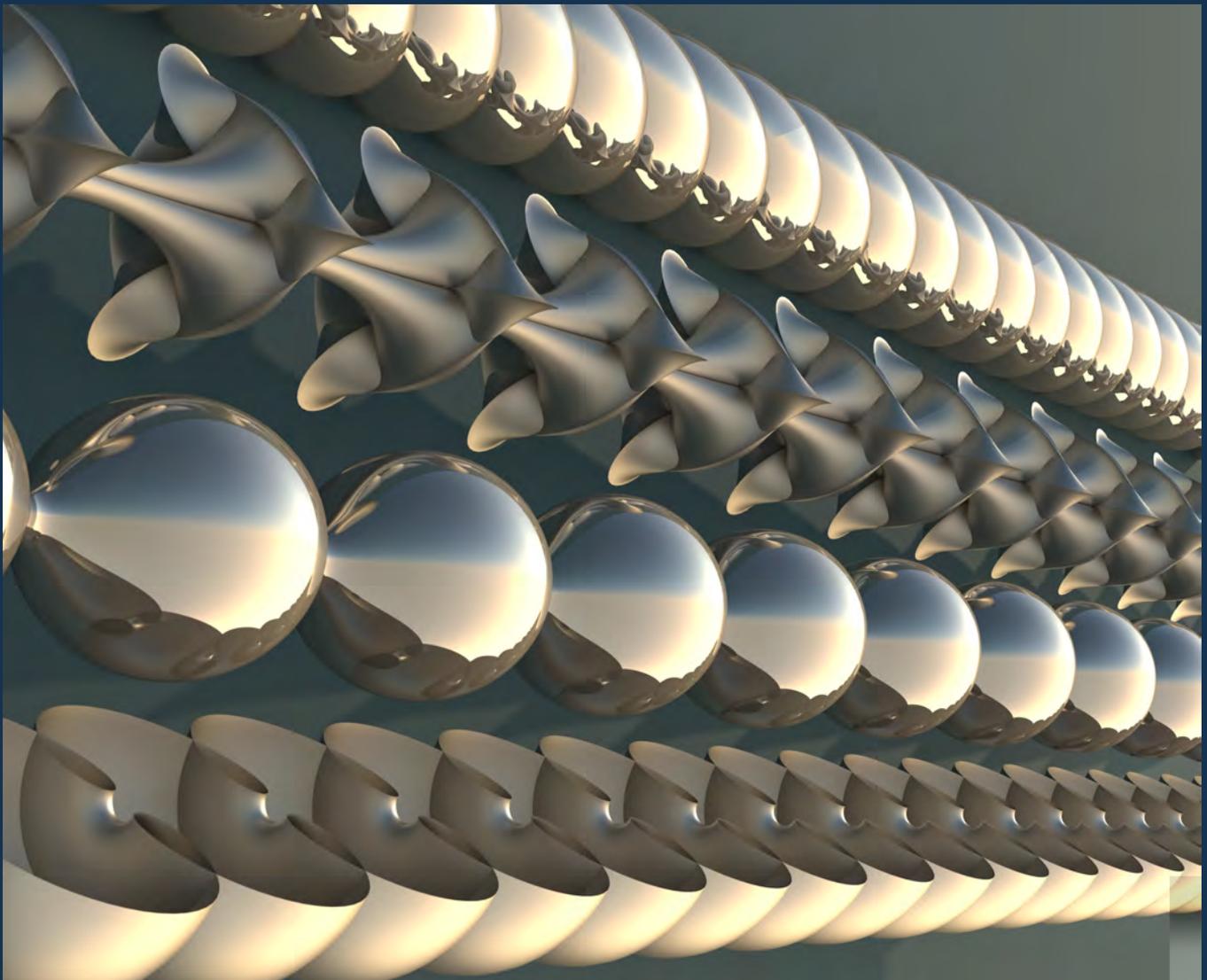




LONDON  
MATHEMATICAL  
SOCIETY  
EST. 1865

# NEWSLETTER

*Issue: 499 - March 2022*



SEMIGROUP  
THEORY

WATSON'S  
BESSEL  
FUNCTIONS

NOTES OF  
A NUMERICAL  
ANALYST

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newsletter.editor@lms.ac.uk

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## CORRESPONDENTS AND STAFF

LMS/EMS Correspondent: David Chillingworth  
Policy Digest: John Johnston  
Production: Katherine Wright  
Printing: Holbrooks Printers Ltd

## EDITORIAL OFFICE

London Mathematical Society  
De Morgan House  
57-58 Russell Square  
London WC1B 4HS  
newsletter@lms.ac.uk

Charity registration number: 252660

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Joining the LMS is a straightforward process. For membership details see [lms.ac.uk/membership](http://lms.ac.uk/membership).

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The Newsletter welcomes submissions of feature content, including mathematical articles, career related articles, and microtheses from members and non-members. Submission guidelines and LaTeX templates can be found at [lms.ac.uk/publications/submit-to-the-lms-newsletter](http://lms.ac.uk/publications/submit-to-the-lms-newsletter).

Feature content should be submitted to the editor-in-chief at [newsletter.editor@lms.ac.uk](mailto:newsletter.editor@lms.ac.uk).

News items should be sent to [newsletter@lms.ac.uk](mailto:newsletter@lms.ac.uk).

Notices of events should be prepared using the template at [lms.ac.uk/publications/lms-newsletter](http://lms.ac.uk/publications/lms-newsletter) and sent to [calendar@lms.ac.uk](mailto:calendar@lms.ac.uk).

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## LMS NEWS

## Printing and Distribution Costs for the LMS *Newsletter*

The purpose of this article is to let members know about the debate around whether the Society should continue printing and distributing physical copies of the LMS *Newsletter*, or switch it to an online-only publication.

As readers of the *Newsletter* will be aware, due to changes in the publication landscape resulting from the move to the Open Access publishing model, the LMS will be seeing a significant drop in annual income from 2022/23. The LMS continues to be well-founded, having significant capital investments, but if our annual operating expenses exceed our income (which comes broadly from publishing, investment income, membership fees and letting income) we will need to call on our capital. This will reduce the resources available to future generations of mathematicians. Council has debated this and is firmly of the view that it would be wrong to take resource away from future mathematicians if this can be avoided. Instead it has looked to see where cost-savings can be made so as to balance our income and expenditure while having the least impact on mathematics, using our charitable aims as a guide to what we should prioritise.

One possible area of cost-saving is through ceasing the printing and distribution of physical copies of the LMS *Newsletter*. The *Newsletter* costs approximately £4,000 per issue to print and distribute, and with 6 issues per year this comes to around £24,000 annually. It is also published online as a PDF document. In this age where many are comfortable with receiving information electronically, it is sensible to ask whether there is still a need for print. For comparison, £24,000 could support two Research Schools, four conferences or 20 'Research in Pairs' collaborations.

Another consideration is the environmental impact of printing and distributing up to 1400 copies of the *Newsletter* per issue. The Society is very mindful of environmental sustainability and wishes to minimise the environmental impact of its activities. Print

copies, if not read or if they could be substituted with an effective online version, are wasteful of energy.

Against this background, the Society has been considering whether it is sustainable (both financially and environmentally) to continue to produce printed copies of the *Newsletter* after this financial year.

The Newsletter Board has discussed the value of printing the *Newsletter* at length. There is a strong feeling that although some readers are perfectly comfortable with online only, there is a significant proportion of the membership who do prefer print and put considerable value on receiving their copy of the *Newsletter*. We have no scientific basis for this view at present, but propose to ask members to think again about how they would like to receive the *Newsletter* in future. There is no suggestion that members will be asked to pay more to receive a printed copy of the *Newsletter*. While the *Newsletter* continues to be printed (which depends on Council allocating a budget in July each year for printing and distribution in the financial year, which starts in August), a printed copy will continue to be available as a membership benefit.

Please do consider when asked whether you would still prefer to have a print copy, notwithstanding the arguments above. We do not expect to significantly reduce costs at this stage through reducing the number of members who receive print (as overall costs do not scale strongly with the number of copies printed) but it will help the Board and Council to gauge whether print is important to you or whether you consider an online version to be a suitable alternative.

If you would like to update your preferred format now, you can do so by logging in to your online LMS membership record at [lms.ac.uk/user](https://lms.ac.uk/user) and going to the 'My LMS Membership' tab; alternatively you can contact [membership@lms.ac.uk](mailto:membership@lms.ac.uk).

If you have any comments or feedback about this issue, please contact [newsletter@lms.ac.uk](mailto:newsletter@lms.ac.uk).

Professor Cathy Hobbs  
LMS Vice-President

## Annual Elections to LMS Council

The LMS Nominating Committee is responsible for proposing slates of candidates for vacancies on Council and vacancies on its own membership. The Nominating Committee welcomes suggestions from the membership.

Anyone who wishes to suggest someone for a position as an Officer of the Society or as a Member-at-Large of Council (now or in the future) is invited to send their suggestions to Professor Tara Brendle, the current Chair of Nominating Committee ([nominations@lms.ac.uk](mailto:nominations@lms.ac.uk)). Please provide the name and institution (if applicable) of the suggested nominee, their mathematical specialism(s), and a brief statement to explain what they could bring to Council/Nominating Committee.

It is to the benefit of the Society that Council is balanced and represents the full breadth of the mathematics community; to this end, Nominating Committee aims for a balance in gender, subject area and geographical location in its list of prospective nominees.

Nominations should be received by 15 April 2022 in order to be considered by the Nominating Committee.

In addition to the above, members may make direct nominations for election to Council or Nominating Committee. Direct nominations must be sent to the Executive Secretary's office ([nominations@lms.ac.uk](mailto:nominations@lms.ac.uk)) before noon on 1 September 2022. For details on making a direct nomination, see [lms.ac.uk/about/council/lms-elections](https://lms.ac.uk/about/council/lms-elections).

The slate as proposed by Nominating Committee, together with any direct nominations received up to that time, will be posted on the LMS website in early August.

## New Open Access Policy for UKRI-Funded Authors

A new UKRI policy ([bit.ly/3s8ir8C](https://bit.ly/3s8ir8C)) will apply to peer-reviewed research articles (including reviews) and conference proceedings submitted for publication on or after 1 April 2022.

This policy, announced in August 2020, affects academics who are publishing work that acknowledges funding from one of the seven UK Research Councils. The policy requires open access on publication under the CC BY licence (or, exceptionally, CC BY-ND) for articles and conference papers submitted on or after 1 April 2022. It also requires open access no later than 12 months after

publication for monographs, book chapters and edited collections resulting from a grant from one of the UK Research Councils, published on or after 1 January 2024. The UKRI policy will inform the open access policy for the next REF.

The policy permits two routes to publishing, the first covering fully open access journals and the second subscription journals. Both routes require immediate open access, on publication, under the CC BY licence. Embargoes on open access are no longer permitted. In practice, there are three ways to comply with the new policy.

- (1) Publish in a fully open access journal or platform.
- (2) Publish in a subscription (hybrid) journal that is part of a transformative agreement.
- (3) Publish in a subscription (hybrid) journal that is not in a transformative agreement and make the accepted manuscript available in an open access repository, under the CC BY licence, on publication.

In the first two methods, the paper is published open access under the CC BY licence. The publisher version of record is open access on the publisher's website (Gold open access). An APC may be payable depending on the journal's business model.

UKRI will continue to fund open access Article Processing Charges (APCs), through the provision of block grants to UK institutions, for papers in fully open access journals and in journals as part of 'transformative agreements' between institutions and funders. It will not fund APCs for outputs in subscription (hybrid) journals except those that have transformative agreements.

The third method where the author uploads the final accepted manuscript under a CC BY licence (Green open access) may not be permitted by the license agreement that authors are asked to sign up to by a publisher. Authors may wish to check the journal's self-archiving policy prior to submission ([jisc.ac.uk/sherpa](https://jisc.ac.uk/sherpa)) or discuss this with their institutional open access support team or the appropriate journal editorial office prior to submission.

Details of routes to compliance for LMS-owned and managed journals can be found at [lms.ac.uk/publications/open-access](https://lms.ac.uk/publications/open-access). Potential authors may also contact the LMS Editorial Office at [lmsjournals@lms.ac.uk](mailto:lmsjournals@lms.ac.uk) with any queries.

## Forthcoming LMS Events

The following events will take place in forthcoming months:

**Diverse Perspectives on Alan Turing and Society Meeting:** 4 March, De Morgan House, London

**LMS Midlands Regional Meeting and Workshop, *Mirrors, Moduli and M-theory in the Midlands*:** 4–7 April, Birmingham

**LMS Meeting and Hirst Lecture:** 6 May, De Morgan House, London

**LMS Meeting and Aitken Lecture:** 1 July, BMA House, London

A full listing of upcoming LMS events can be found on page 40.

## OTHER NEWS

### CMS Consultations on National Academy and Connected Centres Network

The Council for the Mathematical Sciences (CMS), with the support of Professor Andrew Noyes, Chair of the Joint Mathematical Council of the UK, has been consulting on proposals for a National Academy and a Connected Centres Network. Members of the LMS were strongly encouraged to submit individual responses to this consultation before the consultation closed on 21 January 2022.

A Townhall meeting took place on 11 January 2022 as part of this consultation, the recording of which can be accessed via [bit.ly/3HqKSFC](https://bit.ly/3HqKSFC).

A small task and finish group, with support from the INI, has been set up to analyse the responses following the consultation and will report to the CMS. There will be an opportunity for input from institutions in the mathematics community later in the process.

### Outreach Activity for Asylum Seekers

LMS Council has agreed to endorse an initiative to reach out to mathematicians who are asylum seekers in the UK. The initiative was begun at the University of Oxford's Mathematical Institute by Peter Grindrod, Sam Cohen and Ian Griffiths (Oxford), and aims to expand its activity to become a national resource for asylum seekers.

A number of asylum seekers in the UK are professional people with mathematics degrees who have already had careers in academic or professional sectors within their home countries, being part

of mathematically qualified communities such as professional and learned societies there. Once in the UK they are often isolated from those communities and may be unable to work in mathematics for long periods of time. The outreach initiative involves a series of online informal and inclusive interest lectures, established together with the Oxford charity Asylum Welcome, which reach out to mathematically qualified refugees and to mathematically-literate asylum seekers. The aim of these lectures is to enable attendees to re-establish an emotional contact with their mathematical pedigrees and their own identities as qualified mathematicians for however long they remain isolated.

During the summer of 2021, two Zoom pilot events were delivered by Peter Grindrod, Ian Griffith and Sam Cohen, which covered overviews and examples of a range of mathematical research fields, as well as information about the wider UK mathematical community and some specifics of how refugees might pursue mathematics-based careers in the UK, including industry sectors, education and teaching.

This mathematics-based initiative is aligned with the University of Oxford's own initiatives to support refugees, and with Mansfield and Somerville Colleges, which support refugees and asylum seekers.

Refugees and asylum seekers with a maths, or maths-related (STEM) degree, that would like to attend these sessions, may email [acadrefuge@maths.ox.ac.uk](mailto:acadrefuge@maths.ox.ac.uk). Some further details will be sought to register for sessions in the first quarter of 2022.

Professor Cathy Hobbs  
LMS Vice-President

## Clay Research Fellows 2022

The Clay Mathematics Institute has awarded the 2022 Clay Research Fellowships to Hannah Larson, Alexander Petrov and Ziquan Zhuang.



