appears six times a year (September, November, January, March, May and July).

The *Newsletter* is distributed to just under 3,000 individual members, as well as reciprocal societies and other academic bodies such as the British Library, and is published on the LMS website at [lms.ac.uk/publications/lms-newsletter](http://lms.ac.uk/publications/lms-newsletter).

Information on advertising rates, formats and deadlines are at: [lms.ac.uk/publications/advertise-in-the-lms-newsletter](http://lms.ac.uk/publications/advertise-in-the-lms-newsletter).

To advertise contact Susan Oakes ([susan.oakes@lms.ac.uk](mailto:susan.oakes@lms.ac.uk)).

## UPDATE FROM THE EPSRC SAT

Meetings of the EPSRC Mathematical Sciences Strategic Advisory Team (SAT) took place online on 9 March and 27 April 2021.

The SAT welcomed new academic members Helen Byrne, Mark Kambites and James Robinson, alongside Chris Daniels of FlareBright Ltd.

Recent additions to the Mathematical Sciences Team at EPSRC include Luis Lopez-Bracey who is leading on the monitoring and evaluation of the Additional Funding Programme, Carys Howell who takes over the mathematical analysis and nonlinear systems research areas, Victoria Lund who is responsible for pure mathematics and the Lead Agency Opportunity between EPSRC and the NSF, and Rachael Pittaway (mathematical physics, and logic and combinatorics). Rachael is also the lead contact for the Small Grants Scheme. Ruqaiyah Patel is moving on to take up the post of Head of Decarbonising Industry and the Environment within the EPSRC Energy team and we are grateful for all that she has done as part of the Mathematical Sciences team.

As usual, full details of the EPSRC team can be found on the UKRI website and the team welcome enquiries about project remits, grant schemes, and generally to hear feedback from the community.

The SAT has moved to a set of shorter, more frequent meetings this year. Discussions have centred on the following issues.

On Equality, Diversity & Inclusion the Mathematical Sciences team are focussing particularly on gender, race, and neurodiversity. The SAT noted that the new Open Fellowships require a 'narrative CV' format inspired by the Royal Society's Resume for Researchers proposal. While this allows considerable flexibility in describing broader contributions to the discipline and non-standard career pathways, it remains to be seen whether it has the intended effect in encouraging greater diversity in Fellowship applicants. The SAT noted the publication of detailed ethnicity data analysis and the recent EPSRC-wide Race Equality Surveys. The neurodiversity strand of work is currently less well developed but will emerge during 2021.

The New Horizons awards have now been formally published, listed under the two headings of mathematical sciences and physical sciences.

The SAT discussed what Discipline Hopping might look like within the mathematical sciences, the purpose of Discipline Hopping and the potential effectiveness of different mechanisms to encourage it. The SAT advised that the Small Grants Scheme might be a suitable vehicle to encourage these activities for mathematical scientists.

At the 9 March meeting Graham Niblo gave the SAT an introduction to the work of the Science, Engineering and Technology Board (SETB) which exists as a cross-EPSRC advisory body that curates 'big ideas' for potential development into large-scale funding programmes, for example the Physics of Life programme, and which has oversight of EPSRC-funded Institutes including the Alan Turing Institute.

A key activity for the EPSRC team in 2021 is to develop methods to record outcomes, and a monitoring and evaluation framework for the Additional Funding Programme now that it is underway. This work builds on the detailed business case for the Additional Funding Programme submitted last year, and requires, for example, careful definitions of the baseline of activity for future outcomes to be measured against.

The Small Grants Scheme has progressed well over the first half of 2021, with two Panels having been run by the time the SAT met in April in response to sufficient proposals received by EPSRC.

Two SAT members attended a cross EPSRC AI and Robotics meeting which brought together representatives from different EPSRC Strategic Advisory Bodies. The meeting explored the current state of EPSRC's plans in the area, and sought input and advice on cross-cutting areas of high strategic importance in order to support ongoing work in building the case for public investment in AI research and innovation.

The EPSRC team is looking forward to restarting visits to universities and meeting members of the community in person, blending this with online delivery, for example through webinars.

Katie Blaney  
Head of Mathematical Sciences, UKRI EPSRC

Jonathan Dawes  
Chair, EPSRC Mathematical Sciences SAT

## OPPORTUNITIES

## LMS Research Grant Schemes

The next closing date for LMS Research Grant Schemes 1-6 and AMMSI is 15 September 2021. Applications will be considered by the Research Grants Committee at its October 2021 meeting. Applicants should be mathematicians based in the UK, the Isle of Man or the Channel Islands. For grants to support conferences/workshops, the event must be held in the UK, the Isle of Man or the Channel Islands.

### Conferences (Scheme 1)

Grants of up to £7,000 are available to provide partial support for conferences. This includes a maximum of £4,000 for principal speakers, £2,000 to support the attendance of research students and £1,000 to support the attendance of participants from Scheme 5 eligible countries.

### Visits to the UK (Scheme 2)

Grants of up to £1,500 are available to provide partial support for a visitor who will give lectures in at least three separate institutions. Awards are made to the host towards the travel, accommodation and subsistence costs of the visitor. Potential applicants should note that it is expected the host institutions will contribute to the costs of the visitor. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents.

### Joint Research Groups in the UK (Scheme 3)

Grants of up to £4,000 are available to support joint research meetings held by mathematicians who have a common research interest and who wish to engage in collaborative activities, working in at least three different locations (of which at least two must be in the UK). Potential applicants should note that the grant award covers two years, and it is expected that a maximum of four meetings (or an equivalent level of activity) will be held per academic year.

### Research in Pairs (Scheme 4)

For those mathematicians inviting a collaborator, grants of up to £1,200 are available to support a visit for collaborative research either by the

grant holder to another institution abroad, or by a named mathematician from abroad to the home base of the grant holder. For those mathematicians collaborating with another UK-based mathematician, grants of up to £600 are available to support a visit for collaborative research either by the grant holder to another institution or by a named mathematician to the home base of the grant holder. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents.

### Collaborations with Developing Countries (Scheme 5)

For those mathematicians inviting a collaborator to the UK, grants of up to £3,000 are available to support a visit for collaborative research, by a named mathematician from a country in which mathematics could be considered to be in a disadvantaged position, to the home base of the grant holder. For those mathematicians going to their collaborator's institution, grants of up to £2,000 are available to support a visit for collaborative research by the grant holder to a country in which mathematics could be considered to be in a disadvantaged position. Applicants will be expected to explain in their application why the proposed country fits the circumstances considered eligible for Scheme 5 funding. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents. Contact the Grants team if you are unsure whether the proposed country is eligible, or check the IMU's Commission for Developing Countries definition of developing countries ([tinyurl.com/y9dw364o](http://tinyurl.com/y9dw364o)).

### Research Workshop Grants (Scheme 6)

Grants of up to £10,000 are available to provide support for Research Workshops. Research Workshops should be an opportunity for a small group of active researchers to work together for a concentrated period on a specialised topic. Applications for Research Workshop Grants can be made at any time but should normally be submitted at least six months before the proposed workshop.

### Computer Science Small Grants (Scheme 7)

The next deadline for applications under this scheme is 15 October 2021. The scheme provides up

to £1,200 to support a visit for collaborative research at the interface of Mathematics and Computer Science either by the grant holder to another institution within the UK or abroad, or by a named mathematician from the UK or abroad to the home base of the grant holder.

### **African Mathematics Millennium Science Initiative (AMMSI)**

Grants of up to £2,000 are available to support the attendance of postgraduate students at conferences in Africa organised or supported by AMMSI. Application forms for LMS-AMMSI grants are available at [ammsi.africa](http://ammsi.africa).

The next closing date for applications under Research Grant Schemes 8-9 and ECR Travel Grants is 15 October 2021. Applications are invited for the following grants to be considered by the Early Career Research Committee at its November 2021 meeting.

### **Postgraduate Research Conferences (Scheme 8)**

Grants of up to £4,000 are available to provide partial support for conferences, which are organised by and are for postgraduate research students. The grant award will be used to cover the costs of participants. In addition, the Society allows a further amount (of up to £200) to cover caring costs for those who have dependents.

### **Celebrating New Appointments (Scheme 9)**

Grants of up to £600 are available to provide partial support for meetings to celebrate the new appointment of a lecturer at a university. Potential applicants should note that it is expected that the grant holder will be one of the speakers at the conference. In addition, the Society allows a further amount (of up to £200) to cover caring costs for those who have dependents.

### **ECR Travel Grants**

Grants of up to £500 are available to provide partial travel and/or accommodation support for UK-based Early Career Researchers to attend conferences or undertake research visits either in the UK or overseas.

For full details of these grant schemes, and to find information on how to submit application forms, visit the LMS website: <https://www.lms.ac.uk/grants/research-grants>.

Queries regarding applications can be addressed to the Grants Administrator Lucy Covington (020 7927 0807, [grants@lms.ac.uk](mailto:grants@lms.ac.uk)), who will be pleased to discuss proposals informally with potential applicants and give advice on the submission of an application.

## **The ‘Research Reboot’ Scheme**

In response to a proposal from its Covid Working Group, the Society’s Council has approved a new ‘Research Reboot’ grant scheme. This scheme aims to help mathematicians restart their research activities following the intense disruption and upheaval of the pandemic.

Researchers may have found themselves with very little time for research due to illness, caring responsibilities, increased teaching or administrative loads, or other factors. This scheme offers funding for accommodation and caring costs for applicants so that they can leave their usual environment to focus entirely on research for a period from two days to a week. The initial period of the scheme will run from June to September 2021 and could, for example, cover the costs of a hotel or holiday apartment.

Applicants should be mathematicians based in the UK, the Isle of Man or the Channel Islands and may be at any career stage. We will prioritise applications from those who, as a result of the pandemic, faced increased caring responsibilities such as homeschooling. However, applications from those with additional professional demands such as teaching and administration are also welcome.

There will be an optional online support network for successful applicants. Applicants who are not able to commit to a period of time away from home can still apply to join the support network.

### **Research Reboot Scheme 2021**

The deadline for applications under the 2021 Research Reboot Scheme is 1 July 2021. The value of the award will be a maximum of £100 per day for accommodation, subsistence, travel and other necessary expenses to enable the research project. An additional £100 per day may be applied for to cover caring costs for those who have dependents. For more information and to download an application form, see [tinyurl.com/4mmt8h8r](https://tinyurl.com/4mmt8h8r). Prospective applicants are advised to consult the guidance available at [tinyurl.com/63tmtc](https://tinyurl.com/63tmtc). If you have any queries, please contact [grants@lms.ac.uk](mailto:grants@lms.ac.uk).

## LMS Undergraduate Research Bursaries 2020–21

The purpose of the LMS Undergraduate Research Bursaries is to give experience of research to undergraduates in order for them to explore the potential of becoming a researcher and encourage them to consider a career in scientific research. The awards provide support for the student at a rate of £215 per week (or £225 per week in London), for a period of between six and eight weeks.

In 2020–21 the Early Career Research Committee considered the 83 applications received and, thanks to the generosity of the Heilbronn Institute for Mathematical Research, awarded Undergraduate Research Bursaries to 54 students based at 30 universities across the UK. A full list of the current awardees is available on the Society's website at [lms.ac.uk/grants/URBList](https://lms.ac.uk/grants/URBList).

Since the Undergraduate Research Bursaries scheme was launched in 2013 the Society has supported 324 undergraduate students from universities in England, Scotland, Wales and Northern Ireland.

Glowing testimonials from previous grant holders about the impact of this support are also available on the website at [tinyurl.com/24tp5mp8](https://tinyurl.com/24tp5mp8).

For anyone looking to apply for the next round, which will be supported by HIMR, applications will open later in 2021 with a deadline of 1 February 2022. Enquiries should be directed to [urb@lms.ac.uk](mailto:urb@lms.ac.uk).

## LMS Early Career Fellowship Awards 2020–21

In 2020–21, the Early Career Research Committee considered 30 applications and, with the valuable support of the Heilbronn Institute for Mathematical Research (HIMR), awarded 15 fellowships. The fellowships will support these early career mathematicians, who are in the transition between PhD and a postdoctoral position, to undertake research at universities both in the UK and overseas, including institutions in France, Germany, Hungary, Poland and Sweden. These fellowships last between three and six months with the grant providing a monthly stipend and a travel allowance. A full list of the current fellows is available on the Society's website at [tinyurl.com/52zjp4h8](https://tinyurl.com/52zjp4h8).

A key aim of the fellowships is to ensure that early career researchers receive support at this critical juncture in their career during the transition between

PhD and postdoctoral roles. Since the Early Career Fellowship Scheme was re-launched in 2019 after the successful 150th Postdoctoral Mobility Grants programme closed in 2017, the Society has supported 60 early career researchers, including 22 during the additional application round in 2020 as part of the Society's covid-19 response, which was generously supported by HIMR. These fellows have all since gone on to secure a postdoctoral position.

Testimonials from previous fellows can be found at [tinyurl.com/yufhk42v](https://tinyurl.com/yufhk42v).

For anyone looking to apply for the next round, which will again be supported by HIMR with funding from UKRI EPSRC, applications will open later in 2021 with a deadline of 14 January 2022 to support Fellowships starting from April 2022. Enquiries should be directed to [fellowships@lms.ac.uk](mailto:fellowships@lms.ac.uk).

## LMS Research Schools and LMS Research Schools on Knowledge Exchange 2022

### Second Round Call for Proposals

Grants of up to £15,000 are available for LMS Research Schools and LMS Research Schools on Knowledge Exchange which provide training for research students in all contemporary areas of mathematics. Normally up to three Research Schools and up to one Research School on Knowledge Exchange are supported annually, and are often partially funded by the Heilbronn Institute for Mathematical Research ([heilbronn.ac.uk](https://heilbronn.ac.uk)). The Schools support participation of research students from both the UK and abroad. Lecturers are expected to be international leaders in their field. Information about the submission of proposals can be found at [tinyurl.com/ychr4lwm](https://tinyurl.com/ychr4lwm) along with a list of previously supported Research Schools.

Applicants are strongly encouraged to discuss their ideas for Research Schools with the Chair of the Early Career Research Committee, Professor Chris Parker ([research.schools@lms.ac.uk](mailto:research.schools@lms.ac.uk)) before submitting proposals. A requirement for proposals is that there is a good gender balance amongst speakers. Proposals should be submitted to Lucy Covington ([research.schools@lms.ac.uk](mailto:research.schools@lms.ac.uk)) by 3 August 2021.

## Ferran Sunyer i Balaguer Prize 2022

Ferran Sunyer i Balaguer (1912–1967) was a self-taught Catalan mathematician who, in spite of a serious



physical disability, was very active in research in classical mathematical analysis, an area in which he acquired international recognition. Each year, the Ferran Sunyer i Balaguer Foundation awards an international mathematical research prize in his honour, open to all mathematicians. It was awarded the first time in April 1993.

The 2022 prize will be awarded for a mathematical monograph of an expository nature presenting the latest developments in an active area of research in mathematics, in which the applicant has made important contributions. The monograph must be original, unpublished and not subject to any previous publication commitment. The prize consists of €15,000 and the winning monograph will be published in Birkhäuser series Progress in Mathematics. The deadline for submission is 3 December 2021. For further information visit the website [ffsb.iec.cat](https://ffsb.iec.cat).

## ICM

The International Congress of Mathematicians (ICM) is the most significant meeting in pure and applied mathematics worldwide, and one of the oldest scientific congresses. The first ICM took place in Zurich, Switzerland, in 1897. ICMs are organised every four years by the International Mathematical Union (IMU) in partnership with the Local Organising Committee (LOC) from the host country.



