LONDON MATHEMATICAL SOCIETY
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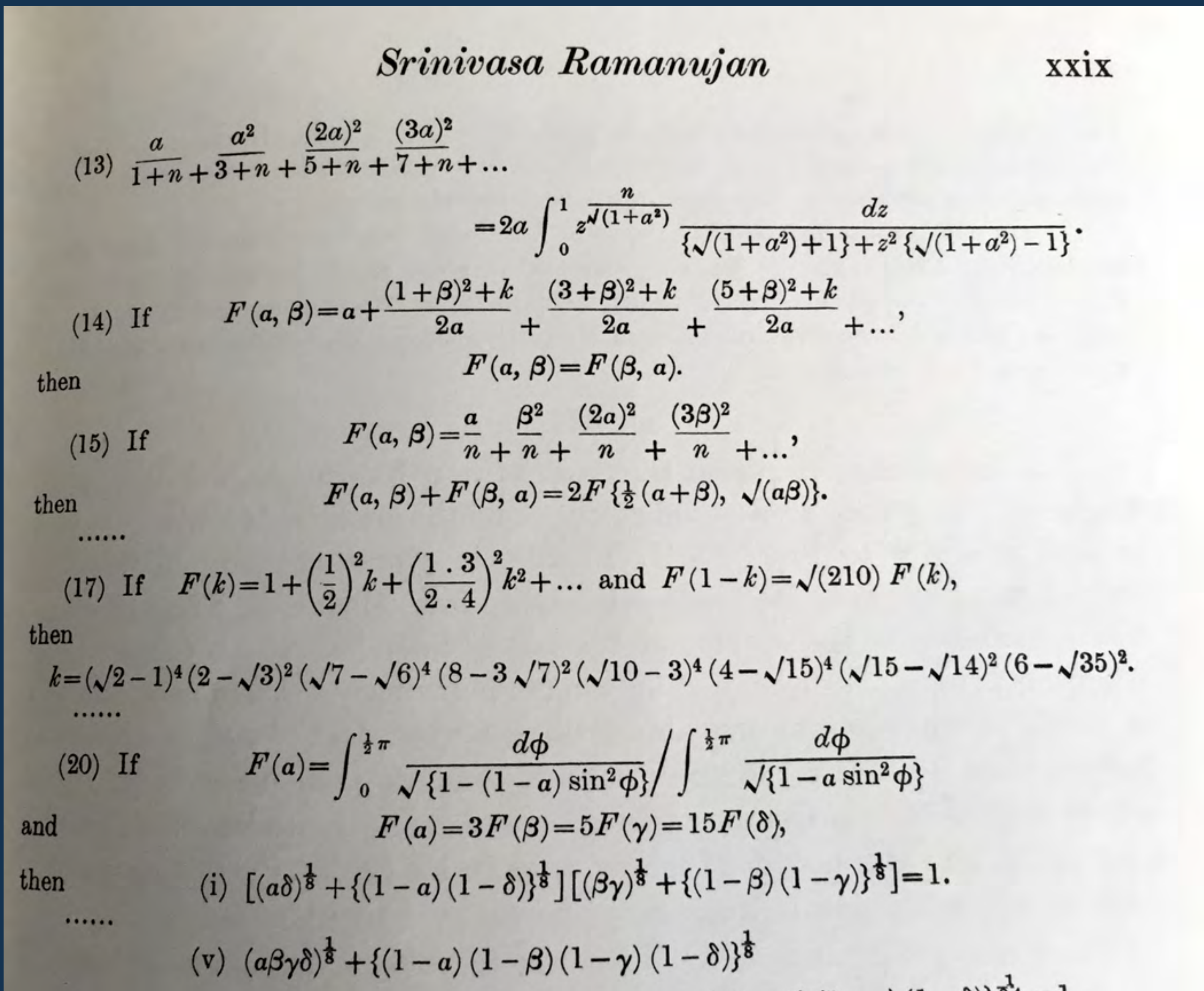
NEWSLETTER

Issue: 497 - November 2021



Srinivasa Ramanujan xxix

(13) $\frac{a}{1+n+}\frac{a^2}{3+n+}\frac{(2a)^2}{5+n+}\frac{(3a)^2}{7+n+}\dots$

$$=2a\int_0^1 z^{\frac{n}{\surd(1+a^2)}}\frac{dz}{\{\surd(1+a^2)+1\}+z^2\{\surd(1+a^2)-1\}}.$$

(14) If $F(\alpha,\beta)=\alpha+\frac{(1+\beta)^2+k}{2\alpha}+\frac{(3+\beta)^2+k}{2\alpha}+\frac{(5+\beta)^2+k}{2\alpha}+\dots,$

then $F(\alpha,\beta)=F(\beta,\alpha).$

(15) If $F(\alpha,\beta)=\frac{\alpha}{n+}\frac{\beta^2}{n+}\frac{(2\alpha)^2}{n+}\frac{(3\beta)^2}{n+}\dots,$

then $F(\alpha,\beta)+F(\beta,\alpha)=2F\{\tfrac{1}{2}(\alpha+\beta),\ \surd(\alpha\beta)\}.$

......

(17) If $F(k)=1+\left(\frac{1}{2}\right)^2k+\left(\frac{1\,.\,3}{2\,.\,4}\right)^2k^2+\dots$ and $F(1-k)=\surd(210)\,F(k),$

then

$$k=(\surd2-1)^4(2-\surd3)^2(\surd7-\surd6)^4(8-3\surd7)^2(\surd10-3)^4(4-\surd15)^4(\surd15-\surd14)^2(6-\surd35)^2.$$

......

(20) If $F(\alpha)=\int_0^{\frac{1}{2}\pi}\frac{d\phi}{\surd\{1-(1-\alpha)\sin^2\phi\}}\bigg/\int_0^{\frac{1}{2}\pi}\frac{d\phi}{\surd\{1-\alpha\sin^2\phi\}}$

and $F(\alpha)=3F(\beta)=5F(\gamma)=15F(\delta),$

then (i) $[(\alpha\delta)^{\frac{1}{8}}+\{(1-\alpha)(1-\delta)\}^{\frac{1}{8}}][(\beta\gamma)^{\frac{1}{8}}+\{(1-\beta)(1-\gamma)\}^{\frac{1}{8}}]=1.$

......

(v) $(\alpha\beta\gamma\delta)^{\frac{1}{8}}+\{(1-\alpha)(1-\beta)(1-\gamma)(1-\delta)\}^{\frac{1}{8}}$

EXTENDING BERTRAND'S POSTULATE

GUESSWORK: INFORMATION THEORY

FLOATING POINT NUMBERS & PHYSICS

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COVER IMAGE

Eye-catching excerpts from the *Collected Papers of Srinivasa Ramanujan*. From the article *Extending Bertrand's Postulate* (I. J. Zucker), page 27.

Do you have an image of mathematical interest that may be included on the front cover of a future issue? Email images@lms.ac.uk for details.

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NEWSLETTER WEBSITE

The Newsletter is freely available electronically at lms.ac.uk/publications/lms-newsletter.

MEMBERSHIP

Joining the LMS is a straightforward process. For membership details see lms.ac.uk/membership.

SUBMISSIONS

The Newsletter welcomes submissions of feature content, including mathematical articles, career related articles, and microtheses from members and non-members. Submission guidelines and LaTeX templates can be found at lms.ac.uk/publications/submit-to-the-lms-newsletter.

Feature content should be submitted to the editor-in-chief at newsletter.editor@lms.ac.uk.

News items should be sent to newsletter@lms.ac.uk.

Notices of events should be prepared using the template at lms.ac.uk/publications/lms-newsletter and sent to calendar@lms.ac.uk.

For advertising rates and guidelines see lms.ac.uk/publications/advertise-in-the-lms-newsletter.

CONTENTS

Methods of Assessment in the Mathematical Sciences

Following the experiences of the past 18 months some universities have proposed the removal of invigilated examinations or allowing them under very restrictive circumstances. This has caused some mathematics departments to ask for support from learned societies to encourage universities to consider this change more carefully. The London Mathematical Society, the Institute of Mathematics and its Applications and the Royal Statistical Society have produced a statement on this important issue, endorsed by the Edinburgh Mathematical Society and Heads of Departments of Mathematical Sciences. The full statement can be read at lms.ac.uk/node/1740.

Compositio Prize

Every three years, the Foundation Compositio Mathematica awards a prize for the best paper published in *Compositio Mathematica* in a three-year period. The Compositio Prize 2021 for the period 2017–19 has been awarded to the two papers:

Daniel Huybrechts, *The K3 category of a cubic fourfold*, Compositio Math. 153 (2017), 58–620.

Colin J. Bushnell† and Guy Henniart, *Local Langlands correspondence and ramification for Carayol representations*, Compositio Math. 155 (2019), 195–2038.

The article *The K3 category of a cubic fourfold* by Daniel Huybrechts establishes a collection of important results about Kuznetsov components of smooth cubic fourfolds. The Kuznetsov component is a subcategory of the derived category. Huybrechts proves that for a smooth cubic fourfold X, there are only finitely many isomorphism classes of fourfolds whose Kuznetsov component is Fourier–Mukai equivalent to that of X, and only one such class when X is very general. It also gives a criterion for the Kuznetsov component to be equivalent to the derived category of a twisted K3 surface. This influential article should pave the way to solving the mysterious rationality problem for cubic fourfolds.

The article *Local Langlands correspondence and ramification for Carayol representations* by Colin Bushnell and Guy Henniart constitutes major progress in the study of local Langlands correspondence. The authors establish a complete classification of Herbrand functions arising from totally wild simple characters of Carayol type. They use this to completely describe the ramification behaviour of the corresponding Weil group representations.

†Colin Bushnell sadly passed away on 1 January 2021.

Gerard van der Geer
President
Foundation Compositio Mathematica

Forthcoming LMS Events

The following events will take place in forthcoming months:

LMS Graduate Student Meeting: 8 November, online (tinyurl.com/hdp8ph8u)

LMS AGM: 12 November, London (lms.ac.uk/events/AGM2021)

LMS Computer Science Colloquium: 17 November, online (tinyurl.com/7u3e6xz7)

LMS–BCS/FACS Evening Seminar: 18 November, London and online (tinyurl.com/9d26u8ep)

LMS South Wales & South West Regional Meeting: 4–6 January, Swansea (tinyurl.com/3a2z36wc)

A full listing of upcoming LMS events can be found on page 51.

OTHER NEWS

Building a Digital Library: Mathematics On YouTube

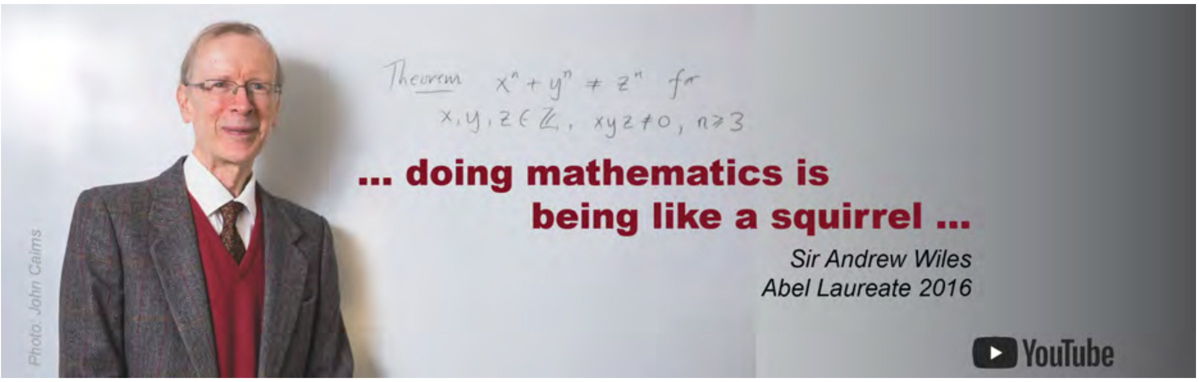


Illustration: Unni I. Kvam

When you have a gold mine of lectures with Abel Prize Laureates, you need a worldwide audience. To reach globally you need to go where people are; in this case, YouTube is the perfect match. In peak times, 1,000 hours are watched daily on 'The Abel Prize'. The most popular video is an interview with Sir Andrew Wiles.

From the early years in the history of the prize, the Abel board decided to film interviews with Abel Prize Laureates and lectures during the Abel week in Oslo. Since 2005, when Peter Lax was awarded the prize, this collection of films has grown to a valuable digital library. All were previously accessible at Abelprize.no, but few people found and watched the videos.

Four months before covid-19 was just a story we read about from Wuhan in China, we started to build a channel for the Abel Prize on YouTube. After Google, YouTube is the second largest search engine on the internet and is a highly regarded platform for digital content among viewers. During lockdown, we concentrated on building the channel with interviews, lectures, announcement of the prize and the award ceremony.

Today there are 700–900 viewers daily on the channel, and they watch on average 100 hours' worth of mathematics and laureates on a daily basis. More than 8,000 have subscribed to the channel in a relatively short period. More people than ever watch the announcement of the prize and the award ceremonies for László Lovász, Avi Wigderson, Hillel Furstenberg and Gregory Margulis have gathered an audience like there is no tomorrow.

The most popular of all films on our Abel Prize channel are those of Andrew Wiles, with his lecture about Fermat's last theorem and the interview by Martin Raussen and Christian Skau. In second place is a lecture by stand-up comedian and mathematician Matt Parker in 2019 when Karen Uhlenbeck received the prize, and in third place comes the acceptance speech by Abel laureate John Nash in 2015.

The videos are most popular with the 18–34 age group, while 93% of the audience is male. Here we have the potential to reach more women. The majority of viewers are located in India and the US. Typically, comments after the videos are similar to "The squirrel metaphor towards the end is simply great", and many questions are asked.

New viewers are fed in from other more popular science channels such as Flammable Math, Mathologer and Science Without the Gobbledygook. When you watch a video the search engine recommends other films based on your profile. Channels like Heidelberg Laureate Forum, Numberphile and the London Mathematical Society.

In future the Abel Prize will continue posting material with laureates and make more lectures available, says project manager for the Abel Prize Håkon Sandbakken. He also hopes to connect with other channels that spread content about mathematics, and hopefully continue to reach a young audience. The interviews with László Lovász and Avi Wigderson conducted by Bjørn Ian Dundas and Christian Skau are now in the editing room and due to be posted this fall on the channel.

Marina Tofting
Head of Communications
The Norwegian Academy of Science and Letters

June Barrow-Green Wins Royal Society Award

The 2021 Royal Society Wilkins–Bernal–Medawar Medal and Lecture has been awarded to Professor June Barrow-Green. Professor Barrow-Green received the prestigious award for 'her research in 19th and 20th century mathematics, notably on historical roots of modern computing, dynamical systems and the three-body problem. Her work places special emphasis on the under-representation of women in historical narratives and in contemporary mathematics. Her recent work includes decolonising of the mathematical curriculum'. The LMS is particularly pleased to see recognition for Professor Barrow-Green's work in support of equality of opportunity in the mathematics community, which the Society also strongly supports .

Throughout her career, Professor Barrow-Green has contributed extensively to the activities of the LMS, including as LMS Librarian and as a member of LMS Council (2008–18). In particular, her position as a renowned member of the history of mathematics community and her valuable historical knowledge has been at the heart of many important projects for the LMS.

The LMS celebrated its 150th Anniversary in 2015 and Professor Barrow-Green was an influential member of the 2015 Celebrations Committee and provided a wide range of ideas and input on a number of major Anniversary projects. These included the production of the LMS timeline from 1865 to the present day (tinyurl.com/bd5325ec) and organising and hosting a De Morgan Day at the Society's headquarters in London, celebrating the life and work of the first LMS President, Professor Augustus De Morgan.

Professor Barrow-Green was also involved in a range of other LMS activities, including as a member of the Website Development Group, LMS Personnel Committee, LMS Prizes Committee and LMS–IMA Zeeman Medal Committee, and she has also been the LMS Council Diarist. She has shared her invaluable experience as a member of the Standing Orders Review Group and the LMS Newsletter Editorial Board.

Past LMS President Wins Royal Society Medal

The Society is delighted to congratulate past LMS President Professor Dame Frances Kirwan on her recent award of the 2021 Royal Society Sylvester Medal.

Dame Frances received the Sylvester Medal for her research on quotients in algebraic geometry, including links with symplectic geometry and topology, which has had many applications. She has made significant contributions in moduli spaces in algebraic geometry, geometric invariant theory (GIT), and the link between GIT and moment maps in symplectic geometry. Her work endeavours to understand the structure of geometric objects by subtle investigation of their algebraic and topological properties, and her research led to the introduction of the Kirwan map. The Society is particularly pleased to see recognition for Professor Kirwan's work and the part she has played in the development and dissemination of mathematical knowledge, including to the global mathematical sciences community, which are core aims of the Society.

Professor Kirwan was LMS President from 2003-05 and she has contributed extensively to the business of the Society including as a member of Council, Prizes Committee, the Women in Mathematics Committee (now the Committee for Women and Diversity in Mathematics), the Publications Nominating Group and as an Editor of the LMS *Journal of Topology*. She was awarded a Whitehead Prize in 1989 and a Senior Whitehead Prize in 2013. Professor Kirwan was also the Society's Mary Cartwright Lecturer in 2002. Professor Kirwan is following in the footsteps of a number of past LMS Presidents who have won the medal, including Mary Cartwright (1964).

More information about the 2021 Royal Society Medal and Award winners is available at tinyurl.com/5v8v78hr.

MATHEMATICS POLICY DIGEST

UKRI Open Access Policy

In August 2021, following consultation with the research community and publishers, UKRI announced its new open access policy for research publications that acknowledge funding from one of its councils. More information is available at tinyurl.com/5ar4hxby.

This updated policy requires immediate open access to peer-reviewed research articles submitted for publication after 1 April 2022. The requirement for monographs, book chapters and edited collections published from 1 January 2024 is that they be made open access within 12 months of publication. Compliance with this policy can be achieved through the author making either the version of record or the accepted manuscript immediately open access under a CC-BY licence. A new Open Access policy for the next REF, expected to mirror the UKRI policy, is currently under development.

Digest prepared by Dr John Johnston
Society Communications Officer

Note: items included in the Mathematics Policy Digest are not necessarily endorsed by the Editorial Board or the LMS.

EUROPEAN MATHEMATICAL SOCIETY NEWS

The latest edition of the EMS Magazine (formerly *Newsletter*) is available now as a single PDF at euromathsoc.org/magazine. It is available to read via individual html pages at euromathsoc.org/magazine/issues/121. Highlights include a somewhat provocative message from the EMS President Volker Mehrmann (do we need large conferences and a multitude of prizes?), an article entitled *Almost impossible E8 and Leech lattices* by Maryna Viazovska and a conversation with Reuben Hersh.

EMS News prepared by David Chillingworth
LMS/EMS Correspondent

Note: items included in the European Mathematical Society News represent news from the EMS are not necessarily endorsed by the Editorial Board or the LMS.