Hannah Larson will obtain her PhD in 2022 from Stanford University, where she has been advised by Ravi Vakil. Displaying remarkable ingenuity, Larson has applied the modern techniques of degeneration and

intersection theory to make significant advances in one of the classical areas of algebraic geometry — the geometry of complex curves and their moduli. Her papers bristle with surprising new ideas that attack classical problems. For example, searching for new perspectives on the space of vector bundles on the Riemann sphere, she proved striking results about the moduli space of curves and about stabilization for branched covers of the sphere (with Canning), and extended Brill-Noether theory (which governs maps of general curves to projective space) to explain seemingly chaotic behaviour in the case of low-gonality curves (with E. Larson and Vogt).

Hannah Larson has been appointed as a Clay Research Fellow for a term of five years beginning 1 July 2022.



Alexander Petrov will obtain his PhD in 2022 from Harvard University, where he has been advised by Mark Kisin. Petrov has demonstrated exceptional creativity in proving surprising theorems concerning Galois representations

and arithmetic local systems on algebraic varieties. Settling a conjecture of Litt, he proved that geometrically irreducible, arithmetic local systems on varieties over  $p$ -adic fields are essentially de Rham. He discovered a deep generalization of Belyi's famous theorem, showing that any irreducible Galois representation which arises in the cohomology of an algebraic variety over a number field, appears in the space of algebraic functions on the fundamental group of the thrice punctured sphere. And he opened a new range of possibilities with counterexamples

to a conjecture of Scholze on Hodge symmetry for rigid analytic varieties.

Alexander Petrov has been appointed as a Clay Research Fellow for a term of five years beginning 1 July 2022.



Ziquan Zhuang obtained his PhD in 2019 from the Princeton University, where he was advised by János Kollár. Since then he has been a Moore Instructor at Massachusetts Institute of Technology. Zhuang is a remarkably prolific

and inventive algebraic geometer who has already made a series of fundamental contributions to higher dimensional birational geometry. These include his landmark solution, with Liu and Xu, of the higher rank finite generation conjecture, which is the final step in the Yau-Tian-Donaldson Conjecture in the case of general Fano varieties. With Xu, Zhuang proved the positivity of the CM line bundle on the  $K$ -moduli space; with Ahmadinezhad, he invented a new framework to verify the  $K$ -stability of a large class of Fano varieties; and with Stibitz he proved striking results on birational superrigidity and  $K$ -stability of Fano varieties.

Ziquan Zhuang has been appointed as a Clay Research Fellow for a term of two years beginning 1 July 2022.

Clay Research Fellowships are awarded on the basis of the exceptional quality of candidates' research and their promise to become mathematical leaders. For more information visit [claymath.org](http://claymath.org).

## 2021 Clay Research Award



The Clay Mathematics Institute announces the 2021 Clay Research Award. The award to Bhargav Bhatt (University of Michigan) recognises his ground-breaking achievements in commutative algebra,

arithmetic algebraic geometry, and topology in the  $p$ -adic setting. His profound contributions include the development, in joint work with M. Morrow and P. Scholze, of a unified  $p$ -adic cohomology theory

(prismatic cohomology) and, in joint work with J. Lurie, a  $p$ -adic Riemann–Hilbert functor. Striking applications of this work include Bhatt’s resolution of longstanding problems in commutative algebra, in particular concerning the Cohen–Macaulay property and Kodaira vanishing up to finite covers. These results have in turn fuelled startling progress on the minimal model programme in mixed characteristic.

The Clay Research Awards, presented annually at the Clay Research Conference, celebrate the outstanding achievements of the world’s most gifted mathematicians. The award will be presented in Oxford in September 2022. For more information visit [claymath.org](http://claymath.org).

## New Scientist Article and Response

The *New Scientist* recently published a response by Caroline Wallace, LMS Executive Secretary, on Michael Brooks’ article on ‘maths anxiety’ among school students. The response (29 December) is below and you can read the original article at [tinyurl.com/2p9bm22s](http://tinyurl.com/2p9bm22s).

### Maths is a Marvel Full of Universal Truths

“I share Michael Brooks’s concerns about low levels of numeracy and high levels of “maths anxiety” (27 November 2021, p 25). But I don’t think the solution simply lies in a “more utilitarian approach” to the subject. This would risk taking the wonder and imagination out of maths, which is what inspires people to keep studying it and pushing back the frontiers of our understanding.

The value of maths to society is clear. Not only have discoveries in it led, for centuries, to applications and achievements in every area of science and technology, but it is also an inherently valuable part of our shared human experience. What’s more, its insights and discoveries don’t just apply through time, but through space too. The same maths truths apply on the other side of the universe, just as they do on Earth.

Ensuring that the pipeline of maths talent remains healthy is why the London Mathematical Society set up the Protect Pure Maths campaign last year ([protectpuremaths.uk](http://protectpuremaths.uk)). This seeks to ensure that university maths departments get the funding they need and society gets the maths graduates necessary to flourish — and to help the next generation of learners avoid maths anxiety.

Asking whether maths belongs with the sciences or humanities may be a provocative and fun diversion, but the subject faces serious challenges around funding and perception. Our campaign exists to make the case for maths. We would welcome Brooks as a supporter.”

## New Years Honours 2022

Yvonne Dawn Baker was appointed an Officer of the Order of the British Empire (OBE) for services to STEM Education. Yvonne is Chief Executive at STEM Learning.

Nira Chamberlain was appointed an Officer of the Order of the British Empire (OBE) for services to mathematical sciences. Nira is an LMS member and was President of the Institute for Mathematics and its Applications from 2020–21.

Catherine Lynne McClure was appointed an Officer of the Order of the British Empire (OBE) for services to education. Lynne is Director at Cambridge Mathematics.

## International Day of Mathematics

As always this is  $\pi$  day: March 14. The theme this year is *Mathematics Unites*, with a photo challenge for schools and universities. See [idm314.org](http://idm314.org) for further details.

There will be a global virtual celebration in English on 14 March 2–3 pm GMT. The celebration will include four lively 10 minute talks by lecturers Wilfred Ndifon (African Institute for Mathematical Sciences, Rwanda), Katie Steckles (Freelance Maths Communicator/Sheffield Hallam University, UK), Steven Strogatz (Cornell University, USA) and Laura Wynter (IBM Research, Singapore). The session is chaired by Sujatha Ramdorai, University of British Columbia, Canada.

The IDM Governing Board hopes to significantly increase the number of schools celebrating the IDM, either in the classroom, or through a larger event. One way to interpret the 2022 IDM theme *Mathematics Unites* is the fact the whole planet shares the same mathematical language. To illustrate this through a school activity, some proofs without words have been added to the material proposed for classroom activities. The IDM Governing Board needs your help for reaching the schools and school networks of your country: invite them to celebrate and to join the IDM community by registering to the

IDM Newsletter in order to keep up to date with all announcements.

The IDM Governing Board is looking for IDM Ambassadors in all countries of the world to help

increase celebrating the IDM. If you are interested to become an IDM Ambassador, please contact: [info@idm314.org](mailto:info@idm314.org).

## MATHEMATICS POLICY DIGEST

### Protect Pure Maths Campaign

*Update from President Ulrike Tillmann and Vice-President Iain Gordon*

The Protect Pure Maths (PPM) campaign, which the LMS helped to found, continues its work in support of the long-term health of all areas of the mathematical sciences. The campaign aims to secure proper funding for the mathematical sciences, as well as ensuring that they are represented and understood in Parliament and that further cuts to mathematics research in universities are prevented. Recently, following contact and advocacy by the campaign, five peers spoke about the importance of mathematical research funding during the Report Stage of the Advanced Research and Invention Agency (ARIA) Bill in the House of Lords. This can be watched from 20:34–20:49 on Tuesday 14 December at [parliamentlive.tv/Lords](http://parliamentlive.tv/Lords). MPs have been encouraged to submit questions on funding for the mathematical sciences for BEIS (Department for Business, Energy & Industrial Strategy) and Education question time, and the campaign made a submission to the House of Commons Science and Technology Committee inquiry into Diversity in STEM. Recently, in the House of Lords, Lord Davies of Brixton tabled an oral question directly concerning the future of mathematics in the UK: ‘To ask Her Majesty’s Government what plans they have to ensure that the UK remains a world leader in the mathematical sciences.’ The topic received significant interest, and the peers who spoke made considered and powerful statements on both the importance of mathematics to research and future developments, and the enjoyment of mathematics and widening accessibility. The Society was very pleased to see mathematics debated in the House of Lords with such enthusiasm, especially as this supports and advances the Society’s aim of promoting the benefits of mathematics and mathematical education to (amongst others) policy makers and funders of research. Prominent members of the academic, business and parliamentary communities have made public statements in support of the PPM campaign, and the supporter base continues to grow. Read more about the campaign at [protectpuremaths.uk](http://protectpuremaths.uk).

### UKRI Consultation on EDI Strategy

UKRI has launched its new Equality, Diversity and Inclusion (EDI) Strategy. The draft strategy sets out UKRI’s ambition for an inclusive research and innovation system and its long-term commitment to achieving this aim. An open consultation will run from 13 January to 28 March on UKRI’s Engagement Hub, the responses to which will be used to refine the draft strategy and inform the development of future action plans and measures. A first edition of the post-consultation strategy will be published in summer 2022. More details at [tinyurl.com/bnwm3fhm](http://tinyurl.com/bnwm3fhm).

### Independent Review of Research Bureaucracy: Interim Report

The BEIS Secretariat team for the Independent Review of Research Bureaucracy has published an interim report, which can be found at [tinyurl.com/2mxns94b](http://tinyurl.com/2mxns94b). A final report and recommendations are to follow in spring 2022. Any questions may be directed to the Review team at [bureaucracyreview@beis.gov.uk](mailto:bureaucracyreview@beis.gov.uk).

### Government Review of UKRI

The Government is currently undertaking a review of UKRI, led by Sir David Grant. The review aims to provide scrutiny of the performance of UKRI and assurances it is achieving the core objectives that led to its creation, as well as an assessment of its readiness to contribute to government ambitions for the future of research and innovation. The review will look to cover the efficacy, efficiency, accountability, and governance of UKRI. See details at [tinyurl.com/2y2zuvt8](http://tinyurl.com/2y2zuvt8). A final report is planned for summer 2022.

Digest prepared by Katherine Wright  
Society Business, Research & Communications  
Officer

*Note: items included in the Mathematics Policy Digest are not necessarily endorsed by the Editorial Board or the LMS.*

## EUROPEAN MATHEMATICAL SOCIETY NEWS

## Société Mathématique de France

In 2022 the Société Mathématique de France will be 150 years old. Celebration days will take place on 16, 17 and 18 March 2022. The programme can be found at [tinyurl.com/5bh8kjsc](http://tinyurl.com/5bh8kjsc).

## European Women in Mathematics

The European Women in Mathematics General Meeting 2022 (EWM GM 2022) will be held 22–26 August, in Aalto University, Espoo, Finland.

- The European Mathematical Society lecturer will be Claire Voisin (CNRS, Institut de mathématiques de Jussieu-Paris, France).
- The plenary speakers will be: Kathrin Bringmann (Cologne), Maria Bruna (Cambridge), Nina Holden (ETH Zürich and Courant Institute of Mathematical Sciences), Kaisa Miettinen (Jyväskylä), Ilaria Perugia (Vienna).
- The Gender speaker will be Jessica Wade (Imperial College London).

### Deadlines:

- February 28, 2022: Submission of Minisymposium Proposals
- March 31, 2022: Application for Travel/Accommodation Grants
- May 31, 2022: Submission of Abstracts

The conference is supported by Aalto Science Institute, Federation of Finnish Learned Societies and Foundation Compositio. More information at [tinyurl.com/42c2mbsx](http://tinyurl.com/42c2mbsx). EWM is supported by G-Research and Smith Institute.

## International Day of Mathematics

- The 2022 International Day of Mathematics (IDM) event map is online at [idm314.org](http://idm314.org).
- Anyone planning an IDM celebration in 2022 is invited to make a pre-announcement on the IDM website; details on the planned event can be added later.

- The IDM Governing Board hopes to increase significantly the number of schools celebrating the IDM, either in the classroom, or through a larger event. One way to interpret the 2022 IDM theme ‘Mathematics Unites’ is the fact the whole planet shares the same mathematical language. To illustrate this through a school activity, some proofs without words have been added to the material proposed for classroom activities. The IDM Governing Board needs your help for reaching the schools and school networks of your country: invite them to celebrate and to join the IDM community by registering to the IDM newsletter.
- The IDM Governing Board is looking for IDM Ambassadors in all countries of the world to help increase celebrating the IDM. If you are interested to become an IDM Ambassador, please contact: [info@idm314.org](mailto:info@idm314.org).
- If you have not yet done so, register to the IDM Newsletter on the IDM website to learn of all announcements.

## Heidelberg Laureate Forum

Calling Outstanding Young Researchers: Apply now for the 9th Heidelberg Laureate Forum (HLF), 18–23 September 2022. Young researchers in computer science and mathematics from all over the world are encouraged to apply for one of the 200 coveted spots. The HLF offers all accepted young researchers the opportunity to personally interact with the laureates of the most prestigious prizes in the fields of mathematics and computer science: the recipients of the Abel Prize, the ACM A.M. Turing Award, the ACM Prize in Computing, the Fields Medal, and the Nevanlinna Prize engage in a cross-generational scientific dialogue with young researchers in Heidelberg, Germany. For more information visit the website at [heidelberg-laureate-forum.org](http://heidelberg-laureate-forum.org).

EMS News prepared by David Chillingworth  
LMS/EMS Correspondent

*Note: items included in the European Mathematical Society News represent news from the EMS are not necessarily endorsed by the Editorial Board or the LMS.*

## OPPORTUNITIES

## Additional LMS Grants

*Application deadline: 15 May 2022*

### Research Reboot Scheme: An Addition to the Research in Pairs (Scheme 4) Grant

In response to a proposal from its Covid working group, the Society's Council has approved a new 'Research Reboot' grant scheme. This scheme aims to help mathematicians restart their research activities following the intense disruption and upheaval of the pandemic.

Researchers may have found themselves with very little time for research due to illness, caring responsibilities, increased teaching or administrative loads, or other factors. This scheme offers funding for travel, accommodation, subsistence and caring costs for applicants so that they can leave their usual environment to focus entirely on research for a period from two days to a week. For applications submitted by the next deadline (15 May 2022), the Reboot Retreats should take place between 1 July and 30 September 2022. Applicants should be mathematicians based in the UK and may be at any career stage.

The value of the award will be a maximum of £100 per day for accommodation, subsistence, travel and other necessary expenses to enable the research project. An additional £100 per day may be applied for to cover Caring Costs for those who have dependents. For more information and the application form visit: [lms.ac.uk/grants/research-pairs-scheme-4](https://lms.ac.uk/grants/research-pairs-scheme-4). Prospective applicants are advised to consult the guidance available at [bit.ly/3HqeYc5](https://bit.ly/3HqeYc5). If you have any queries contact [grants@lms.ac.uk](mailto:grants@lms.ac.uk).