O. A. Ladyzhenskaya

The goal of the ICM is to present a panorama of recent mathematical developments, including all fields of study and different geographic regions, and thus point to the future directions of mathematics. The ICM plenary and invited speakers are mathematicians of the highest quality, able to present current trends of research to a broad mathematical audience. During the congress IMU will announce an array of awards and prizes: Fields Medal for outstanding mathematical

achievement, Abacus Medal (replaces Rolf Nevanlinna Prize) for achievements in information science, Carl Friedrich Gauss Prize for contributions with significant applications outside of mathematics, Chern Medal for outstanding achievements in the field of mathematics, Leelavati Prize for outstanding public outreach. ICM's Emmy Noether Lecture honours women who have made fundamental and sustained contributions to the mathematical sciences. The year 2022 will mark the 100th birthday of Olga Alexandrovna Ladyzhenskaya who was one of the leading figures in the development of the modern theory of Partial Differential Equations. To honour O.A. Ladyzhenskaya, the National Committee of Mathematicians of Russia and St. Petersburg State University established the Ladyzhenskaya Medal in Mathematical Physics. The inaugural prize will be awarded during the ICM 2022 ([icm2022.org/ladyzhenskaya-medal](https://icm2022.org/ladyzhenskaya-medal)).

Beyond plenary, prize and sectional talks, there will be 20 special survey lectures by leading experts in mathematics and theoretical computer science. These lectures will be aimed at a diverse mathematical audience. There will be public lectures covering a variety of topics for an even broader audience of people interested in mathematics, computer science and mathematical physics.

The LOC believes that the ICM should be a celebration of mathematics and its uniting power. As we write in the LOC statement “the universality of mathematics should be a reminder that we all, however different, are the same as human beings. We, the Local Organizing Committee, are creating an ICM where everybody will feel welcome and respected, regardless of gender, ethnicity, nationality, sexual orientation, social or economic status, religion, or any other personal beliefs, customs, or traits.” The full statement is available at [icm2022.org/loc-statement](https://icm2022.org/loc-statement).



Sofya Kovalevskaya

The broad perspective on mathematics to be gained at the ICM and the connections to be formed are especially valuable for the early career researchers (ECR) who could benefit the most from attending ICMs. Unfortunately, the participation of the younger generation in ICMs has been dwindling in recent years. Part of the reason is lack of funding and sometimes lack of information about the ICM and opportunities

to attend. To boost the attendance of early career researchers the LOC have created a new funding initiative: Kovalevskaya grants. These grants are named after Sofya Kovalevskaya who made important contributions to analysis, PDE and mechanics. Kovalevskaya was a prominent figure in the 19th century feminism movement in Russia and played an important role in the fight for women's access to academia. She was the first woman to obtain a doctorate in mathematics (Göttingen 1874) and become a full professor (Stockholm 1889).

The Kovalevskaya grants are a pilot program supported by the IMU with the hope that similar grants will be available for future ICMs. We have announced 1000 grants available for ECRs from developed countries. The Kovalevskaya grant initiative is independent from a separate funding, the Chebyshev grant, which is intended for mathematicians from the developing countries. We reserved about 70 Kovalevskaya grants to ECRs from the UK. These grants will cover registration fee, provide accommodation for the duration of the congress, lunches, invitations to the gala dinner and welcome reception, local transportation, and a cultural program. Additionally, a visa-free entry to Russia will be arranged for all grant recipients. Travel to St. Petersburg is to be covered by UK sources. One possibility is to apply for an LMS travel grant. LMS, with support from EPSRC, has opened a call for ICM travel grants on the LMS website at <https://tinyurl.com/jj577js5>. The Kovalevskaya Grants Committee will award further financial support to successful LMS-EPSRC Travel Grant applicants. Alternatively, applicants with confirmed availability of travel funds will be eligible to apply directly for a Kovalevskaya grant. The travel funds could come from a previous fellowship or a similar grant or from the applicant's home institution. Kovalevskaya grants will be available to UK-based PhD students and researchers who have obtained a PhD or an equivalent degree no earlier than in 2016 (with a possible extension for career interruptions). Full eligibility criteria and the call for applications will appear at [tinyurl.com/4xspzbxh](https://tinyurl.com/4xspzbxh).

We particularly welcome applications from women and other under-represented groups. We will support families and will organise appropriate on-site facilities, including childcare, to simplify the logistics of attending the congress for everyone.

Dmitry Belyaev  
Local Organising Committee



# Call for Proposals

## RIMS Joint Research Activities 2022-2023

**Application deadline : August 31, 2021, 23:59 (JST)**

### Types of Joint Research Activities

- \*RIMS **Satellite** seminars 2022
- \*RIMS **Review** seminars 2022
- \*RIMS **Workshops Type C** 2022
- \*RIMS **Research Project** 2023

**More Information :** RIMS Int.JU/RC Website  
<http://www.kurims.kyoto-u.ac.jp/kyoten/en/>

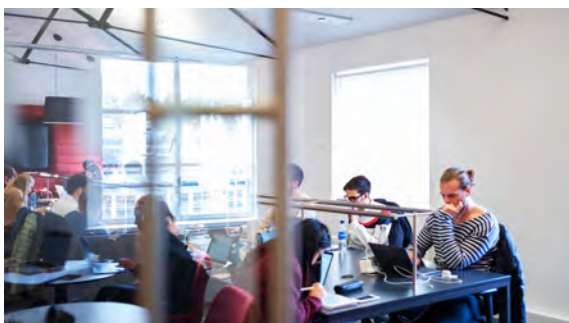


京都大学  
KYOTO UNIVERSITY



Research Institute for  
Mathematical Sciences

## Maximising your LMS Membership: Using the LMS Library at UCL



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The London Mathematical Society throughout its time has acquired a rich collection of mathematical books and periodicals. The Society's Library is housed at the University College London (UCL) Science Library and it is integrated with the Mathematics section. The collection includes:

- periodicals published by other mathematical societies which are received in exchange for the Society's publications
- copies of books and journals published by the Society
- items acquired by the Society as review copies or gifts.

All LMS members may register as a UCL Library user to access all the resources and materials in the UCL family of libraries and storerooms.

### UCL Library Privileges

- Use of all the material available in the reading rooms and stores of the UCL family of libraries.
- Borrowing up to ten items at any one time.
- Placing up to three concurrent reservations on material already on loan.
- Borrowing books by post without service charge (costs for returning the books must be covered by the user).

- Access to MathSciNet and specific electronic journals from designated terminals in the Science Library.
- Use of the *Explore* access points to search for and view electronic publications and save single copies of articles (no more than one article per journal issue) for your own personal use. You can save articles to standard USB sticks, but note that USB sticks containing encrypted software do not work on the *Explore* access points.
- Use of photocopying facilities at UCL libraries (charged at the same rate as UCL staff).
- Rapid photocopying service by post.

### Registering and Renewing

All LMS members may register as UCL Library users or renew their library card free of charge. Refer to our website [www.lms.ac.uk/library/how-register](http://www.lms.ac.uk/library/how-register) for more information about registrations, renewals or replacements of the lost/stolen/damaged library card.

Library cards are valid for 12 months from date of issue and should be renewed each year.

### Visiting the Library

The UCL Science Library is located in the DMS Watson Building on Malet Place, WC1E 6BT. Check opening hours and services available before visiting the UCL Library. You will be able to find all recent updates on the UCL Library website at [tinyurl.com/5yvz74wb](http://tinyurl.com/5yvz74wb).

For more information on the Society's Library see [lms.ac.uk/library/lms-library](http://lms.ac.uk/library/lms-library). Remember that use of UCL Library is just one of many of your Membership Benefits and the complete list can be found at [tinyurl.com/3cdyd7kv](http://tinyurl.com/3cdyd7kv). Feel free to contact a member of our team on [membership@lms.ac.uk](mailto:membership@lms.ac.uk) if you have any questions.

Valeriya Kolesnykova  
LMS Membership Team

## LMS Council Diary — A Personal View

Council met via video conference on Friday 23 April 2021. The meeting began with the President's business, which included an update on the Levelling Up project, funded by a donation of Dr Tony Hill, which has started tutoring pupils and is attracting considerable interest, the recent Society meeting at the BMC-BAMC, which was felt to have been a great success, and the continuing concerning situation at the University of Leicester. Vice-President Gordon then gave updates on the Covid Working Group, including the Research Reboot Scheme that has now been advertised and the fact that the award of Early Career Fellowships, with additional Fellowships as part of the Society's response to covid-19, is going well, and on sustainability issues, in particular the fact that carbon use in Society activities is currently being considered by several Society committees. There was also a discussion about the Egyptian mathematician, Laila Soueif, following which it was agreed that an article about her situation should be included in the *LMS Newsletter*.

The President then reported on two very interesting items that had recently been offered to the Society. The first of these was a low serial number banknote of the new Alan Turing £50 banknote, offered by the Bank of England in thanks for the Society's help in the development of the banknote through allowing use of a mathematical formula and tables from a 1936 paper of Turing published in the *Proceedings of the LMS*. Reflecting the year of the paper's publication, the serial

number of the note that the Society will receive will be AA01 001936. The second item, made in a final bequest to the Society by A. E. L. Davis, a former member of the Society who had recently died, was an original 1650 copy of Maria Kunitz's *Urania Propitia*, of which only nine copies are known to exist. Both these items have been accepted, and it was agreed that the Society should explore holding an event centered around issues of equality, diversity and inclusion to publicise these items alongside a Scottish £10 note depicting the Scottish astronomer and mathematician Mary Somerville that is also owned by the Society.

Other business included an overview of the final report of the Council Communications Review Working Group by Vice-President Hobbs, and an agreement to adopt a draft policy for rapid communications when a Society public position on an issue is needed urgently. We also discussed changes to the Newsletter Board, including a new Editor-in-Chief, committee membership, a proposal to establish a Member-at-Large (Women and Diversity), which will be put to members of the Society at the General Meeting on 2nd July, and the fact that applications for Society membership were slightly down, which will be discussed at the upcoming Society Representatives Day.

The meeting concluded with agreement that the Society should contribute funding to a film project 'Words of Women in Mathematics in the Time of Pandemic', and the President thanking everyone for their contributions.

Elaine Crooks  
LMS Member-at-Large

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### LEVELLING UP

#### Seeking Further Partner Universities

The agenda of the Levelling Up Scheme is to widen participation of those who are under-represented in mathematics. The scheme aims to raise aspirations, with our primary goal being to improve the A-level grades of students from disadvantaged backgrounds so they can access STEM courses in the universities they want to attend. It requires university partners to invest and see themselves as part of the broader ecosystem. Tutoring delivered by one university may inspire a young person to go to that university or another one to study a degree with a high

mathematics content — either way this brings benefits to the mathematics community as a whole.

We are currently in the pilot phase of the Levelling Up programme, and have launched the scheme with the Universities of Durham and Leicester. As we move forward, the aim is to involve more university partners. Dr Tony Hill and the Head of LMS Society Business, Lindsay Walsh, recently presented on Levelling Up to HoDoMS Annual Conference and this has resulted in some strong interest from potential partner universities.



Among the various communities Levelling Up currently aims to reach we include areas with low participation in HE. Often these are in our (former) industrial cities but they also include rural and coastal communities — the lowest participation areas in the UK include areas of South and West Wales, East Yorkshire, Cumbria and the south coast. The scheme needs university partners who can reach those areas. Those with a strong history of civic engagement and local recruitment are well-placed to do this — they often already have deep connections

with schools and communities in the areas we want to reach. The Levelling Up programme particularly encourages universities with connections and profile in low participation areas to make contact and discuss becoming Levelling Up partners.

More information about the Levelling Up Scheme is available at [levellingupscheme.co.uk](https://levellingupscheme.co.uk).

Cathy Hobbs  
LMS Vice-President

## REPORTS OF THE LMS

### Report: BMC–BAMC 2021

The 2021 BMC–BAMC was held online from Glasgow on 6–9 April 2021, having been postponed from the same dates in 2020 owing to the pandemic. It was a wonderful celebration of mathematics in Britain, pure and applied together, as has become the tradition of the conferences every five years. Nearly 1000 participants enjoyed all the advantages that online events can offer, and, it seemed, few disadvantages. The Sococo platform provided us with an excellent meeting venue in which it was easy to get to all the talks (Zoom meetings and webinars) and meet people informally, often (in my case) by chance, in the coffee bars or at the beach. The programme was excellent, the speakers were exceptional, and since many of the lectures have been recorded, we can review them. The organisers, both the academic team from Glasgow, headed by Michael Wemyss, and the support team from ICMS, have to be congratulated on having done such an outstanding job, and produced a wonderful conference, despite all the difficulties caused by the situation we have all found ourselves in this past year.

The LMS lecture *Khovanov homology and four manifolds* was given by Ciprian Manolescu of Stanford University. It was a beautiful, accessible lecture, which focused on the search for exotic 4-manifolds, and in particular for a counter-example to the Smooth Poincaré Conjecture (SPC4); a counter-example would be a smooth 4-manifold that was homeomorphic but not diffeomorphic to the 4-sphere.

Professor Manolescu started with an introduction to 4-dimensional topology and an overview of problems in the field. He explained why dimension four is special, too big to be dealt with using geometry, too small

to allow surgery. It's the first dimension exhibiting a distinction between topological and smooth manifolds. Freedman classified the topological 4-manifolds in 1982, but smooth 4-manifolds remain a mystery, and the Smooth Poincaré conjecture remains open in dimension four.

Starting with Donaldson's 1982 results, significant progress was made in smooth 4-dimensional topology using gauge theory (relating to certain partial differential equations). Recently further successes have followed using algebraic and combinatorial methods.

Professor Manolescu introduced us to Khovanov's 1999 definition of a homology theory for links. This theory is purely combinatorial and algebraic, and the Euler characteristic of Khovanov homology is the well-known Jones polynomial.

Using a deformation of Khovanov homology, in 2004 Rasmussen extracted a numerical knot invariant  $s(K)$ , which gives a lower bound on the slice genus of the knot  $K$ . The slice genus is difficult to compute (no general algorithm is known); when it is zero, the knot is called *slice*. Rasmussen was able to use his invariant to provide a combinatorial proof of Milnor's conjecture (specifying the slice genus of the torus knot), which had previously been proved using gauge theory.

Subsequently, Rasmussen and others used the invariant  $s(K)$  to find new proofs of various results previously proved using gauge theory, including the construction of exotic smooth structures in  $\mathbb{R}^4$ . Completely new results were also obtained, suggesting a new approach to the search for 4-manifold invariants and the building of exotic structures. Professor Manolescu explained Lisa Piccirillo's beautiful 2018 proof that Conway's 12-crossing knot  $C$  is non-slice; she constructed its



trace (an associated 4-manifold) and a partner knot  $C'$  with that same trace, and hence the same sliceness, for which  $s(C')$  was non-zero.

An approach to disprove SPC4 using Rasmussen's invariant was proposed by Freedman, Golpf, Morrison and Walker in 2008; a nontrivial homotopy 4-sphere might be constructed using a non-slice knot. Professor Manolescu described three different attempts to follow the FGMW strategy, and results following. The lecture ended with beautifully illustrated details of an investigation of his with Piccirillo. They searched for pairs of knots with the same 0-surgeries (associated 3-manifolds), but distinct traces; the goal was to prove one of the two knots slice, the other not. They searched using certain 3-component 'RBG' links, ended up with just five candidate pairs of knots, for each of which one of the two knots had Rasmussen's invariant non-zero. If any of those remaining five knots were slice, then SPC4 would be disproved.

Sarah Rees  
Newcastle University

## Report: LMS Women in Mathematics Day 2021

The LMS Women in Mathematics Day took place on Wednesday 24 March 2021 online via Zoom, hosted by the University of Plymouth. This one-day virtual event aimed to promote interest and careers in mathematics for women. The event consisted of six invited talks given by female speakers in mathematics and statistics, a panel discussion on career development, and a poster competition open to female mathematicians at undergraduate and graduate levels as well as early career researchers. The meeting saw 179 online attendees and 20 poster submissions from across the globe. The Vice-Chancellor of the University of Plymouth, Professor Judith Petts CBE, gave a welcoming address to open the event.

The morning session, chaired by Dr Nathan Broomhead, began with a talk by Professor Ulrike Tillmann FRS, from the University of Oxford, who spoke about *Commutative K-theory*. She started by discussing some results about the topology of certain spaces of group homomorphisms and, in particular, the spaces of commuting  $n$ -tuples. She then introduced commuting  $G$ -bundles, defined by imposing commutativity relations on the transition functions for  $G$ -bundles, and described how interesting topology can appear when

studying their classifying spaces. Finally, she explained how these ideas lead to the notion of commutative  $K$ -theory (and some generalisations of it) and surveyed some results about these generalised cohomology theories.