LMS Prizes 2022: Call for Nominations

Deadline: 9 January 2022 (11:59pm)

The LMS invites nominations for the following prizes in 2022, which are intended to recognise and celebrate achievements in and contributions to mathematics. Regulations and nomination forms can be found at tinyurl.com/lmsprizes22.

- De Morgan Medal, which is the Society's premier award and for which the only grounds are the candidate's contributions to mathematics;
- Fröhlich Prize, which is awarded for original/innovative work in any branch of mathematics to a mathematician with fewer than 25 years' experience at post-doctoral level;
- Shephard Prize, for contributions to mathematics with a strong intuitive component which can be explained to those with little knowledge of mathematics;
- Senior Berwick Prize, which is awarded to the author(s) of a piece of research published by the Society between 1 January 2014 and 31 December 2021;
- Anne Bennett Prize, for work in and influence on mathematics, particularly acting as an inspiration for women mathematicians; and
- Whitehead Prizes, which are awarded for work in and influence on mathematics to mathematicians with fewer than 15 years' experience at post-doctoral level (up to six may be awarded).

We strongly encourage nominations for all prizes for women and other underrepresented groups in the mathematical community. The Prizes Committee interprets the criteria for all prizes broadly, so if in doubt please submit a nomination.

Return nomination forms to Katherine Wright, Society & Research Officer: prizes@lms.ac.uk.

The deadline for nominations is 9 January 2022 (11:59pm). Any nominations received after that date will be considered in the next award round.

Christopher Zeeman Medal 2022: Call for Nominations

The Councils of the LMS and the IMA invite nominations for the 2022 award of the Christopher Zeeman Medal, which is the UK award dedicated to recognising excellence in the communication of mathematics.

The IMA and the LMS wish to honour mathematicians who have excelled in promoting mathematics and engaging with the general public. Nominees may be academic mathematicians based in universities, mathematics school teachers, industrial mathematicians, those working in the financial sector or indeed mathematicians from any number of other fields.

Most importantly, these mathematicians will have worked exceptionally to bring mathematics to a non-specialist audience, whether it is through giving public lectures, writing books, appearing on radio or television, organising events or through an entirely separate medium. The LMS and IMA aim to celebrate the achievements of mathematicians who work to inspire others with their work.

The award is named after Professor Sir Christopher Zeeman, FRS, President of the LMS between 1986 and 1988. His notable career has been pioneering not only in the fields of topology and catastrophe theory but also through his ground breaking work in bringing his beloved mathematics to the wider public. Sir Christopher was the first mathematician to be asked to deliver the Royal Institution Christmas Lectures in 1978, a full 160 years since they began. His Mathematics into Pictures lectures have been cited by many UK mathematicians as their inspiration.

A form for nominations is available at tinyurl.com/zeeman22. Email any enquiries to Katherine Wright, Society & Research Officer, London Mathematical Society: prizes@lms.ac.uk. Nominations must be received by 28 February 2022.

Louis Bachelier Prize 2022: Call for Nominations

The Louis Bachelier Prize is a biennial prize jointly awarded by the London Mathematical Society, the Natixis Foundation for Quantitative Research and the Société de Mathématiques Appliquées et Industrielles. The Prize will be awarded to a mathematician who, on the 1st January of the year of its award, has fewer than 20 years (full time equivalent) of involvement in mathematics at postdoctoral level, allowing for breaks in continuity, or who in the opinion of the Bachelier Prize Committee is at an equivalent stage in their career.

The Prize will be awarded to the winner for their exceptional contribution to mathematical modelling in finance, insurance, risk management and/or scientific computing applied to finance and insurance. The prize winner will receive €20,000 including £5,000 to organise a scientific workshop in Europe on their area of research interests.

Nominations are now open for the 2022 Louis Bachelier prize; further details are at lms.ac.uk/prizes/louisbachelierprize. The closing date for nominations is 31 January 2022 (11:59pm). Nomination forms should be sent to prizes@lms.ac.uk.

LMS Grant Schemes

The next closing date for research grant applications (Schemes 1,2,4,5,6 and AMMSI) is 22 January 2022. Applications are invited for the following grants to be considered by the Research Grants Committee at its February 2022 meeting. Applicants for LMS Grants should be mathematicians based in the UK, the Isle of Man or the Channel Islands. For grants to support conferences/workshops, the event must be held in the UK, the Isle of Man or the Channel Islands:

Conferences (Scheme 1)

Grants of up to £5,500 are available to provide partial support for mathematical conferences held in the UK. This includes a maximum of £4,000 for principal speakers, £2,000 to support the attendance of research students and £1,000 to support the attendance of participants from Scheme 5 eligible countries.

Visits to the UK (Scheme 2)

Grants of up to £1,500 are available to provide partial support for a visitor who will give lectures in at least three separate institutions. Awards are made to the host towards the travel, accommodation and subsistence costs of the visitor. Potential applicants should note that it is expected the host institutions will contribute to the costs of the visitor. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents.

Research in Pairs and Research Reboot (Scheme 4)

The Research in Pairs grant is for those mathematicians inviting a collaborator, grants of up to £1,200 are available to support a visit for collaborative research either by the grant holder to another institution abroad, or by a named mathematician from abroad to the home base of the grant holder. For those mathematicians collaborating with another UK-based mathematician, grants of up to £600 are available to support a visit for collaborative research either by the grant holder to another institution or by a named mathematician to the home base of the grant holder. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents.

The Research Reboot grant is for those mathematicians who have found themselves without the time to engage in research due to personal circumstances, illness, caring responsibilities, increased teaching or administrative loads or other factors. Grants of up to £500 are available for accommodation, subsistence and travel to support a two to five day retreat, outside of their usual environment, to help restart research activity. An additional £500 can be applied for to cover Caring Costs for those who have caring responsibilities.

Collaborations with Developing Countries (Scheme 5)

For those mathematicians inviting a collaborator to the UK, grants of up to £3,000 are available to support a visit for collaborative research, by a named mathematician from a country in which mathematics could be considered to be in a disadvantaged position, to the home base of the grant holder. For those mathematicians going to their collaborator's institution, grants of up to £2,000 are available to support a visit for collaborative research by the grant holder to a country in which mathematics could be considered to be in a disadvantaged position. Applicants will be expected to explain in their application why the proposed country fits the circumstances considered eligible for Scheme 5 funding. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents. Contact the Grants team if you are unsure whether the proposed country is eligible, or check the IMU's Commission for Developing Countries definition of developing countries (tinyurl.com/y9dw364o).

Research Workshop Grants (Scheme 6)

Grants of up to £10,000 are available to provide support for Research Workshops. Research Workshops should be an opportunity for a small group of active researchers to work together for a concentrated period on a specialised topic. Applications for Research Workshop Grants can be made at any time but should normally be submitted at least six months before the proposed workshop.

African Mathematics Millennium Science Initiative (AMMSI)

Grants of up to £2,000 are available to support the attendance of postgraduate students at conferences in Africa organised or supported by AMMSI. Application forms for LMS-AMMSI grants are available at ammsi.africa.

The next closing date for research grant applications (Schemes 8–9 and ECR Travel Grants) is 22 February 2022. Applications are invited for the following grants to be considered by the Early Career Research Committee at its March 2022 meeting:

Postgraduate Research Conferences (Scheme 8)

Grants of up to £2,500 are available to provide partial support for conferences held in the United Kingdom, which are organised by and are for postgraduate research students. The grant award will be used to cover the costs of participants. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents.

Celebrating New Appointments (Scheme 9)

Grants of up to £400–£500 are available to provide partial support for meetings to celebrate the new appointment of a lecturer at a university. Potential applicants should note that it is expected that the grant holder will be one of the speakers at the conference. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents.

ECR Travel Grants

Grants of up £500 are available to provide partial travel and/or accommodation support for UK-based Early Career Researchers to attend conferences or undertake research visits either in the UK or overseas.

For full details of these grant schemes, and to find information on how to submit application forms, visit the LMS website: lms.ac.uk/content/research-grants. Queries regarding applications can be addressed to the Grants Administrator Lucy Covington (020 7927 0807, grants@lms.ac.uk), who will be pleased to discuss proposals informally with potential applicants and give advice on the submission of an application.

LMS–Bath Mathematical Symposia 2023: Call for Proposals

The London Mathematical Society is pleased to announce its Call for Proposals for the LMS–Bath Mathematical Symposia to be held at the University of Bath in 2023.

Further information, in particular regarding available funding, will be published on the Symposium website bathsymposium.ac.uk.

Formerly known as the LMS–Durham Symposia, the LMS–Bath Mathematical Symposia are being held at the University of Bath between 2020 and 2025. The Symposia are an established and recognised series of international research meetings, founded in 1974, that provide an excellent opportunity to explore an area of research in depth, to learn of new developments, and to instigate links between different branches.

The format is designed to allow substantial time for interaction and research. The meetings are by invitation only and will be held in July/August, with up to 50 participants, roughly half of whom will come from the UK. A novel element of the symposia is that they will be complemented by a summer school, to prepare young researchers such as PhD students, or a 'research incubator', where problems related to the topic of the conference are studied in groups. The entire event, summer school/incubator and workshop, will typically last around two weeks.

Prospective organisers should send a formal proposal to the Grants Team (Grants@lms.ac.uk) by 15 December 2021. Proposals are approved by the Society's Research Grants Committee after consideration of referees' reports.

Proposals should include:

- A full list of proposed participants, divided into specific categories:
  Category A – Scientific Organisers
  Category B – Key Overseas Participants
  Category C – Key UK-based Participants
  Category D – Important Overseas Participants
  Category E – Important UK-based Participants
- A full list of proposed participants, divided into specific categories:
  Category A – Scientific Organisers
  Category B – Key Overseas Participants
  Category C – Key UK-based Participants
  Category D – Important Overseas Participants
  Category E – Important UK-based Participants

- An indication that proposers have actively sought to include women speakers and speakers from ethnic minorities, or explain why this is not possible or appropriate.
- A detailed scientific case for the symposium, which shows the topic is active and gives reasons why UK mathematics would benefit from a symposium on the proposed dates.
- Details of additional support from other funding bodies, or proposed avenues of available funding.
- Indicative plans for the summer school or research incubator. Where appropriate, prospective organisers should consider the possibility of an 'industry day'.

For further details about the LMS Mathematical Symposia, visit the Society's website: lms.ac.uk/events/mathematical-symposia or the LMS-Bath symposia's website: bathsymposium.ac.uk.

Before submitting: Organisers are welcome to discuss informally their ideas with the Chair of the Research Grants Committee, Professor Andrew Dancer (grants@lms.ac.uk).

LMS Hardy Lectureship Tour 2023: Nominations Sought

The Society is seeking nominations for a Hardy Lecture Tour in 2023. The Hardy Lecturer visits the UK for a period of about 2–3 weeks, and gives the Hardy Lecture at a Society meeting, normally held in London in late June or early July. The Hardy Lecturer will also give at least six other lectures, on different topics, at other venues in the UK. The schedule is decided by the Programme Secretary in consultation with the Hardy Lecturer, and will be designed to allow as many UK mathematicians as possible to benefit from the Hardy Lecturer's presence in the UK.

The holder of the Hardy Lectureship shall be a mathematician who has not been normally resident in the United Kingdom of Great Britain and Northern Ireland for a period of at least five years, at the time of the award. Grounds for the award of the Lectureship include:

- the achievements of the Hardy Lecturer, including work in, influence on, and general service to mathematics;
- lecturing gifts and breadth of mathematical interests;
- the overall benefit the UK mathematical community might derive from the visit;
- the possibility of bringing to the UK a mathematician who might otherwise visit rarely or never.

The Hardy Lectureship is not restricted to mathematicians working in any specific area of mathematics. Previous lecturers include: 2022 Peter Sarnak (IAS Princeton), 2018 Lauren Williams (UC Berkeley), 2016 Jacob Lurie (Harvard) and 2015 Nalini Joshi (Sydney).

The London Mathematical Society will fund:

- travel and accommodation expenses (including travel to/from the UK and within the UK), up to £4,000;
- caring costs (including support towards the costs of an accompanying carer and/or children), up to £3,000;
- a contribution to the host department to hold a dinner for the Hardy Lecturer.

The host department(s) will be expected to provide a partial contribution towards accommodation costs, office accommodation and the academic support normally offered to a distinguished visitor.

Nominations must have the support of the host department(s), and should be sent by the Head of Department to the Chair of the Society, Lectures & Meetings Committee (lmsmeetings@lms.ac.uk). The closing date for proposals is 31 January 2022.

For further details and guidance on how to submit a nomination, please visit the Society's website: lms.ac.uk/events/lectures/hardy-lectureship.

LMS Undergraduate Summer School 2023: Call for Expressions of Interest

Since 2015 the London Mathematical Society has held an annual LMS Undergraduate Summer School aimed at introducing enthusiastic undergraduate students to modern mathematical research. The LMS Undergraduate Summer Schools take place for a two-week period in July and have proved very popular. The Society now seeks expressions of interest in hosting the LMS Undergraduate Summer School in 2023.

What is required of the Summer School host?

The host institution will be responsible for providing the infrastructure for the LMS Undergraduate Summer School to take place as a hybrid event. This will include:

- Catered accommodation for 50–55 undergraduates.
- Accommodation (if necessary) for lecturers.
- Lecture room(s) for the talks.
- Online facilities and engagement for virtual participation for up to 200 undergraduates.
- If necessary, providing PhD students to assist the LMS Undergraduate Summer School e.g. in exercises.
- Organising social activity and weekend excursions.
- Developing the programme of scientific content.

The host organisation would normally establish a local organising committee to manage the LMS Undergraduate Summer School, and must provide a specific local lead organiser who will be an academic in the Mathematics Department of the host institution. The local organising committee should also provide a designated contact in the institution's finance department who can manage payments.

How does the LMS support the organisation of the LMS Undergraduate Summer School?

The LMS Undergraduate Summer Schools are funded by an LMS Grant of £24,000 to be made to the host institution, together with payments made by the university departments of successful applicants (£250 per student attending in-person and £25 per student attending remotely), which are collected by the LMS and transferred to the host in full.

The LMS will administer the entire application process, including the collation of nominations from departments and handling any queries prior to the selection of participants by the host department. The School will be advertised by the LMS in bulletins, the LMS Newsletter and through social media. Host institutions may send 5-6 eligible undergraduates to the LMS Undergraduate Summer School.

The process to select a host

All Expressions of Interests received will be considered by the LMS Undergraduate Summer School Steering Group who will make recommendations to the LMS Early Career Research Committee. Previous hosts have been Loughborough (2015), Kent (2016), Manchester (2017), Glasgow (2018), Leeds (2019) and Swansea (2021 – online event). The 2022 School will be held at QUB.

Departments interested in hosting the LMS Undergraduate Summer School in 2023 are now asked to send a short (2 sides maximum) expression of interest to lmssummerschool@lms.ac.uk by 22 February 2022. Whilst the expression need not be detailed it should include the name of the person in the department who would act as local organiser. The expression of interest should be signed by the head of department.

For the LMS Undergraduate Summer School taking place in 2023, the nomination process would be opened to undergraduates from November 2022.

At this stage, interested departments are asked to provide an approximate budget, outline the facilities, catering and accommodation available and talk about their experience in hosting other similar events. More detailed work would be undertaken with potential hosts in due course. Host institutions are very welcome to make preliminary proposals for the scientific content of the LMS Undergraduate Summer School.

We hope that departments will be interested in working with the LMS to continue the level of success that the LMS Undergraduate Summer School programme has enjoyed in the years so far. Further details about the programme are available on the LMS website at tinyurl.com/3vsdcktx.

LMS Invited Lecture Series 2023: Call for Proposals

Proposals are invited from members and their departments to host the next annual LMS Invited Lectures Series in 2023. This annual lecture series consists of meetings held in the UK at which a single speaker gives a course of about ten expository lectures, examining some subject in depth, over a five day period (Monday to Friday) during a University vacation. The meetings are residential and open to all interested. Funding of up to £6,000 is offered to the host department to support the Invited Lecturer's costs and participants' attendance at the lectures.

Proposals for the Invited Lectures 2023

Any member who would like to suggest a topic and lecturer and be prepared to organise the meeting at their own institution or a suitable conference centre can submit a proposal. For further details visit the Society's website at tinyurl.com/3ewss6tw. The deadline for proposals is 1 February 2022.

LMS Invited Lecturer 2022

The LMS Invited Lecture Series 2022 on *The Mathematics of Deep Learning* will be given by Professor Gitta Kutyniok (Munich) at Cambridge University from 28 February to 4 March 2022. Further details available at tinyurl.com/3vr64rvs.

In addition, the postponed LMS Invited Lecture Series 2021 on equations in groups and complexity will be given by Professor Olga Kharlampovich (CUNY Graduate Center and Hunter College) at the Newcastle University (dates to be determined).

Recent previous Invited Lecturers:

- 2020: Yulia Mishura (University of Kyiv) *Fractional Calculus and Fractional Stochastic Calculus, including Rough-Paths, with Applications,* Zoom via Brunel University, 15–19 June 2020.
- 2019: Professor Søren Asmussen (Aarhus University), ICMS in Edinburgh, 20-24 May 2019.
- 2018 A. Owen (Stanford University) *From the Foundations of Simulation to Quasi Monte Carlo,* Warwick University, 9–13 July.

Enquiries about the Invited Lectures may be addressed to the Chair of the Society's Lectures and Meetings Committee, Brita Nucinkis (lmsmeetings@lms.ac.uk).

Atiyah UK-Lebanon Fellowships for 2022–23

Set up in 2020 in memory of Sir Michael Atiyah (1929–2019), whose father was Lebanese and who retained strong links with Lebanon throughout his life, the LMS Atiyah UK–Lebanon Fellowships operate in partnership with the Centre for Advanced Mathematical Sciences at the American University of Beirut (tinyurl.com/x5zhzpff).

The LMS Atiyah UK–Lebanon Fellowships provide for an established UK based mathematician to visit Lebanon as an Atiyah Fellow for a period of between one week and 6 months, or alternatively for a mathematician from Lebanon of advanced graduate level or above to visit the UK to further their study or research for a period of up to 12 months.

For visits from the UK to Lebanon, the Atiyah Fellowship will cover:

- From the LMS, up to £2,000 towards actual expenses for travel and related expenses, and accommodation and subsistence of £1,000 per month pro rata for up to 6 months.
- In addition, CAMS will cover accommodation and provision of office space and logistical support. This will be independent of the host institution.
- There is the possibility of additional subsistence/payment for agreed teaching.
- Consideration may be given for additional support to Fellows travelling with a family.

For visits to the UK from Lebanon, the Atiyah Fellowship will cover:

- From the LMS, up to £2,000 towards expenses for travel and related expenses, and accommodation and subsistence of £500 per month pro rata for up to 12 months.
- Additional support will be available for PhD and/or promising MSc candidates in either mathematics or mathematical physics.

