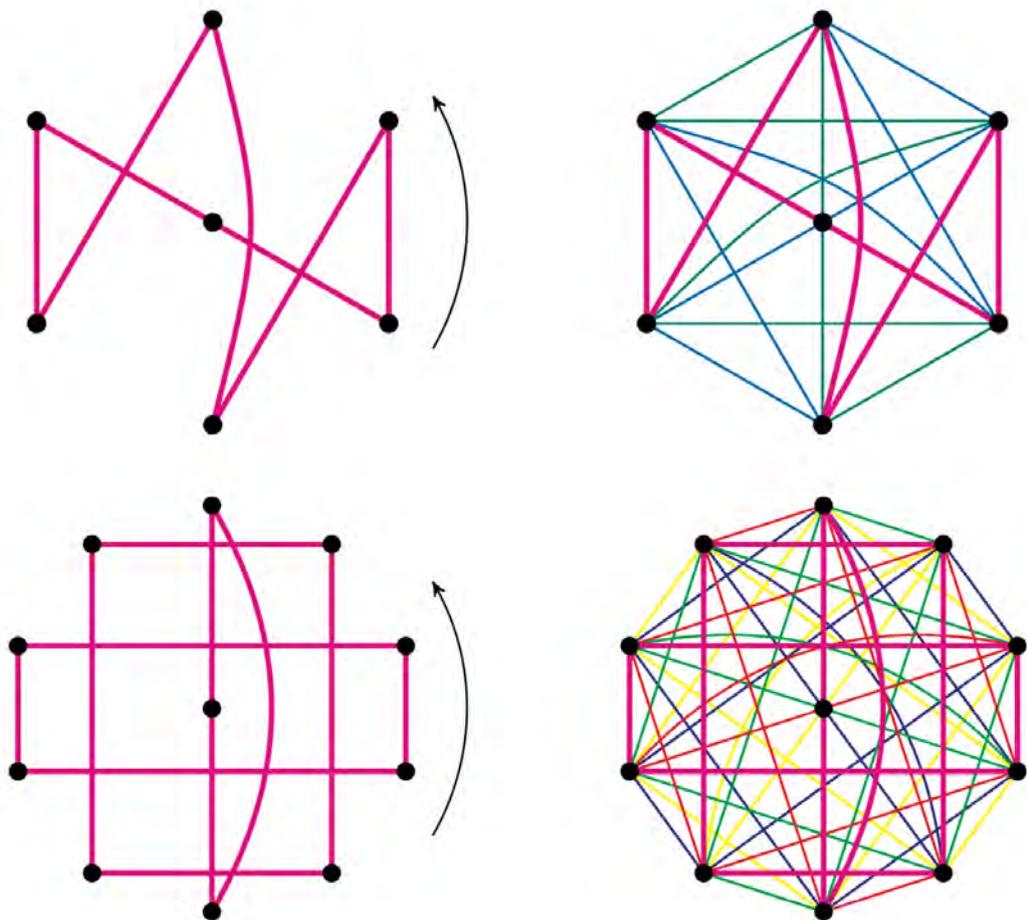




LONDON  
MATHEMATICAL  
SOCIETY  
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# NEWSLETTER

*Issue: 501 - July 2022*



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MEDAL  
LECTURE

NOTES OF  
A NUMERICAL  
ANALYST

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## COVER IMAGE

Images from Katherine Staden's article *A Mathematical Jigsaw: Graph Decomposition*; see the article on page 19.

Do you have an image of mathematical interest that may be included on the front cover of a future issue? Email [images@lms.ac.uk](mailto:images@lms.ac.uk) for details.

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News items should be sent to [newsletter@lms.ac.uk](mailto:newsletter@lms.ac.uk).

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## LMS NEWS

## The Russian Invasion of Ukraine: What the Society is Doing

In March the Society's President, Professor Ulrike Tillmann, made a public statement (see [lms.ac.uk/news/ukraine-statement](https://lms.ac.uk/news/ukraine-statement)) deploring the invasion of Ukraine by Russian government forces. Since then, the Society has launched several strands of work in its areas of activity to respond to these devastating developments.

### Working Group

The Society has set up a Working Group to coordinate its response, chaired by Vice-President Iain Gordon, with membership that includes the Society's President, Treasurer, Publications Secretary, Programme Secretary and the Chair of the Society's Committee for Women and Diversity in Mathematics.

### Council's Position

At its April meeting, LMS Council reaffirmed its support for all individual mathematicians within the international community, irrespective of nationality. It distinguished this from institutional collaborations, however, and the Society therefore suspended some activities with Russian partners and, at the time of writing, is in ongoing discussions about how to achieve this in other cases.

### Support for Mathematicians

The Society launched two grant schemes in May for members of the mathematical community who are seeking refuge in the United Kingdom.

The Society's grant schemes are co-ordinated with the Isaac Newton Institute's (INI's) substantial solidarity programme (see [www.newton.ac.uk/event/slm/](https://www.newton.ac.uk/event/slm/)). Research-active mathematicians at all career stages, including students working towards a doctoral degree, can apply for these INI grants that offer up to twelve months subsistence. In addition, the Society is offering:

- 'Welcome Grants' to give them some immediate financial support while they are travelling to the UK and/or incurring costs in the first month following their arrival. Applications for up to £3,000 will be

considered though it is expected that an individual grant is likely to be in the region of £1,000.

- 'Supplementary Grants', to provide further financial support in addition to the funding provided by the INI (or others) for those in greater need. The value can vary from a few hundred pounds (for furniture or a computer, for example) to a regular supplement to other monthly income, for the applicant and/or any dependants.

Applications can be made for these grants by following the process set out on the INI website at [newton.ac.uk/event/slm](https://newton.ac.uk/event/slm).

In addition to this, the Society will ensure that, in promoting and advertising its existing relevant grant schemes, it makes clear that these are available to mathematicians who have recently reached the UK after fleeing their home country. These include the Society's 'Research in Pairs', 'Celebrating New Appointments' and 'Early Career Researcher Travel' grant schemes which, the Society hopes, will help new arrivals establish themselves in the UK mathematics community and develop their networks.

### Publications

In considering the situation of individual mathematicians who are affected by the Russian invasion of Ukraine, often in terrible ways, the Society has been guided by the principles of the Committee on Publication Ethics (COPE) (see [tinyurl.com/3zsbhu9j](https://tinyurl.com/3zsbhu9j)). The Society's policy is that content published in the Society's journals and books will continue to be selected based on merit alone and irrespective of nationality, geographic location, ethnicity, political beliefs, race, or religion of the authors. Publication of individual authors' work by the London Mathematical Society does not constitute endorsement of the policies of any government or other agencies.

In particular, the Society will continue to publish articles in its core journals written by authors based in Russia. The Society has, however, excluded Russian and Belarusian institutions from its journal exchange programme. It is also in frequent discussion with its publishing partners about all relevant issues related to the Russian invasion.

## Communications

The Society aims to be clear about both the actions it is taking, and the reasons for those actions. The Society also wants to understand the views of its members and the wider mathematical sciences community, and to listen to ideas and suggestions for ways in which the Society – and the community more broadly – can respond in an effective and meaningful manner.