The second speaker was Dr Vicky Hoskins from Radboud University with a talk titled *The Ubiquity of Quiver Moduli*. Having motivated the idea of moduli spaces using examples from linear algebra and geometry, she introduced quivers and the moduli spaces of quiver representations. She then described different constructions of such spaces, coming from invariant theory and symplectic geometry and explained, with explicit examples, how they appear in many areas of mathematics, including representation theory, algebraic geometry, mathematical physics, symplectic and hyperkähler geometry.

The final speaker of the morning session was Professor Karin Baur from the University of Leeds, who discussed *Friezes in Algebra and Combinatorics*. She defined finite frieze patterns of integers which were first studied by Coxeter and Conway-Coxeter in the 70s and explained a correspondence between such frieze patterns and triangulations of polygons. She mentioned how the study of cluster algebras and cluster categories has renewed interest in friezes and led to some generalisations. She finished by talking about more recent work on infinite friezes, explaining that periodic infinite friezes arise from triangulations of an annulus, and presenting some results about the growth of such friezes.

The online poster viewing took place during the lunch time, with an individual breakout room for each poster presenter. Attendees chose which breakout room to join in order to view the posters and discuss with the presenters.

The afternoon session was chaired by Dr Daniel Robertz. Dr Shirley Coleman from the Newcastle University discussed statistics in industrial research. Shirley discussed the fundamental role that Statistics play in the smooth functioning of industrial processes, in their continuous improvement and increasingly in innovative approaches to challenging new procedures. She showed a number of examples of industrial applications, including statistics in pipeline research, reliability modelling in manufacture of cables and natural language processing of help-desk emails in facilities management. She also gave her suggestions and advice, that being able to communicate clearly, working effectively with experts from different areas, understanding the research questions are as important

as the statistical technical skills, for applying statistics in industrial research.

The fifth speaker was Ms Sarah Littler from Select Statistics. Sarah talked about her experience of working as a statistical consultant in industry, covering her background and how she came to work in this area. Sarah discussed the use of statistical modelling, machine learning and survey analysis, using a number of examples in medicine, telecommunication businesses, marketing and schools. She also discussed the use of statistical software, especially the use of R for data visualisation and analysis, as well as turning analyses conducted in R to dynamic and interactive web-applications using the package Shiny. Sarah also shared her experiences of consultancy, in particular, what is needed to be a successful consultant.

The sixth speaker was Professor Christiane Rousseau, from the Université de Montréal. Christiane discussed the equivalence problem in analytic dynamical systems, showing how the local analytic geometry when, embedded in the complex domain, forces the divergence of the normal form. She then showed how this geometry can be used to identify the analytic equivalence classes of formally equivalent systems. When this is done for families of systems unfolding a singularity, a rich behaviour arises with many intricate phenomena, including 'parametric resurgence', where the non-integrability of the singularity is reflected in the resonances of the unfolded systems.

In the final session of the day, all invited speakers were panellists in the career development panel discussion, chaired by Dr Yinghui Wei. It was an engaging discussion with many questions from attendees, and thoughtful comments from the panel. The discussion points included how to attract more young women to study mathematics, their professional progression, the characteristics and opportunities of academic life, as well as work and life balance.

The event closed with concluding remarks and announcement of best poster awards, chaired by Dr Colin Christopher. The three best poster winners were:

**Best undergraduate poster:** Gender Imbalance in Mathematics Citations: Do Female Mathematicians Get Cited Less than Men? Katharina Limbeck, University of Glasgow, UK.

**Best postgraduate poster:** *Mathematically Modelling Three-Dimensional Plan Growth for Use in Additive Manufacture*, Amy Tansell, University of Birmingham, UK.

**Best poster by an early career researcher:** *Legendrian Non-Simple Two-Bridge Knots*, Viktória Földvári, Alfréd Rényi Institute of Mathematics, Hungary.

Yinghui Wei, Nathan Broomhead, Colin Christopher,  
Daniel Robertz  
LMS Women in Mathematics Day Organisers,  
University of Plymouth

## Records of Proceedings at LMS Meetings

### Ordinary Meeting at the University of Glasgow: 8 April 2021

The online meeting was hosted by the University of Glasgow with support from the International Centre for Mathematical Sciences (ICMS), as part of the Joint British Mathematical Colloquium–British Applied Mathematical Colloquium (BMC-BAMC). Over 120 members and guests were present for all or part of the meeting.

The meeting began at 4.00 pm with The President, Professor Jon Keating FRS, in the Chair.

There were 186 members elected to Membership at this Society Meeting.

Eleven people were elected to Ordinary membership: Dr Konstantinos Gourgoulis, Dr Cristina Manolache, Dr Kenichi Mori, Professor Nigel Mottram, Dr Ian Petrow, Dr Dr. Mohd. Rizwanullah, Professor Nicholas Shepherd-Barron, Mr Michael Tosdevin, Mr Seshu Kumar Velagapudi, Mr Oyesetan Waheed Prince and Dr Shahid Zaffar.

Four people were elected to Reciprocity membership: Mr Juan José Granier Torres, Mr Eduard Stefan Grigoriuc, Dr Lubomir Markov and Dr Manikandan Rangaswamy.

Six people were elected to Associate membership: Mr Ibrahim Alsendid, Mr Bishnu Ghosh, Mr Tom Goodman, Dr Florian Klimm, Mr Nick Poullos and Dr Bolys Sabitbek.

Three people were elected to Associate (Undergraduate) membership: Ms Christina Baker, Mr Ryan Graham and Miss Mahsa Manzari.

162 people were elected to Associate (Teacher Training Scholarship) membership: Miss Sheila Abdulrasul, Mr Junaid Adia, Miss Estelle Agyeman, Miss Mariam Ahmed, Mr Mohamed Ahmed, Miss Anisha Akther, Mr Dylan Allport, Mr Thomas Allum-Watts, Ms Annabel Amin, Ms Diane Armitage, Mr Vincent Atigla, Miss Hameeda Azam, Miss Alix Bailey, Miss Jettie Baker, Miss Rosie Ballard, Mr Joseph Barnes, Mr Max Baron, Mr Lewis Bates, Mr Liam Bath, Mr Jack Bedford, Miss Aysha Begum, Mr Tyrell Bernard, Mr Joseph Blackmur, Mrs Tamla Bowdler, Dr Simon Brain, Miss Naomi Bryant, Miss Beth Buckland, Miss Katie Burke, Mr Conor Burrows, Miss Emily Carmichael, Mr Mark Carrick, Mr Gregory Chapman, Miss Subhana Choudry, Mr Henry Cole, Miss Lauren Cooper, Mr Tom Copeland, Miss Natalie Costello, Mr Nick Cowley, Miss Alyssa Critchley, Miss Eilidh Cruse, Miss Sonia Dari, Mr Daniel de Falbe, Mr Amzad Derdiwala, Mr Jack Devereux, Ms Alexandra Dobozy, Ms Elizabeth Dolphin, Miss Rebecca Duffy, Mr Ben Eastwood, Miss Amy Edmond, Miss Ruth Ejigayehu, Miss Sanaa Elrrakik, Mr William Farrow, Dr Samuel Ford, Mrs Nadeera Gamage, Mr Matthew Gandy, Miss Crystal Gardner, Miss Imogen Garton, Mr Charles Gatehouse, Ms Helen Gerrard, Mr Matthew Glover, Miss Siobhan Gnanakulendran, Miss Imogen Gough, Miss Sian Gould, Miss Tabitha Goulty, Dr Rebecca Haestier, Miss Ella Hanley, Dr Sharon Harris, Miss Sophia Harrison, Mrs Sophie Hearle, Mrs Nadine Heseltine, Miss Stephanie Higgins, Mr Rhidian Hill, Miss Olivia Home, Mr Adeel Idrees, Miss Millie Jeffery, Miss Leonie Jenkins-Nwagbo, Miss Jessica Jones, Miss Catherine Jones, Miss Rasihha Lalithasan, Miss Hannah Lavin, Mrs Helen Li, Miss Katie Liddell, Miss Molly Lillicrap, Miss Ellie Lumb, Mr Jaimy John Hargreaves Lunn, Miss Ysabelle Luty, Mr Theo Lynds, Mr Jonathan Mallatratt, Dr Robert Manning, Miss Africa Paloma Martinez Denning, Miss Vanessa Martins de Lima, Dr Rachel McCarthy, Mr Davy McCracken, Mr Jamie Mcilroy, Mrs Emma Mellor, Miss Francesca Merritt, Miss Zoe Miller, Mr Raoul Milner, Miss Katie Mitchell, Miss Hannah Morris, Mr Jack Newall, Mr Michael Newton, Miss Eleanor Norris, Mr Basil Olewa, Mr Luke Owen, Miss Meg Owen, Mr Joseph Page, Miss Ella Panepinto, Mr Sam Parkin, Miss Olivia Parsons, Mr Simon Parsons, Mrs Naomi Pendleton, Mr Matthew Perkins, Mr Harry Phillips, Miss Rhian Player, Miss Laura Pollard, Miss Julia Prout, Dr Jonathan Puddicombe, Miss Manisha Rattoo, Miss Chloe Raw, Mr Rahib Rehman, Miss Jacqui Rhodes, Mr Yousaf Said, Miss Maryam Sajid, Ms Sinead Sarsfield, Mr Thomas Scrivener, Mr George Seager, Mr Alexander Sergiou, Miss Rebecca Sharpe, Miss Tahira Sherif, Mr Lee Skillern, Mr Aaron Smith, Mr Alex Smith, Miss Chloe Smith, Miss Paige Smith, Mrs Sharan Spall, Mr Mark Tallett, Mr Alexander Taylor, Miss Zoe Taylor, Miss Amie Thackeray, Mr Ben Thomas, Miss Sarah Timson, Miss Laurel Umo, Miss Charlotte Upton, Miss Penni Vidler, Dr Carolina Villalonga-Barber, Miss Sadaf Wadoodi, Miss Elena Wang, Mr Matthew Ware, Mr Aaron Waring, Mr Christopher Warner, Miss Dana Whitby, Miss Heather Whyte, Mr Ayub Williams, Mr Thomas Williams, Ms Alys Wilman, Miss Joanna Wilson, Miss Rosie Wilson, Mr Stephen Wolstenholme, Miss Hayley Worrall, Miss Sobitha (Sobi) Yogeswaran and Ms Isha Yusuf.

Because the meeting was held online, no members signed the Members' Book.

Dr Brendan Owens, University of Glasgow, introduced the LMS Plenary lecture given by Professor Ciprian Manolescu (Stanford University) on *Khovanov homology and four-manifolds*.

Professor Keating thanked the speaker for his excellent lecture and then expressed the thanks of the Society to the organisers, Dr Brendan Owens, the University of Glasgow and the ICMS, for a wonderful meeting and Joint BMC-BAMC.

# Allan J. C. Cunningham (1842–1928): a Life of a Computer

TOMOKO L. KITAGAWA

**Abstract.** Allan Joseph Champneys Cunningham is known for the successful factoring of large numbers. His mathematical research has continued as the Cunningham Project, and various computers have taken over the calculations that he attempted. This article traces his footsteps and recaptures his life before and after becoming a mathematician.

## Studying Mathematics at Addiscombe

Allan Cunningham's father was Alexander Cunningham (1814–93), army officer in the East India Company, who was born in Westminster as the second son of a renowned Scottish poet and author, Allan Cunningham (1784–1842). Alexander's older brother, Joseph Davey Cunningham (1812–51), showed great aptitude for mathematics when he was a child, and was going to be sent to Cambridge to study mathematics. But Joseph wished to be a soldier. The British Empire was expanding at the high point of European imperialism. In 1828, Joseph went to Addiscombe, an elite military college, which provided young officers for the East India Company's private army.<sup>1</sup> The subjects of study included mathematics, land surveying, fortification, engineering, as well as the use of various weapons. Joseph graduated with the first prize in mathematics and the first nomination to the Bengal Engineers in 1831.

Like Joseph, Alexander was educated at Addiscombe. He arrived in India in June 1833 and spent his first three years in Delhi. From 1836 to 1840, he served as an A.D.C. (aide-de-camp) to Lord Auckland, the Governor-General of India and married Alicia Maria Whish (b. 1821) on 30 March 1840. He then quit his A.D.C. position and became Executive Engineer to the King of Oudh.

Alicia and Alexander had their first son, Allan, on October 29th, 1842. The family was often separated due to Alexander's participation in military campaigns. From September 1842 to May 1843, he took part in suppressing the rebellion in the Battle of Punniar (1843). He became Executive

Engineer at Gwalior from 1844 and saw the first Sikh War as a field engineer in 1846. Alexander also participated in the Battle of Chillianwala and Gujrat from 1848 to 1849.

## What is the Cunningham Project?

The Cunningham Project took its name from the book written by Cunningham and Woodall in 1925 [1]. Its goal is to factor numbers of the form  $y^n \pm 1$  for integers  $2 \leq y \leq 12$ . Most earlier works listed all the prime factors of  $y^n \pm 1$  for each  $n$  for bases  $y = 2$  and  $y = 10$ . The Cunningham–Woodall book was the first to list the primitive factors for the other six bases ( $y = 3, 5, 6, 7, 11, 12$ ).

Allan was with his mother while his father was away; Allan's brother, Francis Hope Cunningham, was born in 1845 but died in the same year. Another brother, Alexander F. D. Cunningham (1852–1935), was born much later when Allan was ten years old.

As a teenager, Allan left for England and attended King's College School, London. The sons of military men often took this path of going to a boarding school in England. For Allan, a military career became more realistic when he entered Addiscombe, the same as his father and uncle Joseph, in 1859. He was seventeen.

At Addiscombe, a Cambridge-trained mathematician, Jonathan Cape (1793–1868) had served as Senior Professor of Mathematics since 1822. Cape was "the most remarkable member

<sup>1</sup>The college was formally known as the East India Company Military Seminary (1809–55), and it was later called the Royal India Military College from 1858 to 1861. Because it was located at Addiscombe, Surrey, the school was also known as Addiscombe Seminary, Addiscombe College, or Addiscombe Military Academy.

of the staff of the College during the whole course of its [Addiscombe's] existence. . . . He was Addiscombe," a historian of the College, Henry Vibart, wrote in 1894. Allan met Cape, who had taught mathematics to both his father and uncle.



Figure 1. Nine cadets posed in front of Addiscombe Military Academy (c.1859).

Cape wrote a 2-volume textbook, *A Course of Mathematics Principally Designed for the Use of Students in the East India Company's Seminary at Addiscombe* (1839). On the first page of the first volume of this textbook he wrote that the mathematics taught at the college was "Arithmetic, the Use of Logarithms, Algebra, Geometry, the application of Algebra to Geometry, and Mensuration," and "Plane and Spherical Trigonometry, Conic Sections, a short account of Projection, Mechanics, and Hydrostatics, and the doctrine of the Differential and Integral Calculus, a short Treatise of Nautical Astronomy, and its application to the Trigonometrical Survey".

Cape also taught "mixed mathematics", which was "the application of pure mathematics to certain established physical principles, and comprehends all the different branches of Natural Philosophy such as Mechanics, Hydrostatics, and Astronomy". Allan's academic excellence rewarded him with the Pollock Medals, which were given to the most distinguished cadets annually. Allan's appointment to the East India Company's Bengal Engineers (later the Royal Engineers) was decided by June 1860. He was homeward bound.

#### 242. PROP. XII.—To find the least common multiple of three quantities.

Let  $a$ ,  $b$ , and  $c$  be the three quantities; take  $m$ , the least common multiple of  $a$  and  $b$ ; and  $n$ , the least common multiple of  $m$  and  $c$ ; then  $n$  is the least common multiple sought.

For every common multiple of  $a$  and  $b$  is a multiple of  $m$  ( $m$  itself being included); therefore every common multiple of  $a$ ,  $b$ , and  $c$  is a multiple of  $m$  and  $c$ ; also, since  $m$  is a multiple of  $a$  and  $b$ , every multiple of  $m$  and  $c$  is a multiple of  $a$ ,  $b$ , and  $c$ ; consequently, the least common multiple of  $m$  and  $c$  is the least common multiple of  $a$ ,  $b$ , and  $c$ .