### Online Lecture Series: An addition to the Joint Research Groups (Scheme 3) Grant

The value of the award will be £1,000 per financial year for those who would like to organise an Online Graduate Lecture Series. The Society built on its Scheme 3 research networks and expertise to enhance the education of beginning postgraduate students and to support early career researchers. The introduction of the online lecture series element to the Joint Research Groups follows the successful Online Lecturer Series grant scheme, which was run in 2020 in response to the impact of the Covid-19 pandemic on the mathematical community. Application for this element of the Scheme 3 grants is now open both to Joint Research

Groups (new and current) and to mathematicians who are not part of a Joint Research Group. For more information and the application form, please visit: [lms.ac.uk/grants/joint-research-groups-uk-scheme-3](https://lms.ac.uk/grants/joint-research-groups-uk-scheme-3). Prospective applicants are advised to consult the guidance available at [bit.ly/34vQCpt](https://bit.ly/34vQCpt). If you have any queries contact [grants@lms.ac.uk](mailto:grants@lms.ac.uk).

## Breakout Graduate Fellowship Program of the IMU CDC

Thanks to a generous donation by the winners of the Breakthrough Prizes in Mathematics (Ian Agol, Jean Bourgain, Simon Donaldson, Alex Eskin, Christopher Hacon, Martin Hairer, Maxim Kontsevich, Vincent Lafforgue, Jacob Lurie, James McKernan, Takuro Mochizuki, Terence Tao and Richard Taylor), IMU with the assistance of FIMU ([friends-imu.org](https://friends-imu.org)) is opening a new call of the IMU Committee for Developing Countries (CDC) Breakout Graduate Fellowship program to support postgraduate studies, in a developing country, leading to a PhD degree in the mathematical sciences. The IMU Breakout Graduate Fellowship Scheme offers a limited number of complete grants, with duration of up to four years, for excellent students from developing countries.

Professional mathematicians are invited to nominate highly motivated and mathematically talented students from developing countries who plan to complete a doctoral degree in a developing country, including their own home country. Nominees must have a consistently good academic record and must be seriously interested in pursuing a career of research and teaching in mathematics.

For a nomination to be eligible, the country of citizenship of the student, the country of residency and the country where the study will take place must be contained in the list of Developing Countries ([tinyurl.com/57mp9kcx](https://tinyurl.com/57mp9kcx)) as defined by IMU for the period 2019–2022.

The 2022 call opened on 1 February and the deadline for nominations is 30 May 2022. More information at [tinyurl.com/jav8t5ws](https://tinyurl.com/jav8t5ws).

Details of the current IMU Breakout Graduate Fellows are available on the IMU website at [mathunion.org/cdc/awardees-2021](https://mathunion.org/cdc/awardees-2021).

Olga Gil-Medrano  
Secretary for Policy of the CDC

## LMS ECR Professional Development Panel Discussions

A new initiative from the London Mathematical Society's Early Career Research (ECR) Committee has been the introduction of free online panel discussions for ECRs with the aim of supporting their professional development.

Taking place on Zoom over lunchtime, approximately once a month during term-time, these sessions are covering the following topics for ECRs in 2021-22:

- (1) The academic job search
- (2) Starting your first post-doc
- (3) Writing and Publication
- (4) Building your research community
- (5) All topics considered: For this last session, suggestions are welcomed from the ECR Community about what they would like to hear about to help with Professional Development and should be sent to [ecr.chair@lms.ac.uk](mailto:ecr.chair@lms.ac.uk)

So far, the first three sessions have taken place and proven to be quite popular with attendance of close to 100 ECRs at each session. Copies of the slides from these sessions are available on the LMS website at [bit.ly/3IRr26u](http://bit.ly/3IRr26u).

The next session, 'Building Your Research Community', will take place on 24 March 2022 from 12.00-1.00 pm (GMT) on Zoom. Further details, including how to register, are available on the LMS website at [bit.ly/3IRr26u](http://bit.ly/3IRr26u).

Professor Diane Maclagan, University of Warwick  
Convener for LMS ECR Professional Development  
Panel Sessions

## Emmy Noether Fellowships 2022

The LMS will award several Emmy Noether Fellowships with a value between £2,000 and £10,000 in 2022, up to a total of £25,000. The amount awarded for each fellowship will reflect the individual requirement of the applicant. The fellowships are designed to enhance the mathematical sciences research, broadly construed, of holders either re-establishing their research programme after returning from a major break associated with caring responsibilities or those requiring support to maintain their research programme while dealing with significant ongoing caring responsibilities. See application details at [lms.ac.uk/grants/lms-emmy-noether-fellowships](http://lms.ac.uk/grants/lms-emmy-noether-fellowships).

## Mastermind

BBC2's Mastermind is currently scouring the UK to find contenders for the next series which will be filmed in Belfast. Although applications close on Monday 9 May 2022 at midnight, apply as soon as possible as casting is currently under way. Entry is open to UK residents (including Channel Islands & the Isle of Man) who are aged 18 or over. Hat Trick/Hindsight Productions are committed to making programmes as inclusive as possible. Email [mastermind.hth@hattrick.com](mailto:mastermind.hth@hattrick.com) to request an application form.

## International Mathematics Competition for University Students

Conditions permitting, the 29th International Mathematics Competition (IMC) for University Students will be held from 1 to 7 August 2022, in Blagoevgrad, Bulgaria, organised by University College London and hosted by the American University in Bulgaria, Blagoevgrad. Universities are invited to send several students and one teacher as Team Leader; individual students without Team Leaders are welcome. The competition is planned for students in any stage of their undergraduate university education and will consist of two sessions of five hours each. Problems will be from the fields of Algebra, Analysis (Real and Complex), Geometry and Combinatorics. The maximum age of participants is normally 23 at the time of the IMC, although exceptions can be made. The working language will be English.

The IMC in Blagoevgrad is a residential competition, and all student participants are required to stay in the accommodation provided by the hosts. It aims to provide a friendly, comfortable and secure environment for university mathematics students to enjoy mathematics with their peers from all around the world, broaden their world perspective and be inspired to set mathematical goals for themselves that might not have been previously imaginable. Past participants have gone on to distinguished careers in mathematics. Most notably, in 2000 Caucher Birkar participated in the 7th IMC held at UCL. In 2018, he received mathematics' most prestigious award, the Fields Medal.

Over the past 28 competitions the IMC has had participants from over 200 institutions from over 50 countries. For further information and online registration visit the website at [www.imc-math.org.uk](http://www.imc-math.org.uk). Further details may be obtained from Professor John Jayne ([j.jayne@ucl.ac.uk](mailto:j.jayne@ucl.ac.uk)).

## Maximising your LMS Membership: International Connections with other Mathematical Societies

### Reciprocal Agreements with 23 International Mathematical Societies

The Society has reciprocal agreements with the following mathematical societies through which LMS members can join those societies at a 50% discount on the full membership fee of each society, if they

are not normally resident in the same country as the society. For example, a UK-based Ordinary member could join the Mathematical Society of Japan at their reciprocal membership rate. For further information about these societies through links to their webpages visit [tinyurl.com/yu6px2mj](http://tinyurl.com/yu6px2mj).

Reciprocal Societies		
Allahabad Mathematical Society	Société Mathématique de France	Nigerian Mathematical Society
American Mathematical Society	Indian Mathematical Society	Norsk Matematisk Forening
Australian Mathematical Society	Indonesian Mathematical Society	Real Sociedad Matemática Española
Belgian Mathematical Society	Irish Mathematical Society	Singapore Mathematical Society
Canadian Mathematical Society	Unione Matematica Italiana	South East Asian Mathematical Society
Dansk Matematisk Forening	Mathematical Society of Japan	Svenska Matematikersamfundet
Deutsche Mathematiker-Vereinigung	Koninklijk Wiskundig Genootschap	Swiss Mathematical Society
Finnish Mathematical Society	New Zealand Mathematical Society	

In return, members of the above societies who are not normally resident in the UK can join the London Mathematical Society as Reciprocity member and receive a 50% discount on the Ordinary membership fee only (other subscription rates e.g., Associate membership are already discounts on the Ordinary membership rate and the Society does not offer double discounts). If you have any queries, please contact [membership@lms.ac.uk](mailto:membership@lms.ac.uk).

LMS members based in either Northern Ireland or the Republic of Ireland and who are also members of the Irish Mathematical Society can choose whether to be a Reciprocity member of either the LMS or the Irish Mathematical Society.

### Discounted Membership of the European Mathematical Society (EMS)

Members of the London Mathematical Society can join the European Mathematical Society at a 50% discount of the full EMS membership fee: currently

€25.00 instead of €50.00. Further information about the European Mathematical Society is available at: <https://euro-math-soc.eu>.

To join the EMS, members can contact the EMS direct and mention their LMS membership or add EMS membership to their LMS membership record when signing in via the LMS website at [www.lms.ac.uk/user](http://www.lms.ac.uk/user), or can complete or return the LMS subscription form. Payment for EMS membership can also be made via the LMS and over 150 LMS members already pay for EMS membership alongside their LMS membership. If you have any queries contact [membership@lms.ac.uk](mailto:membership@lms.ac.uk).

LMS members who are students may be interested in the EMS' free membership for students and LMS members who are aged 60+ may be interested in the EMS' lifetime membership. Further details about EMS membership rates are available here: <https://euro-math-soc.eu/individual-members>.

### Option to Pay for Membership of European Women in Mathematics (EWM)

Members of the London Mathematical Society who are also members of the European Women in Mathematics have the option to pay for their EWM membership via their LMS Membership account and over 30 EWM members already pay for their EWM membership alongside their LMS Membership.

To do so, you must first be a member of the EWM and further details about EWM membership and

how to join are available at [tinyurl.com/2p9bpbzc](https://tinyurl.com/2p9bpbzc). If they wish, EWM members can then add their EWM membership to their LMS membership account either by logging in via the LMS website: [lms.ac.uk/user](https://lms.ac.uk/user) or can complete or return the LMS subscription form. If you have any queries, please contact [membership@lms.ac.uk](mailto:membership@lms.ac.uk).

Elizabeth Fisher  
Membership & Grants Manager

## LMS Council Diary — A Personal View

Council met in Goodenough College, London, in a hybrid format, with some members in person and others joining remotely via videoconference, on the morning of Friday 12 November. The meeting began with the President's business, including an update on the Protect Pure Maths Campaign, which had resulted in letters being sent to the House of Commons Education Select Committee proposing an inquiry into mathematics education and to the Chancellor of the Exchequer asking for the release of the remaining additional funding for mathematics that had been promised by government, recent discussions at the CMS Board about the proposal for a National Academy, and the continued success of the Levelling Up initiative.

In the next item of business, Vice-President Gordon presented a very interesting paper on environmental sustainability, which led to a lively discussion and agreement that it would be useful to identify a small number of priorities to be pursued in the first instance. Council then agreed to endorse and help advertise both an initiative to reach out to mathematicians who are asylum seekers in the UK, which had originated at the University of Oxford and is now being expanded nationwide, and an initiative proposed by the ICMS to deliver online colloquia to undergraduate students around the country with a view to strengthening the research pipeline. Other business included updates from the Publications Secretary and Treasurer and an extended discussion on what Council would like a possible Select Committee inquiry into mathematics education to focus on if such an inquiry were to

go ahead, with wide agreement that recruitment and retention of teachers and thriving mathematics departments in universities across the UK were both crucial.

The meeting concluded with the President giving thanks on behalf of Council to outgoing Council Members, wishing luck to those members up for election, and thanking all Council members for the collegiate atmosphere during his time as President. On behalf of Council, Vice-President Hobbs thanked the President for the way in which he had chaired Council over what had been a very challenging two years.

Elaine Crooks  
Member-at-Large

## Long-Standing LMS Members

The London Mathematical Society greatly values the contributions made by all its members and would particularly like to acknowledge the following members who will be celebrating membership milestones of more than 50 years in 2022.

Elizabeth Fisher  
Membership & Grants Manager

**75 years membership:** Eric L. Huppert.

**Over 70 years membership:** Bernard Fishel, Godfrey L. Isaacs.

**Over 65 years membership:** John C. Amson, John F. Bowers, Daniel E. Cohen, David E. Edmunds, David A. Edwards, Hanafi K. Farahat, Ioan M. James, Roger

Penrose, Roy L. Perry, Francis Rayner, John R. Ringrose, G.E. Wall.

**65 years membership:** Ronald Brown, Aldric L. Brown, David Monk, Michael F. Newman, Dennis C. Russell.

**Over 60 years membership:** David A. Burgess, Roger W. Carter, Hallard T. Croft, Roy O. Davies, Gearoid De Barra, Ian M.S. Dey, Vlastimil Dlab, Roland F. Hoskins, Glenys Ingram, Donald Keedwell, I.G. Macdonald, Joseph F. Manogue, Alun O. Morris, Frank Rhodes, Stewart A. Robertson, John Rutter, Eira J. Scourfield, Dona Strauss, Ronald F. Turner-Smith, C. Terence C. Wall, Alan West.

**60 years membership:** Benjamin Baumslag, John H.E. Cohn, James O.C. Ezeilo, John F.C. Kingman, Albert A. Mullin, John E. Peters, Joseph B. Roberts, Anthony C. Thompson, Sheila O. Williams.

**Over 55 years membership:** J. Clifford Ault, John C.R. Batty, Alan F. Beardon, Homer Bechtell, Thomas S. Blyth, M.C. Bramwell, William Brown, Charles W. Curtis, M.A.H. Dempster, M.M. Dodson, J. Keith Dugdale, Roger H. Dye, L.C. Eggan, Barry G. Eke, K. David Elworthy, David Epstein, John Erdos, Edward A. Evans, W. Desmond Evans, Cyril F. Gardiner, David J.H. Garling, Peter Giblin, Robin E. Harte, Philip Heywood, Keith E. Hirst, Terence H. Jackson, Otto H. Kegel, J. David Knowles, E. Christopher Lance, David G. Larman, Malcolm T. McGregor, Ian M. Michael, Hugh Morton, Frederick C. Piper, Oliver Pretzel, John S. Pym, George A. Reid, Derek J.S. Robinson, H. Peter Rogosinski, James Edward Roseblade, Colin P. Rourke, Stephan M. Rudolfer, Brian F. Steer, Nelson M. Stephens, Bill Stephenson, W. Brian Stewart, Anthony E. Stratton, Graham F. Vincent-Smith, Grant Walker, John F. Watters, Bertram Wehrfritz, Alfred Weinmann, David J. White, Joyce E. Whittington.

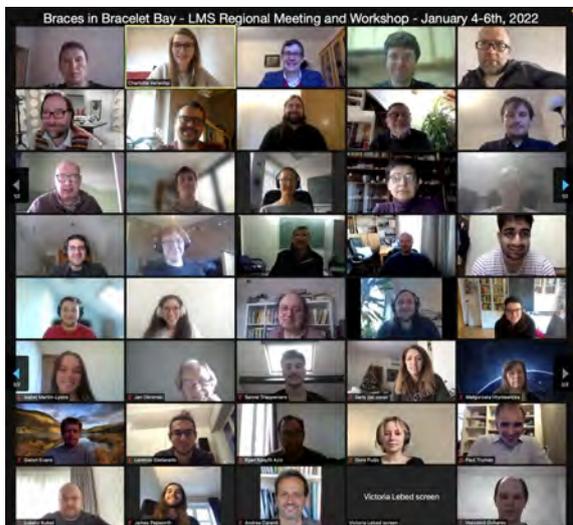
**55 years membership:** Roger M. Bryant, Allan G.R. Calder, Munibur R. Chowdhury, Michael J. Collins, P. Laurie Davies, Martin J. Dunwoody, Roger A. Fenn, James W.M. Ford, W.B. Raymond Lickorish, John McKay, John F. Rennison, Keith Rowlands, Rodney Sharp, David Tall, Martin Antony Walker, Christopher M. Williams, Geoffrey V. Wood.