With this in mind, the Society is:

- Updating the Society's membership, the UK mathematical community and the global mathematical community about the actions that the Society is taking and the reason for these actions.
- Coordinating and sharing good practice with other learned societies and mathematical institutes in the UK and globally.

If you have ideas, suggestions or comments, please do email the Society at [membership@lms.ac.uk](mailto:membership@lms.ac.uk).

Caroline Wallace  
LMS Executive Secretary

## Printing of the *LMS Newsletter*: Update

Following my article in the March 2022 issue about the financial and environmental impact of printing and posting the *LMS Newsletter* to members who wish to receive it, we received a number of responses. Most were strongly in favour of retaining print for those that wish to receive the *Newsletter* this way, though we also received positive support for our wish to reduce the Society's environmental impact. It was noticeable that the number of members opting for online only increased following the article. Many thanks to those members who made this choice, meaning that resources do not need to be unnecessarily used sending print copies to those who are happy to read the *Newsletter* online.

LMS Council recently debated the issue of retaining print as an option and agreed to continue to print the *Newsletter* in 2022/23. During this year a membership survey is planned, which will seek members' views about the priority members place on the printed

issue compared to other membership benefits and other Society activities given that we need to live within our means.

Professor Cathy Hobbs  
LMS Vice-President

## LMS Vacancies: LMS Librarian and LMS Development Committee

The Society is seeking members to serve on its Development Committee, and a member to succeed Mark McCartney as Society Librarian and Chair of the LMS Library Committee. Information about the Development and Library Committees including terms of reference can be found at [lms.ac.uk/about/committees](http://lms.ac.uk/about/committees). If you are interested please contact James Taylor at [james.taylor@lms.ac.uk](mailto:james.taylor@lms.ac.uk), specifying which role(s) is of interest, by 2 September 2022 in order that Council can make decisions on these roles in time for the autumn 2022 Annual General Meeting.

## Forthcoming LMS Events

The following events will take place in forthcoming months:

**LMS Meeting and Aitken Lecture:** 1 July, BMA House, London

**LMS Meeting at the International Congress of Mathematicians 2022:** 7 July, University of Copenhagen and online

**LMS-INI-Bath Symposium: K-Theory and Representation Theory:** 18–22 July, University of Bath

**LMS Invited Lectures: Equations in Groups and Complexity:** 18–22 July, Newcastle University

**LMS-Bath Symposium: New Directions in Water Waves:** 18–29 July, University of Bath

**LMS-Bath Symposium: Combinatorial Algebraic Geometry:** 1–5 August, University of Bath

**LMS-IMA-BSHM: Women in Astronomy Meeting** 16 September, De Morgan House, London, and online

A full listing of upcoming LMS events can be found on page 47.

## More Than the Sum of Our Parts

**Paul Glendinning, President of the Institute of Mathematics and its Applications**

**Ulrike Tillmann, President of the London Mathematical Society**

The Institute of Mathematics and its Applications and the London Mathematical Society share many key values. Both are committed to supporting mathematics and mathematicians, and both seek to influence policy around mathematics. Through the Council for the Mathematical Sciences our institutions work with the Royal Statistical Society, the Operational Research Society and the Edinburgh Mathematical Society to represent mathematics and the potential benefits of mathematics to policy makers. It is natural for our two societies to work together; it creates a better understanding of needs and it provides us with a stronger and more unified voice. The LMS-IMA relationship has gone from strength to strength under the two immediate past Presidents, Nira Chamberlain and Jon Keating, and we are committed to continuing this tradition of positive engagement.

The close links between the IMA and LMS are not necessarily obvious to those not involved with the nuts and bolts of the societies. So we thought it would be useful to outline the areas in which our societies work together and where we expect to work together in the next few years.

The conflict in the Ukraine has highlighted the importance of providing support for displaced mathematicians. Both societies are committed to providing information and access to society events to mathematicians who are displaced by wars and political unrest. The Newton Institute Solidarity programme and list are good examples of the type of activity that can be advertised through our networks and both the IMA and LMS are looking at other forms of support.

Equality, diversity and inclusion are central precepts of both our institutions. The LMS has long had an excellent Good Practice Scheme for mathematics, and the IMA has had a member on the LMS Committee for Women and Diversity in Mathematics. The IMA is currently expanding its support for the IMA Diversity Champion and both the LMS and IMA will continue to exchange best practice and work together to promote these core values and the actions needed to support them. The Black

Heroes of Mathematics conference founded by Nira Chamberlain in 2020 is another good example of the initiatives the societies support jointly.

Education at all levels is vital to the health of our discipline. The IMA and LMS both have active Education Committees and input to national bodies; one of us (UT) is currently chair of the Royal Society Education Committee to which ACME reports. Both Societies are also involved with the Levelling Up: Maths programme that the LMS helped found. The scheme involves universities across the country running online tutorial programmes to help pre-university maths students from under-represented groups to meet their potential. The IMA also administers the Mathematics Teacher Training Scholarships scheme, while the LMS makes grants to support CPD for teachers.

Both of our societies seek to recognise and celebrate achievements in and contributions to mathematics. The LMS and the IMA award two joint prizes: the IMA-LMS David Crighton Medal (for services both to mathematics and to the mathematical community) and the IMA-LMS Christopher Zeeman Medal (which recognises the contribution of mathematicians who promote mathematics to the public).

Engagement and influence with policy makers and media coverage has long been a weak area for the mathematical community. Events like STEM for Britain provide some exposure within Parliament, but deserve wider recognition. Last year the societies ran a very successful joint seminar on mathematics and its impact in Parliament, but these events need to be more regular to have chance at creating change. Last year the LMS co-sponsored the Protect Pure Mathematics (PPM) campaign, and this has provided extra reach into Government and the media. This campaign continues, and both of us are on its Steering Committee.

The Bond Review of Knowledge Exchange in the Mathematical Sciences had a number of recommendations that are central to both of our institutions. Two of the recommendations have

been at least partially acted upon. There has been recognition of the transformative power of mathematics through the announcement of an additional investment of £300 million for mathematical research. Only one third of this is confirmed and the societies will be working with each other, EPSRC, PPM and other stakeholders to hold the government to its initial promise. Another key recommendation of the Bond Review was that tax credits should be available to cover mathematics research within industry. We were delighted that this was announced in the Spring Budget (March 2022) and we expect to see the implications for the mathematical sciences community explained more widely in the near future.

The Bond Review also recommended the creation of a National Academy of Mathematical Sciences and a systematic and co-ordinated approach to knowledge exchange. Proposals for both of these are under review and a next-steps paper from CMS is under consideration by the governing bodies of all the CMS societies. The Newton Institute and ICMS have funding to support the early phases of the Academy and the creation of stronger and more accessible links between academia and industry.

The societies have a responsibility to foster an inclusive environment and to encourage further diversity among our mathematical community. Collaborating with mathematicians with different interests, skills, outlooks and experiences is the way to achieve the greatest and most valuable outcomes. Encouraging people from different backgrounds and ensuring that students who want to study maths can do so to an excellent standard at their local university will help keep the pipeline as wide as possible. Enthusing those from different communities about the wonder and value of maths is both a shared aim and an important duty for us both.