243. SCHOLIUM.—From these principles is derived the rule for finding the least common multiple of several numbers. (See arithmetic, art. 38.)

Let  $axy$ ,  $bxyz$ ,  $cyz$ , be any numbers of which the factors  $a$ ,  $b$ ,  $c$ ,  $x$ ,  $y$ ,  $z$ , are all prime to each other. In dividing by  $x$ , we take out a factor which is common to two terms, and reserve it to be multiplied at the end.

Again, if we divide by  $y$ , we take out a factor which is common to three terms, and multiply by this factor at the end, and so on. By this means we avoid repeating any factor common to different terms more than once.

Note.—In dividing by any factor, it must either be a prime number, or at least prime to all those numbers which are not divided by it. For if we had divided by  $xy$ , for example, and afterwards by  $z$ , we should have obtained a number,  $abcxy^2z$ , which is a common multiple, but not the least common multiple of these three numbers.

$$\begin{array}{l} x \mid axy, bxyz, cyz \\ y \mid ay, byz, cyz \\ z \mid a, bz, cz \\ \hline a, b, c \end{array}$$

Hence the least common multiple is  $abcxyz$ .

$$\begin{array}{l} xy \mid axy, bxyz, cyz \\ z \mid a, bz, cyz \\ \hline a, b, cyz \end{array}$$

Figure 2. A page from Cape's textbook, explaining prime factorization (1857 edition, Volume I, p.178). This would have been one of the first sources from which Cunningham learnt formally about factorization.

## Experiments on the River Ganges

Cunningham's father, Alexander, retired from the Royal Engineers in 1861 as Major-General, and became the first full-time archaeological surveyor to the Government of India from 1861 to 1865. Allan joined the Bhutan Field Force from 1865 to 1866 and became the Assistant Principal of the Thomason College of Civil Engineering located in Roorkee, Uttarakhand, India. This College was founded in 1845 to train engineers for the construction of the Ganges Canal (Figure 3). It preceded the establishment of universities in India under British governance; to train civil servants, universities in Calcutta, Madras, and Bombay were built in 1857.

Allan married Bessie Hannah (d.1876) in 1870, and taught mathematics at Thomason College of Civil Engineering for ten years from 1871. He and Bessie had their first son, Allan H. C. Cunningham (1871–1932) on May 24th, 1871. Both their daughter Mary Emmaline Cunningham (1873–77) and another son Francis Alexander Cunningham (1876–1919) were born in Roorkee. Only a month after the birth of Francis in 1876, however, his wife Bessie passed away. And another tragedy hit the family in a span of one year; Allan lost his daughter at the age of four in 1877. Raising two sons by himself, he continued teaching at the Thomason College of Civil Engineering.





Figure 3. A Photograph titled “Head of Ganges Canal, Hardwar” (1860). The canals were built and used for irrigation. © British Library Board (Photo 15/1(28)).

During this time, Allan conducted hydraulic experiments in the Ganges Canal, funded by the Indian Government under British rule. Interest in the canals was high; the Suez Canal, completed in 1869, facilitated steam transport by reducing the sea passage between Britain and India from three months to three weeks. The canals in general were useful not only for navigation, but also for water-power connected to the wheels of manufacturing establishments.

Large-scale engineering projects to study hydraulics had begun in many parts of the world, and the results were published quickly. It was in 1838 when William George Armstrong (1810–1900) announced the result of his experiment with hydraulics and developed a rotary engine. Armstrong invented the hydraulic accumulator with water towers, which caused a sensation among the engineers in England. Large-scale experiments in Metz, France, were led by the French physicist, P. Boileau in 1844. His paper, published in *Journal de l'École polytechnique*, was influential as he presented novel formulae on hydraulic mechanisms, and later conducted sets of experiments on the flow of water in open channels on the Mississippi River, U.S.A., made at the expense of the French government.

Allan's hydraulic experiment in India was a welcome addition to the international literature on this topic. His project involved a “large expenditure”, which he completed with “enormous industry”. Allan published the results as *Roorkee Hydraulic Experiments* (Thomason College Press, 1880–81). They were promptly reviewed by a specialist in *Nature* in 1882, and the reputation of this work spread all over the world. Allan received the Telford Medal and Premium in 1881. This medal was the highest honour that the British Institution of Civil Engineers (ICE) could bestow, and Premium was an additional

monetary or book award, added to the medal. In this year, in 1881, Allan remarried; his second wife was Lauretta Frances Bartrum (1841–1900). He then moved back to England with his family for duty.

### Collaboration with Constance Marks

Allan worked at the Royal School of Military Engineering in Chatham (headquarters), and became Instructor in Construction from 1885 to 1890. With Lauretta, Allan returned to India at least once; Lauretta and Allan had a girl, Alice Mary Josephine Cunningham (1884–1966), en route. She was born at sea off Cape Town, South Africa. His oldest son, Allan H. C. Cunningham, had grown up by this time and entered the Royal Engineers on July 27th, 1889. Allan himself was appointed as C.R.E. (Commander, Royal Engineering) in Dublin and Shorncliffe from 1890 to 1891.

He retired from the army in 1891 as Lieutenant Colonel, and started studying pure mathematics. In 1892, at the age of 49, his academic life began when he became a Fellow of King's College, London, and in the same year a Member of the London Mathematical Society. He was then a Member of Council of the Society from 1893 to 1902 and served as a Vice-President in 1898. During this time, Allan lost his second wife, Lauretta, in 1900. He lived in his Kensington home with his daughter, Alice.

Allan's early publications as a mathematician are seen in *Mathematical Questions and Solutions from “The Educational Times,” with Many Papers and Solutions in Addition to Those Published in “The Educational Times”*. *The Educational Times* (ET) was a journal of the league of Preceptors, an organisation founded in 1846, which specialised in the discussion of pedagogical issues. The section dedicated to “Mathematical Questions and Solutions” was added in 1849 “in response to the rapid influx of mathematical questions” [2, p.222]. When the section started to be called *Mathematical Questions with their Solutions Taken from the “Educational Times”* (MQ), a call was made for participation by “distinguished mathematicians” and the British mathematicians Arthur Cayley, James Joseph Sylvester, Augustus De Morgan, William Kingdon Clifford, Thomas Archer Hirst, and Isaac Todhunter, as well as foreign contributors Pierre Marie Eugène Prouhet and Luigi Cremona, submitted their entries.

Allan Cunningham made his debut in the ET in 1898, and continued contributing until the demise of the MQ in 1918; he was one of the most active participants, posting 209 problems and solving 426 [2, p. 227; 3].

The editor of the mathematical section of ET was Constance Isabelle Marks (1860–1940), the first woman to be appointed to this position (Figure 4).



Figure 4. Constance Marks's appointment as an editor of the mathematical journal made news. This is the October 23, 1901 issue of *The Morning Leader*. © British Library Board (MFM.M44648 [1901]).

In this journal, Cunningham first announced the publication of *Binomial Factorizations Giving Extensive Congruence-Tables and Factorization-Tables* in 1912, by saying that it was "now in press". The war and its aftermath, however, delayed its publication until 1923.

For making such extensive reference tables, he employed skilled assistants to carry out the computations. Out of the nine assistants listed in the publication in 1923 and 1924, there were seven women. For female graduates of mathematics, becoming a "computer" was a widely pursued career option. The editor of the MQ, Constance Marks, also appeared in Cunningham's book as "author's assistant (Miss C. I. Marks)", but she was treated differently from other assistants who worked under Cunningham's "direct superintendence". Marks was credited with having extended his congruence table to "much higher limits" [4, p. xix].

### The Factorization of Large Numbers

Cunningham was interested in finding the factors of large numbers of the form  $a^n \pm b^n$ , which include Mersenne numbers ( $2^p - 1$ ) and Fermat numbers ( $2^{2^n} + 1$ ), and he often collaborated with Herbert J. Woodall, Associate of the Royal College of Science.

In 1911, Woodall announced his discovery that  $2^{181} - 1$  (a Mersenne number) is divisible by 43441 (a prime), and he thanked Cunningham for verifying the result [5]. This result was published in the section of 'Shorter Notices' in *The Bulletin of the American Mathematical Society*, as well as the July 1911 issue of *Nature* (Figure 5).

Another collaborator of Allan Cunningham's was Thomas Gasquoine Creak (1860–1922), who was educated at Emmanuel College, Cambridge, and graduated as 27th Wrangler in 1883. Creak served as honorary secretary of the North Wales Branch of the Mathematical Association, and he had worked at the Newcastle-upon-Tyne Grammar School [6]. He had worked with mathematicians and was known to be "a most painstaking and willing collaborator". His area of expertise was broad as he helped Alfred George Greenhill (1847–1927) in "troublesome algebra which arises in the division of the period in elliptic functions", George Hartley Bryan (1864–1928) in "the evaluation of the coefficients in some Fourier expansions", George Ballard Mathews (1861–1922) in "calculating polynomials

which occur in lemniscate-function theory”, and William Edward Hodgson Berwick (1888–1944) in “the complex multiplication of elliptic functions”. But his work with Cunningham stood out. Creak helped the computation of Cunningham’s book, *Binomial Factorisations* [4, p.xix]. Together with Woodall, Cunningham and Creak wrote the book, calculating polynomials which occur in lemniscate-function theory. It was titled *Haupt-exponents, Residue – Indices, Primitive Roots, and Standard Congruences*, published by Francis Hodgson in 1922. They also wrote a paper, ‘On Least Primitive Roots’, which appeared in *Proceedings of the London Mathematical Society* in 1923.

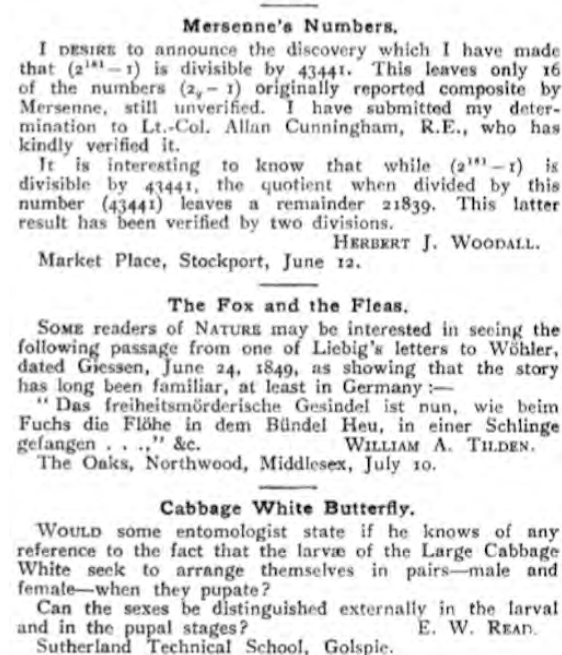


Figure 5. Woodall's announcement of his result in *Nature* (top right). His mathematical result was listed along with other news in science.

Derrick Norman Lehmer (1867–1938), Professor of Mathematics at the University of California, Berkeley, was a regular reviewer of Cunningham's work. Reading *Binomial Factorizations*, *Giving Extensive Congruence-Tables and Factorization-Tables*, Vol. 1 (1923) and *Binomial Factorizations* Vol. 2 (1923), Lehmer stated, "[t]hese two volumes are part of the outcome of thirty years of labor of the veteran computer" [7]. He knew that there would be seven volumes to be published by Cunningham, and applauded his work by writing, "[t]here is no room for doubt that Lt.-Col. Cunningham has undertaken and carried out an immense task, of value in the problem of

identifying large primes, and in the breaking down of numbers of special forms into their prime factors". Five out of seven volumes of *Binomial Factorizations* were published by 1924 and Lehmer wrote a review in *The Bulletin of the American Mathematical Society*, and pointed out the level of difficulties of calculations performed in Cunningham's book [8].

Creak died in 1922, but Cunningham and Woodall continued their work and published the booklet titled *Factorization of  $y^n \pm 1$ ,  $y = 2, 3, 5, 6, 7, 10, 11, 12$  up to high powers ( $n$ )* in 1925 [1], which became a major contribution to the field of number theory. Again it was published by Francis Hodgson in London, and summarised the work they had pursued for thirty years. In the review, Lehmer praised the "veteran computers", and wrote [9]:

This little volume gives the latest account of the efforts of arithmeticians in factoring numbers of the form  $y^n \pm 1$ . ... No doubt in the next few years explorers armed with high-power computing machines will fill up many of these white spaces. ... Every year witnesses the invention of some new device for examining the primality of "large numbers" and doubtless what today are considered as "large numbers" will be "small numbers" for the next generation. In the meantime this little book will serve to point out the gaps.

Allan Cunningham died three years after the publication of this book, on February 8th, 1928. He outlived his second wife by twenty-eight years, and his third son by nine years. His first son, Allan, and his second daughter, Alice, survived him.

His mathematical library was bequeathed to the London Mathematical Society by his will. He hoped that the future would bring more progress in the method of factorizing large numbers. His long-term female collaborator, Constance Marks, edited and published the final two volumes of *Binomial Factorizations* in 1927 and 1928. The Cunningham Project continues his study on the theory of numbers, and Lehmer himself invented some computing devices for factoring numbers (Figure 6). The study of the factoring of large numbers has advanced significantly since Cunningham's time.





Figure 6. A Number Sieve Machine using 16 mm film (1936). Dimensions: 18 x 19 ½ x 10 ¼ in. Courtesy of the Computer History Museum.

## The Prime Factorizations

While the advancement of computing technology has helped the factorization of large numbers, number theorists have worked on improving the methods of factorizations over the years. For a more detailed historical account of the counting methods and their recent development in the field of analytic number theory, see Carl Pomerance's 'A Tale of Two Sieves' (*Notices of the AMS*, Dec 1996), Ian Stewart's 'The Sifting Sands of Factorland' (*Scientific American*, June 1997), and Samuel S. Wagstaff Jr.'s 'The Cunningham project' in *High primes and misdemeanours: lectures in honour of the 60th birthday of Hugh Cowie Williams* (Fields Inst. Commun. 41, AMS, 2004, pp.367–78). A lecture given by James Maynard at Princeton IAS gives further explanations on the applications of sieve methods: 'Sieve methods: what are they, and what are they good for?' <https://www.youtube.com/watch?v=peWcw8rCE3A>

## Acknowledgements

Pieter Moree introduced me to Cunningham's work during my summer research visit at the Max Planck Institute for Mathematics in 2020. I am grateful to Pieter for his feedback and suggestions. I would also like to thank Christopher Hollings, Bernard Lightman, and Keith Hannabuss for their kind assistance at various stages of writing this article.

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## Tomoko L. Kitagawa

Tomoko L. Kitagawa is a historian specialising in the mathematical cultures of Europe, East Asia, and South Africa. Her research often focuses on minority groups. She is a Happiness Coordinator of the mathematicians' social groups, the Prime Powers (Oxford) and the Riemann Rockers (Bonn).

# A Mathematical Approach to Service Apportionment

IAN FLOOD, JOHN PARKER AND GLYN CARTER

The apportionment of interference protection criteria between different services is a problem in radio spectrum sharing and compatibility studies. These apportionments are often unnecessarily conservative. In this article we discuss a mathematical approach which delivers precise, spectrally efficient, service apportionments.

## Service apportionment

Radio spectrum engineers are concerned with studying interference between different radio services operating in the electromagnetic spectrum such as mobile and satellite services, for example. These investigations often consider coexistence between exactly two services in the same frequency band; this is known as a *sharing* study. However, emissions may have the potential to cause excess interference to receivers in adjacent frequency bands and when investigating a coexistence problem of this kind, we run a *compatibility* study.

Such studies will usually aim to calculate the interference sourced to the transmitters associated with one service and incident to the receivers of a second service. If the interference levels satisfy protection criteria then sharing is considered viable but if the criteria are exceeded then sharing may be problematic.

In this article, we express parameters using the decibel terms conventional in radio engineering (see the box on the next page for a brief discussion).