Further information, including on how to apply, is available on the LMS website at tinyurl.com/e2u4wbek. The deadline is 31 January 2022. Queries should be addressed to fellowships@lms.ac.uk. The Chair of the Fellowship Panel is Professor Caroline Series FRS.

LMS Undergraduate Research Bursaries in Mathematics 2022

The Undergraduate Research Bursary scheme provides an opportunity for students in their intermediate years to explore the potential of becoming a researcher. The award provides support to a student undertaking a 6–8 week research project over Summer 2022, under the direction of a project supervisor.

Students must be registered at a UK institution for the majority of their undergraduate degree and may only take up the award during the summer vacation between the intermediate years of their course. Students in the final year of their degree intending to undertake a taught Masters degree immediately following their undergraduate degree may also apply.

Applications must be made by the project supervisor on behalf of the student.

For further information please contact Lucy Covington (urb@lms.ac.uk). Applications will be open from November 2021, with an application deadline of Tuesday 1 February 2022.

LMS Research Schools and Research Schools in Knowledge Exchange 2023

Grants of up to £15,000 are available for LMS Research Schools, one of which will be focused on Knowledge Exchange. The LMS Research Schools provide training for research students in contemporary areas of mathematics. The Knowledge Exchange Research Schools will primarily focus on Knowledge Exchange and can be in any area of mathematics.

The LMS Research Schools take place in the UK and support participation of research students from both the UK and abroad. The lecturers are expected to be international leaders in their field. The LMS Research Schools are often partially funded by the Heilbronn Institute for Mathematical Research (Heilbronn.ac.uk). Information about the submission of proposals can be found at tinyurl.com/ychr4lwm along with a list of previously supported Research Schools. Applicants are strongly encouraged to discuss their ideas for Research Schools with the Chair of the Early Career Research Committee, Professor Chris Parker (research.schools@lms.ac.uk), before submitting proposals. Proposals should be submitted to Lucy Covington (research.schools@lms.ac.uk) by 22 February 2022.

Clay Mathematics Institute Enhancement and Partnership Program

To extend the international reach of the Research School, prospective organisers may also wish to consider applying to the Clay Mathematics Institute (CMI) for additional funding under the CMI's Enhancement and Partnership Program. Further information about this programme can be found at tinyurl.com/y72byonb. Prospective organisers are advised to discuss applications to this programme as early as possible by contacting the CMI President, Martin Bridson (president@claymath.org). There is no need to wait for a decision from the LMS on your Research School application before contacting the CMI about funding through this programme.

Heilbronn Research Fellowships

in Pure Mathematics, Data Science, and Quantum Computing

Starting salary: £38,587-£43,434 (or local equivalent) depending on previous experience, plus the London weighting where appropriate. There is a salary supplement of £3.5K pa, in recognition of the distinctive nature of these Fellowships. In addition, a fund of at least £2.5K pa to pay for research expenses will be available to each Fellow.

3 years fixed term, full-time, with a preferred start date of 1 October 2022. Extensions of up to three years may be available, which can be held at a wider range of universities in the UK.

The Heilbronn Institute for Mathematical Research invites applications for Heilbronn Research Fellowships. Fellows divide their time equally between their own research and the classified research programme of the Heilbronn Institute. Research areas of interest include, but are not restricted to, Algebra, Algebraic Geometry, Combinatorics, Data Science, Number Theory, Probability, and Quantum Computing. Fellows have previously been appointed with backgrounds in most areas of Pure Mathematics and Statistics, and Mathematical/Theoretical Physics.

We expect to make up to eight appointments in **Pure Mathematics** at the University of Bristol, two at Imperial College London, three at King's College London, two at University College London, and four at the University of Manchester**.** In addition, we expect to make two appointments in **Data Science** and one in **Quantum Computing** at the University of Bristol.

Some of our Fellowships are funded through the UKRI/EPSRC 'Additional Funding Programme for Mathematical Sciences'.

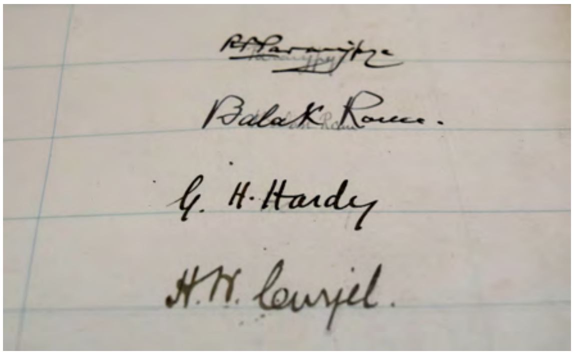
More information about the Heilbronn Institute and the application procedure may be found at our website http://heilbronn.ac.uk and the webpage https://heilbronn.ac.uk/about/fellowships/

Due to the nature of the Institute's work, Fellows must apply for a national security clearance before appointment. UK resident UK nationals will normally be able to meet this condition: other potential applicants should consult the Heilbronn Manager at himr-recruitment@bristol.ac.uk before applying.

The Heilbronn Institute is a supporter of the LMS Good Practice Scheme aimed at advancing women's careers in mathematics and we particularly welcome applications from women.

The application deadline is 11.59pm GMT, Sunday 14th November 2021.

Maximising your LMS Membership: Signing the LMS Members' Book



A unique benefit of London Mathematical Society membership is the opportunity for members to sign the Society's Members' Book. After election to membership, and once they have paid their subscription fees and been formally admitted to the Society, members can literally sign their name into the history of the London Mathematical Society. In a leather-bound volume that dates from the Society's founding in 1865, the Members' Book provides a physical link between current members and the first members of the Society such as Augustus De Morgan whose signatures appear in the early pages. Leafing through, one finds the signatures of other well-known mathematicians such as M.L. Cartwright, G.H. Hardy, Felix Klein and Henri Poincaré inscribed within the first half of the Members' Book.

In just over 150 years since the first autographs, just over half the pages have been filled, which leaves plenty of room for any members who have not yet signed to add their name. The LMS Members' Book appears at every in-person Society Meeting, as far as is practicable, and it has travelled across the globe so members based outside the UK are included. During the recent pandemic the LMS Members' Book has been kept safe and secure at De Morgan House. With the Annual General Meeting due to take place as a hybrid event, we invite those members who can attend in person on Friday 12 November at Goodenough College in London, and who have not yet done so, to come forward when invited by the President and sign the London Mathematical Society's Members' Book.

Valeriya Kolesnykova
Membership & Fellowships Assistant

Annual LMS Subscription 2021–22

Members are reminded that their annual subscription, including payment for publications, for the period November 2021 to October 2022 becomes due on 1 November 2021 should be paid no later than 1 December 2021. In September, the Society sent a reminder to all members to renew their subscription for 2021–22. If you have not received a reminder, please email membership@lms.ac.uk.

Members can now view and pay their membership subscriptions online via the Society's website: www.lms.ac.uk/user. Further information about subscription rates for 2021–22 and a subscription form may also be found on the Society's website at lms.ac.uk/content/paying-your-subscription.

The Society encourages payment by direct debit. If you do not already pay by this method and would like to set up a direct debit (this requires a UK bank account), please set up a direct debit to the Society with GoCardless.com via your online membership record: www.lms.ac.uk/user.

The Society also accepts payment by credit or debit card and by cheque. Please note card payments are now accepted online only and can be made via your online membership record: www.lms.ac.uk/user.

Benefits of LMS membership include free online access to selected Society journals, a complimentary bi-monthly Newsletter, discounts on selected Society publications and much more: lms.ac.uk/membership/member-benefits.

Elizabeth Fisher
Membership & Grants Manager

LEVELLING UP

The latest updates about the Levelling Up: Maths scheme being developed by the LMS, made possible by a generous donation from Dr Tony Hill. The scheme seeks to widen participation of those who are under-represented in mathematics. It is part of a broader Levelling Up: STEM project which also covers Physics and Chemistry.

Expanding the scheme

Major progress continues to be made as plans are put in place for signing up the next cohort of school students for the scheme. The LMS has developed the scheme from its inception, through the pilot phase, working with the universities of Leicester and Durham. The aim now is to strengthen and expand the scheme. The Society has been working with other interested universities and is particularly keen to broaden the geographical representation across England. The Society has now obtained firm commitments from three further universities to participate in the scheme.

The involvement of other mathematics organisations will also contribute greatly to the continued success of the scheme. In a very positive development, the Institute of Mathematics and its Applications (IMA) has expressed an interest in joining the LMS in administering the scheme. It will act as a mathematics 'hub' (ie coordinating the scheme with university 'spokes', including provision of tutorial material and recruitment of universities) alongside the LMS in the next phase of the scheme. It has been agreed that the most effective model going forward will be for the LMS and IMA to run separate hub and spoke models in parallel, i.e. the LMS and IMA will work with different groups of universities, rather than working together with the same universities. The Society is delighted to now be working in collaboration with the IMA, combining the strengths and connections of each organisation to reach a larger number of school students.

The IMA is anticipating coordinating the involvement of four additional universities, which would bring the total current number of universities in the Levelling Up: Maths scheme to nine. This increased commitment from institutions will provide an opportunity for over 270 school students to be part of the scheme.

The Society will continue to work closely with the IMA on this major collaboration and on communicating the commitment of both organisations to widening participation of those who are under-represented in mathematics.

More information about the Levelling Up Scheme is available at levellingupscheme.co.uk.

John Johnston
Society Communications Officer

REPORTS OF THE LMS

Report: LMS Prospects in Mathematics

On 9–10 September 2021, the Prospects in Mathematics meetings for final-year undergraduates went ahead in the 'new normal' fashion: a vibrant variety of research mathematicians giving presentations about postgraduate research opportunities in their respective fields to a Zoom meeting. Each half-hour presentation took the now-familiar form of the presenter sharing their screen with the group and discussing with flair their topic of expertise.

In every instance, Vanessa Miemietz and Shaun Stevens of UEA were friendly and interested hosts, taking active parts in the discussions both in the meetings and in the social events hosted on gather.town. Gillian Kerr of the ICMS provided invaluable technical and administrative support throughout the event.

The event began on Thursday with an overview of the structure, function and history of the LMS, with particular emphasis on its role in assisting PhD students and other researchers to travel to conferences (in better times) and its role as a publisher for research. With the institution introduced, the presentations could begin.

Radha Kessar of City, University of London outlined the dizzying variety of research topics within algebra. She began by helpfully dividing problems in the subject into two categories: existence and classification of different algebraic structures, and the description of these structures as representations. She followed with the examples of classifying basic abelian and non-abelian groups, and an overview of the representations of simple finite groups. Dmitry Korshunov of Lancaster gave an overview of Probability research, emphasising by example its importance in the insurance industry and, soberingly, the prediction of exponential growth or extinction of populations, including the spread of COVID-19. He made clear that probability is a key topic at the intersection of applied and pure mathematics, with bases and applications in both. Mark Blyth of UEA lifted the lid on the huge number of topics covered by Mathematical Biology, from the theory of PDEs behind the formation of leopard spots to the dynamics of tumour growth, giving special attention to his own specialism of arterial blood fluid mechanics. After an informal chat in the gather.town virtual environment, which was designed to mimic a conference experience, Erica Thompson of the Institute of Mathematics and its Applications summarised the career benefits of a PhD. She placed particular emphasis on the marketability of independent problem solving and the ability to tackle challenging tasks on a long-term basis. Sara Lombardo of Loughborough made clear the urgency of viewing the world as a collection of non-linear systems, in particular when predicting extreme weather events, such as this year's European floods. Surajit Ray of Glasgow presented machine learning research done in conjunction with several NHS Trusts to predict survivability of COVID patients, and stressed the benefits of statistical skills both in industry and academia. Next, Tim Dokchitser of Bristol introduced a range of topics within Number Theory, including its applications in cryptography and the potential for research opportunities and funding with the Heilbronn Institute, whose research focusses on national security applications of number theory.

Two panels during the event, one composed of supervisors and the other of PhD students, answered prospective students' questions about the application process and what it means to do a PhD. Both panels emphasised the importance of communication and relationship-building with one's supervisor, both before applying and while completing the PhD. The first day wrapped up with a humorous quiz given by Shaun in the gather.town environment.

With the format by now well established, Nicola Gambino of Leeds began Friday by outlining the various areas of Logic, including set and proof theory. He highlighted Gödel's Completeness Theorem, and its puzzling consequence that ZF set theory neither proves the continuum hypothesis, nor proves its complement. Frances Kirwan of Oxford gave an overview of the full spectrum of Geometry, and emphasised how it interacts, overlaps and bleeds into other areas of mathematics. Peter Topping of Warwick presented on curve-shortening flows, including the asymptotic 'Grim Reaper' curve which, when the flow is applied, shrinks every curve in its path. Sarah Whitehouse of Sheffield gave an introduction to Topology, beginning with Euler characteristics, Jones polynomials and other topological invariants before moving onto the more abstract algebraic side of continuous deformations of topological objects. Tri-Dung Nguyen of Southampton explored the origins of Operations Research in military operations, particularly resupplying the besieged city of Leningrad. He explored some large modern-day applications, from retail to healthcare, and outlined the specialised courses available. Francoise Tisseur of Manchester explored Numerical Analysis with a view of numerically computing solutions to systems of linear equations and eigenvalues of large matrices. Angela Mihai of Cardiff presented research into liquid crystal elastomers, and made clear that a continuum mechanic must have many mathematical skills in their toolbox. Paul Sutcliffe of Durham, appearing on Zoom as Shrek, talked about his interest in topological solitons. He used the example of an infinitely large, infinitely thin magnetic material, and explored the shape of magnetic skyrmions in relation to the position of a magnet. Paul explained the applications of these particles to data storage, describing the sheer volume of research published on the topic, and then extended the concept to magnetic hopfions. For me, after two long days of talks, a Dreamworks character discussing Mathematical Physics was a beautiful sight.

The entire event was wonderful. Watching people talk about their passions is a recipe for infectious excitement. Through the illuminating presentations, thoughtful questions and insightful answers, this year's event served to advertise to us the rich variety of mathematical research in the UK. The first steps for the next generation of mathematics PhD students are not only much clearer, but seem that much brighter.

Archie B. Coombe
Durham University Student

Report: Mathematics Communication Workshops

Earlier this year, I attended the first advanced sessions of the Mathematics Communication Workshops organised and hosted by the LMS. The workshops were part of a set of four interactive training sessions, two at a beginners level and two at an advanced level, which focused on the skills and techniques needed to communicate mathematics effectively. This year, they were designed and delivered by Katie Steckles and Ben Sparks, both part of *Talking Maths in Public* (see talkingmathsinpublic.uk).

Over the years, I'd given talks in local schools and at science cafés in and around Southampton, where I'm based, but while I felt the talks had gone over well (no pints thrown in the talks in pubs, for instance), it would be helpful to have a bit more structure around my planning and delivering those talks.

The workshop took place over two days, via Zoom (as we were still in lockdown back in late April and early May), with about 2.5 hours each day. There were 15 participants from a wide range of backgrounds and experience, though we all had done some public talks before. In the first session, we worked through different aspects of presentation, from the similarities between public maths talks and the structure of stand-up comedy, through to the very topical issue of particular challenges and opportunities that arise in giving a talk in a virtual environment e.g. via Zoom. Katie and Ben were very engaging, deeply experienced and able to handle all of the queries that the assembled crowd threw at them.

At the end of that first part of the workshop, I could already see some of the gaps in my current design and delivery of talks, and ways to fill and bridge those gaps. The second part of the workshop a couple of weeks after the first gave us the opportunity to do precisely that: take the discussion and information from that first session and put it to work. We were paired up and given the task of designing and delivering a short public-type talk to the rest of the participants. The exercise of working through that design and delivery, and the discussion after each of the talks, was very helpful indeed.

A couple of months after the workshops, at the end of June, I was scheduled to give a talk, again by Zoom (as were all things then) to a group of school children, as part of a local outreach project. While it was a talk I'd given a few times before, I took the opportunity to go back through the workshop materials and reflect on that experience, and rework the talk in light of what we'd done, and it was a better talk.

Whether you're a beginner at delivering public talks (and there is a beginner's version of the workshop as well) or you have significant experience, I recommend keeping an eye out for this workshop to come around again. Engaging, helpful and eminently worthwhile, has given me new things to think about, both for my public talks but also for my regular teaching, and it's always a pleasure to take advantage of the experience of others to do such things better.

Jim Anderson
LMS Education Committee

Report: LMS Northern Regional Meeting 2021

The 2021 LMS Northern Regional Meeting took place on the afternoon of Tuesday 7 September, as part of an online conference, which ran from 1–10 September, in celebration of the work of Bill Crawley-Boevey. The speakers were Ulrich Bauer (Technical University of Munich) and Karin Baur (University of Leeds).

There were somewhat over 80 people in (virtual) attendance.

Professor Ulrich Bauer talked about *Persistence in Functional Topology and Data Analysis*. Beginning with examples of holes in data and how one can deduce homology of shapes in R^d by replacing sets of sample points by balls of increasing radii around those points, this led to an exposition of persistent homology and hence persistence modules and persistence barcodes, including results of Crawley-Boevey on direct-sum decomposition of such modules. Limitations on extending these very general results beyond the 1-dimensional case were noted. The exposition then moved to stability of barcodes for functions, interleavings and discrete Morse theory, with an interesting look back to early results of Morse and others which can be seen as beginnings of persistent homology. This linked again to representation theory in that it focusses attention on certain, namely q-tame, persistence modules, where there are structure theorems by Chazal, Crawley-Boevey and de Silva. These themes were brought together with a recent result of Bauer, Medina-Mardones and Schmahl on conditions for the sublevel set filtration of a function to have q-tame persistent homology.

The talk was very nicely presented, making excellent use of the online environment, allowing the speaker, whose image was superimposed at the side of the 'slides', to point to parts of the slides.

Professor Karin Baur gave a lecture on *Surface Combinatorics and Module Categories*. This was a clear and wide-ranging survey of the use of combinatorics of surfaces to model algebraic objects. Starting with a triangulation of a marked surface one obtains, following Fomin, Shapiro and Thurston, an associated cluster algebra, generalising results of Fomin and Zelevinsky. Then the exposition moved to the cluster category of a n-gon, used by Caldero, Chapoton and Schiffler to give a geometric description of the cluster category introduced by Buan, Marsh, Reineke, Reiten and Todorov (defined more generally for quivers without oriented cycles). In this approach, algebraic objects — indecomposable representations, irreducible maps, extensions etc. — correspond to geometric/combinatorial objects and operations, and may be easier to describe in this way; for instance extensions correspond to crossings of curves on the surface. Many examples of the successful use of such correspondences were pointed to. There have been many extensions and further examples of these connections, for instance the connections between tilings and gentle algebras.

The latter part of the lecture described Scott's categorification of the coordinate rings of Grassmannians (Plücker coordinates corresponding to some cluster variables), leading to results of Baur, King and Marsh, which use the alternating strand diagrams of Postnikov, to give a combinatorial approach to the Grassmannian cluster categories of Jensen, King and Su.