This diversity across the different areas of the mathematical sciences, the places mathematics is done and the people involved is a great strength of our community. Our societies are not the same, but there are so many areas of common interest that combining our various strengths and working together makes complete sense.

We look forward to working with each other and with the broader mathematical communities in the next few years.

Paul and Ulrike



# Call for Proposals

## RIMS Joint Research Activities 2022-2023

**Application deadline : August 31, 2022, 23:59 (JST)**

### Types of Joint Research Activities

- \*RIMS **Satellite** seminars 2023
- \*RIMS **Review** seminars 2023
- \*RIMS **Workshops Type C** 2023
- \*RIMS **Research Project** 2024

**More Information :** RIMS Int.JU/RC Website  
<https://www.kurims.kyoto-u.ac.jp/kyoten/en/>



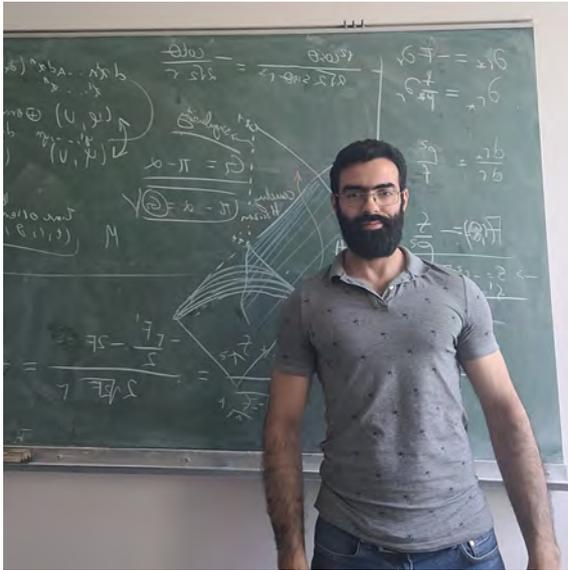
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Mathematical Sciences

## OTHER NEWS

## LMS Atiyah UK-Lebanon Fellow 2022–23



The LMS is delighted to announce that the 2022-23 LMS Atiyah UK–Lebanon Fellow will be Dr Rémi Mokdad Mokdad, currently a postdoctoral researcher at the Institut de Mathématiques de Bourgogne in the University of Burgundy, Dijon, France. Dr Mokdad is Lebanese; following undergraduate studies in mathematics in Lebanon, he obtained his PhD in Brest, France.

Dr Mokdad works on geometric and spectral analysis of partial differential equations in mathematical physics, specifically in general relativity. He is particularly interested in the cosmic censorship Penrose conjecture. He will be visiting Queen Mary, University of London for several months in 2022-23 to work with Dr Juan Valiente Kroon, Head of the Geometry and Analysis group of the School of Mathematical Sciences, and a world-expert in conformal and spinorial methods in general relativity. They will be working on scattering problems in the context of the Penrose conjecture.

During his stay, Dr Mokdad also hopes to visit Edinburgh to interact with Dr Pieter Blue and Dr Jan Sbierski, and Oxford to see Professor Lionel Mason. He also expects to visit the Centre for Advanced Mathematical Sciences in Beirut to strengthen his ties with researchers there.

The Atiyah UK–Lebanon Fellowships were set up in 2019 as a lasting memorial to Sir Michael Atiyah (1929–2019) in the form of a two-way visiting programme for mathematicians between the UK and the Lebanon, where Sir Michael had strong ties. For further information about the Fellowships and information on how to apply visit [lms.ac.uk/grants/atiyah-uk-lebanon-fellowships](https://lms.ac.uk/grants/atiyah-uk-lebanon-fellowships). Applications for Fellowships to be held in the academic year 2023–24 will open on 1 September 2022.

Professor Caroline Series FRS  
Chair of the Atiyah Fellowship Panel

## Azat Miftakhov Days Against the War 2022

The second Azat Miftakhov Days Against the War event will take place on Tuesday 5 and Wednesday 6 July 2022, in solidarity with Azat Miftakhov, a mathematics graduate student from the Moscow State University who has been arbitrarily detained by Russian state authorities since February 2019.

The first day will focus on the new generation of Russian mathematicians committed to human rights. The speakers are Ilya Dumanski (MIT), Alexander Petrov (Harvard) and Slava Rychkov (IHES). Their mathematical lectures will be broadcast live on Zoom and YouTube. On the second day, co-organised with the Memorial Human Rights Center, a panel session will be held to assess the human rights situation in Russia through the cases of persecuted academics. The panel will be held online and in person at the École Normale Supérieure in Paris, and will be open to academics, human rights organisations and journalists.

Registration is mandatory both for participation online or in person. The registration form is available at [tinyurl.com/yc4rxkzs](https://tinyurl.com/yc4rxkzs). Further information is available at [caseazatmiftakhov.org](https://caseazatmiftakhov.org).

## COVIDiary



COVIDiary of Mathematicians is a collection of personal diaries from all around the world written at the beginning of the coronavirus pandemic.

The Serbian mathematical society *Archimedes* organised its annual seminar for mathematics teachers in February of 2020, where the lead guest was Portuguese mathematician Tiago Hirth. The audience was impressed with his talks. Shortly after his visit (and after lockdown was introduced in Serbia) a daily newspaper invited Mr Hirth to contribute to their weekly rubric with the simple title ‘A Diary’ by writing short entries for seven consecutive days. The newspaper wanted an account of the pandemic from somebody living outside Serbia. His diary is now the prologue of the book.

As the article was a hit, *Archimedes* decided to check if Mr Hirth would be willing to continue with the entries. Unfortunately for us, in the meantime, he started working on other projects, so we asked our friends from all around the world to join.

Six other mathematicians accepted the challenge. Besides Tiago Hirth from Lisbon, the authors are Guido Ramellini (an Italian living in Barcelona, Spain), James Tanton (an Australian living in Phoenix, AZ), Kiran Bacche (from Bangalore, India), Sergio

Belmonte (from Altafulla, Spain), and two Serbian mathematicians, Tijana Markovic and Jovan Knežević. The diaries contain some everyday thoughts and photos from authors’ archives. They also include a couple of stock photos, some original illustrations, and a few puzzles.

During the editing process of the Serbian version, Aleksandra Ravas and Dragana Stošić Miljkovic firstly included footnotes with explanations of some unfamiliar references. While working on the last entry of Mr Bacche’s diary, they searched YouTube for the songs mentioned and, after watching them, decided to include the links so the reader could experience the same. That is how the QR codes got into the story! They will reveal some things about the entry to which they belong: the reader might read, watch, or even cook something from the authors’ lives.

Then the proof reader asked if the solutions to the puzzles would be a part of the book. Not being a mathematician, she did not know if her answers (she was solving the puzzles while proofreading the book) were correct, but she was eager to find out. So, although not planned to be part of the book, the solutions were eventually included in the last chapter and written by Ms Ravas.

Original illustrations for the book were made by Anna Muñoz de la Fuente and Giuseppe Bellavia. Eric C. Wilder designed the cover page. For more information about the book visit [covidarymath.com](http://covidarymath.com).