A typical interference protection criterion is a threshold for aggregate interference, denoted here by  $\Sigma I^T$ , incident to a receiver in a specified frequency bandwidth. This criterion can be expressed in decibels relative to a unit of signal power. Hence, if our reference unit of power is 1 W and the specified bandwidth at the receiver is 1 MHz,  $\Sigma I^T$  is expressed in dBW/MHz.

In a scenario involving one interfering service  $i$  and one victim service  $k$ , we can model aggregate interference sourced to  $i$  and incident to the receivers of  $k$ . Let  $\Sigma I_i^C$  denote the calculated aggregate interference sourced to  $i$ . Then if the

inequality

$$\Sigma I_i^C \leq \Sigma I^T$$

is satisfied at the receivers of  $k$ , we may say that sharing appears to be viable.

However, a persistent complication is consideration of a third, fourth or even fifth radio service. These may be incumbent in the frequency band under consideration or they may be emerging services requiring access to the spectrum.

The studies often take account of these other services by apportioning  $\Sigma I^T$  between the different sources of interference incident to a victim receiver; that is, a *service apportionment* is required. Such apportionments are usually very simple and with little supporting analyses. With this in mind, we will discuss a more elaborate approach, developed in a recent study funded by the mobile communications industry association GSMA [1].

In the present article, by way of example, we expand on our hypothetical problem to describe a scenario where services  $i$  and  $k$  are well-established and that a study requires a model of aggregate interference sourced from a new service  $j$  and incident to the receivers of  $k$ . Since  $i$  already shares the spectrum with  $k$  and is a source of interference, an apportionment of  $\Sigma I^T$  is required between  $i$  and  $j$ .

Service apportionments may be subject to some regulatory considerations but, in general, for an apportionment between interfering services with the same regulatory status, an equal apportionment of the protection criterion is applied; see [2] for example.

Hence a service apportionment  $A$ , expressed in decibels (dB), is calculated using

$$A = -10 \cdot \log_{10}(n), \quad (1)$$



## Some decibel terms

In spectrum engineering, signal power levels, gains and losses encountered on the radio path and interference protection criteria, including various adjustments, are denoted using decibel terms that express the ratio between power levels on a logarithmic scale.

Signal power levels and aggregate interference thresholds, such as those discussed in this article, can be expressed in decibels relative to a reference unit of power; typically 1 W (1 watt) or 1 mW (1 milliwatt). Hence a linear value of signal power  $p$  can be expressed in decibels relative to 1 W (dBW) using

$$P = 10 \cdot \log_{10} \left( \frac{p}{1} \right).$$

If an interferer radiates 1 W of power then  $P = 0$  dBW.

Signal power and interference thresholds are normally associated with a specified frequency bandwidth (see the table of results in the Example problem section).

Changes to a signal power level or adjustments to interference protection criteria are expressed in decibels (dB). One such adjustment is given by equation (3) where we exercise decibel terms only. However, some calculations, such as the calculation of total aggregate interference delivered by equation (8), involve linear terms. If we wish to sum two signal levels  $P_1$  and  $P_2$  we use

$$P_1 + P_2 = 10 \cdot \log_{10} \left[ 10^{(P_1/10)} + 10^{(P_2/10)} \right]. \quad (2)$$

Why not simply sum  $P_1$  and  $P_2$  using the decibel terms? Let us say that  $P_1 = P_2 = 0$  dBW. Then attempting to sum the decibel terms gives us  $0 + 0 = 0$ . This is incorrect because we are attempting to sum ratios of these signal powers to 1 W expressed in dBW. Hence, for this summation and following the rule established in equation (2), we require

$$0 + 0 = 10 \cdot \log_{10} (10^{(0/10)} + 10^{(0/10)})$$

which delivers a value of 3 dBW. This is the correct ratio of our summed signal powers to 1 W and we can check this by calculating the linear value using  $10^{(3/10)} = 2$  W.<sup>a</sup>

<sup>a</sup>Note that our 3 dBW value is a non-exact integer but usually written as shown in the engineering notation. Using this non-exact integer within  $10^{(3/10)}$  delivers exactly 2 W.

where  $n$  is the number of services subject to service apportionment. This apportionment can be denoted  $A_i$  when applied to interfering service  $i$ . Then the adjusted aggregate interference threshold that  $i$  must satisfy at the receivers of  $k$  is given by

$$\Sigma I_i^T = \Sigma I^T + A_i. \quad (3)$$

Clearly, such an approach could lead to an overly conservative assessment of the sharing problem in cases where  $\Sigma I_i^C < \Sigma I_i^T$  or an overly optimistic one should  $\Sigma I_i^C > \Sigma I_i^T$ . Rather than relying entirely on equations (1) and (3), our method analyses the existing interference sourced from  $i$  and incident to  $k$  and calculates precise apportionments for  $i$  and  $j$ .

If interference from  $i$  is less than the threshold given by equation (3), we can increase the apportionment available to  $j$ , perhaps radically, and so relax the interference criterion that must be satisfied by  $j$  for sharing to be considered feasible.

This is a typical backdrop to many sharing and compatibility problems. Modelling networks that do not yet exist, such as the emerging  $j$ , can be challenging and must involve uncertainties. However, results from an analysis of the existing radio interference environment, such as that defined by  $i$  and  $k$ , can deliver precise results for service apportionment and a more realistic appraisal of the allowable interference available to future networks.

### Introducing an inequality between service apportionments

Let us say that  $i$  is a well-established service with a high level of certainty regarding future network deployment. Then, assuming data is available, we can build an accurate model of aggregate interference  $\Sigma I_i^C$  at the receivers of service  $k$ . We discuss one approach, using Monte Carlo simulations, in the next section. Having obtained  $\Sigma I_i^C$ , we calculate  $i$ 's utilisation of the equally apportioned aggregate interference threshold delivered by equation (3), at a receiver of  $k$ , via

$$u_i = \left( \frac{10^{(\Sigma I_i^C/10)}}{10^{(\Sigma I_i^T/10)}} \right). \quad (4)$$

Here  $u_i$  is the ratio of calculated aggregate interference sourced to  $i$  and  $i$ 's equally apportioned aggregate interference threshold. If the threshold  $\Sigma I_i^T$  is satisfied exactly,  $u_i = 1$  and equal apportionments should be applied to  $i$  and the new service  $j$ . Otherwise, if  $u_i < 1$  we can introduce an inequality between the service apportionments for  $i$  and  $j$ . If  $u_i < 1$ , then  $j$  can be assigned a larger apportionment factor but if  $1 < u_i < n$  then  $j$  must be assigned a smaller apportionment factor in order for  $\Sigma I^T$  to be satisfied. If  $u_i \geq n$ , then  $i$  has already utilised or over-utilised all of the interference threshold; that is  $\Sigma I^T$  has already been satisfied exactly or exceeded by interference sourced from  $i$ , before  $j$  is even considered.

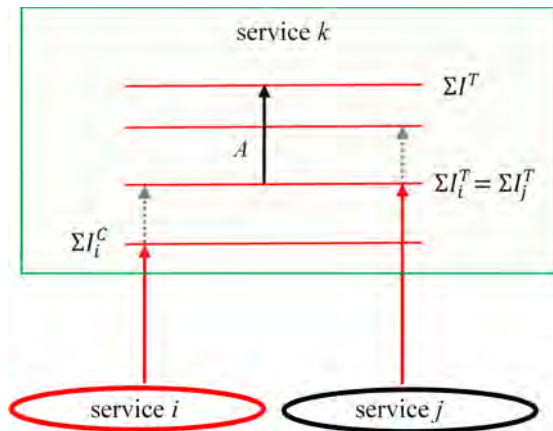


Figure 1. If  $u_i < 1$  then the interference threshold for the emerging service  $j$  can be relaxed.

Since  $u_i$  tells us precisely how our equal apportionment of the aggregate interference threshold  $\Sigma I_i^T$  has been utilised by  $i$ 's interferers, it

forms the basis for a revised service apportionment for both  $i$  and  $j$  in our methodology. Because equations (1) and (3) have been applied to services  $i$  and  $j$  in the first instance, our start point is  $A_i = A_j$ . We first of all revise our service apportionment for  $i$ :

$$A_i^u = 10 \cdot \log_{10} \left[ u_i \cdot 10^{(A_i/10)} \right]. \quad (5)$$

Then for  $0 < u_i < n$  a revised service apportionment for  $j$  can be calculated:

$$A_j' = 10 \cdot \log_{10} \left[ 10^{(A_j/10)} \cdot (1 + (1 - u_i)) \right], \quad (6)$$

noting that  $A_j' = A_i^u$  when  $u_i = 1$ . Then revised apportionments of the aggregate interference threshold  $\Sigma I^T$  are obtained for services  $i$  and  $j$ . The revised threshold for  $i$  can be calculated using

$$\Sigma I_i^{T'} = \Sigma I^T + A_i^u.$$

Since  $A_i^u$  expresses  $i$ 's utilisation of  $\Sigma I_i^T$ , our revised threshold  $\Sigma I_i^{T'}$  is, in effect, the aggregate interference sourced from  $i$  and incident to  $k$ ; that is,  $\Sigma I_i^{T'} = \Sigma I_i^C$ .

Then the revised threshold for  $j$  is given by

$$\Sigma I_j^{T'} = \Sigma I^T + A_j'. \quad (7)$$

Assuming that  $j$  utilises all of the revised threshold available, we can calculate the total aggregate interference  $\Sigma I_{\text{total}}$  incident to  $k$ :

$$\Sigma I_{\text{total}} = 10 \cdot \log_{10} \left[ 10^{(\Sigma I_i^C/10)} + 10^{(\Sigma I_j^{T'}/10)} \right], \quad (8)$$

checking that  $\Sigma I_{\text{total}} = \Sigma I^T$  exactly.

In order to quantify any efficiencies obtained through the use of our method we calculate the increase to the aggregate interference threshold available to the emerging service  $j$ , expressed in decibels, via

$$\Delta \Sigma I_j^T = \Sigma I_j^{T'} - \Sigma I_j^T, \quad (9)$$

and as a percentage increase in the allowable interference available to  $j$ :

$$\Delta \Sigma I_j^T (\%) = 100 \cdot (10^{(\Delta \Sigma I_j^T/10)} - 1). \quad (10)$$

This method can be applied to a range of sharing and compatibility problems where service apportionments are required.

### Example problem

The scale and complexity of software simulations varies considerably and, to some extent, this depends on the precise nature of the sharing or compatibility problem; see [3] for a detailed discussion of the different methodologies used in such study work. It may be appropriate to model interference from a mass of interferers in service  $i$  incident to just one representative receiver of  $k$ . Other problems may require that a larger number of receivers are modelled in which case equation (4) would be exercised at each receiver; then it may be appropriate to consider the maximum value, say, obtained for  $u_i$  over a set of  $k$ 's receivers.

Our example problem involves an established network of microwave fixed links denoted by  $i$ , an established satellite service  $k$ , operating in a frequency band adjacent to  $i$ , and the emerging mobile service  $j$  which requires access to spectrum adjacent to  $k$ . Our compatibility problem is such that a study must be made of interference sourced from  $i$  and incident to  $k$  and with service apportionments applied to  $i$  and  $j$ .

In [1], we give a detailed description of the software simulations performed, using *Visualyse Professional* software [4], in order to obtain the calculated aggregate interference  $\Sigma I_i^C$  sourced from  $i$  and incident to  $k$ . We give a brief summary here: Our Monte Carlo simulations involve the use of real-world deployment data for service  $i$  [5] and a single mobile satellite receiver of the Earth exploration satellite service, representative of  $k$ , is modelled using information from [6]. At each step in our simulations, the orbital location of  $k$ 's satellite, the orientation of its antenna and, therefore, the footprint of its antenna on the surface of the Earth, are random. The location of the terrestrial interferers is fixed including the orientation of their antennas. Hence, the simulations constitute an extensive search of possible interference geometries between a static deployment of interferers on Earth and a representative satellite. We are searching for worst-case scenarios where high levels of aggregate interference are incident to the satellite's receiver and these are the inputs for  $\Sigma I_i^C$  in our service apportionment calculations.

The aggregate interference threshold  $\Sigma I^T$  for  $k$ 's satellite receiver is specified by a Working Party of the International Telecommunication Union (ITU) and is set at  $-166$  dBW/200 MHz [2].

Having obtained a worst case  $\Sigma I_i^C$  from our simulations and  $\Sigma I^T$  from the ITU, we calculate precise service apportionments for the established service  $i$  and the emerging mobile service  $j$ .

We summarise our example calculations:

	parameter	units/description	value
1	$\Sigma I_i^C$	dBW/200 MHz	$-176.85$
2	$\Sigma I^T$	dBW/200 MHz	$-166$
3	$n$	number of services	2
4	$A$	dB	$-3$
5	$\Sigma I_i^T, \Sigma I_j^T$	dBW/200 MHz	$-169$
6	$u_i$	ratio	0.16
7	$A_i^u$	dB	$-10.9$
8	$A_j'$	dB	$-0.37$
9	$\Sigma I_j^{T'}$	dBW/200 MHz	$-166.37$
10	$\Sigma I_{\text{total}}$	dBW/200 MHz	$-166$
11	$\Delta \Sigma I_j^T$	dB	2.64
12	$\Delta \Sigma I_j^T (\%)$	percent	84

Item 1 is the output from a software simulation set up to calculate aggregate interference sourced to service  $i$  and incident to  $k$ . Item 2 is the aggregate interference threshold at the receiver of  $k$ . Item 3 is the number of interfering services subject to service apportionment. Item 4 is an equal service apportionment given by equation (1). Item 5 is the equally apportioned aggregate interference threshold for services  $i$  and  $j$  calculated using equation (3). Item 6 is  $i$ 's utilisation of the equally apportioned aggregate interference threshold delivered by equation (4). Item 7 is the revised service apportionment for  $i$  given by equation (5). Item 8 is the revised service apportionment for  $j$  given by equation (6). Item 9 is the revised aggregate interference threshold for  $j$  calculated using equation (7). Item 10 is the total aggregate interference incident to  $k$ 's receiver, calculated using equation (8); this allows us to check that the aggregate interference threshold is satisfied. Item 11 is the increase to the aggregate interference threshold available to service  $j$ , expressed in decibels, given by equation (9). Item 12 is the percentage increase in the allowable interference available to  $j$  calculated using equation (10).

These results show a radical under-utilisation of an equal apportionment by the established service  $i$ . Item 6 shows that  $u_i = 0.16$ , allowing for an 84% increase in  $j$ 's interference threshold  $\Sigma I_j^{T'}$ ; item 9 shows that  $\Sigma I_j^{T'}$  is just 0.37 dB lower than the overall aggregate interference threshold  $\Sigma I^T$ .

## Conclusions

The practical mathematical method set out here can be applied to many sharing and compatibility problems where a service apportionment is required and where it is possible to undertake a detailed evaluation of the existing radio interference environment including the use of deployment data.

Scope for further work includes the investigation of more sharing and compatibility problems requiring service apportionments. Such investigations could include more complex scenarios involving a larger number of interfering or emerging services. In a scenario, for example, where one established service and two emerging services are subject to service apportionment, we may be able to build a model leading to a precise apportionment for the established service, assigning equal apportionments of the remaining allowance to the two emerging services. Other problems might involve more than one established interfering service where we are required to evaluate utilisation of the equally apportioned aggregate interference threshold for each. Whatever the precise apportionment problem, our mathematical approach can be considered and, where appropriate, exercised in order to calculate service apportionments that are not arbitrarily equal.

Our example calculation is based on a study where we were able to access detailed deployment data for the established service, allowing for a precise calculation. The results show just how radical the impact of such an assessment can be. Because the incumbent interfering service under-utilises an equal apportionment of the aggregate interference threshold, we are able to relax the criterion for the emerging service whilst ensuring that the overall aggregate interference threshold at the victim receiver is satisfied. Effectively, the entire study constitutes a pre-study that aims to set realistic interference thresholds for a compatibility problem.

## FURTHER READING

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### Ian Flood

Ian Flood is a consultant with Transfinite Systems, London, where his work involves modelling spectrum engineering problems in the radio interference environment, including the development of spectrally efficient methods. He is a Chartered Engineer and holds a PhD in graph-theoretic studies from Cardiff University.