Throughout the talk the ideas were clearly illustrated with many examples. At the end, a number of new directions were pointed to and questions raised.

Mike Prest
University of Manchester

Report: LMS Women in Mathematics Day

The University of Strathclyde was delighted to organise the London Mathematical Society *Women in Mathematics Day* on 16 June 2021. The event was run as an online event, hosted by the International Centre for Mathematical Sciences (ICMS) in Edinburgh. The event was hugely successful and well received by the organisers, speakers and attendees. The programme featured five invited talks, by female researchers at different stages of their careers, from different walks of mathematics, science and technology. The speakers included Claire Miller (Professor of Statistics, University of Glasgow), Paola Iannone (Senior Lecturer in Mathematics Education, University of Loughborough), Laura Ciobanu (Associate Professor at Heriot Watt University specialising in group theory), Jill Miscandlon (Senior Manufacturing Engineer, Advanced Forming Research Centre, University of Strathclyde) and Tiffany Wood (Senior Knowledge Transfer Fellow, School of Physics and Astronomy, University of Edinburgh). The speakers discussed their career trajectories, the opportunities and the challenges, their research programmes and outreach activities. The talks gave a holistic view of careers in mathematics, in academia and outside academia, particularly at the interface of mathematics, industrial research and knowledge exchange.

The talks were particularly stimulating for young researchers, who could see women thriving and succeeding in mathematics, and experience the camaraderie in the community. The research talks were accompanied by two short presentations by Diane MacLagan (Professor of Mathematics, University of Warwick) on funding opportunities by the London Mathematical Society and Jane Walker (Centre Manager, ICMS Edinburgh) on funding opportunities available at the ICMS. There are new funding opportunities, some of which are particularly tailored to people with caring responsibilities, people in under-represented groups or people who have been disproportionately affected by the pandemic. It is hugely important that young researchers are made aware of these opportunities.

Last but not the least, there were postgraduate talks and a lively virtual poster session, with ample scope for virtual networking. One of the postgraduate speakers was from Kansas State University in the USA!

The event garnered substantial interest from all around the world, with over 100 registered participants from the UK, Germany, Russia, Saudi Arabia and India. This was certainly one of the distinctive successes - the internationally diverse set of participants brought together by their passion for mathematics and its applications, not only for science but also for an equal and progressive society. The talks were recorded and will be made publicly available in due course. More details about the event, the talks and abstracts can be found at tinyurl.com/5hajnykb.

The organising committee at the University of Strathclyde were absolutely delighted that the event could go ahead in a fully virtual environment, and that the event could reach out to so many all around the world, and the committee thanks the London Mathematical Society profusely for its generous funding and support that facilitates mathematical activity around the UK.

Apala Majumdar
Organiser, University of Strathclyde

Report: LMS Spitalfields History of Mathematics Meeting

A Zoom meeting was held on 14 May 2021 to mark the launch of the *Educational Times* Digital Archive, held in partnership with University College London Special Collections and the LMS.

The *Educational Times* (ET) was a monthly newspaper acting as the mouthpiece of the College of Preceptors, an organisation set up in 1846. Concerned with raising the level of education in the country, the College voiced education concerns which existed before the landmark *Elementary Education Act* of 1870 came into force.

ET ran from 1847 to 1923. For volumes 1–69, 1847–1916, it had an active mathematics section and during its lifetime posted over 18,000 problems and published solutions to many of them. A companion, *Mathematical Questions and their Solutions* from the *Educational Times* (MQ) ran from 1864 to 1918, and was brought about by the energetic editor W.J.C. Miller serving 1862–1897.

In the digitised pages of ET, present day readers will find offerings from across the mathematical community, from writers of all social strata and academic positions. A leader such as J.J. Sylvester, for example, in at the beginning of the ET contributed over 400 problems, the last being in the month of his death in 1897, but equally there were prolific contributions from people we have never heard of, but deserve an airing.

Tony Rawlins (Brunel University) took us through a history of the ET and then gave a personal selection of leading mathematicians who posed and solved problems published in the ET. These were from the 'modern era' and included G. H. Hardy, M. W. Crofton, G. N. Watson, H. Bateman, T. J. I'a Bromwich and Constance Marks (who was editor of MQ 1902–1918), all of whom were members of the LMS.

Sarah Aitchison (University College London), Head of the Special Collections at UCL described how the jointly financed digitisation project began in 2018. The UCL College of Preceptors Archive consists of 93 boxes of which the ET is a part. With digitalisation, the run of ET (with a few gaps) is now freely available. (Readers interested in the history of education generally will also find a rich collection of resources in the Special Collections).

Norman Biggs (London School of Economics) singled out the Rev. Thomas P. Kirkman as one who made over a hundred contributions to ET. Famous in his day for the '15 Schoolgirls Problem' Kirkman was an ingenious mathematician who contributed significant work on group theory, enumerative theory of polyhedra and mathematical design (this including innovative work on finite geometries). To those mathematicians living in remote parts of the country, like Kirkman from a Lancashire village, ET was a lifeline. Indeed, ET saved Kirkman from total isolation in his latter years.

Sloan Despeaux (Western Carolina University, USA) made a comparison between ET and *Nouvelles Annales de Mathématiques* (NA) a journal which existed 1842–1927. Between the two journals there was a 'back and forth' communication with many authors contributing to both ET and NA. The aim of NA was similar to ET and it too contained a problem section. Sloan Despeaux's research (with Jim Tattersall) includes the construction of an electronic 'ET database'. This searchable databank complements the UCL Archive can be used as index to the full ET.

Both ET and NA are great resources for mathematically inclined readers, and for those interested in the social history of Britain and France. Digitalisation has put them onto the desks of readers from anywhere in the world. This is especially significant for ET as the few paper copies in existence are rare and are often in a poor state. Moreover, readers have a resource more complete than exists in national libraries (for example, the British Library). The digitalisation project is now operational and is still being updated. And don't forget, it is free!

Web resources

(1) Educational Digital Archive (at UCL): ucl.ac.uk/library/digital-collections.

(2) The ET database (WC US). Searchable 'index' to ET with cross-reference to MQ: educational-times.wcu.edu.

(3) *Nouvelles Annales de Mathématiques:* numdam.org/journals/NAM.

(4) Recordings of presentations of the 14 May 2021 meeting (time limited): icms.org.uk/events/event/?id=1137.

Tony Crilly

Report: 8ECM, Portorož, Slovenia

In July 1992 the newly founded European Mathematical Society held its first European Congress of Mathematics at the Sorbonne in Paris. Since then the week-long ECM meetings, designed 'to be a forum for discussion of the relationship between mathematics and society in Europe, and to enhance cooperation among mathematicians from all European countries', have been held every four years, usually midway between the International Congresses of Mathematicians (ICM).

Approval for the Eighth European Congress (8ECM) was given at the 7ECM meeting in Berlin in 2016, and the meeting was due to be held at the attractive seaside resort of Portorož in Slovenia in June 2020. But with the rise of the pandemic, it had to be postponed and was eventually held, with all talks presented online but with local presence also, from 20–25 June 2021.

Despite the difficulties, the meeting was a success. Under the control of the 8ECM Organising Committee Chair, Prof Tomaž Pisanski, it was organised locally by the University of Primorska and other Slovenian institutions with a supporting local team of 70 staff and volunteers, and attracted 1,771 participants from 77 countries, including 350 student members. The programme included over 1,000 talks, with 10 plenary speakers, 30 invited speakers and 6 public lectures, supplemented by 62 minisymposia and 9 satellite conferences.

The 8ECM postage stamp issued by the Slovenian Post Office

There were also exhibitions ranging from European Women of Mathematics and mathematical art to the St Andrews MacTutor history of mathematics website and a display of over 250 mathematical postage stamps with historical commentary, including a Slovenian stamp issued especially for this meeting. The film *Secrets of the Surface*, on the life and achievements of the Fields Medal winner Maryam Mirzakhani, was presented every evening throughout the week, and in addition to the academic programme there were social events for those who were able to attend in person.

Among the 'star' attractions at the meeting was a lecture by the 2021 Abel Prize winner László Lovász, and the founders of the EMS were acknowledged by an interview with Jean-Pierre Bourguignon and the Hirzebruch lecture given by Sir Martin Hairer of Imperial College London.

At these meetings the EMS traditionally awards prizes to young European mathematicians. This year's twelve prize winners, selected in 2020 by a prestigious international team chaired by Martin Bridson of Oxford University and the Clay Mathematics Institute, included two from the UK, Ana Caraiani (Imperial College London) and Jack Thorne (Cambridge). All prize winners gave lectures on their work.

The UK was well represented at the meeting with invited lectures given by Daniela Kühn (Birmingham), Richard Nickl (Cambridge), and Alison Etheridge, Nick Trefethen and Stuart White (Oxford), and a public lecture by myself (OU); Tim Gowers (Cambridge) took part in a public panel discussion. The London Mathematical Society was also much in evidence, with a daily lunchtime Q&A session and an LMS meeting that included a lecture by James Maynard of Oxford, introduced by the LMS President, Jonathan Keating.

The next European Congress of Mathematics (9ECM) will be held from 15–19 June 2024 in Seville, Spain.

Robin Wilson
Open University

Records of Proceedings at LMS meetings
Ordinary Meeting: 7 September 2021

This meeting was held virtually on Zoom, organised by the University of Manchester and hosted by the University of Bielefeld, as part of the Northern Regional Meeting & Workshop which was a celebration of the work and 60th birthday of Bill Crawley-Boevey. Over 100 members and guests were present for all or part of the meeting.

The Society meeting began at 1.00 pm BST with the LMS Vice-President, Professor Iain Gordon, in the Chair.

Due to the online nature of the meeting, no members signed the Member's Book and were admitted to the Society.

Professor Mike Prest (University of Manchester) introduced the first lecture given by Professor Ulrich Bauer (TU Munich) on *Persistence in Functional Topology and Data Analysis*.

Before the break between talks, a group photo was taken.

After the break, Professor Prest introduced the second lecture, given by Professor Karin Baur (University of Leeds) on *Surface Combinatorics and Module Categories*.

Professor Gordon thanked the speakers for their excellent lectures and then expressed the thanks of the Society to the organisers, Professor Mike Prest, Professor Henning Krause and Professor Sebastien Eckert (both of Bielefeld University) for a wonderful meeting and workshop. Professor Gordon conveyed his, and the Society's, appreciation of Bill Crawley-Boevey's inspirational mathematical work.

After the meeting closed, there were breakout rooms in which participants could meet and talk with the speakers.

Records of Proceedings at LMS meetings
Ordinary Meeting: 22 June 2021

This meeting was held virtually on Zoom, hosted by the 8th European Congress of Mathematics. 18 members and visitors were present for the Society meeting session.

The Society meeting began at 4.30 pm BST on 22 June with the LMS President, Professor Jon Keating, in the Chair. Professor Keating welcomed guests and thanked the organising parties. No new LMS members were elected at this meeting.

Due to the online nature of the meeting, no members signed the Member's Book and were admitted to the Society.

Professor Keating then introduced Professor James Maynard (University of Oxford) who spoke about *Approximating real numbers by fractions*.

Professor Keating concluded the meeting by thanking the speaker, organisers and meeting attendees on behalf of the LMS.

Records of Proceedings at LMS meetings
Ordinary Meeting: 14 May 2021

This meeting was held virtually on Zoom, hosted by ICMS, at the LMS Spitalfields History of Mathematics Meeting, in partnership with UCL Special Collections. 31 members and visitors were present for the Society meeting session.

The Society meeting began at 2.00 pm BST on 14 May with the LMS President, Professor Jonathan Keating FRS, in the Chair. Professor Keating welcomed guests, thanked the organising parties, and welcomed the election of 22 new LMS Members.

Twelve people were elected to Ordinary Membership: Mr Duncan Adamson, Miss Sinead Baker, Dr Tobias Barker, Dr Carl Barton, Dr Writambhara Chakraborty, Dr Jamshid Derakhshan, Dr Christopher Hampson, Professor Jan Obloj, Professor Cyprien Saito, Dr Iain Smears, Dr Entmont Stamatis and Professor Marie-Therese Wolfram.

Five people were elected to Reciprocity Membership: Mr Spencer Cobbs, Dr Can Hatipoglu, Professor Aaron Lauda, Dr Turlough Lynch and Mr Selvaraju Munandy.

Three people were elected to Associate Membership: Dr Bobby Cheng, Mr Benedikt Petko and Mr Michael Rosbotham.

Two people were elected to Associate (Undergraduate) Membership: Mr Darren Loroy and Mr Gabriel Smakaj.

Professor Keating then handed over to Dr Mark McCartney (LMS Librarian) to introduce Professor Tony Rawlins (Brunel), who spoke about *Some Mathematicians who Published and Solved Problems in the Educational Times*. Dr McCartney then introduced Sarah Aitchison (UCL), who spoke about UCL Special collections. During the break a PowerPoint presentation gave the attendees a flavour of the items held in the UCL Special Collections.

After the break Dr McCartney introduced Professor Norman Biggs (LSE) who spoke about *Kirkman and the Educational Times: Groups and Designs*. Dr McCartney then introduced a presentation given by Professor Sloan Despeaux (Western Carolina) on *Questions and Answers, Questions et Résponses: Exchange Between the Educational Times and the Nouvelles Annales de Mathématiques*.

Professor Keating concluded the meeting by thanking the speakers, organisers and meeting attendees on behalf of the LMS.

Records of Proceedings at LMS meetings
General Meeting: 2 July 2021

This meeting was held virtually on Zoom. Over 80 members and visitors were present for all or part of the meeting. The meeting began at 3.30 pm with the President, Professor Jon Keating FRS in the Chair.

Minutes of the Annual General Meeting, held on 20 November 2020, had been made available 21 days prior to the General Meeting. The President invited members to vote by an electronic poll, to ratify these Minutes. The Minutes were ratified by a majority.

On a recommendation from Council it was agreed to elect Professor Charles Goldie and Professor Chris Lance as scrutineers in the forthcoming Council elections. The President invited members to vote by an electronic poll to ratify Council's recommendation. The recommendation was ratified unanimously.

The President, on Council's behalf, proposed that the following four people be elected to Honorary Membership of the Society: Professor Bao Châu Ngô, University of Chicago, Professor Laure Saint-Raymond, Institut des Hautes Études Scientifiques (IHES), Professor Peter Sarnak, Institute for Advanced Study and Professor Ya-xiang Yuan, Chinese Academy of Sciences. This was approved by acclaim. The President read a short version of the citations, to be published in full in the *Bulletin of the London Mathematical Society*.

The President then announced the awards of the prizes for 2021:

Pólya Prize: Professor Ehud Hrushovski FRS (University of Oxford)
Senior Whitehead Prize: Professor Tara Brendle (University of Glasgow)
Naylor Prize and Lectureship: Professor Endre Süli (University of Oxford)
Berwick Prize: Dr Ailsa Keating (University of Cambridge)
Anne Bennett Prize: Dr Viveka Erlandsson (University of Bristol)
Whitehead Prizes: Dr Jonathan Evans (Lancaster University); Professor Patrick Farrell (University of Oxford); Dr Agelos Georgakopoulos (University of Warwick); Dr Michael Magee (Durham University); Dr Aretha Teckentrup (University of Edinburgh); Professor Stuart White (University of Oxford)

The President then announced the award of the joint LMS–BSHM Hirst Prize 2021: Professor Karine Chemla (Centre National de la Recherche Scientifique)

There were 20 nominations for elections to membership at this meeting.

Four Associate (undergraduate): Malek Alhajkhouder, Romeo King, Leonard Mushunje, Stefan Roberts.

Five Associate: Dr Gemma Crowe, Dr Farhana Akond Pramy, Dr Nathan Kirk, Dr Megchung Zhang, Dr Larry Read.

Nine Ordinary: Tarig Abelgadir, Emmanuel Akaligwo, Dr Dafydd Gwion Evans, Dr John Evans, Professor William Lee, Dr Ian Mackie, Lawrence Reilyy, Dmytro Tupchiienko, Dr Markus Upmeier.

Two Reciprocity: Dr James Haselman, Adnane Snoussi.

Because the meeting was held online, no members signed the Members' Book.

The President introduced a lecture given by Professor Emily Riehl (Johns Hopkins) on Elements of ∞-Category Theory.

At the end of the meeting, the President thanked Emily Riehl for her brilliant lecture, and also thanked Markus Land for his lecture *Infinity-categories in Algebra and Topology* at the Graduate Student Meeting on 24 June.

Extending Bertrand's Postulate

I.J. ZUCKER

In 1845 Joseph Bertrand originally conjectured that for any integer $n > 3$, there always exists at least one prime number p with $n < p < 2n - 2$. A less restrictive formulation is: For every $n > 2$ there is always at least one prime p such that $n < p < 2n$. It is this latter formulation that is now accepted as Bertrand's Postulate. This has been proved but it seems that no one has considered whether a smaller number might replace 2. Here it is suggested that $\frac{5}{3}$ is the smallest number that should replace 2 in the original conjecture.

I have reached an age where one may look back on past failures and regrets with some equanimity, but one lost opportunity has upset me to this day. So what is the source of this long-lasting annoyance? It had its origins as a sixth former studying for a higher school certificate — the A-level of my day — in mathematics, physics and chemistry. In those days, the late 1940s, the sole source of extra-curricular knowledge was the public library. Luckily one of the better such libraries was available to me in Whitechapel, where I spent many happy hours trawling through the abundant mathematics and science sections. I thought at one time I knew every book there when one day I noticed a book entitled *'Collected Papers of Srinivasa Ramanujan'* [1]. Nowadays, I suspect that anyone with some interest in mathematics has heard of Ramanujan, what with books, films and television programmes about him being accessible. But in 1949, this was not the case. I certainly had never met the name. Out of curiosity, I opened the book and in five minutes was entranced by his story and work. To this very day, whenever I open my own copy of that book the strangeness and beauty of some of the results still overwhelm me.

At the time of this first reading I doubt whether I understood any of the mathematics in the book, and how any of those beautiful relations were obtained. There were, however, two items which especially caught my eye (Figure 1). The first was from one of the letters sent by Ramanujan to Hardy [1]. It is the following statement. If

$$F(k) = 1 + \left(\frac{1}{2}\right)^2 k + \left(\frac{1\cdot 3}{2\cdot 4}\right)^2 k^2 + \cdots \quad \text{and} \quad F(1-k) = \sqrt{210}\,F(k) \tag{1}$$

then

$$k = \left(\sqrt{2}-1\right)^4 \left(2-\sqrt{3}\right)^2 \left(\sqrt{7}-\sqrt{6}\right)^4 \left(8-3\sqrt{7}\right)^2 \left(\sqrt{10}-3\right)^4 \left(4-\sqrt{15}\right)^4 \left(\sqrt{15}-\sqrt{14}\right)^2 \left(6-\sqrt{35}\right)^2 \tag{2}$$

The second was a tiny two page paper, *'A proof of Bertrand's postulate.'* [2]. I have no idea why these two items particularly caught my eye, but both played a part in my future. First let me dispose of the first item as quickly as possible. At the end of the 1940s, after the termination of WWII with the dropping of two nuclear bombs, physics was all the rage, and I was guided into doing a degree in that subject. Later on working for a PhD, while investigating the intermolecular force law between argon atoms using experimental data from argon frozen as a crystalline solid, I met with the following expression:

$$\sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \sum_{p=-\infty}^{\infty}{}' (m^2 + n^2 + p^2)^{-s}, \tag{3}$$

where $'$ excludes the case when (m, n, p) are simultaneously equal to zero. In general, s is a complex number, but my interest in it was restricted to s being a positive integer. Even so (3) seemed to me to be a three-dimensional analog of the zeta function, and to investigate that seemed to be a bit of jump. Instead, I began to look at two-dimensional versions of equation (3) and 20 years later, this led in [3] to a simple proof of (1).