**Over 50 years membership:** R.B.J.T. Allenby, David H. Armitage, Mark A. Armstrong, Irene A. Ault, Roger C. Baker, Anthony D. Barnard, Earl R. Berkson, Thomas G.K. Berry, W. Meurig Beynon, Nicholas H. Bingham, David A. Brannan, F. Trevor Brawn, Sheila Carter, Donald J. Collins, Roger J. Cook, Alan Cornish, H.G. Dales, Richard Delanghe, Peter Dixon, John D. Dixon, John Duncan, Colin R. Fletcher, John B. Fountain, T. Alastair Gillespie, Charles Goldie, Paul R. Goodey, John A. Haight, F. Mary Hart, Wilfrid A. Hodges, Jonathan Hodgson, A. Geoffrey Howson, Graham J.O. Jameson, David L. Johnson, Cherry Kearton, Michael E. Keating, Monsur A. Kenku, Arthur Knoebel, P. Ekkehard Kopp, Thomas J. Laffey, Earl E. Lazerson, Charles Leedham-Green, David W. Lewis, John H. Loxton, Bernard L. Luffman, Bob Margolis, Vassilis Mavron, Peter McMullen, Paul R. Meyer, William Moran, Kung-Fu Ng, Peter J. Nicholls, Adam J. Ostaszewski, David R. Page, Fredos Papangelou, Jon V. Pepper, Timothy Porter, Hilary A. Priestley, Andrew Rae, Helen D. Robinson, J. Christopher Robson, Harvey Rose, Peter Rowlinson, Philip Samuels, Klaus Schmidt, John Silvester, Allan M. Sinclair, David Singerman, David B. Singmaster, Patrick F. Smith, Linda R. Sons, Gabrielle A. Stoy, R.F. Streater, Elizabeth Taylor, Donald Taylor, R. Kenneth Thomas, Brian Thorpe, David Tipple, R.C. Vaughan, Michael R. Vaughan-Lee, Richard Walton, Jeffrey R.L. Webb, Dominic J.A. Welsh, Christopher Wensley, Anthony Wickstead, Douglas R. Woodall, J.D.M. Wright, Abraham Zaks.

**50 years of membership:** Béla Bollobás, Colin Campbell, C.T.J. Dodson, Brian Fisher, Leslie R. Fletcher, Allan Hayes, Anthony J.W. Hilton, William Holcombe, Chris Houghton, Tiao-Tiao Hsu, Ignacio Iribarren, John E. Jayne, Hendrik W.E. Leemans, T.H. Lenagan, Frederick Long, Zafer Mahmud, Harold V. McIntosh, Paul Milnes, Peter E. Newstead, Roger J. Plymen, John R. Quine, Nigel Ray, David L. Salinger, Jean F.H. Schmets, G. Peter Scott, Dan Segal, John R. Shackell, David B. Webber, Graham D. Williams, Stephen Wilson, R.M.W. Wood, Christopher F. Woodcock.

## REPORTS OF THE LMS

## Report: Braces in Bracelet Bay — South Wales and South West Regional Meeting, PhD Workshop and Workshop on Braces



The Braces in Bracelet Bay online meeting and workshops took place on 4–6 January 2022, with approximately 70 registrants. It began on Tuesday morning with a PhD Students' Workshop with speakers, Lorenzo Stefanello (Università di Pisa) and Andrew Darlington (University of Exeter), and an audience of around 40. The 2022 LMS South Wales and South West Regional Meeting followed in the afternoon, with up to 47 participants, during which Shahn Majid (QMUL), Anastasia Doikou (Heriot-Watt University) and Leandro Vendramin (Vrije Universiteit Brussel) spoke. The specialist Workshop on Braces then took place on 5 and 6 January, with about 35 to 45 participating in each talk. The speakers were, in order of speaking, T. Gateva-Ivanova, J. Okninski, Ł. Kubat, C. Verwimp, I. Colazzo, V. Gubarev, A. Pilitowska, V. Lebed, P. Truman, A. Ghobadi.

Lorenzo Stefanello began the PhD Students' Workshop by presenting his joint work with Andrea Caranti on *An Explicit Construction of Brace Blocks*, generalising some results of Alan Koch on brace blocks. The audience appreciated his very interesting and clear talk and raised a number of questions, which indicated some new future directions.

Andrew Darlington, in his second year of his PhD course, then gave his first presentation at a mathematics workshop on *Transitive Subgroups of the Holomorph and Hopf-Galois Structures on Separable Field Extensions*. He also gave very clear indication of his future goals, and answered several questions competently. The audience enjoyed his talk very much also.

Following lunch, the host and principal organiser, Professor Tomasz Brzezinski (Swansea University), introduced the first speaker of the Regional Meeting, Professor Shahn Majid, as one of the inventors of braided geometry. Professor Majid gave a fascinating colloquium style overview of *Yang-Baxter Equations and Braided Geometry*, illuminating the significance of the Yang-Baxter equation (YBE) and its variants in category theory and quantum algebra, and reporting on recent results relating to quantum groups, knot theory and braided geometry.

Professor Anastasia Doikou gave a clear and detailed exposition on the *Yang-Baxter Equation, Quantum Integrability and Braces*. She began with some motivation from a physical point of view by reviewing quantum integrability and explaining the mathematical connections with the YBE and their roles in quantum algebras, (quasi) Hopf algebras, braid groups etc. Her main objectives were to apply the theory of braces to derive new quantum algebras, construct a new class of quantum integrable systems, find symmetries and open transfer matrices. She also discussed quasi-triangular Hopf algebras and expressed set theoretic solutions as Drinfeld twists.

Professor Leandro Vendramin's talk on *Radical Rings, Braces and the Yang-Baxter Equation* provided more insight into one of the main focusses of the Workshop, namely the role braces and their generalisations play in the study of set theoretic solutions to the YBE. Several open problems were discussed in some detail.

With the Regional Meeting talks having motivated and introduced the use of algebraic structures to study set theoretic solutions of the YBE, the scene was set for the specialist Workshop. An enjoyable, informative and stimulating two days of a talks and discussions, facilitated by virtual breakout rooms, on the latest developments in the

interplay between brace theory, (YB-)semi-trusses, Hopf-Galois structures, Rota–Baxter groups and solutions of the YBE ensued. There was a nice mix of speakers from various algebraic communities that ranged from close-to-completion PhD students, post-doctoral researchers and well-established researchers, including pioneers in the algebraic study of set theoretic YBE and brace theory, such as Professor Tatiana Gateva-Ivanova and Professor Jan Okninski. The grouping of talks according to cognate themes facilitated discussions. Several insights were made and new research directions and the potential for new collaborative projects began to emerge.

All three meetings ran very smoothly under the friendly stewardship of Professor Brzezinski, who was assisted by Charlotte Verwimp (a PhD student at Vrije Universiteit Brussel, who introduced YB-semitrusses as a means to construct solutions of the YBE) and Dr Bernard Rybolowicz (a post-doc at Heriot-Watt University). Links to the slides of each talk along with video recordings of most talks were made available at [sites.google.com/view/lmsmeetingbracesinbraceletbay](https://sites.google.com/view/lmsmeetingbracesinbraceletbay).

Gwion Evans  
Aberystwyth University

## Records of Proceedings at LMS meetings

### Ordinary Meeting: 4 January 2022

This meeting was held virtually on Zoom, hosted by Swansea University, as part of the South West & South Wales Regional Meeting. Up to 47 members and visitors were present for all or part of the Society meeting session. This one day meeting took place during the LMS Workshop on Braces in Bracelet Bay from 4 to 6 January 2022.

The meeting began at 2.00 pm with Vice-President, Professor Catherine Hobbs, in the Chair.

Due to the online nature of the meeting, no members signed the Members' Book and were admitted to the Society.

Professor Tomasz Brzezinski, Swansea University introduced the first lecture given by Professor Shahn Majid (QMUL) on *Yang–Baxter Equations and Braided Geometry* and the second lecture by Professor Anastasia Doikou (Heriot-Watt University) on *Yang–Baxter Equation, Quantum Integrability and Braces*.

After tea, Professor Jan Okninski (University of Warsaw) introduced the third lecture by Professor Leandro Vendramin (Vrije Universiteit Brussel) on *Radical Rings, Braces and the Yang–Baxter Equation*.

Professor Hobbs thanked the speakers for their excellent lectures and then expressed the thanks of the Society for a wonderful meeting to the organiser, Professor Tomasz Brzezinski of Swansea University, together with the three external scientific advisers who helped select the speakers: Professor Jan Okninski (University of Warsaw), Professor Agata Smoktunowicz (University of Edinburgh) and Professor Eric Jespers (Vrije Universiteit Brussel).

# Semigroup Theory Comes In From the Cold

MARK V. LAWSON

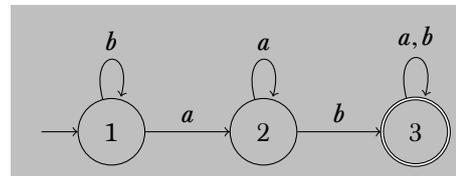
A semigroup is just a set equipped with an associative binary operation whereas a monoid is a semigroup with a (perforce) unique identity. How is it possible to have a *theory* about things so general?

In fact, one no more studies *all* semigroups than one studies *all* groups. Instead, classes of semigroups are studied which are both interesting in their own right and have applications to other subjects. The aim of this article is to describe some of those classes and touch on their applications.

To begin, at the beginning. The monoid of the natural numbers  $\mathbb{N}$  equipped with addition is the first algebraic structure we meet and is the simplest algebraic structure tout court. But questions about finitely generated submonoids of the natural numbers immediately lead to real mathematics — the reader will have come across such submonoids before, because questions about them are often phrased in terms of the values that can be made from a collection of stamps of different denominations. If we are given  $r$  relatively prime positive integers  $a_1, \dots, a_r$ , then the submonoid of  $\mathbb{N}$  they generate, denoted by  $\langle a_1, \dots, a_r \rangle$ , consists of all numbers of the form  $m_1 a_1 + \dots + m_r a_r$  where  $m_1, \dots, m_r$  are natural numbers. It is not immediately obvious — remember that we are working only with natural numbers — but every natural number from some value onwards belongs to this submonoid. This means there is a largest natural number, called the Frobenius number, that does *not* belong to this submonoid. In the case where  $r = 2$ , the Frobenius number is easy to calculate: it is  $a_1 a_2 - (a_1 + a_2)$ , a result known to Sylvester. But determining the Frobenius number for  $r \geq 3$  is, in general, highly non-trivial. The submonoids of the natural numbers generated in this way are called *numerical semigroups*; their theory is not only deep but also has a range of applications [1].

Amongst the first general class of semigroups to be intensively studied were the finite ones, a theory particularly associated with the names of John Rhodes and Bret Tilson. The motivation for studying them did not come only from finite group theory, as might be expected, but also from the theory of finite-state automata. As an example, the picture below shows a simple finite-state automaton that

searches for the string  $ab$  in an input string of  $as$  and  $bs$ . You can think of it as part of a simple text editor:



The states of this finite-state automaton are the numbered circles; the initial state, which is state 1, is indicated by the inward pointing arrow; the terminal state, which is state 3, is indicated by the double circle. This machine operates as follows: it starts in its initial state and reads an input string of  $as$  and  $bs$  from left-to-right by following the labelled arrows; if at any point the machine enters the terminal state then the input string is said to be recognized, otherwise it is rejected. The set of strings recognized by a finite-state automaton is called the language recognized by the machine and, more generally, a language is said to be recognizable if there is some finite-state automaton that recognizes it in this way. The theory of recognizable languages is of interest to computer scientists and even group-theorists since they can be used to define the class of automatic groups. Their significance to us lies in the fact that finite-state automata determine finite monoids. The basic point is that the composition of functions is associative and so the set of all functions on a set with  $n$  elements (say,  $1, 2, 3, \dots, n$ ) forms a monoid  $\mathcal{T}_n$ , called the full transformation monoid. In our example, each input string of  $as$  and  $bs$  determines a function of the set  $\{1, 2, 3\}$  to itself and so the totality of such input strings determines a submonoid of  $\mathcal{T}_3$ . Thus a finite-state automaton determines both a recognizable language and a finite monoid. This might lead you to expect a correspondence between finite semigroups and recognizable languages. In fact, one has to work with suitable *families* of monoids and suitable *families* of recognizable languages formalized in what is called Eilenberg's variety theorem. See

[20] for the basics and [23] for a modern approach. The great open problem of this theory is rooted in the fundamental distinction between reversible and irreversible computations: namely, to determine the group-complexity of a finite semigroup. This question takes us back to the origins of finite semigroup theory in automata theory and the construction of such automata in series-parallel terms from irreversible components — flip-flops — and reversible components — which can be interpreted in terms of groups.

Despite this connection between finite semigroups and finite-state automata, the theory of finite semigroups became increasingly algebraic over the year, shedding its automata-theoretic trappings. However, the theory of finite semigroups has a different flavour from that of finite groups. Whereas in group theory, it is homomorphisms that are important, in the case of finite semigroups it is the so-called relational morphisms. A special case of such a morphism is the notion of division: we say that a semigroup  $S$  divides a semigroup  $T$  if  $S$  is a homomorphic image of a subsemigroup of  $T$ . This definition is a legacy of the idea of one automaton *simulating* another; I shall describe an important application of the notion of division towards the end of this article.

Just as with finite groups, the theory of representations of finite monoids by means of finite matrices is also mathematically interesting. Two examples to illustrate this will have to suffice here. First, with every language, one may associate a zeta function. Using the theory of virtual characters of finite monoids, one can prove that the zeta function of a so-called cyclic recognizable language is a rational function [24, Theorem 12.5]. The sofic shifts yield examples of such languages. Second, random walks on finite semigroups — specifically those in which every element is idempotent — are important in the theory of certain Markov processes; such finite monoids arise from hyperplane arrangements and were effectively known to Tits himself, but the associative law was disguised in terms of projections [24, Chapter 14]. The difference between the representation theory of groups and monoids has been cited as something positive from the perspective of cryptography [11]. Unlike the case of finite groups in good characteristic, monoid algebras are rarely semisimple meaning that modern tools, such as quivers, are needed to understand them.

I have already touched on free monoids since they are involved in the study of finite-state automata, and subsets of free monoids are just the (formal) languages. If  $A$  is a set then the set of all strings over  $A$ , denoted by  $A^*$ , is the free monoid on  $A$ , where the binary operation is concatenation and the identity is the empty string. There is a natural monoid homomorphism  $\lambda: A^* \rightarrow \mathbb{N}$  given by the length function  $x \mapsto |x|$ . If the set  $A$  contains exactly one element then the free monoid is actually isomorphic via this homomorphism to the natural numbers under addition. More generally, free monoids can be characterized in terms of this length function thanks to a theorem of Levi: they are the left cancellative monoids  $M$  with the property that incomparable principal right ideals are disjoint and equipped with a monoid homomorphism  $\lambda: M \rightarrow \mathbb{N}$  such that  $\lambda^{-1}(0)$  is only the set containing the identity. Free monoids have numerous applications [15], but here I will focus on an algebraic fact that at first sight looks problematical: submonoids of free monoids need not be free. We now turn this observation into a definition. Those finite subsets that *do* generate free submonoids are called codes, the terminology arising from the fact that such codes really can be used to encode information, with the Huffman codes of lossless data-compression being examples.