### John Parker

John Parker is a founder and director of Transfinite Systems Ltd. He works as a consultant in the technical aspects of radio regulations. His work focusses on spectrum sharing studies and promotion of more efficient use of the spectrum through improvement of numerical methods and simulation. He has a PhD in theoretical physics from the University of Manchester.



### Glyn Carter

Glyn Carter is Senior Spectrum Advisor at the GSMA, where his main focus is on future spectrum, including work in ITU-R and CEPT on sharing and compatibility studies. He has been involved in spectrum management for over 20 years, and has also worked as a consultant and for mobile operators and a radio equipment vendor. He has a BSc in mathematics from Westfield College and a PhD in cryptography from Royal Holloway, University of London.

## Notes of a Numerical Analyst

# The Universe Speaks in Numbers?

NICK TREFETHEN FRS

Not long ago Graham Farmelo gave a public lecture in our Mathematical Institute about his book *The Universe Speaks in Numbers: How Modern Maths Reveals Nature's Deepest Secrets*. I've just finished reading this book with great pleasure, and I want to share with you the results of a rather quirky literature search it has prompted.

My subject isn't Farmelo's book *per se*, but the view it compellingly represents, which is a widespread one these days. Mathematics and physics drifted apart in the mid-20th century, according to this view, but something big has happened since then. The two fields are now extraordinarily close, with influences in both directions between some of the most exciting mathematics and some of the most exciting physics. Two of the leaders of this new synergy have been Ed Witten in physics and Michael Atiyah in mathematics.

But then there is controversy. The physics in question is string theory in its many forms. Is string theory physics? Is it mathematics?

Farmelo's title got me thinking. Of course, he doesn't mean "numbers" literally; this is a colourful way of referring to mathematics. Yet the question I found myself asking was, what actual numbers appear in some of the great works of theoretical physics?

So I've taken a sample. It turns out many papers mention the integers from 0 to 10, as well as  $e$ ,  $\pi$ ,  $i$  and  $\infty$ . I will exclude these from the tallies, just as Desert Island Discs grants everyone the Bible and the complete works of Shakespeare. What follows are exact lists of all the other numbers that appear in seven special papers.



*Maxwell's great paper on electromagnetism* (1865) contains these numbers:  $10^{-50}$ , 0.0008, 0.00083,  $1/1000$ , 0.00126,  $1/500$ , 0.01841, 0.02010, 0.0236, 0.158194, 0.22437,  $1/4$ ,  $1/2$ , 1.0618, 4.13, 13, 16, 24, 27, 32, 45, 48, 77, 100, 193, 313, 475, 7345, 13000, 44997, 410000, 430165, 436440, 456748, 60000000, 298000000, 308000000 and 310740000.

*Einstein's Nobel Prize-winning paper on the photoelectric effect* (1905):  $6 \cdot 10 \cdot 10^{-56}$ ,  $1.62 \cdot 10^{-24}$ ,  $4.866 \cdot 10^{-11}$ ,  $10^{-8}$ ,  $1.9 \cdot 10^{-5}$ ,  $2/3$ ,  $3/2$ , 1.7, 4.3,  $9.6 \cdot 10^3$ ,  $10^7$ ,  $6.4 \cdot 10^{12}$ ,  $9.6 \cdot 10^{12}$ ,  $1.03 \cdot 10^{15}$  and  $6.17 \cdot 10^{23}$ .

*Einstein's 3-page announcement of  $E = mc^2$*  (1905):  $9 \cdot 10^{20}$ . (That's right! — just  $9 \cdot 10^{20}$ , along with 1 and 2. Einstein wrote the equation as  $\text{Mass} = L/V^2$ .)

*Einstein's main paper on general relativity* (1916):  $1.87 \cdot 10^{-27}$ ,  $6.7 \cdot 10^{-8}$ ,  $1/4$ ,  $1/2$ , 0.02, 1.7, 11, 12, 24 and 43.

*Schrödinger's Nobel Prize-winning theory of quantum mechanics* (1926):  $10^{-18}$ , 0.0004849, 0.0008312, 0.001573, 0.003433, 0.04418, 0.08975,  $5/36$ ,  $3/16$ ,  $21/100$ ,  $2/9$ , 0.2386,  $1/4$ ,  $1/2$ , 1.281,  $\sqrt{2}$ , 4.5, 11–20, 22, 26, 28, 30, 32, 64, 10000 and 100000, plus four tables of data.

*Penrose's Nobel Prize-winning proof of stability of black holes* (1969):  $1/2$ , 0.7, 1.3,  $10^8$ ,  $3 \cdot 10^{10}$ ,  $10^{11}$  and  $4 \cdot 10^{38}$ .

Finally, the joint paper by Atiyah and Witten, a work of 106 pages.

*Atiyah and Witten, "M-theory dynamics on a manifold of  $G_2$  holonomy"* (2001):  $\sqrt{6}$ , 11, 24, 27, 36, 48, 72 and 144.



### Nick Trefethen

Trefethen is Professor of Numerical Analysis and head of the Numerical Analysis Group at the University of Oxford.



## Mathematics News Flash

Jonathan Fraser reports on some recent breakthroughs in mathematics.

### Proof of the Kalai-Meshulam conjecture

**AUTHORS:** Maria Chudnovsky, Alex Scott, Paul Seymour and Sophie Spirkl

**ACCESS:** <https://arxiv.org/abs/1810.00065>

Consider a finite simple graph. An *induced cycle* is a cycle such that no two vertices of the cycle are connected by an edge which is not part of the cycle. Such cycles are sometimes called chordless cycles, where one thinks of a cycle as a circle and an edge violating the definition of induced cycle as a chord. A *stable set* is a collection of vertices such that no two are adjacent in the graph.

Playing around with some simple examples reveals a connection between induced cycles and stable sets. Consider the simple example where the graph is a 3-cycle (a complete graph on 3 vertices). Then there are only three non-empty stable sets, each containing only one vertex. In particular, the number of stable sets with even cardinality and the number with odd cardinality differ by 2 (counting 0 as an even number). This phenomenon persists for induced cycles with length divisible by 3. I enjoyed working this out for induced cycles of length 6, but leave cycles of longer length to the interested reader!

Rather remarkably, this observation has a strong converse. In the 1990s, Kalai and Meshulam conjectured that if a graph does *not* contain an induced cycle with length divisible by 3, then the number of stable sets with even cardinality and the number with odd cardinality differ by at most 1. This paper proves the Kalai-Meshulam conjecture, and was published in the *Israel Journal of Mathematics* in 2020.

### Geometric anomaly detection in data

**AUTHORS:** Bernadette Stolz, Jared Tanner, Heather Harrington and Vidit Nanda.

**ACCESS:** <https://doi.org/10.1073/pnas.2001741117>

When presented with a large data set, one often seeks to understand a low-dimensional set of phenomena which underpin the complicated output.

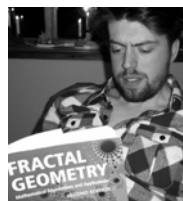
This is referred to as ‘dimensionality reduction’ and has powerful applications across many fields including statistics, machine learning and signal processing. A common approach is to assume that the data lives near a low-dimensional manifold. This paper, published in *Proceedings of the National Academy of Sciences* in 2020, introduces an intriguing new approach to this general problem, which takes into account spatial variation in the topology of the data.

### Purely unrectifiable metric spaces and perturbations of Lipschitz functions

**AUTHORS:** David Bate

**ACCESS:** <https://arxiv.org/abs/1712.07139>

A set in Euclidean space is *n-rectifiable* if it can be covered up to a set of *n*-dimensional Hausdorff measure zero by a countable collection of Lipschitz images of  $\mathbb{R}^n$ . A set is *purely n-unrectifiable* if all of its *n*-rectifiable subsets have *n*-dimensional Hausdorff measure zero. The Besicovitch-Federer projection theorem characterises this latter notion in terms of projections: a subset of  $\mathbb{R}^N$  (with  $N > n$ ) with finite *n*-dimensional Hausdorff measure is purely *n*-unrectifiable if and only if almost all projections of the set onto *n*-dimensional subspaces of  $\mathbb{R}^N$  have *n*-dimensional Hausdorff measure zero. This theorem fails in every infinite dimensional Banach space. However, this paper, published in *Acta Mathematica* in 2020, provides a replacement for the Besicovitch-Federer projection theorem which characterises purely *n*-unrectifiable subsets of a complete metric space in terms of (perturbations of) Lipschitz functions.



**Jonathan Fraser** is a Reader of Mathematics at the University of St Andrews and an Editor of this Newsletter. His research interests centre on fractal geometry.

Microtheses and Nanotheses provide space in the Newsletter for current and recent research students to communicate their research findings with the community. We welcome submissions for this section from current and recent research students. See [newsletter.lms.ac.uk](http://newsletter.lms.ac.uk) for preparation and submission guidance.

## Microthesis: Heterogeneity and Concurrency in a Heterosexual Dynamic Network Model

TRYSTAN LENG

Understanding the impact of network structure on the outcome of epidemics is vital in designing mathematical models to inform public health. This microthesis presents a network model of a heterosexual population to elucidate the importance of concurrency in models for the control of sexually transmitted infections (STIs).

The spread of an epidemic can be thought of as a process on a network. A network is an ordered pair  $(V, E)$ , where  $V$  is a set of vertices and  $E$  is a set of edges between vertices, and can be represented by an *adjacency matrix*,  $A$ :

$$A_{ij} = \begin{cases} 1 & \text{if individual } j \text{ is connected to individual } i, \\ 0 & \text{otherwise} \end{cases}$$

In the context of the spread of STIs, vertices denote individuals while edges denote an individual's unprotected sexual contacts. The *degree* of an individual  $i$  in a network is the number of contacts  $i$  has, and is given by:

$$k_i = \sum_{j=1}^n A_{ij}$$

By calculating the proportion of individuals of each degree, we obtain the *degree distribution* of a network. We can define degree distributions at different time-scales - e.g. we can consider the degree distribution of partners individuals currently have, or we can consider the degree distribution of new partners individuals have had in the previous year.

If everybody formed new partnerships at the same rate, then degree distributions would be Poisson distributed. This is not what we observe in UK data [1]. For instantaneous degree distributions, a large proportion of individuals are of degree 1, indicating a strong preference for monogamous relationships. Despite this preference, a substantial proportion of

individuals engage in *concurrent* partnerships each year, i.e. sexual partnerships that overlap in time.

Yearly degree distributions have a heavy tail of highly-connected individuals. This 'core group' of active individuals plays a key role in sustaining the transmission of STIs.

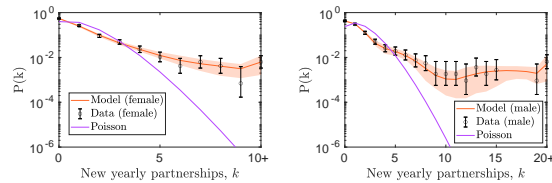


Figure 1. Yearly degree distributions of new heterosexual partnerships in the UK for females (left) and males (right) aged 16-24 [1], and fitted distributions from a dynamic network model compared to Poisson distributions with the same mean.

### Dynamic network models

Modelling helps us predict the course of STI epidemics and the potential impact of control measures. We wish to describe a dynamic network model of a heterosexual population that can satisfy the above properties for both sexes.

To model a network where partnerships change through time, individuals must be able to form partnerships, and partnerships must be able to dissolve. We assume that a single individual of sex  $s$  and risk group  $r$  forms new partnerships at a rate  $f_{sr}$ . We assume that individuals who form partnerships

at a high rate also dissolve them at a high rate, so we take the rate at which partnerships dissolve to be  $b(f_{Mr} + f_{Fq})$ , where  $b > 0$ . What should we assume about the rate at which individuals in relationships form new (additional) partnerships?

At one extreme, we could assume that they form partnerships at the same rate as if they were single. Doing so allows us to obtain degree distributions analytically, but potentially leads to high levels of concurrency. At the other extreme, we could assume that individuals are strictly monogamous, ignoring concurrent partnerships. Doing so, the partnership dynamics of the population can be expressed as a *pair-formation* model [2].

An alternative approach is to assume that individuals in relationships form additional partnerships at a reduced rate  $c_s f_{sr}, c_s \in [0, 1]$ . By introducing these parameters, we obtain a model that can be fitted to yearly concurrency data, but we lose some of the tractability of the alternative assumptions.

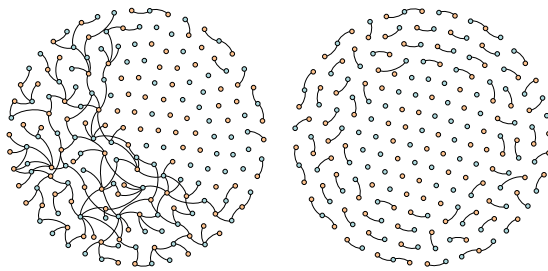


Figure 2. A model assuming  $c_M = c_F = 1$  (left) and a model where  $0 < c_M, c_F < 1$  fitted to yearly concurrency data (right) can both match yearly degree distributions closely, but result in very different instantaneous network structures.

Each of these assumptions results in a different dynamic network model. How do epidemiological outcomes compare between models?

### Comparing models

Using the Deviance Information Criterion [3] to select the optimal number of risk groups, we find that relatively few risk groups (3 or 4) are required for each model to match yearly degree distributions. However, these different assumptions result in very different instantaneous network structures (Fig. 2).

We can simulate the spread of epidemics on each of these models. At a fixed probability of infection across contacts, the size of epidemics differs considerably between models. However, if we set this probability to satisfy the same endemic prevalence in each model, the impact of control measures such as vaccination are similar across models (Fig. 3), echoing a result I obtained using a different approach [4].

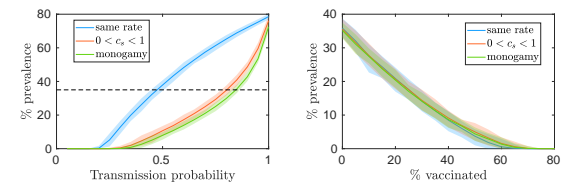


Figure 3. Left: prevalence between models at a fixed probability of transmission. Right: the impact of vaccination on transmission when models are matched to the same endemic prevalence without vaccination.

Capturing levels of concurrency is important if the probability of STI infection across contacts is known, but the model must be simulated explicitly. If models are matched to prevalence data, the impact of concurrency on control measures is modest, and therefore a model with greater analytical tractability may be preferred. The ‘best assumption’ depends on the model’s purpose, and is always a balance between accuracy, interpretability, and tractability.

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### Trystan Leng

Trystan is a PDRA at the University of Warwick. His research focusses on constructing models that incorporate contact structure to answer questions for public health. Outside of mathematics, he has a keen interest in the history and philosophy of science.

# A Tool Kit for Groupoid $C^*$ -Algebras

by Dana P. Williams, American Mathematical Society, 2019, US\$ 129,  
ISBN: 978-1-4704-5133-2

Review by Mark V. Lawson



The course of the theory of  $C^*$ -algebras was irrevocably altered by Alain Connes' insight [1] that each  $C^*$ -algebra should be viewed as a noncommutative topological space, an insight inspired by the Gelfand representation theorem

for commutative  $C^*$ -algebras. This approach to  $C^*$ -algebras has been enormously fruitful, but it is important to understand that the  $C^*$ -algebra replaces the topological space and so there is no actual noncommutative topological space in the background. However, it is possible to construct  $C^*$ -algebras from objects which are honest-to-goodness noncommutative topological spaces: specifically,  $C^*$ -algebras can be constructed from suitable topological groupoids in a way that generalizes the Gelfand representation. The book under review is about just such  $C^*$ -algebras.