However, it was Bertrand's postulate that is the origin of my abiding regret. First some facts about it. Joseph Bertrand, a French mathematician, made the conjecture in 1845 and it is usually stated as:

Conjecture (Bertrand's postulate). *For any number $n \geq 2$, there is at least one prime, p, such that $n < p \leq 2n$.*

Chebyshev proved the result in 1852. In 1932, Paul Erdos at the age of 18 first made a name for himself publishing a much simpler proof [4]. It was only then that Erdos was informed by a colleague of Ramanujan's proof. So Erdos looked it up and was taken by its beauty. It does not matter how small the number between 1 and 2, there is at least one prime, p, such that $n < p < (1+\epsilon)n$ for some $n \geq n_0$.

Surprisingly there seem to be only two such definite results known, namely:

Theorem (Nagura [5]). *For $n \geq 25$ there is always a prime, p, such that $n < p < (1 + \frac{1}{5})n$.*

Theorem (Schoenfeld [6]). *For $n \geq 2010760$, there is always a prime, p, such that $n < p < (1 + \frac{1}{16597})n$.*

Now I had always wondered what is the smallest number between 1 and 2 for which Bertrand's postulate was true, and a great opportunity to ask Paul Erdos himself arose in the following way. In 1981, I had a sabbatical to the Department of Physics at the Haifa Technion, Israel at the invitation of Joshua Zak. (He confessed to me that my major attraction to him was that in any paper published together his name would for once not be last — his desire was satisfied.) One day a notice appeared that Paul Erdos was going to give a talk in the Department of Mathematics, and this seemed a chance to ask him. So I went to his talk which was wide-ranging on problems in number theory. There were questions at the end and ready with my question I reminded him of Bertrand's postulate. He responded immediately:

> Chebyshev said it and I say again,
> There is always a prime between n and
> 2n.

I sat down — I had lost my nerve. I was in the middle of over thirty mathematicians and did not want to make a fool of myself asking a possibly trivial question. But another chance then occurred. The Technion is, or was then, a little off the beaten track and not very accessible by public transport. Erdos needed to get to Tel-Aviv and it fell to me to drive him to the Haifa bus station. I expected Erdos to talk mathematics but for once in his life this did not happen. Instead all he wanted to talk about were refuseniks — an unofficial term for individuals typically, but not exclusively, Soviet Jews who were denied permission to emigrate, primarily to Israel, by the authorities of the Soviet Union and other countries of the Eastern Bloc. You have to understand that in 1981 the Cold War had not ended. Many Jewish mathematicians at that time had been deprived of their jobs and had suffered in other ways. For example, Grigory Margulis was awarded a Fields Medal in 1978 and was refused permission to travel to Helsinki to receive it. Erdos must have known many of these refuseniks personally and was concerned about their safety. Indeed I believe Erdos was going to Tel-Aviv to visit Piatetski-Shapiro, one of the few refuseniks who had somehow managed to leave Russia. Thus once again, for the second time in an hour, I did not ask him my question, something I have forever regretted. For I am sure that Erdos would have found the answer and would have proved it, and I might have obtained the prestigious Erdos number of one instead of being stuck with the more pedestrian three.

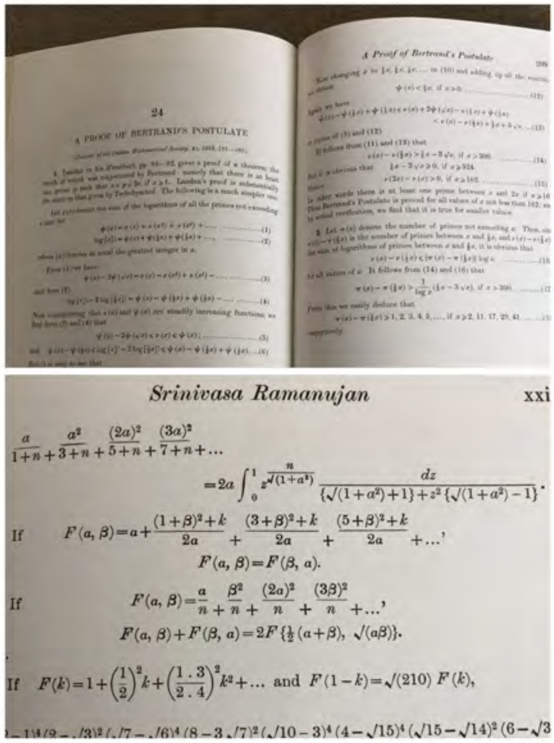


Figure 1. Eye-catching excerpts from *Collected Papers of Srinivasa Ramanujan*

Why is it that I wish to share this little bit of personal grief now? Well over the past few weeks in lockdown it suddenly occurred to me that the ratio of consecutive primes must be the factor determining what Bertrand type statements may be made. Thus starting with ratio $\frac{3}{2}$ we find that $2 \cdot \frac{3}{2} = 3$ just gets the prime 3, but fails with 3 itself since $3 \cdot \frac{3}{2} = \frac{9}{2}$ and no prime is captured. On to the next pair of primes, we look at $\frac{5}{3}$. Between 2 and $2 \cdot \frac{5}{3}$ there is the prime 3. Between 3 and $3 \cdot \frac{5}{3}$ we just capture 5. From then on it would appear that for any number one might replace 2 in Bertrand's conjecture by $\frac{5}{3}$. I believe the smallest number between and 1 and 2 for which Bertrand's original conjecture is valid is $\frac{5}{3}$.

What occurs with other prime ratios? The next is $\frac{7}{5}$ and this fails to yield a prime between 7 and $\frac{49}{5}$. Whether this is the smallest number for which $\frac{7}{5}$ fails I do not know, but perhaps one should look for when the gaps between primes are large and the next three largest are $\frac{11}{7}$, $\frac{17}{13}$ and $\frac{23}{19}$. So this suggests to me the following:

Conjecture (Author's conjectures). *For $n \geq 2$ there is always a prime, p, such that $n < p < \frac{5}{3}n$.*

For $n \geq 8$ there is always a prime, p, such that $n < p < \frac{7}{3}n$.

For $n \geq 11$ there is always a prime, p, such that $n < p < \frac{11}{7}n$.

For $n \geq 17$ there is always a prime, p, such that $n < p < \frac{17}{11}n$.

For $n \geq 23$ there is always a prime, p, such that $n < p < \frac{23}{19}n$.

This last statement is of interest since $\frac{23}{19} = 1.2105\cdots$ starting at 23, which is close to Nagura's 1.2 starting at 25 which has been *proved*. And that is the point of this communication. When it comes to numbers anyone can make conjectures as above, but proof is everything. However, much as I have tried I am unable to prove any of the above assertions. So is anyone out there who can prove or disprove at least the first of those statements? The road to follow I believe would be Ramanujan's proof which was the method Nagura used.

Acknowledgments

I would like to thank my eldest grandson, Joshua Zucker, a mathematics student at Bristol University with whom I discussed this matter, and who persuaded me to write this.

FURTHER READING

[1] G. H. Hardy, "Obituary of Srinivasa Ramanujan", p xxix, Collected Papers, Ramanujan, C.U.P., (1927).
[2] S. Ramanujan, "A Proof of Bertrand's Postulate", J.Indian Math.Soc., XI, p181–182, (1919).
[3] I. J. Zucker, "The evaluation in terms of Γ-functions of the periods of elliptic curves admitting complex multiplication", Math.Proc.Camb.Phil.Soc.82, p111–118, (1977).
[4] P. Erdos, "Beweis eines Satzes von Tschebyschef", Acta Litt. Sci. (Szeged) (in German), 5, p194–198, (1932).
[5] J. Nagura, "On the interval containing at least one prime number", Proceedings of the Japan Academy, Series A. 28 (4), p177–181, (1952).
[6] L. Schoenfeld, "Sharper Bounds for the Chebyshev Functions θ(x) and ψ(x), II", Mathematics of Computation. 30 (134), p337–360, (1976).

John Zucker

John Zucker has spent his working life in academia, in the Physics departments of City University, University of Surrey, and King's College, London, and is now retired. He is a mathematician *manqué*, and on his retirement collaborated with two similar souls — Geoff Joyce and Richard Delves — excavating abandoned 19th century mathematical mines and finding gems here and there.

Guesswork

KEN R. DUFFY

How hard is it to guess an encryption key, a password, or the locations of errors in data? Understanding guesswork has led to new measures of cryptographic security, new error correcting codes, a new proof of Shannon's capacity theorem, and, ultimately, algorithms that will find their way into your devices.

Guessing games

The best known guessing game in Information Theory is a version of "20 questions" [4]. In it, Alice selects an object, Z, at random from a finite list, $\mathscr{L} = \{a_1, \ldots, a_M\}$, following a distribution, $\mathbb{P}(Z = a_i) = p_i$, that is known to Bob. Bob is allowed to ask questions of the form "Is Z in the set $S \subset \mathscr{L}$?" and gets truthful yes or no answers. Bob's goal is to ask as few questions possible, on average, to identify Z. It is not hard to convince yourself that an optimal solution will involve creating a decision tree of set-based queries and, indeed, one approach that proves to be optimal is to create a Huffman tree.

In a Huffman tree, all of the elements of the list $\mathscr{L}$ are set up as leaf nodes, and, akin to agglomerative clustering, the two nodes with smallest probabilities are identified, and a new internal node is created that has those nodes as children. The probability of the new node is set to the sum of the probabilities of its children. With the new node taking the place of its children, this process is repeated until the tree is complete. In the guessing game, one asks queries following the tree from its root to the leaf that represents Z.

One of the interesting properties of this guessing game is that the Shannon entropy [10] of the random variable Z, given by $H(Z) = -\sum_{i=1}^{M} p_i \log_2(p_i)$, arises. Shannon entropy is a commonly used summary statistic of how "random" a distribution is, where its maximum is $\log_2(M)$ when the distribution is uniform, $p_i = 1/M$ for all i, and its minimum is 0 when $p_i = 1$ for some i. Note that Shannon entropy is solely a function of the probabilities of the objects in the list and not their values. For the guessing game, it can be established that, on average, the minimum number of queries that Bob needs to make to identify Alice's selection is $H(Z) + 1$, which is achievable by use of a Huffman tree. While a lot more can be said about 20 questions, here we are instead interested in a guessing game that places a significantly more severe restriction on Bob: what if he can only ask questions of the sort "Is $Z = a_i$?" for individual objects in the list of possibilities, one at a time?

Guesswork

The security of many systems is predicated on a user or application, Alice, selecting an object, Z, a password or key, from a list. If Bob can only query one possibility at a time, then so long as the length of the list, M, is large and Alice's choice is not too predictable, the number of guesses Bob must make in order to identify the selected object, its *guesswork*, is also likely to be large.

A guessing strategy is a bijective map, $S : \mathscr{L} \mapsto \{1, \ldots, M\}$, that determines the order in which objects in the list are queried. In particular, $S(Z)$ is a random variable that counts how many queries would be made before Z is identified. If the object is selected uniformly at random, $p_i = 1/M$ for all i, as happens with cryptographically secure keys, then no query-order strategy is better, probabilistically, than any other. Indeed $\mathbb{P}(S(Z) = i) = 1/M$ for all $i \in \{1, \ldots, M\}$ and for all strategies and, for example, the average number of queries until the selected object is identified, the strategy's expected guesswork, is $\mathbb{E}(S(Z)) = \sum_{i=1}^{M} i/M = (M+1)/2$. In practical terms, older, shorter encryption keys are now vulnerable to brute force guessing attacks, so for common cryptographic purposes $M = 2^{128} \approx 10^{38}$ or $2^{256} \approx 10^{76}$.

A mathematically more interesting question arises when Z is not selected uniformly at random, $\mathbb{P}(Z = a_i) = p_i$, and the ever-inquisitive Bob knows the distribution. Indeed, since the earliest days of code-breaking, deviations from uniformity have been exploited. Human-selected passwords, for example, are highly non-uniform, embarrassingly so [6], and cryptographical implementations can leak side-information, which is tantamount to having non-uniformity.

In what I disingenuously tell people is a one page paper[1], the late James Massey sought to address the question of how the distribution of Z impacts its average guesswork [7]. The first matter to be addressed is Bob's guessing strategy. If, breaking ties arbitrarily, Bob queries entries in decreasing order of likelihood (i.e setting $G : \{a_1, \ldots, a_M\} \mapsto \{1, \ldots, M\}$ so that $G(a_i) < G(a_j)$ if $p_i > p_j$), then for any other strategy, S, we have that $\mathbb{P}(G(Z) \leq i) \geq \mathbb{P}(S(Z) \leq i)$, hence G is a probabilistically optimal strategy. Indeed, the guesswork random variable $G(Z)$ experiences first-order stochastic dominance by any other strategy, $S(Z)$, which ensures that its average guesswork is smaller, $\mathbb{E}(G(Z)) \leq \mathbb{E}(S(Z))$. Note that Bob's strategy, G, experiences a law of diminishing returns: the likelihood that each query in turn will identify the randomly selected object is decreasing with query number.

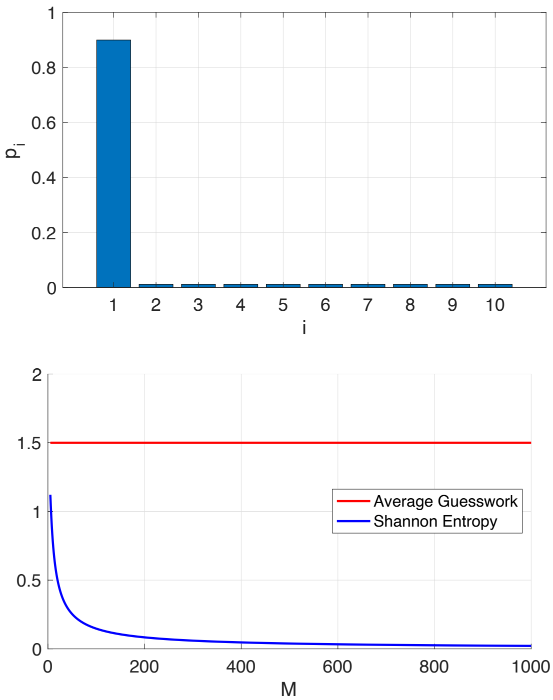


Figure 1. Shannon entropy does not characterize guesswork. Consider the distribution $p_1 = 1 - 1/M$, $p_i = 1/(M(M-1))$ for $i \in \{2, \ldots, M\}$. The expected guesswork is constant, $\mathbb{E}(G(Z)) = 1.5$, but the Shannon entropy goes to zero as M increases, $H(Z) \to 0$.

Massey asked the natural question of whether the Shannon entropy of the random variable Z tells you how hard it is to guess. The short answer is not really. You can readily derive a lower-bound for the average number of queries in terms of the Shannon entropy, but there's no matching upper bound. The example in Fig. 1 gives an indication as to why. Even though probability is concentrating on the first query as M increases, the average guesswork is a constant independent of M because if the first query is not correct, Bob is left with a uniform distribution that is hard to guess. The Shannon entropy, on the other hand, tends to zero as M increases. This example will form the basis of our intuition when we consider asymptotes.

The sci.crypt FAQ

David Malone, who introduced me to guesswork, came to its study after wondering about the veracity of this statement in the sci.crypt FAQ of the early 2000s https://tinyurl.com/38fuvae9

> We can measure how bad a key distribution is by calculating its entropy. This number is the number of "real bits of information" of the key: a cryptanalyst will typically happen across the key within 2^E guesses. E is defined as the sum of $-p_K \log_2 p_K$, where p_K is the probability of key K.

Asymptotic guesswork

For a deeper mathematical investigation, it's evident that additional structure is needed, and in 1996 Erdal Arikan introduced a natural, revealing asymptotic regime [1]. His investigation was motivated by a practical concern rather than mathematical curiosity alone, and if you buy a 5G phone, you will be availing yourself of an error correction technology, polar codes, whose origins can be traced back to his seminal consideration of guesswork [2].

Instead of considering a single random variable, Z, taking values in a finite list, $\mathscr{L} = \{a_1, \ldots, a_M\}$, Arikan considered a string of length n, $Z^n = Z_1 \cdots Z_n$ constructed from independent and identically distributed characters, $\{Z_i\}$, each having the same distribution as Z. Alice picks the string Z^n at random, and Bob's goal is to identify it by asking truthfully answered questions of the form "Is $Z^n = b_1 b_2 \cdots b_n$?". The asymptotic regime considered is as the string becomes long, $n \to \infty$, and it is reasonable

[1]All papers at the IEEE International Symposium on Information Theory were one page long at the time.

to anticipate that the average amount of guessing needed to identify Z^n should grow exponentially, but how quickly? And how does it depend on the distribution of Z?

For didactic simplicity, let's assume that the alphabet is binary $\mathscr{L} = \{0,1\}$, and $P(Z=1) = p < 1/2$ and so that $P(Z=0) = 1-p > 1/2$. Hence Alice is picking a random binary string, Z^n, of length, n that Bob is trying to guess. Following tradition, let us not fret about detailed epsilontics and trust that everything can be made rigorous.

As $p < 1/2$, the most likely string consists of n zeros, the next most likely string has a single one and $n-1$ zeros, and so forth. Thus Bob's guesswork order, G, follows strings with an increasing number of ones (i.e. Hamming weight), breaking ties arbitrarily. Before analysing the average guesswork, let's concern ourselves with probabilities. The likelihood that the string Z^n contains $i \in \{0,\ldots,n\}$ ones is

$$\mathbb{P}\left(\sum_{j=1}^{n} Z_j = i\right) = \binom{n}{i} p^i (1-p)^{n-i}.$$

Using the laziest version of Stirling's approximation, $n! \approx n^n e^{-n}$, a bit of simplification shows that the number of strings with about nx ones satisfies

$$\binom{n}{nx} \approx 2^{-n\left(x\log_2(x)+(1-x)\log_2(1-x)\right)}.$$

Hence the probability of having nx ones in the string

$$\binom{n}{nx} p^{nx}(1-p)^{n(1-x)} \approx 2^{-nI(x)}$$

where

$$I(x) = x\log_2\left(\frac{x}{p}\right) + (1-x)\log_2\left(\frac{1-x}{1-p}\right).$$

It is easy to see that I is strictly convex and its minimum occurs when $x = p$, hence the most likely number of ones is np and, using our approximation to the binomial coefficient, the number of strings with np ones (which Information Theorists call the Typical Set) is approximately $2^{nH(Z)}$, where H is Shannon entropy. Thus one might imagine that Shannon entropy does play a role in guesswork, because by the time Bob has made

$$\binom{n}{0} + \binom{n}{1} + \cdots + \binom{n}{np} \approx 2^{nH(Z)}$$

queries, he has acquired almost all of the probability. It is not, however, the full story.