## Rollo Davidson Award 2022



Konstantin Tikhomirov (left) and Amol Aggarwal

The Rollo Davidson Trustees have announced the award of the Rollo Davidson Prize for 2022 jointly to Amol Aggarwal (Columbia University) for fundamental contributions to random matrix theory and integrable probability and Konstantin Tikhomirov (Georgia Institute of Technology) for deep new results on the singularity of random Bernoulli matrices. For further information visit [tinyurl.com/msy77kaj](http://tinyurl.com/msy77kaj).

## Clay Research Award 2022

The Clay Mathematics Institute has announced the winners of the Clay Research Award 2022. A joint award is made to Søren Galatius (University of Copenhagen) and LMS member Oscar Randal-Williams (University of Cambridge) for their profound contributions to the understanding of high

dimensional manifolds and their diffeomorphism groups, which has completely reinvigorated the subject.

An award is also made to John Pardon (Princeton University) for his wider-ranging and transformative results in geometry and topology, including his ground breaking work in symplectic topology

### EUROPEAN MATHEMATICAL SOCIETY NEWS

## EMS Magazine

Issue No. 124 of the EMS Magazine is available to read and download free here. It contains messages from EMS President Volker Mehrmann and the Magazine Editor-in-Chief Fernando P. da Costa focussing on the Russian invasion of Ukraine, together with a letter from Ukrainian mathematicians, and the Society's (and Magazine's) response. There is also a memorial article Sir David Cox: A Wise and Noble Statistician

(1924–2022) by Christos P. Kitsos, and much else besides.

EMS News prepared by David Chillingworth  
LMS/EMS Correspondent

*Note: items included in the European Mathematical Society News are not necessarily endorsed by the Editorial Board or the LMS.*

### MATHEMATICS POLICY DIGEST

## Protect Pure Maths Campaign: Latest Activities

### Science and Technology Committee asks: where is additional £300 million for mathematics?

As many members will know, in January 2020 the government and UKRI announced £300 million in additional funding for mathematical sciences, of which £124 million has been allocated so far. In March of this year, via the Protect Pure Maths campaign, the UK's mathematical societies called for the £300 million to be honoured.

At a Science and Technology (S&T) Committee hearing held on 15 June, Professor Dame Ottoline Laser (Chief Executive of UKRI) and George Freeman MP (Minister for Science, Research and Innovation, Department for BEIS) were asked about the status of the £300 million. Dame Ottoline informed the Committee that the majority of the funding had still not been received.

Greg Clark MP, Chair of the S&T Committee, noted his concern at this development and questioned whether the government had deprioritised mathematics, in contradiction to its statement made in 2020 in support of the discipline. Graham Stringer MP, a member of

the Committee, also asked for reassurance that the £300 million was still new money that will be allocated. The minister agreed to discuss the matter further with Dame Ottoline and return to the Committee with a detailed answer.

You can watch the full session at [bit.ly/3ba0oKt](https://bit.ly/3ba0oKt). The discussion on the £300 million is held from 12:01 to 12:10.

The Society is very pleased that the S&T Committee recognises the importance of this funding to mathematics, and is delighted to be part of the Protect Pure Maths Campaign that has helped highlight this issue.

### Oral Evidence Session on Diversity in STEM

Via the Protect Pure Maths Campaign, LMS President Ulrike Tillmann gave evidence to the Science and Technology Select Committee on Wednesday 18th May. She addressed the chronic underrepresentation of women at every level of mathematics education, and talked about the challenges in tackling misconceptions of mathematics. She also called on the government to reaffirm its commitment to the additional funding for mathematics. The call for evidence was in part prompted following recent high-profile comments made

to the Science and Technology Committee by Katharine Birbalsingh, Chair of the Social Mobility Commission, and Headmistress at Michaela Community School, in which she spoke about girls not liking 'hard maths'. Watch the whole session at [bit.ly/303jSyQ](https://bit.ly/303jSyQ).

### **Guardian Letter**

Professor Tillmann also wrote a letter, which was published in The Guardian on 27 May, in response to the suggestion by Andy Haldane, former head of the government levelling-up taskforce, that mathematics could be rebranded as 'numeracy'. Read the letter at [bit.ly/3aDtUld](https://bit.ly/3aDtUld).

### **Additional Funding**

The Protect Pure Maths campaign has written to George Freeman MP about the allocation of the remaining £300m in additional funding and are awaiting a response.

Further information about the Protect Pure Maths campaign can be found at [protectpuremaths.uk](https://protectpuremaths.uk).

### **REF 2021 results**

The REF 2021 results were announced on 12 May and can be found at [bit.ly/3037puM](https://bit.ly/3037puM). The Society is delighted to see the outstanding quality of UK mathematical sciences research reflected in these results, which recognise work from institutions of all sizes and from all across the UK.

Digest prepared by Katherine Wright  
Society Business, Research & Communications Officer

*Note: items included in the Mathematics Policy Digest are not necessarily endorsed by the Editorial Board or the LMS.*

## OPPORTUNITIES

### **LMS Grant Schemes**

The next closing date for research grant applications (Schemes 1, 2, 3, 4, 5, 6 and AMMSI) is 15 September 2022. Applicants for LMS Grants should be mathematicians based in the UK, the Isle of Man or the Channel Islands. For grants to support conferences/workshops, the event must be held in the UK, the Isle of Man or the Channel Islands. Applications are invited for the following grants to be considered by the Research Grants Committee at its October 2022 meeting.

#### **Conferences (Scheme 1)**

Grants of up to £5,500 are available to provide partial support for conferences. This includes travel, accommodation and subsistence expenses for principal speakers, UK-based research students, participants from Scheme 5 countries and Caring Costs for attendees who have dependents.

#### **Visits to the UK (Scheme 2)**

Grants of up to £1,500 are available to provide partial support for a visitor who will give lectures in at least three separate institutions. Awards are made to the host towards the travel, accommodation and subsistence costs of the visitor. Potential applicants

should note that it is expected the host institutions will contribute to the costs of the visitor. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents.

#### **Joint Research Groups in the UK (Scheme 3) Application deadline 30 September 2022**

Grants of up to £1,500 are available to support joint research meetings held by mathematicians who have a common research interest and who wish to engage in collaborative activities, working in at least three different locations (of which at least two must be in the UK).

#### **Online Graduate Lecture Series (Scheme 3) Application deadline 30 September 2022**

Grants of up to £1,000 can be applied for by those who would like to organise an Online Graduate Lecture Series. The introduction of the online lecture series element to the Joint Research Groups follows the successful Online Lecturer Series grant scheme, which was run in 2020 in response to the impact of the Covid-19 pandemic on the mathematical community. Applications for this element of the Scheme 3 grants are open both to Joint Research Groups (new and current) and to mathematicians who are not part of a Joint Research Group.

### Research in Pairs (Scheme 4)

For those mathematicians inviting a collaborator, grants of up to £1,200 are available to support a visit for collaborative research either by the grant holder to another institution abroad or by a named mathematician from abroad to the home base of the grant holder. For those mathematicians collaborating with another UK-based mathematician, grants of up to £600 are available to support a visit for collaborative research either by the grant holder to another institution or by a named mathematician to the home base of the grant holder. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents.