Before we can say more about the book, we have to answer the following question: what is a topological groupoid? We begin with groupoids. Groupoids are (small) categories in which every arrow is invertible. We shall need a very small amount of notation and terminology in what follows. Each element  $g$  of a groupoid is equipped with a domain  $\mathbf{d}(g) = g^{-1}g$  and a range  $\mathbf{r}(g) = gg^{-1}$  and we shall call the set of all elements of a groupoid  $G$  with range  $e$  a *fibre (over  $e$ )*. In this setting, groupoids should be viewed as algebraic structures (whence my use of the word 'small') and not as categories of structures. In particular, a group is a groupoid with a single identity, a set can be regarded as a groupoid in which every element of the set becomes an identity, equivalence relations are groupoids, and group actions give rise to groupoids. Significantly, groupoids contain both geometric as well as algebraic information. Just as groups can be furnished with extra structure, so too can groupoids. Structured groupoids of various

complexions were first studied by Charles Ehresmann [2] but it is the topological groupoids which are important here. A *topological groupoid* is simply a groupoid equipped with a topology in which composition and the taking of inverses are both continuous maps.

The correct perspective to understand where this book is coming from is to regard a topological groupoid as being a noncommutative topological space, where the composition in the groupoid introduces the necessary element of noncommutativity. The specific connection between topological groupoids and  $C^*$ -algebras which this book builds upon goes back to the monograph by Renault [5] but, as we shall see, a topological groupoid needs to be equipped with extra structure before a  $C^*$ -algebra can be constructed from it.

It is difficult to say anything meaningful about arbitrary topological groupoids but two classes have turned out to be important: the *open topological groupoids*, in which the domain and range maps are open, and the more restrictive *étale topological groupoids*, in which the domain and range maps are local homeomorphisms. The étale topological groupoids are particularly prevalent in the theory of  $C^*$ -algebras since they commonly arise when a  $C^*$ -algebra is constructed from combinatorial data such as aperiodic tilings [3] or higher-rank graphs [4]. This book deals with  $C^*$ -algebras constructed from locally compact Hausdorff topological groupoids equipped with a Haar system which is (essentially) a set of measures living on each fibre. The provision of such a Haar system forces the groupoid to be open. It is important to observe that although étale topological groupoids come equipped with a Haar system, in this case consisting of counting measures, the author is not merely interested in étale groupoids; in fact, their theory is usually rather straightforward. In any event, given a suitable topological groupoid  $G$  equipped with a Haar system two  $C^*$ -algebras can be constructed: the *universal groupoid  $C^*$ -algebra*  $C^*(G)$  and the *reduced groupoid  $C^*$ -algebra*  $C_r^*(G)$ . How these two  $C^*$ -algebras are constructed is thoroughly

described in Chapter 1. The first obvious question is how dependent are the  $C^*$ -algebras on the choice of Haar system. This question is discussed in detail in Chapter 6. For étale groupoids, the answer is straightforward: the universal groupoid  $C^*$ -algebras are isomorphic. For other kinds of open groupoids, there is not such a clear-cut answer, but the author investigates this question from a number of points of view. The simplicity of the associated  $C^*$ -algebras is then surveyed in Chapter 11. Chapters 1, 6 and 11 form what this reviewer would see as the core of the book, but the remaining chapters deal with special topics such as the important one of amenability in Chapters 9 and 10.

The book is written as a text-book with exercises at the end of each chapter, which is ideal for experts, but for the rest of us this is a superb reference for particular topics that are currently only to be found scattered throughout the literature. One of the rôles of book reviewers used to be helpfully pointing out typos. This is no longer needed since books are now associated with online errata pages and the ones for this book can be found at <https://math.dartmouth.edu/~dana/groupoids/>. My only suggestion for a future edition once the typos are dealt with would be the inclusion of an appendix on étale topological groupoids summarizing the main results pertinent to them obtained in the book; étale groupoids are so important in the construction of  $C^*$ -algebras even if they are topologically simple-minded.

## FURTHER READING

- [1] A. Connes, *Noncommutative geometry*, Academic Press, 1994. Available for download at <https://www.alainconnes.org/en/downloads.php>
- [2] Ch. Ehresmann, *Oeuvres complètes et commentées*, Amiens, 1984. Available for download at [https://ehres.pagesperso-orange.fr/C.E.WORKS\\_fichiers/C.E\\_Works.htm](https://ehres.pagesperso-orange.fr/C.E.WORKS_fichiers/C.E_Works.htm)
- [3] J. Kellendonk, The local structure of tilings and their integer group of coinvariants, *Comm. Math. Phys.* 187 (1997), 115–157.
- [4] A. Kumjian, D. Pask, Higher rank graph  $C^*$ -algebras, *New York J. Math.* 6 (2000), 1–20.
- [5] J. Renault, *A groupoid approach to  $C^*$ -algebras*, *Lecture Notes in Mathematics* 793, Springer-Verlag, Berlin, 1980.



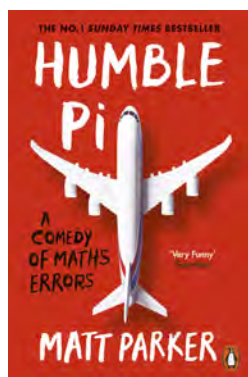
## Mark V. Lawson

Mark V. Lawson is professor of mathematics at Heriot-Watt University. His main research interest is in algebraic semigroup theory. He was awarded the Muhony-Neumann Prize by the Australian Mathematical Society in 2017 for a paper he wrote on noncommutative Stone duality.

# Humble Pi: A comedy of maths errors

by Matt Parker, Penguin, 2019, £9.99, ISBN: 978-0141989143

Review by Noel-Ann Bradshaw



All mathematicians know what it is like to make mistakes in their calculations and thinking. Sometimes we spot these quickly ourselves but often it takes someone else to find them. In *Humble Pi*, Matt Parker gathers a delightful mix of mathematical errors and

intriguing related snippets of information, from as far back as 3,000 BC to the present day, weaving them together in his own entertaining and comedic style.

Parker, winner of the 2020 IMA-LMS Christopher Zeeman Medal for excellence in mathematics communication, includes examples ranging from the small and easily overlooked (such as fixing a shed padlock incorrectly) to hard-coded computer errors that affect much of the software and machinery that we all use. Areas that come under his scrutiny include combinatorics, probability, engineering,



programming, counting and random numbers. Some of the research for this book has seemingly been quite straightforward; leading to delightfully unusual facts: for example, did you know that the oldest person whose name we know was an accountant from around 3,000 BC? Other research is more bizarre: such as spending three days flipping a coin to determine how likely it is that it lands on its edge. Well-known featured examples of public-facing mathematical errors include diagrams of interlocking cogs that can't actually turn, and the symbol used to denote a football on UK road signs that mathematically couldn't form a suitable sphere.

Although this book is easy to read, and not mathematically taxing, it took me a surprising amount of time to get through it as I found it was almost impossible to go past examples involving phone and computer calculators without trying them on my own devices. As a result, I have learned how to force Microsoft Excel to show that it really thinks that  $0.5 - 0.4 - 0.1 = -2.77556E - 17$ , which is peculiarly satisfying.

I particularly like the way that Parker groups the errors in this book. Rather than have all engineering examples in one chapter and all computer errors in another, Parker arranges them by his view of the underlying mathematics, which is not always obvious. So an initial example about the days of the week can lead into fence posts and the life of a London tube escalator before culminating with discussion of intervals in music theory. This variety in one chapter means that even if one example does not appeal to you, another will soon come along illustrating the same fundamental error.

Some of the instances that Parker relates come about due to theoretical knowledge lagging behind applications, such as steam engines coming into use before the theory of thermodynamics was fully developed. This can lead to problems but, as Parker says, the important thing is that we learn from past errors and don't repeat them.

Parker says at the outset that he has deliberately left three errors in the book. However, it is not really clear what he considers to be an error. By most people's standards the page numbering could be thought to be an error as it delightfully starts at 314 and goes back to 0. Personally, I think this should be adopted by other authors as it shows the reader at a glance how many pages they have left to go. Another possible error could be the page numbers of the index and other end-matter, but again I am sure this is by design. I have discovered a missing apostrophe but, as I suspect this is unintentional, I am left disappointed that I have failed to spot any of Parker's intended errors. Unless of course the statement that the book contains errors is itself a deliberate error!

Like many popular maths books this is a book that should appeal to people with little mathematical knowledge and understanding, as well as to mathematicians. Indeed, this book was a present from one of my sons, who sadly thinks he has little interest in maths. He had discovered and enjoyed Parker's videos and thought we would both enjoy reading this book. So, whilst it would be wonderful if we didn't make any more mathematical errors, if they enable more people, like my son, to appreciate the usefulness and power of mathematics then it just might be worth it.



### Noel-Ann Bradshaw

Noel-Ann Bradshaw is Head of School of Computing and Digital Media at London Metropolitan University. She has published on a wide variety of topics

in mathematics education and also on Florence Nightingale's use, management and understanding of data. She is currently spending her spare time creating new cocktail recipes which has made the recent months a little more bearable.

# Against All Odds: Women's Ways to Mathematical Research Since 1800

Edited by Eva Kaufholz-Soldat and Nicola M. R. Oswald, Springer, 2020,  
£99.99, US\$139.99, ISBN: 978-3030476106

Brigitte Stenhouse



*Against All Odds* raises the question: What did, and does, it take for a woman to become a mathematician?

Although most chapters focus on individuals, this book is more than an anthology of biographies. Rather, we are presented with a selection of case studies bracketed into

three themes — Institutions, Collaborative Couples, and Approaches. The final article comes under the heading Perspectives, and argues for the importance of teaching students the political and social implications of science. The book focuses almost exclusively on Europe and the USA, with a bias towards the 20th century.

Through a consideration of the universities of Göttingen, Würzburg, and Prague the first section demonstrates how a lack of educational opportunities for women at the turn of the 20th century constrained women who wished to study mathematics, and introduces us to those who nevertheless did so. Key trends recognised here were the necessity of a supportive and middle-class family background, and the importance of secondary education; even once women were legally permitted to study at university, without schools they could not satisfy the entry criteria. Thus the first women to enter these universities often came from outside the Empires in which the institutions were based.

Building on the extensive literature on collaborative couples in the sciences, the second section applies this 'unit of analysis' to mathematicians

more precisely. Vogt's chapter considers how the collaborative relationship between Emma S. and Wladimir S. Woytinsky evolved during their respective careers, suggesting that "the question of co-authorship depend[ed] deeply on the social situation of women" [1, 147]. Similarly to Grace Chisholm Young (treated in Mühlhausen's chapter), Emma Woytinsky did not claim authorship credit on her joint work with her spouse until he had achieved a permanent post and provided the couple with a stable income.

The pitfalls and difficulties of writing biographies of female mathematicians forms the focus of the Approaches section. Boucard has conducted an extensive investigation into how biographies and representations of Sophie Germain were mobilised by opposing sides in the numerous debates on the intelligence of women during the French Third Republic (1870–1940). Germain's biography was used as evidence that women were capable of original mathematical thought, whilst others used phrenological studies of her skull to contend that her talents lay in mere calculation and stubbornness. Even when it was conceded that women *could* study mathematics, the debate persisted over whether they *should*. Fajstrup, Gjerløff, and Kjeldsen summarise and analyse interview responses from four professors of mathematics working in Denmark, focusing on early life, career, and how gender has affected their experience of the mathematical community.<sup>1</sup> Their detailed methodological reflections section should be invaluable to future scholars, as they cogently argue the need for authors to reflect on how their own experiences, prejudices, and situation affect the histories they write.

<sup>1</sup>Content note for readers: Fajstrup et al state that their focus is on experiences of those who were "one of the very few research mathematicians with two X-chromosomes" [1, 251]. I found this emphasis unhelpful and divisive, especially as it is quickly replaced with a focus on gender and womanhood, not genetics or biology.

Although the title of the work under review suggests a consideration of *all* the barriers which women have faced when pursuing mathematical research, there is a glaring lack of black and brown women throughout. This is especially notable when one considers that black and brown university students continue to face barriers such as the awarding gap, and in 2018 there were only 25 black female professors in the UK (out of 13,535) *in any subject* [2]. There is an expanding literature on such mathematicians, such as [3] and [4], but much remains to be done to incorporate these stories into general history of mathematics scholarship. Whilst Govoni does briefly allude to barriers faced by racialized minorities in science, she does not make use of the extensive literature on decolonising the curriculum which similarly calls for a holistic re-evaluation of the science and mathematics curricula at universities [5].<sup>2</sup>

One of the self-professed aims of the book is to encourage more women to choose a career in mathematics, through providing role models. Such role models have been visible for over 100 years — and yet we still have systemic under-representation today. Throughout this book numerous barriers to mathematical research are showcased, many of which hold true today in some form: discrimination in secondary education; unequal opportunities for graduate research; discriminatory hiring and promotional practices; and the difficulties of balancing personal and professional aspirations when domestic labour falls predominantly on women. However, the book also showcases the ways in which women actively navigated and circumvented these barriers: through the building of professional networks; finding and becoming mentors; even sharing living arrangements with a second family to distribute childcare and household labour.

As *Against All Odds* clearly demonstrates, even once a woman has chosen a career in mathematics, systemic barriers remain which cannot be solved by role models alone. It is therefore a shame that the authors did not take this opportunity to advocate for more radical ways to make a career in research *viable* for women. Women need more than to be inspired by the resilience of those whose lives they cannot possibly emulate, they need equal access to education and professional networks, and the space and funding to successfully conduct research.

*Against All Odds* is an important contribution to the history of women in mathematical research, noting however that it fails to acknowledge the paths to research built and taken by women of colour. As well as names, faces, and stories of those whose contributions to knowledge have often been overlooked in the history of mathematics, the book provides an expansive view of both historical and persistent barriers to mathematical research.

## FURTHER READING

[1] E. Kaufholz-Soldat, N. M. R. Oswald (eds), *Against All Odds: Women's Ways to Mathematical Research Since 1800*, Springer, 2020.

[2] N. Rollock, *Staying Power: The career experiences and strategies of UK Black female professors*, Feb 2019, [https://www.ucu.org.uk/media/10075/Staying-Power/pdf/UCU\\_Rollock\\_February\\_2019.pdf](https://www.ucu.org.uk/media/10075/Staying-Power/pdf/UCU_Rollock_February_2019.pdf)

[3] M. Lee Shetterley, *Hidden Figures: The untold story of the African American women who helped win the space race*, William Collins, 2016.

[4] *Mathematicians of the African Diaspora*: [www.mathad.com](http://www.mathad.com)

[5] Open University, 'Innovating Pedagogy 2019. Exploring new forms of teaching, learning and assessment, to guide educators and policy makers', 2019.



**Brigitte Stenhouse**

Brigitte Stenhouse is a PhD student in History of Mathematics at the Open University, UK. Her research looks at the work of Mary

Somerville (1780–1872), and considers questions around translations; differential calculus in early-19th-century France and Great Britain; and gendered access to knowledge. She has recently discovered *She-Ra* and the *Princesses of Power* and thinks *Entrapta* is the best representation of a scientist that she has ever seen on TV.

<sup>2</sup>Thank you to Dr Jenny Douglas, Open University, for bringing this literature to my attention at her 'Decolonising the STEM Curriculum' seminar, October 2020: slides available here [tinyurl.com/y668xh5e](https://tinyurl.com/y668xh5e).

## Obituaries of Members

### Lucien P. Foldes: 1930–2021



Professor Lucien Foldes, who was elected a member of the London Mathematical Society on 15 February 1973, died on 10 February 2021, aged 90.

*Adam Ostaszewski writes:*  
Lucien is remembered as

a pioneering innovator of mathematical finance, mediating between a fledgling mathematics theory and established economics theory, with martingale utility-gradient methods his legacy. Engaging and collegial, impeccable in both style and mathematics, this was a true gentleman of the old order — born in Vienna of Austrian–French–Hungarian parentage. He emigrated to England in summer 1938 with his mother, ordained by an employment-bound visa to a post as cook to Professor Morris Ginsberg (LSE Sociology). He attended Bunce Court School in Faversham, Kent (1938–1945) and Monkton Wyld School in Charmouth, Dorset (1945–1947), both progressive, then as a second-year entrant into LSE (from Regent Street Polytechnic) he read for the B. Com. (Industry and Trade). In 1950 he gained First Class Honours and in 1952 an MSc (Econ.) in Business Administration. Appointed in 1951–52 Assistant in Economics at LSE, then in 1954, on returning from National Service, Assistant Lecturer, he was promoted to Lecturer in 1955, Reader in 1961 and Professor in 1979, retiring as Emeritus Professor of Economics in 1996.