Sometimes monoids hide in plain sight. The theory of self-similar group actions has become the focus of considerable attention in recent years, particularly since the Grigorchuk group is just such a group and was the first example of a group of intermediate growth [17]. Self-similar group actions arise from a group  $G$  and a free monoid  $A^*$  together with what amounts to a two-sided action: of  $G$  on  $A^*$  and  $A^*$  on  $G$ . This two-sided action is an example of what is called a *Zappa-Szép* product of  $A^*$  and  $G$  and leads to a monoid structure defined on the direct product  $A^* \times G$ , denoted by  $A^* \bowtie G$ . The monoid  $A^* \bowtie G$  is analogous to a free monoid but with a non-trivial group of units  $G$ . It is, in fact, an example of what we term a left Rees monoid: that is, a left cancellative monoid  $M$  in which incomparable principal right ideals are disjoint and equipped with a monoid homomorphism  $\lambda: M \rightarrow \mathbb{N}$  with the property that  $\lambda^{-1}(0)$  is the group of units of  $M$ . Thus every self-similar group action gives rise to a left Rees monoid, but the converse is also true: every left Rees monoid gives rise to a self-similar group action. It follows that self-similar group actions can be packaged into a class of left cancellative monoids [13] characterized by their arithmetic properties — by which we mean, by properties of their ideals.

Left Rees monoids were discovered long before self-similar group actions were introduced by group theorists since they are defined in Chapter 6 of [19] and arise for purely semigroup-theoretic reasons.

Groups describe symmetries but they say nothing about the way in which the parts of a structure are integrated into the whole. This requires inverse semigroups. A semigroup is said to be inverse if for each element  $s$  in the semigroup there is a unique element, denoted by  $s^{-1}$ , such that  $s = ss^{-1}s$  and  $s^{-1} = s^{-1}ss^{-1}$ . The elements  $s^{-1}s$  and  $ss^{-1}$  are idempotents and an inverse semigroup with a unique idempotent is exactly a group. The correct way to think about inverse semigroups is as the abstraction of the symmetric inverse monoids  $\mathcal{F}_X$ ; that is, the set of all bijections between the subsets of the set  $X$ . The idempotents of  $\mathcal{F}_X$  are the identity functions on the subsets of  $X$  and the product of two idempotents is determined by the intersection of the subsets involved. It follows that the idempotents of  $\mathcal{F}_X$  commute, something that is true for arbitrary inverse semigroups [12]. Inverse semigroups are ubiquitous: they arise naturally in the theory of  $C^*$ -algebras [18]; they can be used as the basis of a definition of a ‘non-commutative lattice’ and so lead to a non-commutative generalization of classical Stone duality [14]; they can be used to provide an abstract setting for the Banach-Tarski paradox and in this guise there is natural connection between inverse semigroups and certain commutative monoids [25]; and they are fundamental to understanding a class of toposes known as étendues [6].

Let me now describe two theorems and their related subject areas that show the depths of semigroup theory.

It is easy to find examples of finite semigroups whose idempotents commute. For example, the finite symmetric inverse monoid  $\mathcal{F}_n$ , the set of all bijections between the subsets of  $\{1, 2, \dots, n\}$ , is just such an example, as are all semigroups that divide  $\mathcal{F}_n$ . Ash proved the remarkable result that every finite semigroup whose idempotents commute must divide a finite symmetric inverse monoid. His proof used Ramsey theory in an essential way [2]. Ash went on to generalize this result in his ground-breaking theorem [3]. Like any deep mathematical result, this has consequences in many unlikely places, such as the structure of the profinite topology on free groups [8]. Furthermore, it is shown in [9], that Ash’s theorem is related to the basic mathematical problem of extending partial automorphisms to automorphisms;

a special case of this problem lies behind the setting up of classical Galois theory, for example.

As is evident from this article, the set of idempotents of a semigroup is an interesting object. Remarkably, the set of idempotents of an arbitrary semigroup can be formalized by means of what is termed a biordered set [16] and every biordered set is the biordered set of the set of idempotents of some semigroup [5]; specifically, the semigroup freely generated by the biordered set (in a well-defined way). In recent work, the structure of such semigroups has been scrutinized. In particular, it was natural to suppose that their maximal subgroups would be free, but this turns out to be false [4]. Not only that, but every group can appear as such a maximal subgroup [7]. More generally, the theory of idempotent generated semigroups is important in the theory of reductive algebraic monoids, whose biordered sets are natural generalizations of buildings [21, 22].

I have omitted any discussion of history in this article, but it is revealing to read the account given in [10] which takes us up to 1970; this will show semigroup theory’s debt to Eastern European mathematicians. Semigroup theory was largely forged during the Cold War, but now it has come in from the cold.

### Acknowledgements

I have, of course, been guided by my own interests in writing this article, but I would like to thank a number of colleagues who commented on the first draft, none of whom can be blamed for any of the infelicities that remain: John Fountain (York), Victoria Gould (York), Stuart Margolis (Bar Ilan), Volodymyr Mazorchuk (Uppsala), and Benjamin Steinberg (CUNY).

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### Mark V. Lawson

Mark is a professor of mathematics at Heriot-Watt University, Edinburgh. His main research interests are in algebraic semigroup theory. In 2017, he won

the Mahony-Neumann-Room prize of the Australian Mathematical Society for his paper *A noncommutative generalization of Stone duality*. Mark was born in southeast Essex, failed his 11-plus and received his formative education in a secondary modern and a technical college.

# One Hundred Years of Watson's *Bessel Functions*

P. A. MARTIN

Inevitably, Watson's awesome book is a mine of useful information on special functions. But it is also a superb source for all mathematicians interested in the development of classical analytical methods. We give some background on Watson and his work, and celebrate 100 years of his masterpiece.



Figure 1. G. N. Watson, with winged collar

George Neville Watson (1886–1965) served as the 36th President of the London Mathematical Society (1933–1935) [7] (Figure 1). He was born in Westward Ho!, a seaside village in the west of England. He became a student at Trinity College, Cambridge, in 1904; Fellows there included E. T. Whittaker, G. H. Hardy and E. W. Barnes. Watson stayed in Cambridge for ten years. Then, after four years at University College London (UCL), he moved to the University of Birmingham, where he spent the rest of his career, from 1918 until he retired in 1951 as the Mason Professor of Pure Mathematics.

Watson is perhaps best known as one half of 'Whittaker and Watson'; they co-authored what became a standard book on mathematical analysis and special functions [13]. Whittaker wrote the first edition alone in 1902 (378 pages) but Watson contributed much to later editions, including new material and much more rigour [9, p. 553], [14, p. 524]; the fourth edition [13] has 608 pages. After he retired,

Watson envisaged a much expanded edition, but he did not complete it [9, p. 553], [11, p. 256]; after his death, numerous manuscript pages were deposited in the archives of the University of Birmingham.

Watson is well known to applied mathematicians for several contributions:

- The Watson transform, which is a method for replacing a slowly convergent infinite series by another that converges rapidly. He used this method in two papers on the propagation of electromagnetic waves around the Earth, including calculations showing that the postulated Heaviside layer was consistent with experimental results [14, p. 522].
- Watson's lemma, which gives the asymptotic expansion of Laplace-type integrals. He proved this useful result in the context of his study of parabolic cylinder functions [9, p. 554].
- Watson integrals, which are certain triple integrals of trigonometric functions [14, p. 527], [15].

These can be seen as examples of Watson's skill as a 'problem solver': he was 'most ready to help a colleague in difficulties and was willing to go to great trouble over problems of applied mathematics in which he was not basically interested' [14, p. 527].

However, it is Watson's *Treatise on the Theory of Bessel Functions* (WBF) that stands out as a singular scholarly achievement (Figure 2).

WBF was first published in 1922, with a second edition in 1944 [12]. From the Preface:

This book has been designed with two objects in view. The first is the development of applications of the fundamental processes of the theory of functions of complex variables. ... The second object is the compilation of a collection of results which would be of value to

the increasing number of Mathematicians and Physicists who encounter Bessel functions in the course of their researches.

Both objects were achieved: WBF is still in print, and it has been cited about 20,000 times. (Some reviews of both editions have been collected in [8].) Nowadays, one might consult the *NIST Digital Library of Mathematical Functions* for a quick summary of relevant formulae, but WBF remains as the prime resource for serious study of Bessel functions and their many relatives.

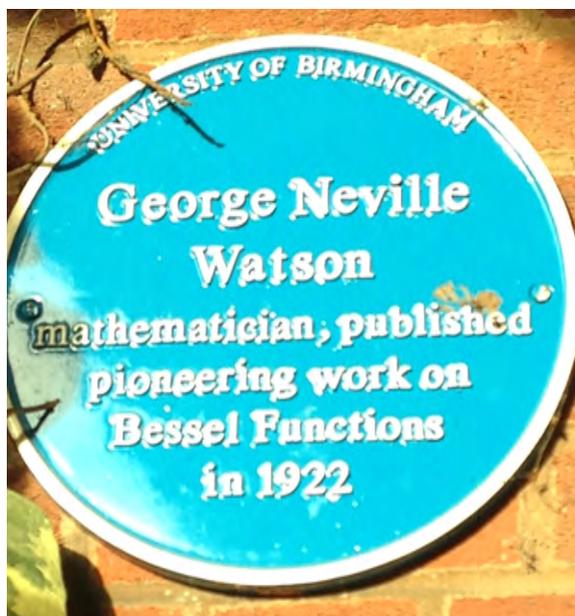


Figure 2. Blue plaque at the University of Birmingham

Watson is clear that he is aiming for a complete survey of everything known about Bessel functions: there is an extensive bibliography. (For more on the history of Bessel functions, see [6].) However, in many cases, he supplies new proofs of known results. He fixes definitions and notations. For example, he uses  $J_\nu(z)$  and  $Y_\nu(z)$  for Bessel functions of the first and second kinds, where  $z$  and  $\nu$  are arbitrary complex variables. ‘The book contains not only formulae and theoretical investigations, but also extensive tables, some of which Watson had himself calculated. Throughout his life he found relaxation in numerical work, for which he used a Brunsviga calculating machine’ [9, p. 555]. That may be true, but we also know that Watson declined a request to do ballistics calculations in Karl Pearson’s laboratory at UCL during World War I; see the letter from A. V. Hill to Pearson quoted by Barrow-Green [4, p. 97], where some indication of Watson’s character in 1917 may

be gleaned: ‘I have known Watson these last 12 years [Hill and Watson were exact contemporaries at Trinity College], and am interested (and distressed) to hear he is exactly like he used to be. I didn’t know that the War, and coming to London, had not made him more humble and human, but from what you say it obviously has not.’

After publication of the first edition of WBF in 1922, Watson completed his work on the fourth edition of Whittaker and Watson in 1927 [13]. The following decade ‘might be described as Watson’s “Ramanujan period”’ [14, p. 525]. ‘He had received from G. H. Hardy copies of Ramanujan’s famous note books’ [9, p. 555] and he then endeavoured to supply proofs: this generated about 30 papers. (For a complete list of Watson’s papers, up to 1962, see [9]; he also co-authored a long paper 20 years after his death [2].) According to B. C. Berndt, ‘Watson invested at least ten years to the editing of Ramanujan’s notebooks. He never completed his task, but fortunately his efforts have been preserved’ and many proofs in Berndt’s book are due to Watson [5, p. vi]. Rankin [10] has given a detailed description of Watson’s involvement with Ramanujan’s notebooks.

A second edition of WBF was published in 1944: why? Rankin notes that, during World War II, WBF ‘was in great demand in government scientific establishments, both in [the UK] and abroad. It became difficult to acquire and unofficial copies were circulated in some quarters. It was no doubt largely for this reason that a second edition appeared in 1944’ [9, p. 555]. R. A. Askey offers an American perspective, stating that he had ‘been told that when the work on the first successful atomic pile was being done at the University of Chicago, a copy of [WBF] was chained to a table and always open’ [3]. Bessel functions arise in various problems of applied mathematics, and many of these had to be solved quickly. However, in the preface to the second edition, Watson famously admits that his ‘interest in Bessel functions has waned since 1922’, and so his revisions are confined to the ‘correction of minor errors and misprints and to the emendation of a few assertions’. He does not even cite his own occasional papers from the 1930s on Bessel functions.

In his lectures at the University of Manchester on asymptotic methods, Fritz Ursell [1] liked to remark that he knew of only two light-hearted remarks in WBF [12]. One concerns the Stokes phenomenon and its discovery: on p. 202, Watson notes that ‘it was apparently one of those which are made at three o’clock in the morning’. Ursell did not identify the

second remark, but the author is fond of this one on p. 523: ‘The construction of the Neumann series when the Maclaurin series is given is consequently now merely a matter of analytical ingenuity’.

Concerning Watson the man, there are some evocative remarks in [9] and [11]; see also [8]. For example, in ‘manner and appearance (he always wore a wing collar) he recalled the professors of an earlier generation’ (see Figure 1) and he ‘did not like telephones and regarded them as “an invention of the devil”’ [11, p. 256]. He had similar strong aversions to motor-cars and fountain pens [8]. Although he ‘had a great admiration for his friend and co-author, Sir Edmund Whittaker [who was Professor in Edinburgh], he only visited Scotland twice, once in June 1939 to receive his Honorary LL. D. from Edinburgh University, and in July 1914 to attend the Napier Tercentenary Congress. He used to say that he feared to make a third visit ... as each of his two previous visits had precipitated a major European catastrophe’ [9, p. 552]. Evidently, Watson had a sense of humour!

### Acknowledgements

I thank Tony Rawlins for permission to use his picture of the blue plaque in Figure 2 and June Barrow-Green for pointing out the letter about Watson in [4]. I thank them, David Abrahams and Stefan Llewellyn Smith for comments and encouragement.

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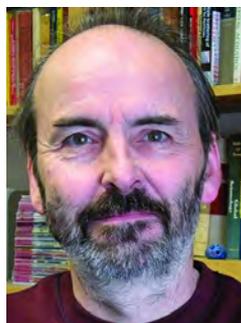
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### Paul Martin



Paul is a professor in the Department of Applied Mathematics and Statistics at the Colorado School of Mines. Born in London, he emigrated to the USA in 1999 after 24 years at the University of Manchester. His

main research interests concern waves and their interaction with objects. Apart from playing with Bessel functions and his four grandchildren, he also enjoys all aspects of the fluid mechanics of beer.

## Notes of a Numerical Analyst

## Is Everything a Rational Function?

NICK TREFETHEN FRS

There's an old idea that I call the Kirchberger Principle, since it was expressed by Hilbert's student Paul Kirchberger in 1903. (It almost surely goes back further, and if you know a 19th-century source, I'd be glad to hear from you.)

Since the only operations that can be carried out numerically are the four elementary operations of addition, subtraction, multiplication and division, it follows that we are only masters of more general functions insofar as we can replace them by rational functions, that is, represent them approximately.

This observation seems so basic that one could hardly doubt it. But it leads to a puzzle. If Kirchberger's Principle is valid, then numerical analysis should be more or less synonymous with rational approximation theory. This is patently not so. Why not? I've been thinking about this a good deal, and I'd like to offer three explanations.