Honorable intentions & epsilontics

My only grievance with the late, great probabilist Joseph Doob was that he lost a coin-toss to William Feller:

> While writing my book I had an argument with Feller. He asserted that everyone said "random variable" and I asserted that everyone said "chance variable." We obviously had to use the same name in our books, so we decided the issue by a stochastic procedure. That is, we tossed for it and he won. [11]

That chance event has led generations of English-speaking students to believe there is randomness in Probability Theory when Kolmogorov's insight was, instead, to build it on uncertainty through lack of invertibility.

Claude Shannon had greater cause for upset, however. In his Mathematical Reviews of Shannon's seminal paper introducing Information Theory [10], Doob wrote (trolled?):

> ...it is not always clear that the author's mathematical intentions are honorable. (MR0026286)

That upset Shannon, and later Doob expressed regret for its tone. I have never heard comment, however, on Doob's review of another important paper in Information Theory by Brockway McMillan [8]:

> Following tradition, the "detailed epsilontics" of the proof of the fundamental theorem are omitted. (MR0055621)

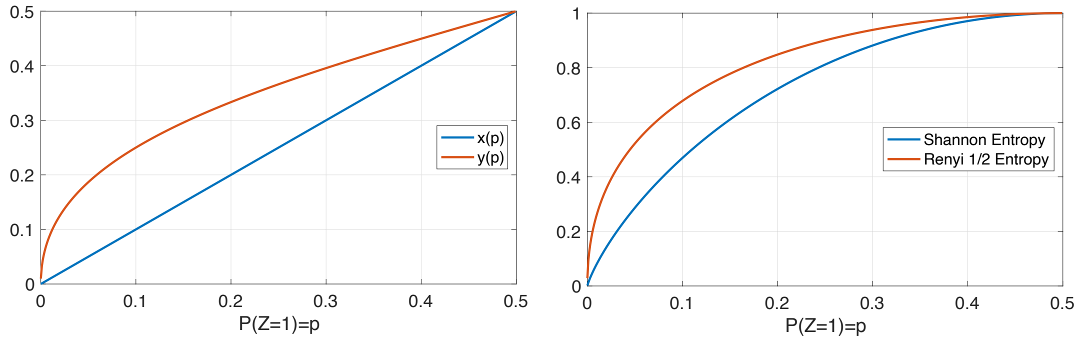


Figure 2. For binary strings made of independent and identically distributed characters with $P(Z=1)=p$, most of the probability will be acquired by the time all strings with $nx(p)=np$ have been queried, and the expected number of queries needed to do so scales as 2 to the power of Shannon entropy. The number of bit flips that dominates the expected guesswork is the larger value $ny(p)$ and the expected guesswork scales as 2 to the power of Rényi entropy with parameter 1/2, $H_{1/2}(Z)$ which is a larger value.

To evaluate the average guesswork, using the Law of Total Expectation we have that $\mathbb{E}(G(Z^n))$ equals

$$\sum_{i=0}^{n} \mathbb{E}\left(G(Z^n)\middle|\sum_{j=1}^{n} Z_j = i\right)\mathbb{P}\left(\sum_{j=1}^{n} Z_j = i\right),$$

and the average amount of querying Bob must do given that the true string contains i ones is

$$\mathbb{E}\left(G(Z^n)\middle|\sum_{j=1}^{n} Z_j = i\right) = \sum_{j=0}^{i-1}\binom{n}{j} + \frac{1}{2}\binom{n}{i},$$

so that

$$\mathbb{E}(G(Z^n)) = \sum_{i=0}^{n}\left(\sum_{j=0}^{i-1}\binom{n}{j} + \frac{1}{2}\binom{n}{i}\right)\binom{n}{i}p^i(1-p)^{n-i}.$$

Thus to understand how $\mathbb{E}(G(Z^n))$ grows with n, we are interested in finding the largest term in a sum of elements that looks like

$$\binom{n}{ny}^2 p^{ny}(1-p)^{n(1-y)},$$

which differs from our earlier consideration by the square. Using the approximation to the binomial coefficient introduced earlier, differentiation reveals this expression is maximized when $y(p) = \left(-p+\sqrt{p(1-p)}\right)/(1-2p)$, which is plotted in Fig. 2. Note that $y(p)$ is larger than p for all $p \in (0,1/2)$ so that the number of ones that dominates the average guesswork is larger than the number of ones where the probability concentrates. Taking a little care, one can show that

$$\lim_{n\to\infty}\frac{1}{n}\log_2 \mathbb{E}(G(Z^n))$$

$$= \lim_{n\to\infty}\frac{1}{n}\log_2\left(\binom{n}{ny(p)}^2 p^{ny(p)}(1-p)^{n(1-y(p)))}\right)$$

$$= \log_2\left(\left(p^{1/2}+(1-p)^{1/2}\right)^2\right).$$

This final expression can be recognised as the Rényi entropy with parameter $1/2$ of Z, of which we will talk about more later, which is a less common measure of how "random" a random variable is.

Both the Shannon and Rényi entropies are plotted against p in Fig. 2. Notably, the Rényi entropy is always larger than Shannon. Thus, even though Bob is almost guaranteed to have identified Alice's string by the time he has made $2^{nH(Z)}$ queries, the average number of queries he must make grows exponentially in n with a larger exponent. This occurs because most of the time Bob identifies Alice's string quickly. If Alice picks something unusual, however, then owing to the law of diminishing returns in the probability obtained per query, how many queries Bob takes to identify it more than makes up for its rareness, similarly to the one dimensional example in Fig. 1.

Guessing our way back to Shannon

Before returning to questions of epsilontics, let's consider a classic theorem viewed through a guesswork lens. Shannon's landmark 1948 paper [10] is a truly readable magnum opus. One of the many important questions that was addressed for the first time was how much redundancy is needed to reliably communicate over a noisy channel.

Alice wishes to send Bob a message, a binary string of length n, X^n, but all she has at her disposal is an unreliable channel that flips bits independently with probability p and leaves them unchanged with probability $1-p$. If Z^n is the binary string of length n that indicates the bits that have been flipped, then Bob receives

$$Y^n = X^n + Z^n,$$

where the addition is bit-wise in the Boolean algebra so that $1+1=0$.

If Alice could have sent any one of the 2^n possible binary strings of length n, Bob has no way of knowing whether what he received was corrupted by noise on the channel. If, instead, Alice and Bob agree in advance to restrict meaningful messages to those in a shared code-book, a subset of 2^{nR} of the 2^n possible strings, where $R<1$, then if Bob receives Y^n and it is not in the code-book, he knows noise has corrupted the transmission – error detection. Moreover, he can make a best guess as to what was sent – error correction, or decoding.

To describe decoding, let $\mathscr{C}_n = \left\{c^{n,i} : i \in \{1,\ldots,2^{nR}\}\right\}$, where $c^{n,i} \in \{0,1\}^n$, be the collection of strings in the code-book. If Alice picks a string uniformly at random, then, using Bayes' rule, Bob's optimal decoding is the most likely message to be sent given what was received

$$\begin{aligned}\hat{X}^n &\in \arg\max_{c^n \in \mathscr{C}_n} \mathbb{P}(Y^n = c^n + Z^n)\\ &\in \arg\max_{c^n \in \mathscr{C}_n} \mathbb{P}(Z^n = Y^n - c^n).\end{aligned}$$

This appears to require Bob to evaluate a likelihood for each of the 2^{nR} code-words for every transmission received, which is computationally burdensome.

Instead, Bob could use Guessing Random Additive Noise Decoding [5]. Note that Bob's decoding, c^n, corresponds to a noise effect, Z^n, that has the highest likelihood. By subtracting putative noise effects from Y^n in order from most likely to least likely and querying if what remains is in the code-book, the first time Bob finds something in the code-book, he will have identified his optimal decoding. From our earlier considerations, we know Bob will get the right answer with high likelihood after $2^{nH(Z)}$ queries and, on average, after $2^{nH_{1/2}(Z)}$.

How many queries Bob will have to make before he identifies an erroneous decoding, with his querying finding a code-word that was not sent, depends on the code-book construction. Following Shannon, imagine that the set of 2^{nR} elements of the code-book $\mathscr{C}_n$ was chosen uniformly at random. Then the chance that a query will identify an incorrect code-book element is $2^{nR}/2^n = 2^{-n(1-R)}$ and so, on average, Bob will make $2^{n(1-R)}$ queries before he identifies an erroneous decoding.

What is the densest code-book, i.e. the highest R, that Alice and Bob can use? If $2^{n(1-R)} < 2^{nH(Z)}$, Bob's querying is likely to identify an erroneous decoding before he identifies a correct one. On the other hand, if $2^{nH(Z)} < 2^{n(1-R)}$ then he is likely to identify the correct decoding before he finds an erroneous one. Thus they require that $H(Z) < 1-R$ or, as is more commonly written, $R < 1-H$. We have rediscovered Shannon's channel coding theorem, which tells us that to communicate reliably over a noisy channel, Alice and Bob must use a code-book rate that is less than one minus the Shannon entropy of the noise, which is dubbed the channel's capacity.

As in real-world systems bit-flip probabilities are small, often 10^{-5} or below giving $H(Z) \approx 1.8 \times 10^{-4}$, guessing the noise can be much more efficient than other decoding approaches. This line of reasoning can be made rigorous and broader, including for what is called a soft-detection system in which each bit is flipped with a different likelihood that is known to Bob. Determining the guesswork order for that circumstance is much more challenging, and the most computationally practical approach found to date relies on the creation of integer partitions with non-repeating parts; but that's a story for another time. Translating these ideas on how to efficiently decode arbitrary moderate-redundancy codes from theory into practice is an active line of research that has already led to implementations in chips, and a number of tough mathematical questions remain.

From moments to probabilities

I dread to think the scathing remarks Doob would have rightly used to describe the loosey goosey approach taken so far, but everything can be made rigorous, and much more established besides. Those results are, admittedly, best achieved with methods that do not rely on combinatorial estimates.

With

$$H_\beta(Z) = \frac{1}{1-\beta}\log_2\left(\sum_{i=1}^{M} p_i^\beta\right)$$

being the Rényi entropy of order $\beta > 0$ and $\beta \neq 1$ of Z (H_1 is defined to be Shannon entropy), Arikan established how moments beyond the mean (e.g. the variance and skewness) of guesswork scale in the limit as the string becomes long

$$\lim_{n\to\infty} \frac{1}{n} \log_2 \mathbb{E}(G(Z^n)^\alpha) = \alpha H_{\frac{1}{(1+\alpha)}}(Z), \text{ for } \alpha > 0.$$

While Arikan worked with strings created from independent and identically distributed letters, equivalent results have since been established for a much broader class of processes, including by Pfister and Sullivan [9] who extended the remit to any $\alpha > -1$. Considering moments, $\alpha > 0$ makes sense, so why $\alpha > -1$? Given the opportunity to query each of the authors, I did. Alas neither could recall their rationale, so I will attribute my best guess: their arguments worked for $\alpha > -1$, so being professional mathematicians that's what they stated.

Why it matters is that the Laplace transform is a standard device in the probabilist's toolbox. Given a random variable A, if the Laplace transform of its distribution $\mathbb{E}(\exp(\alpha A))$, its moment generating function, is finite for all α in a neighbourhood of the origin, knowing it is equivalent to knowing its distribution. Akin to that, if you have a stochastic process, say $\{A^n\}$, and can determine how its moment generating functions scale

$$\lim_{n\to\infty} \frac{1}{n} \log_2 \mathbb{E}\left(e^{\alpha n A^n}\right), \text{ for } \alpha \in \mathbb{R},$$

then Large Deviation Theory, for which S.R.S. Varadhan won the Abel Prize in 2007, will, in many circumstances, provide estimates on how the probabilities of extreme values of A^n behave as n increases.

Identifying A^n with $n^{-1} \log G(Z^n)$ and using that most powerful of mathematical tools, the isomorphism, we see that results on how the moments of guesswork scale can instead be considered as how the moment generating function of the logarithm of guesswork scales

$$\lim_{n\to\infty} \frac{1}{n} \log_2 \mathbb{E}\left(e^{\alpha \log G(Z^n)}\right).$$

Arikan realised this possibility in the early 2000s, and that programme was completed in 2013 [3], establishing a Large Deviation Principle (LDP) for $\{n^{-1} \log G(Z^n)\}$, which is a powerful mathematical enabler in deriving other results.

While we have skimmed the surface of guesswork, the literature contains many variants. Guesswork has been used to quantify cryptographic security, to ask questions of many-user systems, to wonder how side-information impacts how hard it is to query things, and much more besides. All you require is that your mathematical intentions be honourable, and to be willing to engage in epsilontics.

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Author profile

Ken R. Duffy

Ken is a Professor at Maynooth University in Ireland where he directs its applied mathematics research centre, the Hamilton Institute. He likes to work in multi-disciplinary teams to design, analyse and realise systems using tools from probability and statistics. Algorithms he has co-designed have been implemented both in silicon and in DNA.

Notes of a Numerical Analyst

Floating point numbers and physics

NICK TREFETHEN FRS

The laws of classical continuum mechanics describe the motion and deformation of fluids and solids. They involve quantities such as density, pressure, and temperature, and they are written as partial differential equations. Of course, these laws are approximations, for the world is not continuous but is made of discrete atoms and molecules. Density is an average, pressure is an average, temperature is an average. But this is the right thing to do for most applications in science and engineering: to ignore the molecules and regard the physical world as continuous.

Physicists understand very well the "implementation details" by which the continuum is built up from discrete particles. For example:

• *If you halve the volume of a box, keeping temperature constant, the pressure of a gas inside doubles. Reason: twice as many impacts of molecules per unit cross-section per unit time.*

• *If you double the temperature of a box, keeping volume constant, the pressure doubles. Reason: the momentum of each particle increases by a factor of $\sqrt{2}$, since energy and temperature scale with velocity-squared; the number of impacts per unit cross-section per unit time also goes up by $\sqrt{2}$.*

How fine is the physical continuum? The famous Avogadro's number, about 6×10^{23}, is the number of molecules in a mole. There are about 50 moles of gas in a cubic meter at ordinary conditions, so this comes to about 3×10^{25} molecules per cubic meter in a gas (Loschmidt's constant). The cube root of 3×10^{25} is about 3×10^{8}. Thus there are about 3×10^{8} molecules per linear meter in an ordinary gas. For a solid, the figure is about ten times higher: 3×10^{9}. Thus, roughly speaking,

A gas or solid has around 10^9 particles per meter.

This is how fine the discretisation is in our physical world. It's interesting to compare it with the floating-point arithmetic on our computers. In the IEEE double-precision standard that has prevailed since the 1980s, the real line is discretized by $2^{52} \approx 10^{16}$ numbers between 1 and 2, the same between 2 and 4, and so on. Thus we find:

Computer arithmetic is a million times finer than physics.

If I gave floating point arithmetic coordinates to the desk I'm sitting at, for example, I would find there were around a million coordinate points between each adjacent pair of molecules. In fact, 10^{16} is more or less the number of molecules I'd encounter in a line going all the way through the earth from here to New Zealand.

Figure 1. Turing Award winner William Kahan of UC Berkeley, the man behind the IEEE floating-point arithmetic standard.

Another angle on the extraordinary resolution of floating point numbers is the fact that in the physical world, essentially nothing is known to 16 digits of accuracy. Quantities like the electron mass or the gravitational constant are known to between 5 and 12 digits. Well, the speed of light is *exactly* 299,792,458 meters per second! — but only because the meter is defined thereby.

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Nick Trefethen
Trefethen is Professor of Numerical Analysis and head of the Numerical Analysis Group at the University of Oxford.

Mathematics News Flash

Jonathan Fraser reports on some recent breakthroughs in mathematics.

The rectangular peg problem

AUTHORS: Joshua Evan Greene and Andrew Lobb
ACCESS: https://arxiv.org/abs/2005.09193

The *Square Peg Problem*, first asked by Toeplitz in 1911, asks whether every Jordan curve contains the vertices of (that is, inscribes) a square. Recall that a *Jordan curve* is a non-self-intersecting continuous loop in the plane. This notorious problem remains open to this day although it was proved for *smooth* Jordan curves as early as 1929 by Schnirelman. Further special cases are now known requiring less regularity. The reader may enjoy considering certain simple cases: how many inscribed squares can you find when the Jordan curve is a circle? How about an ellipse? Or perhaps a right-angled triangle?

The *(smooth) Rectangular Peg Problem* is a variant on this theme and asks if every smooth Jordan curve contains the vertices of a rectangle with every possible aspect ratio. This paper, published in *Annals of Mathematics* in 2021, solves the smooth rectangular peg problem in the affirmative. The proof uses the result of Shevchishin and Nemirovski that the Klein bottle does not admit a smooth Lagrangian embedding in $\mathbb{C}^2$.

Convergence of Gaussian Process Regression with Estimated Hyper-parameters and Applications in Bayesian Inverse Problems

AUTHOR: Aretha Teckentrup
ACCESS: https://arxiv.org/abs/1909.00232

Gaussian process surrogate models have been used to great effect in the statistical literature to approximate data likelihood with much greater computational efficiency than the previously favoured (fully) Bayesian approach. Convergence analysis is a key issue and this paper, published in *SIAM/ASA Journal on Uncertainty Quantification* in 2020, makes highly novel contributions to this area. For example, the use of *hierarchical* Gaussian process surrogates to approximate the data likelihood is justified by bounding the error introduced in the posterior distribution.

The *posterior distribution* refers to the distribution one obtains after conditioning on certain observations (perhaps obtained experimentally) and is central to Bayesian inference.

Anosov flows, growth rates on covers and group extensions of subshifts

AUTHORS: Rhiannon Dougall and Richard Sharp
ACCESS: https://arxiv.org/abs/1904.01423

Let X be a regular cover of a compact smooth Riemannian manifold admitting a transitive Anosov flow. The *Gurevich entropy* of X counts the growth of the number of periodic orbits of a given length of the *lifted* flow, that is, the induced flow on X. This paper, published in *Inventiones Mathematicae* in 2021, establishes a general and remarkable equivalence: the Gurevich entropy of X and of the maximal abelian subcover of X coincide if and only if the covering group of X is *amenable*. A key feature of this result is the removal of certain symmetry conditions present in previous work in this area. The proof uses group extensions of subshifts of finite type as a central tool and the lifted flow is assumed to be transitive throughout.