### Research Reboot (Scheme 4)

Grants of up to £500 for accommodation, subsistence and travel plus an additional £500 for Caring Costs are available to assist UK mathematicians who may have found themselves with very little time for research due to illness, caring responsibilities, increased teaching or administrative loads, or other factors. This scheme offers funding so that they can leave their usual environment to focus entirely on research for a period from two days to a week. For applications submitted by the next deadline (15 September 2022), the Reboot Retreats should take place between 01 November 2022 and 30 January 2023.

### Collaborations with Developing Countries (Scheme 5)

For those mathematicians inviting a collaborator to the UK, grants of up to £3,000 are available to support a visit for collaborative research, by a named mathematician from a country in which mathematics could be considered to be in a disadvantaged position, to the home base of the grant holder. For those mathematicians going to their collaborator's institution, grants of up to £2,000 are available to support a visit for collaborative research by the grant holder to a country in which mathematics could be considered to be in a disadvantaged position. Applicants will be expected to explain in their application why the proposed country fits the circumstances considered eligible for Scheme 5 funding. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents. Contact the Grants team if you are unsure whether the proposed country is eligible, or check the IMU's Commission

for Developing Countries definition of developing countries ([tinyurl.com/y9dw364o](http://tinyurl.com/y9dw364o)).

### Research Workshop Grants (Scheme 6)

Grants of up to £10,000 are available to provide support for Research Workshops. Research Workshops should be an opportunity for a small group of active researchers to work together for a concentrated period on a specialised topic. Applications for Research Workshop Grants can be made at any time but should normally be submitted at least six months before the proposed workshop.

### African Mathematics Millennium Science Initiative (AMMSI)

Grants of up to £2,000 are available to support the attendance of postgraduate students at conferences in Africa organised or supported by AMMSI. Application forms for LMS-AMMSI grants are available at [ammsi.or.ke](http://ammsi.or.ke).

The next closing date for early career research grant applications for ECR Travel Grants is 15 October 2022. Applications are invited for the following grant to be considered by the Early Career Research Committee at its June 2022 meeting:

### Postgraduate Research Conferences (Scheme 8)

Grants of up to £2,500 are available to provide partial support for conferences, which are organised by, and are for, postgraduate research students. The grant award will be used to cover the costs of participants. In addition, the Society allows the use of the grant to cover Caring Costs for those who have dependents.

### Celebrating new appointments (Scheme 9)

Grants of up to £400-£500 are available to provide partial support for meetings to celebrate the new appointment of a lecturer at a university. Potential applicants should note that it is expected that the grant holder will be one of the speakers at the conference. In addition, the Society allows the use of the grant to cover Caring Costs for those who have dependents.

### ECR Travel Grants

Grants of up to £500 are available to provide partial travel and/or accommodation support for UK-based Early Career Researchers to attend conferences or undertake research visits either in the UK or overseas.

For full details of these grant schemes, and to find information on how to submit application forms, visit the LMS website: [lms.ac.uk/content/research-grants](https://lms.ac.uk/content/research-grants). Queries regarding applications can be addressed to the Grants Administrator Lucy Covington (020 7927 0807, [grants@lms.ac.uk](mailto:grants@lms.ac.uk)), who will be pleased to discuss proposals informally with potential applicants and give advice on the submission of an application.

## LMS Research Schools 2022

The London Mathematical Society Research Schools provide training for research students in all contemporary areas of mathematics. Students and post-docs from both the UK and abroad can meet a number of leading international experts in the topic as well as other early career researchers working in related areas.

The LMS Research Schools take place in the UK and support participation of research students from both the UK and abroad. The lecturers are expected to be international leaders in their field. The LMS Research Schools are often partially funded by the Heilbronn Institute for Mathematical Research.

The following Research Schools will take place in 2022:

### **Bicategories, Categorification and Quantum Theory**

Leeds, 11–15 July 2022

Main Lecturers: Richard Garner (Macquarie University, Sydney), Marco Mackaaij (University of Algarve), Chris Huenen (Radboud University Nijmegen), Sonia Natale (Universidad Nacional de Córdoba).

### **Unimod 2022**

Leeds, 18–22 July 2022

Main Lecturers: Amador Martin-Pizarro (Albert Ludwig Universität in Freiburg), Assaf Hasson (Ben Gurion University of the Negev), Zoé Chatzidakis (École Normale Supérieure).

### **Elliptic Curves**

Baskerville Hall, Hay-on-Wye, 8–12 August 2022

Main Lecturers: Alina Cojocaru (University of Illinois Chicago), Jennifer Park (Ohio State University), Chao Li (Columbia University), Joseph Silverman (Brown University), Jan Vonk (University of Leiden), Christian Wuthrich (University of Nottingham).

For more information and links to the events visit: [lms.ac.uk/events/lms-research-schools](https://lms.ac.uk/events/lms-research-schools)

## LMS Undergraduate Summer School 2022

The LMS Undergraduate Summer Schools are annual two-week courses, which are held every Summer at a UK university. The aim is to introduce modern mathematics to the best UK undergraduates who will enter their final year the following Autumn and encourage them to think about an academic career in maths. The LMS Undergraduate Summer School consists of a combination of short lecture courses with problem-solving sessions and colloquium-style talks from leading mathematicians. 50 places are available to students per year.

Location: Heriot-Watt and Edinburgh Universities

Date: 15–26 August 2022

Website: [lms.ac.uk/events/lms-summer-schools](https://lms.ac.uk/events/lms-summer-schools)

Course Lecturers:

Keith Ball (University of Warwick)

Alex Bartel (University of Glasgow)

Tara Brendle (University of Glasgow)

Fraser Daly (Heriot-Watt University)

Gianne Derks (University of Surrey)

Charles Young (University of Hertfordshire)

Colloquium Speakers:

Maggie Chen (Cardiff University)

Laura Ciobanu (Heriot-Watt University)

Iain Gordon (University of Edinburgh)

Andy Hone (University of Kent)

Des Johnston (Heriot-Watt University)

David Jordan (University of Edinburgh)

Yuliya Kyrychko (University of Sussex)

Irene Kyza (University of Dundee)

## LMS Council Diary — A Personal View

In my second year as a Trustee I continue to be impressed by the remarkable variety of topics considered by Council. The meeting of Friday 22 April was no exception. It is not possible to summarise all of Council's discussion here. Instead, I will exploit the 'personal view' in the title of this column and mention only a selection of items on our agenda.

Much of the morning was spent discussing the Society's response to the war in Ukraine. In addition to public statements already made by the Society and President Ulrike Tillmann's article in the recent May (and 500<sup>th</sup>!) edition of the *Newsletter*, Council formally approved the formation and membership of a 'Ukraine Working Group' convened by Vice-President Iain Gordon. The group will take the lead in communications on Ukraine including public statements, and liaising with LMS committees and other learned societies and mathematical bodies such as the INI and ICMS. Additionally, the group will make recommendations to Council in support of mathematicians fleeing the war and, more generally, other conflict zones. It will carefully consider the Society's response regarding its journal and publishing activity, especially the English translations of Russian journals with which the Society is involved, and in the handling of papers by authors based at Russian institutions.