The first is an observation about computers. The closer you look at actual machines, the harder it is to argue that  $+$ ,  $-$ ,  $*$ ,  $/$  are the atoms from which other operations are composed. In practice,  $/$  is reduced to the first three by algorithms such as Newton's method. Moreover, very similar reductions are used for  $\sqrt{\phantom{x}}$ , making it hard to justify any difference in status between  $/$  and  $\sqrt{\phantom{x}}$ .

The second observation also has to do with how we, and our machines, actually compute. We *manipulate digits*. This goes well beyond  $+$ ,  $-$ ,  $*$ ,  $/$ , for it requires comparisons and branches. For example, the first step in evaluating  $\sin(x)$  may be to shift  $x$  to the interval  $[0, 2\pi)$ . So Kirchberger must be modified by the footnote that our approximations are not globally rational but *piecewise rational*.

The third observation is the most interesting mathematically. Suppose we do Newton's method, say, to evaluate  $\sqrt{x}$ :

$$t_{k+1} := \frac{1}{2} \left( t_k + \frac{x}{t_k} \right).$$

If  $t_0 = 1$ , the result after  $k$  steps is a *composite* rational function of  $x$  of the form  $r_k(\cdots r_1(x)\cdots)$ , and it has degree  $2^{k-1}$ , not  $k$ . This is far from the usual setup in approximation theory, where we consider approximations in the class of all rational functions of a given degree  $n$ .

The more you dig about these questions, the further you find yourself from Kirchberger. In almost any calculation with real numbers, to get  $d$  digits of accuracy, we must do at least  $O(d)$  work. For addition,  $O(d)$  is enough. For multiplication, division or square root, it's  $O(d \log d)$ . A process involving Newton's method would seem to multiply the cost by another  $\log d$ , but that can be avoided if you adjust the precision as you go.

So in computing with real numbers, no operations are atomic: they all require more work as you demand more accuracy. And once you realize this, the traditional distinction between finite and infinite processes fades away. A linear system of equations  $Ax = b$  is the archetypical *finite* problem of numerical linear algebra, solvable in a finite number of steps, whereas an eigenvalue problem  $Ax = \lambda x$  is *infinite*, requiring an iteration. But in the end they both cost  $O(d \log d)$ .

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**Nick Trefethen**

Trefethen is Professor of Numerical Analysis and head of the Numerical Analysis Group at the University of Oxford.

## Mathematics News Flash

Jonathan Fraser reports on some recent breakthroughs in mathematics.

### A proof of the Erdős-Faber-Lovász conjecture

**AUTHORS:** Dong Yeap Kang, Tom Kelly, Daniela Kühn, Abhishek Methuku, and Deryk Osthus  
**ACCESS:** <https://arxiv.org/abs/2101.04698>

During a party hosted by Vance Faber in 1972, Erdős, Faber and Lovász formulated the following conjecture in combinatorics: the chromatic index of any linear hypergraph on  $n$  vertices is at most  $n$ . This problem grew in esteem over the years, with Erdős eventually offering a prize of \$500 for a solution. This paper proves the conjecture for all sufficiently large  $n$ .

A *hypergraph* is a generalisation of a graph where an edge may ‘connect’ any number of vertices. A hypergraph is said to be *linear* if two distinct edges (interpreted as subsets of the vertex set) intersect in at most one vertex. Finally, the *chromatic index* of a hypergraph is the smallest number of colours needed to colour the edges such that any two edges sharing a vertex have distinct colours.

Kahn conjectured that the bound in the Erdős-Faber-Lovász conjecture can be improved if the hypergraph is “far” from a certain collection of extremal examples. This paper confirms a quantitative version of Kahn’s conjecture by establishing linear and sublinear ‘stability estimates’.

### Equiangular lines with a fixed angle

**AUTHORS:** Zilin Jiang, Jonathan Tidor, Yuan Yao, Shengtong Zhang, and Yufei Zhao  
**ACCESS:** <https://arxiv.org/abs/1907.12466>

A collection of lines through the origin in  $\mathbb{R}^n$  are said to be *equiangular* if every pair is separated by a common angle. An interesting problem, with applications in numerous areas of pure and applied maths and statistics, is to determine the largest cardinality of a set of equiangular lines as a function of the ambient spatial dimension  $n$ . It turns out that this number grows like  $n^2$  and as  $n$  increases, the best angle to choose approaches  $\pi/2$  in all known examples.

This paper, published in *Annals of Mathematics* in 2021, considers the same problem but for a *fixed* angle. That is, they ask for the largest cardinality of a set of equiangular lines as a function of  $n$  and a fixed common angle. This version of the problem has also attracted substantial attention in the literature. Unsurprisingly, the growth (as  $n$  increases) depends on the fixed angle in a subtle way and the solution to the problem is presented in terms of graphs whose spectral radius corresponds to the fixed angle.

### An asymptotically sharp form of Ball’s integral inequality by probability methods

**AUTHOR:** Susanna Spektor  
**ACCESS:** <https://arxiv.org/abs/1708.08106>

A well-known and counter-intuitive result of Keith Ball is that the volume of an  $(n-1)$ -dimensional section of a cube in  $\mathbb{R}^n$  never exceeds  $\sqrt{2}$ : a value, strikingly, independent of  $n$ . In order to prove this result, Ball (Lemma 3, *Cube Slicing in  $\mathbb{R}^n$* , 1986) established the inequality:

$$\frac{1}{\pi} \int_{-\infty}^{\infty} \left( \frac{\sin^2 t}{t^2} \right)^p dt \leq \frac{1}{\sqrt{p}}$$

for  $p \geq 1$ . This paper, published in *Bulletin of the London Mathematical Society* in 2021, introduces a novel, and very simple, *B*-spline approach to estimating the above integral. Various estimates are given and an asymptotic result is established showing

$$\frac{1}{\pi} \int_{-\infty}^{\infty} \left( \frac{\sin^2 t}{t^2} \right)^p dt \sim \frac{\sqrt{3/\pi}}{\sqrt{p}}$$

for  $p \rightarrow \infty$ .



**Jonathan Fraser** is a Professor at the University of St Andrews and an Editor of this *Newsletter*. He is pictured here in the mist with Dylan (3) and some trees (age unknown).

## Reciprocal Societies: Allahabad Mathematical Society

The Allahabad Mathematical Society was founded on 25 November, 1958 by B. N. Prasad, an eminent mathematician together with his close disciples. The publication of a research journal named as *Indian Journal of Mathematics* was started.



B. N. Prasad had obtained the PhD degree of the University of Liverpool in 1931 under the supervision of E. C. Titchmarsh, and worked with legendary names like Emil Borel, Arnold Denjoy and E. C. Titchmarsh. He was

Fellow of the National Institute of Sciences of India, General President of Indian Science Congress, and President of the National Academy of Sciences of India. The President of India conferred upon him the title of 'Padma Bhushan' in the year 1963, in recognition of his services to the cause of Science and Education was nominated Member of the Rajya Sabha, the upper house of the Indian Parliament.



Pramila Srivastava, who obtained a D.Phil. degree in 1956 from the University of Allahabad under the supervision of B.N. Prasad and a D.Sc. degree in the year 1960, made outstanding contributions in areas of Infinite Series and Integrals including Dirichlet's Series and

Riesz Summability. She was a Reader at Banaras Hindu University, Visiting Professor at Marburg Lahn University Germany and Professor at University of Allahabad. In the year 1986 she launched the journal 'Bulletin of the Allahabad Mathematical Society' and also initiated publication of the 'Lecture Notes Series'. Invited by the New York Academy of Sciences in the year 1994 she was also bestowed with the honour of being a member of the New York Academy of Sciences.

The Society is a non-profit scientific organization for promoting the cause of advanced study and research in various branches of mathematics including theoretical physics and mathematical statistics. It provides an outlet for original researches through its publications and organizes lectures, symposia and conferences. The Society has reciprocity arrangement with various societies including the American Mathematical Society, Canadian Mathematical Society, Cambridge Philosophical Society, Edinburgh Mathematical Society and the London Mathematical Society. Exchange agreements exist with several Universities and Scientific/Educational Institutions.

The Society publishes two journals. The *Indian Journal of Mathematics* (IJM) is devoted to original research papers in different branches of mathematics and mathematical statistics; three numbers per volume are published annually. The *Bulletin of The Allahabad Mathematical Society* (BAMS) aims at publishing research papers in all areas of mathematics, pure and applied, and high quality research survey/expository articles. Both journals are refereed/peer-reviewed and are indexed in Mathematical Reviews (USA), Current Mathematical Publications (USA), Zentralblatt MATH (Germany); the IJM is reviewed in Scopus (Elsevier) also. The Society has also published *Proceedings: George Bachman Memorial Conference* (GBMC) and Lecture Note Series on topics of interest.

Manuscripts submitted to the journals are evaluated entirely on the basis of their scientific content. Measures are taken to uphold the highest standards of publication ethics and editorial decisions are based on peer review. The Society has a rich library of Journals received from all over the world in exchange of its publications.

Members of the Society are entitled to get all publications of the Society at discounted price for their personal use. Members of the American Mathematical Society and the London Mathematical Society are entitled to annual membership by paying only half the amount of annual membership fee.

# Microthesis: $RO(C_2)$ -graded Bredon Homology of a Point

IGOR SIKORA

Singular homology of a point is very easy to compute. However, if we try to compute the homology of a point in equivariant topology, it is not so simple anymore.

## Algebraic invariants in topology

One of the first characterisations of topological spaces we learn is that two topological spaces  $X, Y$  are 'topologically identical' if there is a continuous bijection  $f: X \rightarrow Y$  such that  $f^{-1}: Y \rightarrow X$  is also continuous. If such a map exists, we say that  $X$  and  $Y$  are *homeomorphic*.

The relation of being homeomorphic is easy to define. However, in most situations it is very hard to verify if two given topological spaces satisfy this relation. To make this task easier, we use *algebraic invariants*, which are algebraic objects that can be assigned to topological spaces. In general, invariants are tools that potentially could recognise if topological spaces are not the same, that is, if the invariants are different, the spaces are not homeomorphic. However, they are not a perfect tool, for if the invariants are equal we cannot conclude that the spaces are homeomorphic. Defining, computing and exploring such invariants is the domain of *algebraic topology*.

In order to define many of the algebraic invariants in algebraic topology we distinguish topological spaces up to a weaker relation than the homeomorphism, namely a *homotopy equivalence*. Intuitively we can say that two spaces are homotopy equivalent if they can be transformed continuously into one another. In particular, spaces of different dimensions can be homotopy equivalent, e.g., a point and a line or plane, or any Euclidean space. This distinguishes the homotopy equivalence relation from homeomorphism - spaces of different dimensions cannot be homeomorphic.

## Singular homology and the homology of a point

An example of an algebraic invariant on which we will focus here is *singular homology*  $H_*(X)$ . Roughly

speaking, it is a graded abelian group containing information about the count of holes of various dimensions in a topological space  $X$ , which are not filled.

There are many approaches which one can take when defining singular homology. The first one considers the singular chain complex of a topological space  $X$  and defines  $H_*(X)$  to be the homology of this chain complex. Alternatively, one can use the axiomatic approach and define the singular homology as a sequence of functors from the category of topological spaces to the category of abelian groups, subject to *Eilenberg-Steenrod axioms*. Among these axioms there is the *dimension axiom*, stating that the (singular) homology of a point needs to be concentrated in the degree zero. In other words, the only non-zero homology group of a point appears in the degree 0.

An interesting fact is that we can drop this axiom. This leads us to the world of *generalised homology theories*, among which there are K-theory, cobordism and stable homotopy. This observation, together with the possibility of representing generalised homology theories by *spectra*, i.e., topological spaces with stability data, gave rise to the *stable homotopy theory*.

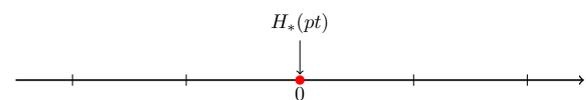


Figure 1. The dimension axiom: the homology of a point is concentrated in degree 0.

## Equivariant topology

The situation gets more complicated when our topological space  $X$  has some symmetries which we want to take into account. Such symmetries are encoded by *an action of a group  $G$  on  $X$* . The branch of topology which consider topological

spaces together with group actions on them is called *equivariant topology*.

To illustrate this, let us consider three actions of the cyclic group with two elements  $C_2$  on the one-dimensional sphere (a.k.a. the circle). Since  $C_2$  has only one non-trivial element, we need to distinguish one symmetry on the circle:

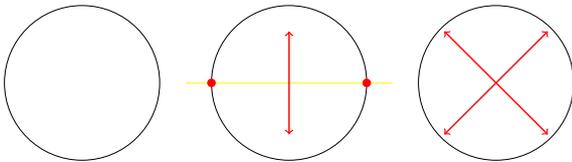


Figure 2. Three actions of  $C_2$  on the circle.

The circle on the left has as a distinguished symmetry the identity, which means that the action is *trivial*. In the middle circle the distinguished symmetry is the axis symmetry, and in the right circle we distinguish the reflection with respect to the center of the circle (the antipodal action).

We can easily see that these three actions are substantially different by counting the points fixed by each symmetry. In the first action all of the points in the circle are fixed. The second action has two fixed points, while the last one has no fixed points. Thus using such a simple topological space as a circle, we produced three objects which are different in equivariant topology.

### Bredon homology and $RO(G)$ -grading

In equivariant topology the role of the ordinary homology is played by *Bredon homology*. It has an interesting property: in the equivariant world, we naturally consider homology theories to be graded over  $RO(G)$ , the ring of representations of the group  $G$ . This is formed by considering isomorphism classes of real representations of  $G$  as a ring with the direct sum and the tensor product with added additive inverses. In other words, for any real  $G$ -representation  $V$  there are homology groups  $H_V^G(X)$  and  $H_{-V}^G(X)$ .

This feature is one of the reasons why computations in Bredon homology are quite complex. For example, even computing  $RO(G)$ -graded homology of a point is not an easy task. As an illustration of this, we can see an example of the  $RO(G)$ -graded homology of a point, given in Figure 3. The horizontal axis corresponds to multiplicities of trivial representations

of  $C_2$ , while the vertical axis corresponds to multiplicities of the sign representation.

On the horizontal axis there is only one non-trivial group - in the degree 0. This reflects the dimension axiom in the equivariant topology (compare with Figure 1). The Bredon homology of a point is concentrated in degree zero, but only when we restrict our grading to trivial representations. However, when we leave the comfortable world of trivial representations (described by the vertical axis in the picture), we get a lot of non-zero entries.

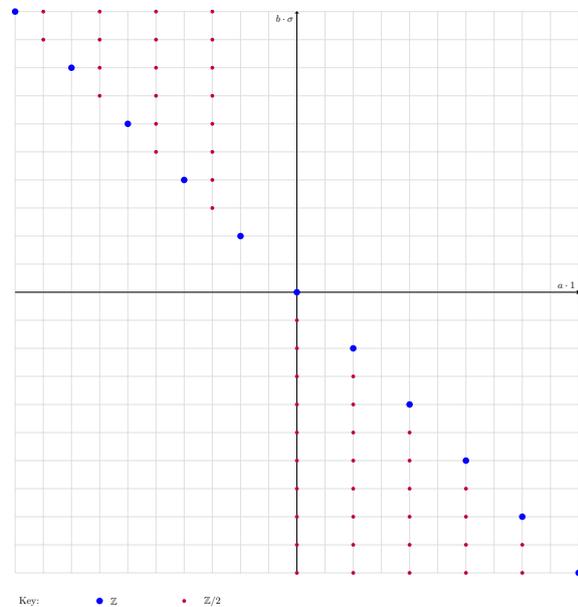


Figure 3. The  $RO(C_2)$ -graded homology of a point with coefficients in  $\mathbb{Z}$ .