It is natural to consider what happens to the periodic orbits when they are projected onto M. It is shown that (in the case when the covering group is amenable) they equidistribute with respect to the equilibrium measure coming from a thermodynamic variational principle. When the original flow is the geodesic flow this equilibrium measure is the measure of maximal entropy.

Jonathan Fraser is a Professor of Mathematics and Statistics at the University of St Andrews and an Editor of this Newsletter. He is pictured with his son Reuben Samuel Fraser, born August 23rd 2021.

Microtheses and Nanotheses provide space in the Newsletter for current and recent research students to communicate their research findings with the community. We welcome submissions for this section from current and recent research students. See newsletter.lms.ac.uk for preparation and submission guidance.

Microthesis: The Student-Project Allocation Problem

SOFIAT OLAOSEBIKAN

Matching problems arise when we need to find an optimal allocation between sets of agents. Typically, agents have preferences over the possible outcome and the goal is to find a matching that takes these preferences into consideration. In my PhD thesis, I presented structural and algorithmic results for problems in the context of assigning students to projects.

Background

Matching problems arise when we need to find an allocation between a set A of agents and another set B of agents, e.g., allocating pupils to schools, junior doctors to hospitals and teachers to regions. A typical trend in all the applications is that the agents in A have ordinal preferences over the agents in B (and vice-versa). Further, each agent in one set has a specified *capacity*, which is the maximum number of agents from the other set that they can accommodate. The goal is to find a *matching*, i.e., an allocation of agents in A to the agents in B that takes their preferences and capacities into consideration.

The Student-Project Allocation problem (SPA)

SPA is a matching problem based on the allocation of students to projects. Students have preferences over projects, where each project is offered by one lecturer; whilst lecturers may either have no preference at all, or they may have preferences over students and/or over projects. When both students and lecturers have some form of preference, the solution we seek is a *stable matching*, which guarantees that no student and lecturer will have an incentive to deviate from the matching by forming a private arrangement involving some project.

SPA with lecturer preferences over Students (SPA-S)

A variant of SPA, where lecturers have preferences over students and the preference lists are strictly ordered, is referred to as SPA-S. A matching M in this setting is stable if there is no student s and lecturer l such that if they decide to form a private arrangement outside M via one of l's projects, then both of them would be better off. It is known that an arbitrary instance I of SPA-S may admit many stable matchings.

Let $\mathcal{M}$ be the set of all stable matchings in I, and let $M, M', M'' \in \mathcal{M}$. We say that M dominates M', denoted $M \preceq M'$, if and only if each assigned student either prefers her project in M to that in M' or is indifferent between them.

Contributions: My thesis characterised the stable matchings in I, in the special case that for each student, all of the projects in their preference list are offered by different lecturers. The following are the main results that led to the characterisation.

(i) $(\mathcal{M}, \preceq)$ is a partial order.

(ii) $(\mathcal{M}, \preceq)$ is a distributive lattice, with

- $M \wedge M'$, the meet of M and M', i.e., a stable matching in which each student is assigned the better of her projects in M and M';
- $M \vee M'$, the join of M and M', i.e., a stable matching in which each student is assigned the poorer of her projects in M and M';
- $(M \wedge M') \vee M'' = (M \vee M'') \wedge (M' \vee M'')$;
- $(M \vee M') \wedge M'' = (M \wedge M'') \vee (M' \wedge M'')$.

SPA-S with Ties (SPA-ST)

SPA-ST is a variant of SPA-S where the preference lists of students and lecturers may admit indifference in the form of ties. As a result of the presence of ties, three different forms of stability arise, namely *weak stability, strong stability*, and *super-stability*.

Suppose M is a matching in an instance of SPA-ST. Then M is (i) weakly stable, (ii) strongly stable, or (iii) super-stable, if there is no student and lecturer l such that if they decide to become assigned to one another outside of M via one of l's proposed projects, respectively,

(i) both of them would strictly improve,

(ii) one of them would strictly improve and the other would not be worse off,

(iii) neither of them would be worse off.

Under weak stability, the problem of finding a stable matching that assigns as many students to projects as possible is NP-hard. A $\frac{3}{2}$-approximation algorithm exists in the literature for this problem.

Contributions: My thesis described the first polynomial-time algorithm to find a super-stable (respectively strongly stable) matching or to report that no such matching exists, given an instance of SPA-ST. I also proved some structural results concerning the set of super-stable (respectively strongly stable) matchings in I. Further, I presented results obtained from an empirical evaluation of the two algorithms based on randomly-generated SPA-ST instances.

SPA with lecturer preferences over Projects (SPA-P)

SPA-P is a variant of SPA where both students and lecturers have preferences over projects. In this setting, a *stable matching* ensures that (i) no student and lecturer who are not matched together would rather be assigned to each other than remain with their current assignment, and (ii) no group of students acting together could undermine the integrity of the matching by swapping their assigned projects, in order to be better off.

Stable matchings in this context can be of different sizes. Moreover, the problem of finding a maximum size stable matching (MAX-SPA-P) is NP-hard. There are two known approximation algorithms guaranteed to produce stable matchings that are at least $\frac{1}{2}$ and $\frac{2}{3}$ the size of the optimal solution.

Contributions: In my thesis, I proved the following results under SPA-P.

- MAX-SPA-P is solvable in polynomial time if there is only one lecturer involved;
- MAX-SPA-P is hard to approximate within some constant $c > 1$ if two lecturers are involved, unless P=NP;
- MAX-SPA-P is NP-hard if the length of each student's preference list is at most 3, with an arbitrary number of lecturers.

I also formulated an integer programming (IP) model to enable MAX-SPA-P to be solved to optimality in the general case. A general construction of my IP model is a follows:

- create binary-valued variables to represent the assignment of students to projects;
- enforce constraints to ensure that the assignment is a matching, and that the matching is stable; finally
- describe an objective function to maximize the size of the stable matching.

Acknowledgements

The author was supported by a College of Science and Engineering Scholarship, University of Glasgow.

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Sofiat Olaosebikan

Sofiat is a lecturer in the School of Computing Science at the University of Glasgow. She has a background in mathematics, and her broad research interests are in graph algorithms and combinatorial optimisation. Inspired by her fascinating journey into computing science, Sofiat established the Programming Workshop for Scientists in Africa initiative, to empower young Africans with computer programming skills.

2022 HEILBRONN FOCUSED RESEARCH GRANTS - Call for proposals

The Heilbronn Institute for Mathematical Research is offering a number of grants of up to £7.5K to fund focused research groups to work on adventurous and challenging mathematical problems, or to discuss important new developments in mathematics. Grants under this scheme will be funded either through the UKRI/EPSRC 'Additional Funding Programme for Mathematical Sciences' (part of the £300M government investment announced in 2020) or by the Heilbronn Institute directly.

Open to all mathematicians and to any department in the UK, these grants will support travel and local expenses for groups to come together to focus intensively on a problem or to discuss a significant new development in mathematics. We expect these groups to be normally 8 or fewer people. Groups are encouraged to include international participants but should also involve a substantial UK-based component.

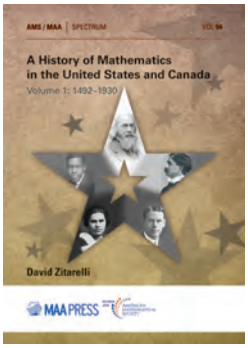
Proposals from these areas of research, interpreted broadly, will be given priority: Pure Mathematics, Probability and Statistics, and Quantum Information.

One A4 page proposals should be sent by **9am, Thursday 27th January 2022** to: heilbronn-manager@bristol.ac.uk For further particulars and additional information, please visit our website: https://heilbronn.ac.uk/frg/

A History of Mathematics in the United States and Canada: Volume 1: 1492–1900

by David E. Zitarelli, American Mathematical Society, 2019, US$120,
ISBN: 978-1-4704-4829-5

Review by Christopher Hollings



There have already been several books dealing with the history of mathematics in North America, each with its own focus: [1], for example, has much to say on mathematics teaching, whilst [2] consists of a series of reminiscences and articles on particular topics and individuals; [3] looks specifically at the birth of the American mathematical research community. This book, the result of a long-standing project by the late David Zitarelli, is the first volume of what promises to be an encyclopaedic history of US and Canadian mathematics from the early European colonisation of North America to the mid-twentieth century. This volume, appearing within the MAA's general interest Spectrum series, covers the period 1492–1900; volume 2, expected within the next couple of years, will continue the story up to 1941.

The book is split into three parts: 'Colonial Era and Period of Confederation, 1492–1800', 'New Republic, 1800–1876', and 'Research Community, 1876–1900', with each part being further subdivided into rather substantial chapters. Each part begins with a general introduction, and ends with a short transitional chapter which eases the reader into the following part and emphasises the continuity of the story over the sudden breaks that the chapter divisions might otherwise suggest. The story begins with the first European mathematical visitors to North America (in particular, Thomas Harriot), before turning to the foundation of the first colleges (beginning with Harvard in 1636) and the early study and application of mathematics by European settlers. Here and throughout, Zitarelli takes a broad view of what it means to be a mathematician: mathematical research in the modern sense would not take root in American soil until the later parts of the nineteenth century, but mathematics was nevertheless present before this point in the hands of enthusiasts, as well as in its applications to navigation, astronomy, surveying, and cryptology (particularly during the US Civil War), as well as in a host of other settings.

Moving through the following centuries, the book presents us with the gradual establishment of centres for mathematical education in North America, and gives us a taste of the level of mathematics that was being taught at each stage. We see the publication of the first American mathematics textbooks, and attempts (the majority of them very short lived) to found mathematical journals — question-and-answer periodicals at first, but then research publications later on. Alongside these details, we are presented with potted biographies of hundreds of mathematical figures, including some well-known names, but the majority will probably be previously unknown to most readers. Much of the story of the development of American mathematics is told through these biographies.

During the later parts of the book, we see the formation of the American mathematical research community, and the introduction of the first PhD programs that meant that Americans were no longer reliant on European institutions for higher study in mathematics. The foundation of the New York Mathematical Society in 1888 (which became the *American* Mathematical Society in 1894) and the holding of the 'zeroeth' International Congress of Mathematicians in Chicago in 1893 are taken as markers of the emergence of a professional and internationally visible mathematical community.

The book is written is a very informal and chatty style. The material is presented with a view to its use in the teaching of mathematics, though this is likely to be most relevant to those in American institutions. Canadian people and places are mentioned, but the focus is more firmly on the United States and its predecessor colonies. The material is arranged according to whatever theme seems most appropriate at that point of the book, thus particular people, institutions, journals, and (less often) mathematical topics come to the fore in different places. In compiling this history, Zitarelli has evidently remained on the look-out for female and African-American mathematical figures, who appear throughout the book, thereby presenting a surprisingly diverse picture.

The book gives the impression of exhaustiveness as to topics covered, but does not try to include absolutely every detail — it is supported by additional online material, available at `https://davidzitarelli.wordpress.com/`, and features many suggestions for further reading. Overall, the book is rather dense, and so is perhaps best regarded as an encyclopaedic source, rather than a book to be read from cover to cover. The fact that key individuals, institutions, and so on, are picked out in bold print in those passages where they feature heavily supports the view of this volume as a reference book. It is very difficult to do justice in such a short review to the sheer amount of material that this book contains, but I fully expect it to be an extremely useful resource to have on my bookshelf.

FURTHER READING

[1] Florian Cajori, The teaching and history of mathematics in the United States, Government Print Office, Washington, 1890.
[2] Peter L. Duren, A century of mathematics in America, 3 vols., AMS, 1988/1989.
[3] Karen Hunger Parshall and David E. Rowe, The emergence of the American mathematical research community, 1876–1900: J. J. Sylvester, Felix Klein, and E. H. Moore, AMS/LMS, 1994.

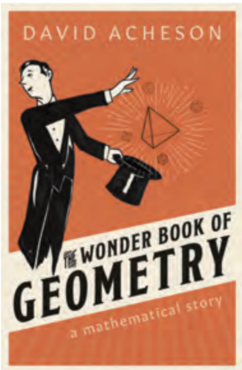
Christopher Hollings

Christopher Hollings is Departmental Lecturer in Mathematics and its History in the Oxford Mathematical Institute, and Clifford Norton Senior Research Fellow in the History of Mathematics at The Queen's College. His research covers various aspects of nineteenth and twentieth century mathematics.

The Wonder Book of Geometry: A Mathematical Story

by David Acheson, Oxford University Press, 2020, £12.99,
ISBN 978-0-1988-4638-3

Review by Jasmine Wootten



The Wonder Book of Geometry by David Acheson makes the bold claim in the introduction that within a mere 30 minutes, by reading this book anyone can experience the wonders that geometry possesses. Whilst this assertion seems rather outlandish and far-fetched, through the use of bitesize chapters, simple explanations and practical applications it is entirely plausible, give or take a few minutes.

This 267-page book contains 32 short chapters. It starts with the basics of parallel lines, angles and triangles; before moving on to discuss Euclid's *Elements* (it wouldn't be a book on geometry without it); Pythagoras' theorem; pi; congruence of triangles; and circle theorems, among other things. Each concept is explained from the ground up to ensure that even if you are not well versed in geometry, the book is still accessible to you. Several of the explanations may be slightly harder to follow if you are not used to reading mathematical explanations, however even skimming over these more complicated parts it is still possible to understand the general themes. The book also caters for those who are more comfortable with mathematical ideas through the use of the notes section which contains a more detailed explanation of certain points. Additionally, there is a challenge on page 106/7 where you are asked to consider how the rearrangement of an 8×8 grid with 3 lines can apparently increase the area. This allows the reader to take a more active role as they try to solve this problem; luckily the answer to this is also provided.

Whilst the book highlights the success of different mathematicians in proving geometrical ideas it also describes several occasions when mathematicians have missed the mark. Chapter 30, titled 'When Geometry Goes Wrong . . .' shines a light on several mathematicians who, in the pursuit of new ideas or in challenging old ones, have made mistakes and in some cases, such as with Malfatti's problem, it took a while for these mistakes to be fixed. I particularly like this chapter because everyone makes mistakes so I think it is important to highlight that when in search of mathematical greatness, not everyone got it right all the time.

Scattered throughout the book are anecdotal stories from Acheson's own experiences learning geometry. For example, how when he was first introduced to spherical geometry and triangles with 270 degrees he was initially unimpressed. This helps to keep the tone of the book light-hearted and engaging. The inclusion of some practical applications of the concepts helps make this book appealing to people who may not usually choose a geometry book. In particular I found the explanation of how the WWII 'Dam Buster' raids made use of the angle-side-angle nature of a triangle to determine the height needed to drop the bomb in order to produce the bouncing effect over water particularly interesting. The book contained multiple images and diagrams, some of which helped clarify a particular point, some which showed extracts from historical books and some which were simply there for the reader's amusement. This is a real strength of the book because it ensures that the reader isn't overwhelmed by too much complex mathematical thought and enables the book to be accessible to a wider audience.

I was pleasantly surprised by this book. Whilst I was familiar with many of the geometrical ideas, I wasn't as familiar with the background behind these ideas and the people who helped contribute to their development. I enjoyed the straightforward nature of this book. I didn't have to spend ages rereading parts in order to follow what was going on. Everything was explained clearly and concisely so that the wonders of geometry could definitely be seen.

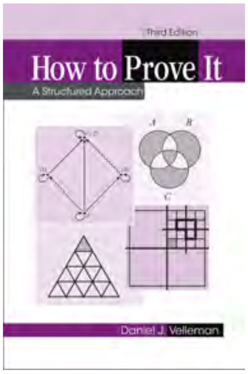
Jasmine Wootten

Jasmine Wootten graduated from the University of St Andrews in 2020 with a BSc in Mathematics. She has a particular interest in the History of Mathematics and more applied topics such as Mathematical Biology and Solar Theory. Jasmine is currently putting her mathematical skills to good use working as an exposure management analyst. In her free time, she enjoys playing sports and going for walks with a picnic.

How to Prove It: A Structured Approach (3rd Edition)

by Daniel J. Velleman, Cambridge University Press, 2019, £29.99, US$37.99,
ISBN: 978-1108439534

Review by Peter Rowlett



This is a textbook on mathematical proof techniques aimed principally at first-term undergraduate students of mathematics and theoretical computer science, which also covers basics of set theory, relations, functions, number theory and cardinality.

The author aims to expose the structure and underlying principles in construction of mathematical proof. Chapters 1 and 2 carefully build concepts and notation around logic and set theory, with some reference to 'common sense' arguments, before chapter 3 begins introducing proof strategies and more formal methods in a systematic way. These techniques are practised in chapters on relations and functions before the introduction of mathematical induction. Finally, the techniques learned are applied to more substantial topics in number theory and infinite sets.

This is the third edition of a book first published in 1994. According to the preface, the third edition includes a new chapter on number theory and more than 150 additional exercises, along with some minor reorganising of content and other small changes.

The book has a friendly way of addressing the reader, commenting on what is happening and hinting at what the reader may have noticed, which is an appealing style. A nice feature of the book is that it follows the statement of a question with a section of 'scratch work', breaking down the question and how what has been learned so far fits into its solution, before going on to state formally the resultant theorem and proof without reference to the scratch work. An aim of the book is to teach students to write proofs "just as mathematicians do", but the presentation of scratch work illuminates useful thinking. For an idea of the writing style and teaching method (p. 94):

> As you can see from the preceding example, there's a difference between the reasoning you use when you are figuring out a proof and the steps you write down when you write the final version of the proof. … When mathematicians write proofs, they usually just write the steps needed to justify their conclusions with no explanation of how they thought of them.

The author makes careful distinction between "explaining your thought processes" and "justifying your conclusions", focusing the purpose of proof squarely on justification but allowing explanation to occur in the scratch work.

The book makes use of carefully chosen examples, then introduces new theory and techniques while discussing these. This means the strategies appear as they are needed, making the development feel quite natural and effective.

The book does not always use my preferred choice of symbols, but this is to be expected. In any case, logic notation is used in the scratch work where it is useful shorthand, but the book recommends using ordinary English as much as possible in the final write-up.

At times I felt I might have benefitted from clearer structure or signposting. The chapter sub-headings did not stand out visually to me if I flicked through looking for something I'd seen previously. The way examples lead naturally into explanations means sometimes it is hard to find those explanations again, and perhaps some sub-sub-headings might have helped.