The day of Council coincided with 'Earth Day' and fittingly Council discussed progress and suggestions for making the Society more environmentally sustainable. Among the proposals agreed was a pilot scheme aimed at reducing travel which is not carbon efficient by asking applicants to some LMS grant schemes to explicitly address this in both the application and reporting phases. Efforts to reduce energy use at De Morgan House were also reported, and it was pleasing to note the building has subsequently received a more efficient energy rating than typical for buildings of its type, but there is undoubtedly more work to do (see also the article by Vice-President Gordon in the May *Newsletter*).

The President updated Council on wide-ranging Society activity including the Protect Pure Mathematics campaign, and the proposal for an Academy of Mathematical Sciences. The former is going from strength-to-strength with the campaign receiving much media attention and recent positive engagement in Parliament. This includes the

announcement by the Chancellor of the Exchequer in his 2022 Spring Statement that R&D tax credits would be extended to include research in pure mathematics, one of the key 'asks' of the campaign.

Council gave careful thought in recommending mathematicians for Honorary Memberships of the Society. They will be announced at the Society General Meeting in July. Council also decided the recipient of the Hardy Lectureship from a high-quality field and endorsed the recommendations of the Prizes Committee. These will also be announced at the General Meeting.

The final item of the day concerned the search for a new Executive Secretary to replace Caroline Wallace. The President updated Council on the recruitment process and its successful outcome, with Simon Edwards (currently at The Institution of Engineering and Technology) being appointed to the role.

Robb McDonald  
General Secretary

## Maximising your Membership: The Verblunsky Members' Room

LMS members are warmly invited to visit De Morgan House in Russell Square, London and make use of the LMS Members' Room, or Verblunsky Room. Named after Samuel Verblunsky, a generous benefactor and long standing LMS member (1929-96), the LMS Members' Room offers a quiet and comfortable space to work and relax in the midst of the London hubbub.

With free wi-fi and complimentary tea and coffee in the nearby kitchen, members are welcome to use the room during office hours; Monday – Friday, 9.30am – 5.30pm. Please contact us by email on [membership@lms.ac.uk](mailto:membership@lms.ac.uk) to arrange a visit. Directions to De Morgan House can be found at [tinyurl.com/2p8e2n4u](http://tinyurl.com/2p8e2n4u).

The Verblunsky Members' Room also houses two special collections: the Hardy Collection and the Philippa Fawcett Collection. The Hardy Collection, named after the Society's former President, contains over 300 volumes from G.H. Hardy's personal library of books, which were used by him at various points throughout his career. As such, one can get a glimpse of the authors who influenced his thinking or caught

his attention. Many of these volumes contain Hardy's signature and in some cases they also contain a dedication. For further information about the Hardy Collection, see [tinyurl.com/2p8679c9](https://tinyurl.com/2p8679c9).

The Philippa Fawcett Collection, named after the first woman to come top in the Cambridge Mathematical Tripos exams and an early member of the LMS, is a wide-ranging library of books written by and about women who studied or worked in mathematical subjects in the nineteenth and first part of the twentieth century, or earlier. The Collection was donated to the London Mathematical Society by one of its members, A.E.L. Davis. It is hoped that the books will be a useful resource to scholars of the history of women in mathematics as well as an inspiration to female mathematicians of the future. See [tinyurl.com/rmru3jhm](https://tinyurl.com/rmru3jhm) for further details.

Other treasures on display in the Verblunsky Members' Room include an Armada chess set, a

collection of 90th birthday messages presented to Sir Christopher Zeeman, a facsimile of the LMS Members' Book from 1865-1990 and gifts from other Societies. If you have not yet signed the original Members' Book, look out for it at our Society Meetings: [lms.ac.uk/events/society-meetings](https://lms.ac.uk/events/society-meetings).

De Morgan House itself is situated on Russell Square within the creative heart of Bloomsbury and a short walk from the West End. With UCL only a 10 minute walk away, members can also take advantage of their complimentary use of the LMS Library that is housed at UCL. For details on how to register/renew as a UCL Library user, see [lms.ac.uk/library/how-register](https://lms.ac.uk/library/how-register).

We look forward to welcoming you at De Morgan House soon.

Valeriya Kolesnykova  
Accounts, Fellowships & Membership Assistant

## REPORTS OF THE LMS

### Report: HoDoMS Annual Conference 2022

The Annual Conference of HoDoMS (Heads of Departments of Mathematical Sciences) was held in person in Birmingham on 28 and 29 April with an online option for speakers and delegates. After two years of online conferences, many delegates expressed how wonderful it was to be in person again. The conference had a range of speakers on topics chosen to help support heads of department in their role.

We began with a presentation from Geoffrey Grimmett from the Heilbronn Institute describing the work of the Institute and the different funding opportunities it provides. Later that day, we also heard from Minhyong Kim, the Director of ICMS, giving an introduction to his vision for its contribution.

The theme of diversity ran through the other talks on the first day. June Barrow-Green demonstrated a new tool for enriching the curriculum through the history of mathematics via original source material, with particular emphasis on the role played by people from underrepresented groups. Peter Clarkson and Gwyneth Stallard gave an outline of the new Athena

SWAN charter and they gave practical advice for departments who are preparing applications for a Silver award. Cathy Hobbs gave a very timely presentation on undergraduate recruitment across a range of universities. The figures she presented show a very stark divergence, with high tariff institutions substantially increasing student numbers (in part caused by over recruitment due to the A level fiascos) and with low tariff institutions struggling with small numbers. This presentation was followed by a workshop led by Cathy and Mary McAlinden, exploring ways we can support departments that may be at risk through declining numbers. There was general agreement that mathematics should be available to students from a variety of backgrounds and with a spectrum of career aspirations.

On the second day, the talks were mostly about different aspects of our future. Katie Blaney gave a presentation about EPSRC and future funding developments, although precise levels of funding are still unknown.

Ulrike Tillmann spoke of future initiatives coming from the Isaac Newton Institute, including the funding available to support refugees from Ukraine, and Alison Etheridge gave an update on the proposed National Academy for the Mathematical Sciences.

There was a talk by Jonathan Dawes on possible future directions for UK mathematical sciences research. Steve Langdon described the research on working patterns in universities during the pandemic and then he led a discussion of the future of hybrid working. Many issues around hybrid teaching were common across different universities, but the issues around hybrid studying varied more, with particular problems in urban universities where students commute long distances and do not have space on campus to study.

The conference concluded with a talk by Matt Saker from the Institute and Faculty of Actuaries, who described the actuarial profession and the skills needed. The planned final talk of the conference, on student mental health, had to be cancelled as the speaker John de Pury was unwell.

John R. Parker  
Head of Department  
Durham University

## Report: LMS Hirst Lecture and Society Meeting



Karine Chemla (left) and the LMS copy of *Urania Propitia*

The LMS Hirst Lecture was held on the afternoon of 6 May 2022 in De Morgan House. There were several attendees across all career stages, from undergraduate students to retired academics.