In the 1950s his thesis-led research concentrated on delegation in budgeting and control of public enterprise. During the 1960s he turned to microeconomics and welfare (investment, redistribution, monopoly), then to quantitative decision models, with focus risk and uncertainty. Deeper immersion into mathematics (analysis and probability theory — self-taught) produced a foundational contribution on expected utility and then a slew of papers in stochastic analysis of risk in investment decisions.

Among the first to use martingale methods in financial theory as optimality conditions, he was the first to model prices with general semimartingales

and to introduce random measures. As recognized expert, he lectured on Semimartingale Calculus in Portfolio Theory at the 1992 Oberwolfach Conference on Mathematical Finance.

Foldes first considered optimal saving and consumption planning in continuous time for risky returns to capital, establishing existence of an optimal plan, characterised via martingale properties of shadow prices and finite welfare conditions.

Later he extended this framework to the Ramsey-style, Arrow-Kurz growth model, encompassing a production function whereby capital and population determine output, introducing Brownian uncertainty into the various components. Here his martingale condition reduces to a pair of first-order nonlinear ODEs, connecting ‘average propensity to consume out of capital’ and ‘elasticity of consumption with respect to capital’. Phase-plane arguments for economically significant parameters establish uniqueness of a trajectory through the unique pair of stable and unstable asymptotic nodes of the system (as capital grows high or low), yielding the optimal consumption function. His geometric approach draws in novel tools from stable manifold theory for evaluating the impact of perturbed economically significant parameters, ‘bearing on important classical questions of economic theory [whose resolution] should supersede various scattered results in the literature...[albeit not] related directly to statistical data...’ (Foldes).

His continued investigations were slowed by the poor condition of his heart, and his passing was precipitated by an accidental fall. He leaves widowed Carol Foldes, his constant mainstay and devoted wife (one may add also his *tex-amanuensis*) in a partnership of over 50 years, also forged within the LSE community.

### Robin L. Hudson: 1940–2021



Professor Robin Lyth Hudson, who was elected a member of the London Mathematical Society on 17 October 1974, died on 12 January 2021, aged 80.

*Dave Applebaum and Martin Lindsay write:*

Born in Aberdeen on 4 May 1940, Robin grew up in Kidderminster, Stourbridge and Disley. He was an



undergraduate at Oxford, contemporary with David Elworthy and Aubrey Truman (subsequent life-long friends), and stayed on for a DPhil under John T. Lewis' guidance. Robin was appointed assistant lecturer at the University of Nottingham in 1964, promoted to a chair in 1985 and served as Head of Department from 1987 to 1990. He spent sabbatical semesters in Heidelberg (1978), Austin Texas (1983) and Boulder Colorado (1996). After taking early retirement in 1997, he held part-time research posts at Nottingham Trent University (1997–2005), the Slovak Academy of Sciences (1997–2000) and Loughborough University (2005–21), and a visiting professorship at the University of Łódź (2002) which awarded him an honorary doctorate in 2013. Robin had 14 PhD students, including the two of us, and has a growing number of descendants; several postdocs and other research students also benefitted immensely from his mentorship. Starting when they were students, and for 20 years thereafter, Robin and his Russian-speaking wife Olga translated articles for Russian Mathematical Surveys.

Robin is best known for his highly creative contributions to mathematical physics and quantum probability. An early result, now known as Hudson's theorem in quantum optics, shows that the pure quantum states with positive Wigner function are the Gaussian ones. Together with PhD students, Robin established one of the first quantum central limit theorems, proved an early quantum de Finetti theorem, and introduced quantum Brownian motion as a non-commuting pair of families of unbounded operators, using the formalism of quantum field theory. From the early 1980s he developed a very fruitful collaboration with K.R. Parthasarathy of the Indian Statistical Institute (Delhi Centre). They established a (bosonic) quantum stochastic calculus of creation, preservation and annihilation processes that simultaneously generalises the Itô calculus for Brownian motion and that of the Poisson process, and incorporates a fermionic stochastic calculus too. The resulting theory provided new insights into the quantum theory of irreversible processes, and significant applications in quantum optics and quantum control theory. It has also led to a wealth of theoretical developments, such as quantum stochastic flows, introduced by Robin himself, and quantum Lévy processes. Much later, he developed a novel theory of quantum stochastic double product integrals which, within his extensive collaboration with Sylvia Pulmannová from Bratislava, he related to quantum Yang-Baxter equations and the quantisation of Lie bialgebras. In his most recent papers Robin

also explored their implications for quantum Lévy area.

Robin was passionate in his love of mathematics; an inspiring teacher and a dedicated PhD supervisor. He loved hillwalking and music, performing Bach piano fugues himself to accompany both his inaugural lecture in Nottingham and a public lecture *Rules of the Glass Bead Game: Reflections on Mathematics and Music* in Greifswald (2003). Robin also sang bass in the choral society in Southwell, where he moved with his family in 1977. His gentle personality and generous spirit endeared him to colleagues and friends worldwide, many of whom were welcomed to Robin and Olga's home. He will be fondly remembered and sorely missed.

Robin is survived by his wife Olga and children Dan, Hugh, Lucy and Michael.

## Brian H. Murdoch: 1930–2020



Dr Brian Murdoch, who was elected a member of the London Mathematical Society on 12 May 1955, died on 9 December 2020, aged 90.

*Elizabeth Oldham writes:*  
Brian was born in

England on 3 April 1930, but grew up in Dublin, Ireland. In 1947, he entered the School of Mathematics in Trinity College Dublin, the university to which so much of his life was devoted. He was an outstanding student, being awarded a Foundation Scholarship during his first undergraduate year. (Scholars of Trinity are selected on the basis of a special examination, typically taken during students' second of four undergraduate years; some very able students achieved "Schol" in their first year.) After graduating in 1951 with a Gold Medal, he went to Princeton, where he studied with William Feller and completed his PhD dissertation on preharmonic functions. Following that, he held posts at Newcastle University and Queen's University Belfast; he then returned to his alma mater as a Junior Lecturer in 1957. The rest of his career was spent in Trinity. Among the topics addressed in his published papers were preharmonic functions and random walks. He gave much time and thought to his teaching, and former students remember his kind and supportive approach. He was elected a Fellow of the College in 1965, and became Erasmus Smith's Professor of



Mathematics in spring 1966, holding that post until his retirement in 1989.

The hallmark of Brian's long period in the chair was dedicated service. When first appointed, he brought welcome stability after each of his two predecessors had come and gone within a couple of years. In autumn 1966, he was joined in the School of Mathematics by David Spearman, who had been appointed as University Professor of Natural Philosophy. Working as a team, they transformed the School of Mathematics from one that focused on the development of a small number of outstanding mathematicians to one that was more accessible and had a wider vision: producing considerably more graduates — many of whom would take their sound mathematical knowledge productively into aspects of life other than academic — while still catering for specifically dedicated mathematicians. The programme was broadened by giving students access to modules in computer science and statistics. A further initiative was the introduction of joint degrees in theoretical physics, in mathematics and economics, and in mathematics and philosophy. These were pioneering individually designed courses which were introduced long before "2-subject Moderatorships" were adopted into the University curriculum.

Brian was a member of the Religious Society of Friends (Quakers), and his Quaker principles underpinned his life. This was reflected, not only in his selfless dedication to the School of Mathematics and its students, but also in his leadership style — he strove tirelessly for consensus — and in his service to various bodies outside the college at local and national level. In the field of school education, for instance, he contributed effectively both to individual schools and to the National Council for Curriculum and Assessment, in the latter case helping to formulate upper secondary mathematics courses. He was also actively engaged with the Young Scientist Exhibition.

Brian was a kind and friendly man and is fondly remembered by his colleagues. He is survived by his wife Winifred (Winnie), their children Hazel, Peter and Fiona, and six grandchildren.

## Death Notices

We regret to announce the following deaths:

- Frank R. Drake, formerly of University of Leeds, who died on 16 May 2021.
- John C. Polkinghorne FRS, formerly of the University of Cambridge, who died on 9 March 2021.
- Lynne H. Walling, University of Bristol, who died on 28 May 2021.

## Cryptography and Coding Conference

Location: Online  
 Date: 14–15 December 2021  
 Website: [tinyurl.com/4k3wf39b](https://tinyurl.com/4k3wf39b)

This is the 18th IMA international conference on cryptography and coding theory. Original research papers on all technical aspects of cryptography and coding theory are solicited for submission by 16 July 2021. The IMA will be hosting the online event.

## Mathematics in Signal Processing

Location: Aston, Birmingham  
 Date: 18–20 May 2022  
 Website: [tinyurl.com/2mraf956](https://tinyurl.com/2mraf956)

This is the 12th IMA international conference on signal and information processing. Contributed papers are invited on all aspects of mathematics in signal processing and will be accepted on the basis of a 300–500 word abstract which should be submitted by 7 January 2022 via [my.ima.org.uk](https://my.ima.org.uk).

## Rational Points on Higher-Dimensional Varieties

ICMS, Edinburgh; 25–29 April 2022

Website: [tinyurl.com/y7en9a4d](https://tinyurl.com/y7en9a4d)

The aim of the meeting is to bring together leading experts and early career researchers to make progress on understanding rational points

on surfaces and higher dimensional varieties. The emphasis in the meeting will be on building upon this bridge and further inspiring collaboration between the analytic and geometric communities.

## LMS Prospects in Mathematics

Organised by UEA, posted online by ICMS, 9–10 September 2021

Website: [icms.org.uk/events/event/?id=1160](https://icms.org.uk/events/event/?id=1160)

All those who are considering applying for a Mathematics PhD in 2022 are invited to attend the 2021 LMS Prospects in Mathematics Meeting.

The meeting will feature a range of speakers from a wide range of mathematical fields across the UK who will discuss their current research and what opportunities are available to you:

### Speakers:

Algebra: Radha Kessar (City)  
 Analysis: Peter Topping (Warwick)  
 Continuum Mechanics: Angela Mihai (Cardiff)  
 Geometry: Frances Kirwan (Oxford)

Logic: Nicola Gambino (Leeds)  
 Mathematical Biology: Mark Blyth (East Anglia)  
 Mathematical Physics: Paul Sutcliffe (Durham)  
 Nonlinear Systems: Sara Lombardo (Loughborough)  
 Number Theory: Tim Dokchitser (Bristol)  
 Numerical Analysis: Françoise Tisseur (Manchester)  
 Operational Research: Tri-Dung Nguyen (Southampton)  
 Probability: Dmitry Korshunov (Lancaster)  
 Statistics: Surajit Ray (Glasgow)  
 Topology: Sarah Whitehouse (Sheffield)  
 PhD Careers in Industry: Erica Tyson (IMA)

For further programme details and to register, see the website. The registration deadline is 1 September 2021.

Covid-19: Owing to the coronavirus pandemic, many events may be cancelled, postponed or moved online. Members are advised to check event details with organisers.

## Society Meetings and Events

### July 2021

- 2 Summer General Society Meeting (online)

### September 2021

- 1-10 Northern Regional Meeting, Conference in Celebration of the 60th Birthday of Bill Crawley-Boevey, University of Manchester

- 9-10 LMS Prospects in Mathematics Meeting (online)

### January 2022

- 4-6 South West & South Wales Regional Meeting, Swansea

## Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website ([www.lms.ac.uk/content/calendar](http://www.lms.ac.uk/content/calendar)). Please send updates and corrections to [calendar@lms.ac.uk](mailto:calendar@lms.ac.uk).

### July 2021

- 7-9 Nonlinearity and Coherent Structures, Loughborough University (492)
- 12-16 New Challenges in Operator Semigroups, St John's College, Oxford (490)
- 16 Harmonic Analysis and Partial Differential Operators, Loughborough University (494)
- 16-17 Python for A-Level Mathematics (494)
- 19-23 Rigidity, Flexibility and Applications, Lancaster University (492)
- 21-23 Research Students' Conference in Population Genetics, University of Warwick (493)
- 26-30 Young Geometric Group Theory X, Newcastle University (493)

### August 2021

- 12-14 Young Functional Analysts' Workshop, Lancaster University (493)
- 16-20 IWOTA, Lancaster University (481)
- 18-20 Young Researchers in Algebraic Number Theory III, University of Bristol (492)

### September 2021

- 1-3 Scaling Limits: From Statistical Mechanics to Manifolds, Cambridge (493)
- 8-10 Mathematics of Robotics Conference (online) (494)
- 9-10 Heilbronn Annual Conference 2021, Heilbronn Institute (493)
- 16-17 Statistics at Bristol: Future Results and You 2021, Heilbronn Institute (494)
- 19-24 8th Heidelberg Laureate Forum, Heidelberg, Germany
- 21-23 Conference in Honour of Sir Michael Atiyah, Isaac Newton Institute, Cambridge (493)

### November 2021

- 18-20 Mathematics in Times of Crisis, online (494)

### December 2021

- 14-15 Cryptography and Coding Conference, online (495)

## April 2022

25-29 Rational Points on Higher-Dimensional Varieties, ICMS, Edinburgh (495)

## May 2022

18-20 Mathematics in Signal Processing, Aston, Birmingham (495)

## July 2022

24-26 7th IMA Conference on Numerical Linear Algebra and Optimization, Birmingham (487)

# William Benter Prize in Applied Mathematics 2022

## Call for **NOMINATIONS**

The Liu Bie Ju Centre for Mathematical Sciences of City University of Hong Kong is inviting nominations of candidates for the William Benter Prize in Applied Mathematics, an international award.

### The Prize

The Prize recognizes outstanding mathematical contributions that have had a direct and fundamental impact on scientific, business, financial, and engineering applications.

It will be awarded to a single person for a single contribution or for a body of related contributions of his/her research or for his/her lifetime achievement.

The Prize is presented every two years and the amount of the award is US\$100,000.

### Nominations

Nomination is open to everyone. Nominations should not be disclosed to the nominees and self-nominations will not be accepted.

A nomination should include a covering letter with justifications, the CV of the nominee, and two supporting letters. Nominations should be submitted to:

#### Selection Committee

c/o Liu Bie Ju Centre for Mathematical Sciences

City University of Hong Kong

Tat Chee Avenue, Kowloon, Hong Kong

Or by email to: [lbj@cityu.edu.hk](mailto:lbj@cityu.edu.hk)

**Deadline for nominations: 30 September 2021**

### Winner of the Prize 2020

The 2020 Prize went to Michael S. Waterman (University Professor Emeritus at the University of Southern California, Distinguished Research Professor at Biocomplexity Institute, University of Virginia). Due to the pandemic of Covid-19, the award ceremony will be held in summer 2022 at the **International Conference on Applied Mathematics**.

### Presentation of the Prizes 2020 and 2022

The recipient of the Prize (2022) will be announced at the **International Conference on Applied Mathematics** to be held in summer 2022. The Prize Laureates (2020 and 2022) are expected to attend the award ceremony and present a lecture at the conference.

The Prize was set up in 2008 in honor of Mr William Benter for his dedication and generous support to the enhancement of the University's strength in mathematics. The inaugural winner in 2010 was George C Papanicolaou (Robert Grimmer Professor of Mathematics at Stanford University), and the 2012 Prize went to James D Murray (Senior Scholar, Princeton University; Professor Emeritus of Mathematical Biology, University of Oxford; and Professor Emeritus of Applied Mathematics, University of Washington), the winner in 2014 was Vladimir Rokhlin (Professor of Mathematics and Arthur K. Watson Professor of Computer Science at Yale University). The winner in 2016 was Stanley Osher, Professor of Mathematics, Computer Science, Electrical Engineering, Chemical and Biomolecular Engineering at University of California (Los Angeles), and the 2018 Prize went to Ingrid Daubechies (James B. Duke Professor of Mathematics and Electrical and Computer Engineering at Duke University).

The Liu Bie Ju Centre for Mathematical Sciences was established in 1995 with the aim of supporting world-class research in applied mathematics and in computational mathematics. As a leading research centre in the Asia-Pacific region, its basic objective is to strive for excellence in applied mathematical sciences. For more information about the Prize and the Centre, please visit <https://www.cityu.edu.hk/lbj/>



Liu Bie Ju Centre for  
Mathematical Sciences

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