There is an understandable attempt to relate logic concepts to everyday examples, which seems to be a sensible pedagogic approach. However, various examples and exercises are not understood or answered solely using information from the question without bringing in additional 'common knowledge' from a certain cultural perspective. For example, there are exercises which rely on knowledge of Canadian provinces and territories, which historical figures were alive in which years in the 1750s or who Elizabeth Taylor was married to, and there is an exercise under "Write definitions using elementhood tests for the following sets..." for which the answer is "$\{ x \mid x$ is an Ivy League School $\}$". Such knowledge can be looked up, of course, but perhaps that isn't the intended skill being developed. Presumably the intention is to make the content easier to digest by relating it to known concepts, but this may cause a barrier or feeling of alienation depending on the background of the reader. Certainly I would re-contextualise some examples and exercises were I to use them with my students. There is also some American spelling and phrasing (e.g. "pants", "majoring in math" and "the freshman class"). There are some examples that use gender and marriage, contexts that are probably better avoided when making arbitrary choices of framing. I didn't find this a major distraction, and anyway many examples are generic ("If it's raining and I don't have my umbrella, then I'll get wet") or more international in outlook ("$C = \{ x \mid x$ is a country in Europe $\}$").

As an aside, since the book was written before the COVID-19 pandemic and I was reading it during, there were some examples that raised a wry chuckle from me; "Both having a fever and having a headache are sufficient conditions for George to go to the doctor" and "If anyone in the dorm has a friend who has the measles, then everyone in the dorm will have to be quarantined" come to mind.

Overall, this is an engagingly-written and effective book for illuminating thinking about and building a careful foundation in proof techniques. I could see it working in an introduction to proof course or a course introducing discrete mathematics topics alongside proof techniques. As a self-study guide, I could see it working as it so well engages the reader, depending on how able they are to navigate the cultural context in some examples.

Peter Rowlett

Peter Rowlett is a Reader at Sheffield Hallam University, where he teaches mathematics and researches higher education teaching and learning practice. He is also interested in recreational mathematics and maths communication. Outside of work he enjoys spending time with his six-year-old son, who is an expert in mathematical play, tree-climbing and Norse myths.

Obituaries of Members

Brian Sleeman: 1939–2021

Professor Brian Sleeman, who was elected a member of the London Mathematical Society on 17 February 1989, died on 19 July 2021, aged 81.

Peter Grindrod and Mark Chaplain write: Brian was educated in London and attended the Battersea College of Technology (now the University of Surrey), being awarded a BSc in 1963. He then undertook postgraduate research at the University of London, supervised by Felix Medland Arscott, an expert in higher special functions. He was awarded a PhD in 1966 for his thesis *Some Boundary Value Problems Associated with the Heun Equation*.

In 1969, Brian was appointed an Assistant Lecturer in Mathematics at Queen's College, Dunde, which was then still part of the University of St Andrews. At the time of his appointment, Professor Douglas Jones was head of the Department of Applied Mathematics, and he encouraged Brian to spend a year at the Courant Institute of Mathematical Sciences at New York University, working in Joe Keller's group. In the same year, Queen's College, Dundee became the new University of Dundee and Brian was promoted to Lecturer in Mathematics. He was subsequently promoted to Reader in Mathematics in 1971. He spent the academic year 1976–1977 as a visiting professor at the University Tennessee, Knoxville, USA, and a series of lectures he gave there in the spring of 1977 became the basis for his book *Multiparameter Spectral Theory in Hilbert Space* (1978). In 1978 he was promoted to Professor at the University of Dundee.

One of Brian's major contributions was assisting in the running of conferences on the *Theory of Ordinary and Partial Differential Equations*. The first, held in Dundee in March 1972, had Proceedings published in the Springer Lecture Notes in Mathematics series with Norrie Everitt and Brian Sleeman as editors, and the Twelfth Conference held in honour of Professor D.S. Jones at the University of Dundee, in June 1992.

Up until 1980 or so, Brian's research was mostly in applied analysis, multiparameter spectral theory, and direct and inverse scattering theory. He had some wonderful collaborators including Pat Brown (Calgary) and David Colton (Delaware — and of the Colton–Sleeman uniqueness theorem). In the 1980s Brian became an early champion and pioneer of mathematical biology. Many of his subsequent PhD students followed that discipline, with several going on to their own academic careers through his inspiration. He had a huge impact both first hand, and second hand through his students, on the initiation and growth of UK mathematical biology. He himself addressed a wide range of important applications, including heart physiology, nerve pulse transmission, chemical reactions, tumour growth, and epidemics. In 1995 Brian left Dundee for the University of Leeds where he continued to thrive on his own terms in a much bigger environment. He retired in 2004 and he was honoured with a conference on his 65th birthday.

Among the many honours given to Brian was that he was elected a Fellow of the Royal Society of Edinburgh in 1976, and was President of the Edinburgh Mathematical Society 1988–1989. He was founding editor of the journal *Computational and Mathematical Methods in Medicine* which adopted this title in 2006 having been founded in 1997 as the *Journal of Theoretical Medicine*. He was intellectually active right to the end, working on applied analysis problems with Professor Iain Stewart, University of Strathclyde. He will be remembered as an inspirational teacher, an enthusiastic researcher, a caring and highly supportive supervisor, a true ambassador for the mathematical sciences.

He leaves his wife Julie, daughter Elizabeth, sons Matt and David, and grandchildren Lucy, Jacob, Charlotte, Joshua and Maya.

Michael Taylor: 1959–2021

Michael Taylor, who was elected an associate member of the London Mathematical Society on 10 November 2017, died on 31 August 2021, aged 62.

Alan Bluck writes: Once Michael had graduated in Chemistry at Sussex University he went on to be a programmer, working on Gas Chromatography and developing Assembler and Machine Code on Z80 microprocessors for laboratory computers.

Mike was passionate about science and went on to study Physics and Mathematics at The Open University. He was a member of many organisations including the Institute of Physics, the Institute of Mathematics and its Applications, the London Mathematical Society and, more recently, the Institute of Engineering and Technology, as well as the Royal Society of Chemistry. He enjoyed travelling to London to attend scientific conferences and talks. Every year he loved to go to the Cambridge Science Festival where he seemed to know many of the speakers and presenters.

He was an enthusiastic chess player and participant in pub quizzes and knew the answers to many of the questions because of his amazing memory for dates, people and places. He held his Scouter's Warrant and liked to organise activities and games for the Scouts. He travelled widely, including to Africa, China and New Zealand, and was a keen linguist, speaking French and Spanish.

In May 2019, Mike was diagnosed with oesophageal cancer and sadly died on 31 August 2021 of metastasis, aged 62.

Victor P. Snaith: 1944–2021

Professor Victor Snaith, who was elected a member of the London Mathematical Society on 15 February 1973, died on 3 July 2021, aged 77.

John Coates writes: I first met Victor in the summer of 1975, when I returned to Cambridge from the United States. In particular, I took over from Victor his undergraduate teaching responsibilities at Emmanuel College, and I still remember the kindness and the care with which he explained to me all that had to be done. Victor then left himself for the United States to spend a year at Purdue University before moving permanently to Canada. He was a professor at the University of Western Ontario from 1976–88, and then held the R.F. Britton chair at McMaster University from 1988–98. He was elected a fellow of the Royal Society of Canada in 1984. I had the pleasure of visiting Victor at McMaster University on several occasions, and found a very stimulating mathematical environment there, with many visitors funded by his personal research grant. Up until his move to McMaster, he had worked mainly in algebraic topology, growing out of his 1969 PhD thesis under Luke Hodgkin at Warwick University. However, many of Victor's visitors to McMaster were working in arithmetic geometry, and it was clear that Victor's own research interests were also shifting in this direction, or rather to the interface between arithmetic geometry and algebraic topology.

In 1998, Victor surprised many of us by deciding to return to work in England. He was appointed to a chair at the University of Southampton in 1998, and in 2004 he moved to the University of Sheffield, working there until his retirement in 2009. He spent the Michaelmas Term 2002 as a Visiting Professorial Fellow at Emmanuel College, and I was struck in our many conversations by the broadness of his mathematical interests and work, not to mention his many other intellectual pursuits, which included chess and music. This became even clearer near the end of his life when, despite his health failing, he began doing innovative work on a circle of questions related to the Langlands Programme, which he published as a book entitled *Derived Langlands* in 2019. Altogether, he was a remarkable mathematician, with wide intellectual interests outside mathematics.

Victor is survived by his wife Carolyn, his daughters Nina, who is a Professor of Mathematical Physics at Bristol University, Anna, who is a Professor of English at King's College London, and his son Daniel, who is now a composer and musician, after first doing a PhD in Mathematics at Imperial College London.

Death Notices

We regret to announce the following deaths:

- Matthew P. Gaffney, who died on 17 September 2015, elected an LMS member 15 November 1962.
- Abe Sklar, who died on 30 October 2020, elected an LMS member 18 May 1961.
- Helge A. Tverberg, who died on 28 December 2020, elected an LMS member 17 October 1975.
- David Borwein, who died on 3 September 2021, elected an LMS member on 20 January 1949.

Biographical Memoirs and LMS Obituaries

Since August 2021, all obituaries (both recent and historical) published in the *Bulletin of the London Mathematical Society* are free to read and can be accessed at tinyurl.com/39hjem8z.

The following obituaries have recently been published in the *Bulletin*:

- John David Philip Meldrum, 1940–2018: bit.ly/3FhneKJ.
- Samuel James Taylor, 1929–2020: bit.ly/3AdRmTA.
- Noel Glynne Lloyd, 1946–2019: bit.ly/3iyPzm5.

Society Meeting

LMS Annual General Meeting

12 November 2021, 3–6pm, Goodenough College, Mecklenburgh Square, Holborn, London WC1N 2AB. This meeting will also be live streamed on Zoom Webinar. More details of how to join online are available on the LMS website at tinyurl.com/cu4u62ry.

Website: tinyurl.com/cu4u62ry

15.00	LMS Annual General Meeting
15.30	Supporting Lecture *Random Lattices* Professor Jens Marklof (Bristol) **Abstract:** Since the fundamental contribution of Minkowski, Siegel, Rogers, Schmidt and others, averages over the space of lattices have become an indispensable tool in number theory. A key breakthrough was the arrival of techniques from ergodic theory in the 1980s and 1990s, which led to the solution of numerous classic problems, including Margulis' proof of the Oppenheim conjecture and Ratner's celebrated measure-classification theorems. This lecture will explain that there is value in thinking of lattices not just 'on average' but as intrinsically random objects — point processes — and survey some recent applications ranging from pure mathematics to statistical physics.
16.30	Tea/Coffee Break
16.55	Announcement of LMS Election Results
17.00	Presidential Address *Random Matrices and the Riemann Zeta-Function* Professor Jonathan Keating (Oxford) **Abstract:** I will speak about yet another example of the wonderful interplay between Mathematics and Physics. In this, the theory of complex quantum systems, in particular random matrix theory, turns out to provide a remarkably accurate description of certain statistical properties of the Riemann zeta-function and the distribution of the primes. I will discuss the history of this unexpected relationship, and some recent developments relating to it.

The meeting will open with Society business, including the presentation of certificates to all the 2021 LMS prize winners. These lectures are aimed at a general mathematical audience. All interested, whether LMS members or not, are most welcome to attend this meeting.

The meeting will be followed by a reception, which will be held at Goodenough College. For further details about the AGM, contact Dr John Johnston (lmsmeetings@lms.ac.uk).

Women and Non-Binary People in Mathematics: Opportunities for the Future

Location: University of Bristol
Date: 9–10 November 2021
Website: tinyurl.com/zmfdtfky

The event is targeted at undergraduates and taught postgraduates in their final two years, with the aim of encouraging them to consider PhD study and careers in research. It is particularly aimed at female or non-binary applicants. The event will include talks by women from academia and industry as well as from current PhD students; there will also be panel discussions, and opportunities for asking questions and networking.

LMS–BCS/FACS Evening Seminar

Location: De Morgan House, London, and online
Date: 18 November 2021, from 6pm
Website: tinyurl.com/9d26u8ep

In association with the British Computer Society Formal Aspects of Computing Science (BCS-FACS), the LMS hosts an annual evening seminar on aspects of the computer science–mathematics interface. These events are free to anyone who wishes to attend. The speaker this year will be Peter Sewell, who will talk on *Underpinning mainstream engineering with mathematical semantics*. This is a hybrid event with availability for 20 people to attend in person at De Morgan House, and unlimited spaces online.

LMS Research School: Adaptive Methods and Model Reduction for PDEs

Location: University of Nottingham
Date: 2–6 May 2022
Website: tinyurl.com/wuwmc2kp

Young researchers will be exposed to state-of-the-art model reduction and adaptivity techniques for PDEs, and their interplay with computational modelling, machine learning and engineering applications. Lecturers and Plenary Speakers: Wolfgang Dahmen (USC), Olga Mula (UPD), Simona Perotto (POLIMI), Serge Prudhomme (EPDM), Gianluigi Rozza (SISSA), Barbara Wohlmuth (TUM). Organisers: Andrea Cangiani (SISSA), Paul Houston and Kris van der Zee (UoN).

LMS Computer Science Colloquium

Location: Online
Date: 17 November 2021
Website: tinyurl.com/7u3e6xz7

The LMS Computer Science Colloquium is an annual online event, which includes themed talks on a topical issue at the interface of mathematics and computer science. The event is aimed at PhD students and post-docs, although others are welcome to attend. The theme for this year's event will be *Mathematical Foundations for Machine Learning*. The event will start at 10am, with introductions from the LMS Computer Science, followed by talks by four speakers.

LMS South Wales & South West Regional Meeting

Location: University of Swansea
Date: 4–6 January 2022
Website: tinyurl.com/3a2z36wc

The lectures are aimed at a general mathematical audience. All interested, whether LMS members or not, are most welcome to attend this event. This is an Ordinary Meeting of the Society. This day meeting takes place during the LMS Workshop on Braces in Bracelet Bay from 4 to 6 January 2022.

LMS Research School: Rigidity, Flexibility and Applications: An LMS Research School on Knowledge Exchange

Location: Lancaster University
Date: 18–22 July 2022
Website: tinyurl.com/5cu8827f

This Research School will involve 4 mini-courses on geometric rigidity, statistical mechanics and sphere packings, topology of linkages, and applications of rigidity to soft matter physics. Alongside the technical sessions, there will be plenary talks by experts in making an impact with mathematics. Application deadline: 31 January 2022 (registration for research students is only £150).

Society Meetings and Events

November 2021

- 8 LMS Graduate Student Meeting, online
- 12 LMS Annual General Meeting, London
- 17 LMS Computer Science Colloquium, online
- 18 LMS–BCS/FACS Evening Seminar, London and online

January 2022

- 4-6 South West & South Wales LMS Regional Meeting, Swansea

Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website (www.lms.ac.uk/content/calendar). Please send updates and corrections to calendar@lms.ac.uk.

November 2021

- 8 LMS Graduate Student Meeting, online
- 9-10 Women and Non-Binary People in Mathematics, Bristol (496)
- 12 LMS Annual General Meeting, London (496)
- 17 LMS Computer Science Colloquium, online (496)
- 18 LMS–BCS/FACS Evening Seminar, London and online (496)
- 18-20 Mathematics in Times of Crisis, online (494)

December 2021

- 14-15 Cryptography and Coding Conference, online (495)

January 2022

- 4-6 South West & South Wales LMS Regional Meeting, Swansea (496)

April 2022

- 11-13 BAMC, Loughborough University
- 25-29 Rational Points on Higher-Dimensional Varieties, ICMS, Edinburgh (495)

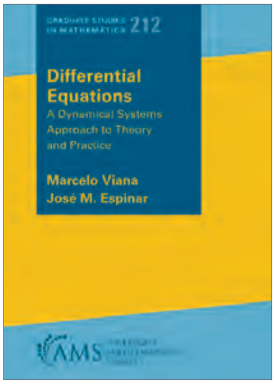
May 2022

- 2-6 Adaptive Methods and Model Reduction for PDEs Research School, Nottingham (496)
- 18-20 Mathematics in Signal Processing, Aston, Birmingham (495)

July 2022

- 18-22 Rigidity, Flexibility and Applications LMS Research School, Lancaster (496)
- 24-26 7th IMA Conference on Numerical Linear Algebra and Optimization, Birmingham (487)

AMERICAN MATHEMATICAL SOCIETY



DIFFERENTIAL EQUATIONS

A Dynamical Systems Approach to Theory and Practice

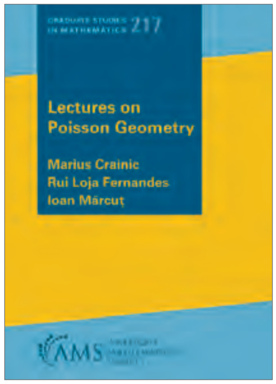
Marcelo Viana, IMPA - Instituto de Matemática Pura e Aplicada & Jose M. Espinar, Universidad de Cádiz

In collaboration with Guilherme T. Goedert and Heber Mesa

This graduate-level introduction to ordinary differential equations combines both qualitative and numerical analysis of solutions, in line with Poincaré's vision for the field over a century ago. Taking into account the remarkable development of dynamical systems since then, the authors present the core topics that every young mathematician of our time - pure and applied alike -ought to learn.

Graduate Studies in Mathematics, Vol. 212

Nov 2021 549pp 9781470451141 Hardback £99.95



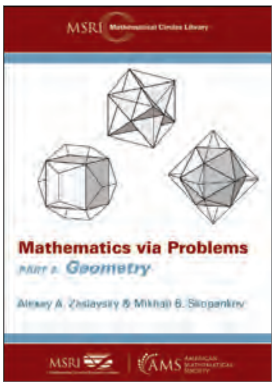
LECTURES ON POISSON GEOMETRY

Marius Crainic, Utrecht University, Rui Loja Fernandes, University of Illinois at Urbana-Champaign & Ioan Mărcuț, Radboud University

"This excellent book will be very useful for students and researchers wishing to learn the basics of Poisson geometry, as well as for those who know something about the subject but wish to update and deepen their knowledge. The authors' philosophy that Poisson geometry is an amalgam of foliation theory, symplectic geometry, and Lie theory enables them to organize the book in a very coherent way". ***—Alan Weinstein, University of California at Berkeley***

Graduate Studies in Mathematics, Vol. 217

Nov 2021 479pp 9781470464301 Hardback £99.95



MATHEMATICS VIA PROBLEMS

Part 2: Geometry

Alexey A. Zaslavsky, Central Economical and Mathematical Institute and Russia and Moscow Power Energetic Institute & Mikhail B. Skopenkov, National Research University Higher School of Economics, and Institute for Information Transmission Problems RAS

Develops important parts of mathematics through problems. The authors put together sequences of problems that allow high school students (and some undergraduates) with strong interest in mathematics to discover and recreate much of elementary mathematics and start edging into more sophisticated topics such as projective and affine geometry, solid geometry, and so on, thus building a bridge between standard high school exercises and more intricate notions in geometry.

A co-publication of the AMS and the Mathematical Sciences Research Institute

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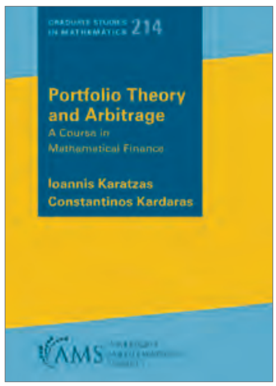

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