The first speaker was Professor Serafina Cuomo, from the Department of Classics and Ancient History at Durham University. The title of Professor Cuomo's talk was *Maths and the City. A Snapshot of Numeracy in Classical Athens*. She presented four areas of life in ancient Greece (the Agora, the Assembly, the Temple and the Household) and gave examples of how people

worked with numbers in these different situations. She explained to us devices used to help count and measure. This included the Salamis Tablet, which is a marble counting board dating from around 300 BCE, as well as a *mensa ponderaria*, which is a block containing five bowl-shaped cavities that was used in a public place as a standard for measures. In the question time we also heard about a story of Cicero, who was a very good accountant, spotting financial fraud!

The second talk was the Hirst Lecture, given by Professor Karine Chemla. Professor Chemla is a Senior Researcher at the Centre National de la Recherche Scientifique (CNRS) working in the laboratory SPHERE (CNRS and Université de Paris). In 2021 she was awarded the Hirst Prize and Lectureship by the LMS and BSHM. Professor Chemla's talk was on *Algebraic Work with Operations in China, 1st Century–13th Century*. She began by outlining to us some of the history of Chinese mathematicians working in this period. This includes Qin Jiushao's *Mathematical Work in Nine Chapters* (1247), which contains the Chinese Remainder Theorem, and Li Ye's *Measuring the Circle on the Sea-Mirror* (1248).

Professor Chemla then explained to us an 11th CE algorithm of Jia Xiang to manipulate polynomial expressions, which is known today under the names of Ruffini and Horner. Xiang's algorithm was used to produce the  $n$ -th root of a number. One of Professor Chemla's ideas was to argue that there were several early Chinese academics that identified that multiplication and division are 'exactly opposed' operators. For example, Professor Chemla showed to us how Xiang's algorithm relied on an alternating process of multiplication and division. She then argued that this algebraic approach to solving such polynomial equations is virtually the same as what Jiushao used to solve the Chinese Remainder Theorem. This illustrates that both results can be seen as coming from a unified school of Chinese thought aimed at understanding algebraic operations. Such a view counters previous work that presented the  $n$ -th root algorithm and the Chinese Remainder Theorem as independent.

The meeting concluded with a wonderful surprise from the LMS's Chief Executive Caroline Wallace who brought out for display the Society's recently acquired book *Urania Propitia* by Maria Cunitz. This book was published in 1650 and fewer than 25 physical copies are known to exist. The book was recently gifted to the Society by A.E.L. Davies, long-time LMS member and supporter of the Society.

Zoe Wyatt  
King's College London

## Report: Learning Mathematics with Lean



Images capturing scenes from hybrid talks

There is a very active community growing around the use of interactive (called also automated) theorem provers such as Lean (<https://leanprover-community.github.io>) both for research and for teaching undergraduate mathematics. The workshop we organised at Loughborough University aimed at reaching those who are interested in using Lean for teaching and wish to learn from the experience of others, to become acquainted with results in education concerning the impact of using Lean for teaching on students' learning, and to access resources that others have constructed and trialled. The title of the workshop was *Learning Mathematics with Lean* and it was held on 6 April in a room equipped for hybrid meetings provided by the Centre for Mathematics Cognition at Loughborough University. The event attracted much attention — a week before it was held, 70 participants had registered to attend online and 13 participants (apart from the speakers and organisers) to attend in person. Unfortunately, several of our colleagues attending in person had to attend online instead due to COVID so on the day we had thirteen colleagues in the room. Throughout the day we had on average 40 colleagues online from the UK, Canada, Europe, the US indicating that this topic is attracting much attention worldwide. We had a very interesting line up of talks:

- *Fools Rush In: How Not to Insert Theorem Provers into a Curriculum*, Prof Kevin Buzzard (Imperial)
- *Lean Games*, Dr Gihan Marasingha (Exeter)
- *Undergraduate Students Shared their Experience with Lean*, James Arthur and Alvaro Tovar Saade (Exeter); Beth Holmes and Dylan Crook (Imperial)

- *Research on Students' Use of Lean*, Dr Paola Iannone (Loughborough) and Dr Athina Thoma (Southampton)
- *Teaching with Lean*, Prof Jeremy Avigad (Carnegie Mellon).

For those in the room the talks were followed by two hands-on workshop. The first where Prof Buzzard showed the Natural Number Game ([bit.ly/3n5QYCD](http://bit.ly/3n5QYCD)) and the second where Dr Marasingha showed how to use the MIU game ([bit.ly/3zTfEWo](http://bit.ly/3zTfEWo)) which he designed and used with his students for the first time at the start of the current academic year.

We designed an evaluation form which we distributed to attendees — 12 colleagues (all of whom had attended online) responded. Participants were colleagues in HE institutions interested in using Lean as a teaching tool. They appreciated the possibility to hear experiences about the use of Lean for teaching as well as summaries of results from educational research. They also commented on how useful it was to see how to use Lean resources and games first-hand and were excited at the possibility of accessing a site which contained the resources for Lean as well as the education findings related to this topic. All indicated that they would be interested in other events like this and after the meeting we were contacted by several colleagues who could not attend but were interested in the topic and wanted to have access to the recordings of the talks (these are available at [bit.ly/3xL0bFp](http://bit.ly/3xL0bFp)).

We are now planning, with Prof Jeremy Avigad, Dr Heather Macbeth (Fordham University), Dr Patrick Massot (Université Paris-Sud) and Prof Kevin Buzzard, the introduction of a 'Teaching Mathematics with Lean' page on the Lean community page on github ([leanprover-community.github.io](http://leanprover-community.github.io)) to store links to resources with a short commentary and links to a site that will contain education reports and academic papers about the impact of using Lean on students' learning as well as to the videos of our event. We are currently constructing the education site which we have called the Lean for Teaching Mathematics page (under construction [bit.ly/3n891bs](http://bit.ly/3n891bs)).

The day was a success — as our participants reported — some suggested to start an annual 'Teaching with Lean' conference to follow this day and we are considering this option. So if you are interested in using Lean for one of your first year modules... watch this space!

Paola Iannone (Loughborough)  
Athina Thoma (Southampton)

## Records of Proceedings at LMS Meetings

### Ordinary Meeting: 4 April 2022

This meeting was held in person and virtually on Zoom on 4 April 2022, hosted by the University of Birmingham, as part of the Midlands Regional Meeting and Workshop. Up to 51 members and visitors were present for all or part of the Society meeting session. This one day meeting took place as part of the LMS Workshop on *Mirrors, Moduli and M-theory in the Midlands* from 5 to 7 April 2022.

The meeting began at 1.30pm with the LMS President, Professor Ulrike Tillmann, in the Chair.

Four members signed the Members' Book and were admitted to the Society.

Dr Cyril Closset, University of Birmingham, introduced the first lecture given by Professor Frances Kirwan (University of Oxford) on *Moduli Spaces, Moduli Stacks and Quotients in Algebraic Geometry*.

After tea, Dr Ana Peón-Nieto, University of Birmingham, introduced the second lecture by Professor Diane Maclagan (University of Warwick) on *Solving Equations with Polynomial Coefficients (with Applications to Toric and Tropical Geometry)*.