Computations in  $RO(G)$ -graded homology theories is the part of algebraic topology which has small number of computed examples. Therefore the impact of my thesis was to compute the Bredon homology of a point using a method based on the *Tate diagram*, as developed by Greenlees and May.



### Igor Sikora

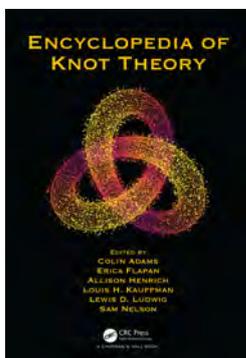
Igor is a finishing PhD student at the University of Warwick. His main research interests are in algebraic topology, especially in equivariant homotopy theory.

Outside of the research, Igor is a keen sailor and a (retired) trombone player in wind/brass bands.

# Encyclopedia of Knot Theory

edited by Colin Adams, Erica Flapan, Allison Henrich, Louis H. Kauffman,  
Lewis D. Ludwig, and Sam Nelson, £190, ISBN: 978-1138297845

Review by Brent Everitt



Knot theory is an area of mathematics that requires no introduction, and while this massive tome is certainly no introductory text, it does give a panoramic — and, well, *encyclopaedic* — view of this vast subject.

The book is divided into 91 short chapters, most of which are 5–15 pages long, and apart from a small number of exceptions, no two chapters have the same authors. This clocks in at a whopping 941 pages. Each chapter is relatively self-contained, independent of the others and individually referenced. All of which makes for pleasant dipping while you are sitting in your favourite armchair.

As knot theory starts from a very intuitive place, a significant amount of the book could be read by a starting undergraduate student in mathematics. There is still plenty here for more mature mathematicians (even professional knot theorists) and some parts — particularly those on quantum and homological invariants — have quite formidable prerequisites.

The chapters are grouped into 15 parts following broad themes. Parts I and II introduce the subject and the various ways to represent knots and links and generally get a mathematical handle on them. There are the, by-now ubiquitous, planar diagrams and Reidemeister moves; various coding mechanisms that could in principle be computerised (Gauss codes, DT codes); knots and links as the closures of braids and summaries of some attempts at tabulations of knots.

Parts III and IV devote themselves to important classes of knots. The torus links (those winding  $p$  times one way and  $q$  times the other way around a torus) are a classic test-bed for the calculation of invariants. The

hyperbolic knots — whose complement in  $S^3$  can be given the structure of a hyperbolic 3-manifold — are famously ubiquitous and have volume that interacts in a mysterious way with quantum invariants. The alternating knots, having a planar diagram where the crossings alternate over and under, have the simplest and nicest theory: the Tait conjectures hold and their homological invariants (such as Khovanov homology) are often determined by simpler polynomial ones (such as the Jones polynomial). There are also the rational knots, satellite knots, and other less well-known classes.

There is a part on the relationship between knots and surfaces and in particular Seifert surfaces and the various genera of a knot. Another part deals with higher dimensional knots with a particular emphasis on knotted surfaces in the 4-sphere — this contains the prettiest pictures in the book of knotted surfaces and the higher dimensional analogues of the Reidemeister moves between them.

Part VII contains myriad ‘knot-like’ objects: abstract knots, knotoids, braidoids, singular knots and virtual knots. There are chapters on spatial graphs, which are embeddings of *graphs* rather than of copies of  $S^1$ , into the 3-sphere. It turns out that one can mimic the basic theory of classical knots in this setting. Part XIV contains topics that might ordinarily be overlooked by mathematicians: constructing actual physical knots from sticks for example, and open-ended knots. These are the kind of knots encountered by the ordinary person in the street in shoe-laces and Hoover cords. The final part makes the inevitable connections with biology and chemistry.

The remaining five parts deal with the most successful area of the subject: knot invariants. There has been a steady development of these over the last century in order to answer the central problem of knot theory: when are two knots different from each other, or occasionally, when are two knots the same? An invariant is just a mathematical object that can be stuck onto

a knot like a post-it note. Maybe one explanation for the enduring appeal of the subject, apart from its intuitive origins, is that there seems to be no limit to the sophistication of the invariants that can arise.

The unknotting, crossing and bridge numbers are just that — simple numerical invariants. Polynomial invariants emerged in the work of Alexander in the 1920s, with both the Alexander and the Jones polynomial (1980s) providing the catalyst for some extraordinary connections to other areas of mathematics. Quantum invariants, motivated by ideas from mathematical physics, thus warrant a part to themselves. Knot theory was one of the first areas of mathematics to be swept-up in the categorification movement of the 1990s–2000s. The two most well-known homological invariants that categorify the Alexander polynomial (knot-Floer homology) and the Jones polynomial (Khovanov homology) deservedly get their own parts.

A book with such an ambitious remit is bound to contain omissions and oddities. I wonder if a more systematic attempt might have been made to define the basic terms; instead, this is left to the individual authors to do, with the result that ‘what is a knot’ and ‘when are two knots the same or different’

are fundamental questions that the reader might have to look up someplace else. But this is a small point compared to what has been achieved by this encyclopaedia, which would make a fine addition to any personal or departmental library, or to a departmental coffee table.

#### FURTHER READING

[1] C.C. Adams, *The knot book: An elementary introduction to the mathematical theory of knots*, W.H. Freeman and Co., 1994.

[2] L.H. Kauffman, *Knots and physics*, Series on Knots and Everything, vol. 53, 4th ed., World Scientific, 2013.

[3] W.B. Raymond Lickorish, *An introduction to knot theory*, Graduate Texts in Mathematics, vol. 175, Springer, 1997.



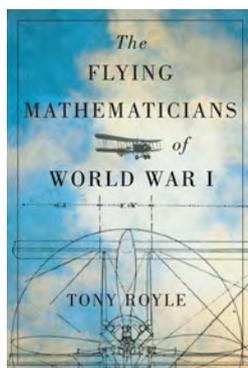
#### Brent Everitt

Brent Everitt is Reader in Mathematics at the University of York. His research interests are in algebra and topology.

## The Flying Mathematicians of World War I

by Tony Royle, McGill-Queen's University Press, 2020, CAD\$ 37.95, ISBN: 978-0228003731

Review by Deborah Kent



mathematics. Partly, the work is so riveting because

Tony Royle has often captivated conference-goers with tales of early aeronautical derring-do. He draws a crowd with research on aircraft design and testing in the early twentieth century, when aerodynamics was an experimental science, teetering between guess work and predictive

of the courageous, dedicated, and curious women and men pushing the boundaries of knowledge. And partly it's because Royle's own experience as a pilot in the Royal Air Force and commercial airlines means he knows all the technical aspects of flight and, significantly, also the real risks and emotions. When he talks of being in a stalled plane, say, plummeting towards earth, it's clear he really *does* know what that feels like. Still, I wondered how would the enthusiasm and humanity of those presentations translate into a book?

Very well, it turns out. In *The Flying Mathematicians of World War I*, Royle presents a richly textured narrative of the heady early days of powered flight integrated with his own personal insights. While the book features men, like Hawker and deHavilland, who are often associated with the development of aeroplanes, it also highlights lesser-known figures with pivotal contributions at the Royal Aircraft Factory. Some went on to scientific output that now overshadows their World War I era aviation work, while many others died during test flights. Notably, Royle also discusses the significant challenges overcome by some pioneering women involved with engineering mathematics in the early 1900s. The book presents the remarkable stories of Hilda Hudson, Letitia Chitty, and Beatrice Cave-Browne-Cave and their work with the Technical Section of the Admiralty Air Department.

A spectacular assortment of archival sources animate the story. Royle not only used official engineering diagrams, aircraft photos, factory correspondence, test flight reports, etc, but also had access to personal correspondence, scrapbooks, and flight logs from private collections. I hope those families know their precious heirlooms could not be in more appreciative hands than those of a trained historian familiar with the wind turbulence of an open cockpit — just like those WWI-era test pilots. The logbooks especially come alive through Royle's reading. Because he *knows* about flight logs, he's able to extract and communicate key features of the experiments and their outcomes. The book has lots of illustrations, and I found them incredibly helpful in understanding new (to me) things like aircraft rigging wires or destructive tests on undercarriage struts. And likewise for appreciating both the appearance and output of particular instruments (photokymograph, anyone?). Overall, Royle does a terrific job introducing the mathematics and flight lingo necessary to follow the arc of the story as well as appreciate the nature of the technical challenges and share a sense of relief at the solutions. There is also helpful appendix further detailing Engineering and Aerodynamic Issues.

As the book explores the technical developments of powered flight, the real people involved are ever close at hand. I marvelled at anecdotes describing superhuman calm during brain-scrambling g-forces of aerobatic flying. One gets a clear sense of workplace tensions between career aviators and the mathematicians who got their wings, better to understand the aircraft they were designing. Participating in any capacity required resilience and commitment to an enterprise that sometimes involved watching friends and colleagues plummet to their deaths. Pictures communicate a vivid sense of the working conditions and the precariousness of those early planes. Those are powerfully interspersed with photos of the mathematicians, engineers, and scientists in domestic scenes — on beach walks, taking tea, or playing yard games with (or in some cases as) children. Always throughout the narrative, Royle communicates great admiration for the bravery and much respect for work of the early pioneers of flight.

In some very few cases, what might be a catchy turn of phrase in person reads as slightly clunky in the book. For example, “once airborne, his downward velocity rather exceeded his horizontal, causing significant bodily trauma upon his reacquaintance with the terra firma” [p. 14]. Still, this readable book tells a compelling story of early aircraft design and testing during WWI in Britain, suited to a wide audience. Royle's blend of personal aviation experience and technical training as an historian of mathematics make him a skilled and enthusiastic guide.



### Deborah Kent

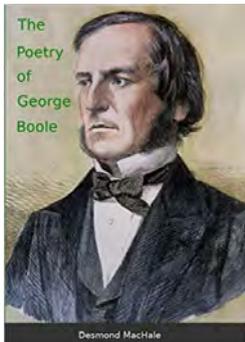
Deborah Kent is a Reader in History of Mathematics at the University of St. Andrews, although pandemic conditions

have dramatically delayed her arrival there. Her main research interests are in mathematical sciences in the 19th and early 20th centuries.

# The Poetry of George Boole

by Desmond MacHale, Logic Press, 2020, £31.89, ISBN: 978-1-71652-027-3

Review by Mark McCartney



In a recent feature article in the *Newsletter* (May 2021, pp.20–25), Desmond MacHale gave a wonderfully readable overview of George Boole's poetry. If after reading that article you found your appetite whetted for more on Boole's versifying, then

MacHale's book *The Poetry of George Boole*, which contains all of Boole's surviving poetry, is the place to go.

Boole's output ranged from juvenile translations of classical Greek and Latin authors, to sonnets, religious verse, and reflections on history. He was not the only mathematician or physicist in the nineteenth century to try their hand at poetry. For example, William Rowan Hamilton, J. J. Sylvester and James Clerk Maxwell all wrote poems, with what might be delicately described as varying levels of success [1, 2]. For my money, Maxwell's poetry is the best of them (and furthermore, in my view, some of Maxwell's poetry is very good indeed). However, as the seventy or so poems collected in this volume show, Boole was, I am sorry to say, *not* amongst the best of them. As MacHale himself admits, 'Boole's works exhibited all the outward trappings of poetry, but most of the time lacked true poetic feeling' (p. 131). He also quotes the Irish academic and poet Sean Lucy who states that 'Boole was a very able versifier who sometimes rose to poetry. His works have an earnest, serious, dutiful quality one expects of the Victorian period . . . These solemnities, with a fondness for abstraction and a lack of originality of language, make most of his verses heavy, and many of them dull.' (p. 127)

With two quotes like that, you are probably thinking that you will give this book a miss. But that would be a mistake. After all, it would be more of a surprise if Boole had been a first class mathematician *and* a first class poet, rather than the combination of abilities that actually arose. It is the fact that poetry was the form which Boole used in his spare hours

to express his thoughts which matters. By reading his poems we find out more about who he was: his thoughts on religion, or on Ireland, or on architecture, or on loss. All of these are as much a part of who Boole was as his work on Boolean algebra, and if we want to piece together the whole man, then every piece of historical evidence is useful.

Thus, if you are interested in the life of George Boole and the wider context of his thought, then this is a book worth reading. The author has done scholars a service by bringing all the known poems together under one cover. On his path though the collection, MacHale gives brief critiques of aspects of the poems and sets them in context, so that by the end of the book the reader has not only gained insight into Boole's poetry, but also has a sketch of his life and worldview. For this reviewer, it had the result of him adding the standard biography of Boole, by, as it turns out, one Desmond MacHale [3], to his list of books to be read by the end of the year.

## FURTHER READING

- [1] Daniel Brown, *The Poetry of Victorian Scientists*, Cambridge University Press, 2013.
- [2] Sam Illingworth, *A Sonnet to Science*, Manchester University Press, 2019.
- [3] Desmond MacHale, *The Life and Work of George Boole: A Prelude to the Digital Age*, Cork University Press, 2015.

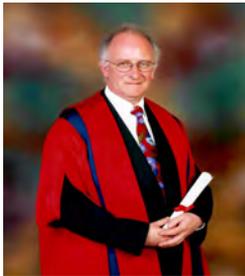


## Mark McCartney

Mark McCartney is a senior lecturer in mathematics at Ulster University. His research interests lie in the areas of discrete chaos and the history of applied mathematics and physics. The multiple chiming and ticking clocks in his house have caused comment (not all of it favourable!) over the last year of online meetings.

## Obituaries of Members

### Colin J. Bushnell: 1947 – 2021



Professor Colin Bushnell, who was elected a member of the London Mathematical Society on 16 May 1975, died on 1 January 2021, aged 73.

*John Silvester writes:* Colin was born and grew up in Gosport, the son

of George, a shipwright, and Rhoda Bushnell. He had a younger sister, Kathryn. He attended Gosport County Grammar School. In 1965 he entered King's College London, attracted by the presence there of Hermann Bondi. He was interviewed by Rex Tims, whose notes read "Amusing chap. Got in a bit of a muddle." Years later Colin told me that Rex had asked him a tricky little question concerning the enumeration of face-colourings of a cube.

Colin graduated with first-class honours, and stayed on to study for a PhD under the supervision of Ali Fröhlich. He obtained his PhD in 1972, for a thesis entitled *Representations of 2-graded Groups*. He was appointed lecturer at the University of Illinois at Urbana-Champaign but returned to King's in 1975 as lecturer, being promoted to Reader in 1985 and Professor in 1990.

After initial seminal work with Fröhlich on the arithmetic properties of Gauss sums for  $p$ -adic division algebras, in a major part of his research Colin led the twenty-five-year development, partly in collaboration with Philip Kutzko (Iowa) and Guy Henniart (Paris), of a distinctive family of methods that revolutionised the representation theory of reductive  $p$ -adic groups and enabled major breakthroughs in the study of the local Langlands correspondence. This work was hugely influential and highly acclaimed, with Colin being an invited speaker at the ICM in 1994, a recipient of the Senior Whitehead Prize of the LMS in 1995 and an elected inaugural Fellow of the AMS in 2013. He also regularly held visiting appointments at institutions such as the IAS in Princeton and the IHES in Paris.

Colin's passion for research was clear to everyone who met him and, even after retirement in 2014, this enthusiasm and energy didn't wane, and he continued to produce major breakthroughs, with an article (jointly authored with Henniart) being published in the *Annals of Mathematics* as recently as 2017.

Despite his magnificent research achievements and international recognition, Colin retained a wonderful affection for King's College, and worked selflessly and proudly to help ensure its success, serving as Head of Department from 1996 to 1997 and as Head of the School of Physical Sciences and Engineering from 1997 to 2004, and then as an Assistant Principal from 2004 until his retirement, during which time he had primary responsibility for overseeing a major redevelopment of the entire historic Strand campus. In recognition of this tremendous service Colin was elected a Fellow of King's College in 2002.

Colin was also an enthusiastic and very successful supervisor of PhD students, and a dedicated and popular teacher of undergraduates, with a distinctive, highly energetic lecturing style that impressed students and colleagues alike.

Colin had a wide-ranging passion for music, with an enviable collection of classical recordings, ranging from Beethoven chamber music and Chopin piano solos to symphonies, lieder and choral works. He borrowed the scores of the late Beethoven quartets so that he could study them in detail. He loved opera and attended many performances at the Royal Opera House, and also La Scala, Glyndebourne, Buxton and Nevill Holt. He was a voracious reader, with eclectic tastes, including history, crime thrillers and Terry Pratchett. He spent many holidays visiting archaeological sites around the world. Latterly, he took up keeping tropical fish with his accustomed thoroughness. Recently he also dabbled in local politics, joining the Liberal Democrats and becoming agent for the local election team.

Colin is survived by his wife of 44 years, Lesley, whom he met at King's College. He is greatly missed. I am grateful to Lesley, and also to David Burns, for supplying much of the material for this obituary.

## Jacques Tits: 1930 – 2021



Professor Jacques Tits, who was LMS Hardy Lecturer in 1983 and elected an Honorary Member of the London Mathematical Society in 1993, died on 5 December 2021, aged 91.

*Martin Bridson and Sarah Rees write:* Jacques Tits was born on 12 August

1930 in Uccle (Brussels) to Léon (a mathematician) and Louisa André Tits (a piano teacher). A prodigy, he entered the Université Libre de Bruxelles at the age of 14, when it reopened in 1945. He earned his doctorate from the ULB in 1950, working with Paul Libois, and rose through the faculty to become professeur ordinaire in 1962, before moving to the University of Bonn in 1964. He married his wife of 60 years, historian Marie-Jeanne Dieuaide (1932–2016), on 8 September 1956.

In 1973 Tits accepted the ‘chaire de théorie de groupes’ created for him at the Collège de France; he was elected to the French Académie des Sciences the same year. In those pre-EU times, this move required him to change his citizenship from Belgian to French. Following his retirement from the Collège de France in 1999, Tits was the inaugural de la Vallée Poussin Professor in Louvain.

In 2008 Jacques Tits and John Griggs Thompson were awarded the Abel Prize “for their profound achievements in algebra and in particular for shaping modern group theory”. His numerous other honours included membership of the Academies of Science of France, USA, Norway, Belgium, and the Netherlands, and honorary doctorates from Utrecht, Ghent, Bonn and Leuven. He won the 1993 Wolf Prize and the 1996 Cantor Medal, and was appointed Chevalier de la Légion d’Honneur in 1995 and Officier de l’Ordre national du Mérite in 2001.

Tits elucidated the nature of groups by realising them as automorphisms of geometric structures, to enormously powerful effect. Among the many

mathematical concepts that bear his name, perhaps the most important is that of a Tits building; these buildings are remarkable combinatorial-geometric objects that he devised to capture the structure of simple algebraic groups over arbitrary fields. His subsequent work with Bruhat also established affine buildings as a powerful analogue of symmetric spaces in the theory of  $p$ -adic Lie groups.

Every building is a union of subcomplexes (‘apartments’), each modelled on a complex associated to a Coxeter group (the Weyl group of the algebraic group, in the classical case). The study of these generalised reflection groups originated in the work of HSM Coxeter, but Tits was hugely influential in developing their theory, particularly as the unacknowledged author of chapters 4–6 of *Groupes et algèbres de Lie in Bourbaki’s Eléments de mathématique*.

The Tits alternative is another of his ideas that now pervades a wide swath of mathematics: the template for this is his proof in 1972 that a finitely generated group of matrices over any field either has a solvable subgroup of finite index or else contains a non-abelian free group.

Tits made significant contributions to the construction of sporadic finite simple groups: the (simple) derived subgroup of the finite group of Lie-type  ${}^2F_4(2)$  is named after him, and his construction of the Fischer-Griess monster (the largest of the sporadic simple groups) exemplifies the elegance of his mathematics.

Beyond his research, Jacques Tits made major contributions to the international mathematical community. He often shared his ground-breaking ideas in series of lectures (including the LMS Hardy lectures in 1983). He was a wise and influential editor who served as Editor-in-Chief of *Publications Mathématiques de l’IHES* from 1979 to 1999. He served on the Fields Medal committee, twice. He supervised 18 graduate students, leading to 240 mathematical descendants, but this vastly understates the number of mathematicians whom he inspired, through his personal attention and kindness as well as his oeuvre — we are two among them.



# IMA–LMS David Crighton Lectures

**Tuesday 10 May 2022, 6.20pm**

followed by a reception at The Royal Society,  
Carlton House Terrace, London SW1Y 5AG

Registration will open at 6.05pm

## *Mathematical science PhDs – past, present and future* **Professor Ken Brown CBE FRSE**

**Abstract:** I'll review changes in UK mathematical science PhDs through the time of my career and beyond, using my personal experiences as a student and supervisor and my interactions with EPSRC to inform the discussion. There will be some mathematics, present in part for its own sake but also to inform our thinking about policy issues. My aim will be to keep the interest of both mathematicians and non-mathematicians.

## *Hyperbolic geometry in data science* **Professor Caroline Series FRS**

**Abstract:** Hyperbolic space is very big: the area of a circle or sphere expands exponentially with its radius. Thus it contains plenty of room for expanding tree-like structures. Indeed, hyperbolic space can be coarsely likened to a tree. For this reason, hyperbolic space lends itself to organising and representing data with hierarchical structure, ranging from biological classification, cell development, communication or social networks, to linguistic relationships.

Features of hyperbolic space can also be used to model other observed behaviours of real world complex networks. Recent years have seen an explosion of both techniques and applications. This talk will attempt to give an historical overview of some of the main ideas.

Professor Ken Brown (2019 winner) and Professor Caroline Series (2021 winner) will each be presented with the David Crighton Medal, which is awarded biennially for services both to mathematics and to the mathematical community by the Institute of Mathematics and its Applications and the London Mathematical Society.

Admission to the lecture and reception is by ticket only.

For tickets please contact Gemma Reeves at the IMA, Catherine Richards House, 16 Nelson Street, Southend-on-Sea, SSI 1EF or email [gemma.reeves@ima.org.uk](mailto:gemma.reeves@ima.org.uk) by 29 April 2022. Tickets are free of charge and will be allocated on a first come, first served basis.

Please confirm whether you wish to attend the lecture and reception, or the lecture only.

 LMS Meeting

## Society Meeting: Diverse Perspectives on Alan Turing

Friday 4 March 2022 at De Morgan House, London and via Zoom

Website: [lms.ac.uk/events/AlanTuring](https://lms.ac.uk/events/AlanTuring)

Opening of the Meeting & Society Business:

- Jonathan Dawes (University of Bath)  
*Unpublished Patterns of Thought: Alan Turing's Later Work on Morphogenesis*
- Janet Foster (LMS Archivist):  
*Introduction to Alan Turing's Legacy within the LMS Archives*
- Andrew Hodges (University of Oxford)  
*A Double Enigma: What did Alan Turing do in 1945 and in 1954?*
- Debbie Marriott (Bank of England):  
*The Creative Journey of the New £50 Banknote Design*
- Stephen H. Muggleton (Imperial College London):  
*Alan Turing and the Development of Artificial Intelligence*

• Sir Dermot Turing:

*Alan Turing's 'Wasted Years': The Mathematics of World War 2*

These lectures are aimed at a general mathematical audience. All interested, whether LMS members or not, are welcome to attend. The meeting will be followed by a reception, which will be held at the De Morgan House.

For further details and to register, see [bit.ly/3L6UVlj](https://bit.ly/3L6UVlj). A Society Dinner will be held after the meeting at the Blue Door Bistro, Montague on the Gardens Hotel. The cost of the dinner will be £35.00, including drinks. To reserve a place at the dinner, note it on the registration form on the website.

 LMS Meeting

## Midlands Regional Meeting and Workshop

4 April 2022, Birmingham

Website: [tinyurl.com/muub3dnk](https://tinyurl.com/muub3dnk)

- 10.00–11.00: Career Development Seminar with Dr Tyler Kelly  
 11.20–12.30: Research talks by Graduate Students  
 12.30–13.15: Lunch  
 13.15–13.30: Opening of the LMS Meeting & Society Business  
 13.30–14.30: Frances Kirwan (University of Oxford)  
*Moduli Spaces, Moduli Stacks and Quotients in Algebraic Geometry*  
 14.30–15.00: Tea/Coffee Break  
 15.00–16.00: Diane Maclagan (University of Warwick)  
 16.15–17.15: Richard Thomas (Imperial College): *Higher Rank Invariants*  
 17.30: Wine Reception and Dinner

This meeting forms part of the Midlands Regional Workshop 'Mirrors, Moduli and  $M$ -theory in the Midlands', on 4–7 April 2022. The lectures are aimed at a general mathematical audience. All interested, whether LMS members or not, are welcome to attend. The meeting will be followed by a reception and Society Dinner.

Funds are available for partial support to attend the meeting and workshop. Requests for support with an estimate of expenses should be addressed to the organisers. Information will be available on the LMS website in due course.

For further details and to register, see the LMS website.

## LMS Meeting

# Hirst Lecture & Society Meeting

6 May 2022 at De Morgan House and via Zoom

Website: [lms.ac.uk/events/Hirst-Lecture](https://lms.ac.uk/events/Hirst-Lecture)

Opening of the Meeting & Society Business:

- Serafina Cuomo (Durham)  
*Maths and the City. A Snapshot of Numeracy in Classical Athens*
- Hirst Lecture: Karine Chemla SPHERE (CNRS & Université de Paris)  
*Algebraic Work with Operations in China, 1st century–13th century*

The meeting features the Hirst Lecture given by the winner of the Hirst Prize and Lectureship 2021.

These lectures are aimed at a general mathematical audience. All interested, whether LMS members or not, are welcome to attend. The meeting will be followed by a reception, which will be held at the De Morgan House.

For further details and to register, see <https://www.lms.ac.uk/events/meeting/hirst>. A Society Dinner will be held after the meeting at a nearby venue. The cost of the dinner will be £35.00, including drinks. To reserve a place at the dinner, note it on the registration form on the website.

## AGGITatE Days: Algebraic Groups and Geometric Invariant Theory at Essex

Location: University of Essex  
Date: 7–8 April 2022  
Website: [tinyurl.com/2p8k2yzn](https://tinyurl.com/2p8k2yzn)

Funded by two LMS Scheme 9 awards, this workshop brings together researchers in algebraic groups and geometric invariant theory to enable interactions. Talks will be given by E. Hamilton (Cambridge), F. Kirwan (Oxford), A. Litterick (Essex), B. Martin (Aberdeen), J. Martinez-Garcia (Essex), T. Papazachariou (Essex) and G. Röhrle (Bochum). Deadline for registration with funding: 7 March.

## British Combinatorial Conference

Location: Lancaster University  
Date: 11–15 July 2022  
Website: [tinyurl.com/y3dn4emh](https://tinyurl.com/y3dn4emh)

BCC 2022 will be held in-person at Lancaster University, from 11–15 July 2022. The event includes nine internationally renowned principal speakers, and six mini-symposia organised by leading researchers. The deadline for registration is 31 March 2022. For full details and registration visit the conference website.

## British Topology Meeting

Location: Durham University  
Date: 12–14 April 2022  
Website: [tinyurl.com/ev5zrxej](https://tinyurl.com/ev5zrxej)

The 35th British Topology Meeting will take place in Durham as an in-person event and bring together the Topology community in the UK. The meeting is supported by an LMS Conference Grant, which allows us to support postgraduate students. We also welcome contributing talks. Visit the website for invited speakers and other information.

## New Trends in Moduli Spaces and Vector Bundles

Location: University of Warwick  
Date: 25–29 July 2022  
Website: [tinyurl.com/bdfpped9](https://tinyurl.com/bdfpped9)

This is a conference on moduli theory in a broad sense. It addresses recent research in moduli theory, the geometry of moduli spaces and interaction with other areas, covering the theory of stability conditions in derived categories and non-reductive geometric invariant theory, higher rank Brill-Noether theory, and Higgs bundles and character varieties.

# Society Meetings and Events

## March 2022

- 4 Diverse Perspectives on Alan Turing and Society Meeting, De Morgan House, London

## April 2022

- 4-7 Midlands Regional Meeting and Workshop, Mirrors, Moduli and M-theory in the Midlands, Birmingham

## May 2022

- 6 Society Meeting and Hirst Lecture, De Morgan House, London
- 10 IMA/LMS David Crighton Lectures, Royal Society, London

## July 2022

- 1 Society Meeting and Aitken Lecture, BMA House, London

# Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website ([www.lms.ac.uk/content/calendar](http://www.lms.ac.uk/content/calendar)). Please send updates and corrections to [calendar@lms.ac.uk](mailto:calendar@lms.ac.uk).

## April 2022

- 4-5 Discrete Mathematics Conference, University of Essex (498)
- 6-8 Probability, Analysis and Dynamics Conference, University of Bristol (498)
- 7-8 AGGITatE Days: Algebraic Groups and Geometric Invariant Theory at Essex, University of Essex (499)
- 11-13 BAMC, Loughborough University (498)
- 12-14 British Topology Meeting, Durham University (499)
- 20-22 UK Association for Computational Mechanics, University of Nottingham (498)
- 25-29 Rational Points on Higher-Dimensional Varieties, ICMS, Edinburgh (495)

## May 2022

- 2-6 Adaptive Methods and Model Reduction for PDEs Research School, Nottingham (496)
- 23-25 Inference for Expensive Systems in Mathematical Biology, Oxford University
- 18-20 Mathematics in Signal Processing, Aston, Birmingham (495)

## June 2022

- 8-10 Mathematics of Finance and Climate Risk Conference, Holiday Inn, Liverpool (498)
- 27-1 Jul Point Configurations: Deformations and Rigidity Graduate Research School, University College London (498)
- 29-1 Jul 7th IMA Conference on Numerical Linear Algebra and Optimization (498)

## July 2022

- 11-15 British Combinatorial Conference, Lancaster University (499)
- 13-15 Maths in Music Conference, Royal College of Music, London (498)
- 18-22 New Challenges in Operator Semigroups, St John's College, Oxford (498)
- 18-22 Rigidity, Flexibility and Applications LMS Research School, Lancaster (497)
- 20-22 Microlocal Analysis and PDEs, University College London (498)
- 24-26 7th IMA Conference on Numerical Linear Algebra and Optimization, Birmingham (487)
- 25-29 New Trends in Moduli Spaces and Vector Bundles, University of Warwick (499)