Dr Michel van Garrel, University of Birmingham, introduced the third lecture by Professor Richard Thomas (Imperial College London) on Counting Sheaves on *Calabi–Yau 4-folds, Part I*.

Professor Tillmann thanked the speakers for their excellent lectures and thanked the organisers, Dr Cyril Closset, Dr Ana Peón-Nieto and Dr Michel van Garrel, all of the University of Birmingham.

## Records of Proceedings at LMS Meetings

### Ordinary Meeting: Hirst Lecture and Society Meeting

This meeting was held on Friday 6 May 2022, in person at De Morgan House and virtually via Zoom. Over 20 members and guests were present in the Hardy Room, and over 30 members and guests were present on Zoom, for all or part of the meeting.

The meeting began at 2.30pm, with the President, Professor Ulrike Tillmann FRS, in the Chair.

10 people were elected to Associate membership: Mr Thomas Patrick Bernert, Mr Michael Garn, Mr Arvind Mishra, Mr Lewis Molyneux, Miss Bhairavi Premnath, Dr Giacomo Roberti, Mr Robert Sansom, Dr Layla Sorkatti, Dr Wenkai Xu, Dr Nam Nguyen.

4 people were elected to Associate (Undergraduate) membership: Mr Langchen An, Miss Alina Budnyuk, Mr James Calvert, Mr Seyed Ali Sadreddin Yasavol.

15 people were elected to Ordinary membership: Dr Jessica Claridge, Dr Necip Erdogan, Dr Chris Guiver, Dr Jiahua Jiang, Mr John Jones, Dr Theodoros Kouloukas, Dr Priscila Leal da Silva, Dr Vasos Pavlika, Dr Omar Rivasplata, Dr Manoj Sahni, Ms Isatou Sarr, Dr Nawal Hussein, Dr Andrew Smith, Dr Ali Taheri, Mrs Ozlem Wit.

3 people were elected to Reciprocity membership: Dr F. Ayça Çetinkaya, Dr Adefunke Familua, Dr Reema Jain.

Two members signed the Members' Book and were admitted to the Society.

The Chair introduced the first talk given in person by Serafina Cuomo (Durham) on Maths and the City. A Snapshot of Numeracy in Classical Athens.

After the break, the Chair introduced the second talk given in person by the winner of the LMS-British Society for the History of Mathematics Hirst prize Karine Chemla (CNRS & Université de Paris) on Algebraic Work with Operations in China, 1st Century—13th Century.

Professor Tillmann thanked the speakers for their excellent lectures and then expressed the thanks of the Society to the organisers, the SLAM Committee and the BSHM for a wonderful meeting and workshop.

Afterwards, a wine reception was held at De Morgan House. The Society dinner was held at the Antalya Restaurant.

# A Mathematical Jigsaw: Graph Decomposition

KATHERINE STADEN

We've all tried to solve a jigsaw puzzle. Suppose instead we are shown the whole picture and given many small pieces, and want to decide if the pieces do in fact fit together to make the picture. This article delves into the mathematical jigsaw of graph decomposition.

## Small puzzle pieces

The Oberwolfach Institute is a famous venue for mathematical conferences and events. Suppose some essential building work combined with coronavirus precautions means that there's only one meeting room and it only allows 3 people. I'm inviting  $n$  (many more than 3) people to a conference, and to allow everyone to share their ideas efficiently, each pair of people should meet in this room exactly once during the conference. Is this possible?

To understand both the problem and why it's a 'mathematical jigsaw', we need to formulate it mathematically. A graph, or network, is a collection of points (vertices), some of which are joined by lines (edges). Form a complete graph  $K_n$  by letting each person be a point and join every pair by an edge (to represent every possible pair of people). Now give each meeting period a colour  $c_1, \dots, c_\ell$ . This means we want to colour the edges of  $K_n$  with  $\ell$  colours so that the edges in each colour form a triangle. In the jigsaw analogy, the whole picture is  $K_n$  and the pieces are  $\ell$  triangles. This is a *graph decomposition* problem.

To count the number of pairs of people, I choose the first person in  $n$  ways, and then the second in  $n - 1$  ways, and divide by 2 because I counted each pair twice, in both orders. So there are  $\frac{n(n-1)}{2}$  pairs, and each meeting accounts for 3 of them. So  $\ell = \frac{n(n-1)}{6}$  and in particular,  $n(n-1)$  must be divisible by 6. Since each person needs to meet  $n - 1$  other people, and they meet 2 at each meeting they're invited to,  $\frac{n-1}{2}$  must be an integer, so  $n$  is odd. Putting these conditions

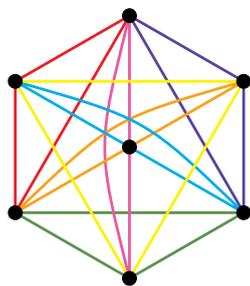


Figure 1. A Steiner triple system of order 7

together, it turns out that  $n$  must leave a remainder of 1 or 3 when divided by 6. We say that such  $n$  are *triangle divisible*. So this is a necessary condition for the problem to have a solution. Are there any other obstacles?

## Graphs and relationships

Apart from their interest to mathematicians, graphs are extremely useful and ubiquitous structures that can be used to model relationships between objects, where the model abstracts a situation to capture key information, sometimes at the expense of losing helpful information. The London underground network is a good example, and in fact the familiar map of the network is a graph, where the stations are vertices and a line represents a direct train between them. This graph contains additional information, such as the relative positions of vertices which approximate the real positions of stations, the names of the stations and the colours of the edges. Some useful information which is not included is a weight on edges, which could be the number of minutes a particular journey takes.



Figure 2. Part of the London underground network

This question dates back to the early nineteenth century, to Plücker, Woolhouse, Kirkman, Steiner

and others. The following question was posed by Woolhouse in 1844 in the *Lady's and Gentleman's Diary*:

*Determine the number of combinations that can be made out of  $n$  symbols,  $p$  symbols in each; with this limitation, that no combination of  $q$  symbols, which may appear in any one of them shall be repeated in any other.*

The problem we considered above is the case  $p = 3$  and  $q = 2$  and is now known as the existence problem for *Steiner triple systems*. It was solved in the affirmative in 1846 by Kirkman – there are no obstacles other than the trivial one of lack of triangle divisibility. For  $q = 2$  and general  $p$ , the problem is to decompose  $K_n$  into copies of  $K_p$ . A still more general problem replaces  $K_p$  by any fixed graph  $F$  on  $p$  vertices. Again one can derive a simple necessary condition based on divisibility. It wasn't until the 1970s that this existence problem was solved for large  $n$  in a transformative series of papers by Wilson, who showed that once again there are no other obstacles to decomposition. We can even allow any collection of small  $F$ ; that is, the puzzle pieces can be different. In other words, when the puzzle pieces are all small, the  $q = 2$  problem is solved. What is not yet well understood is when the number of vertices in puzzle pieces can be comparable with  $n$ .

### The mathematics of seating plans

Let's return to the Oberwolfach Institute for another conference conundrum. The Institute's dining hall contains some circular tables of various sizes; I don't know how big they are but I know they seat  $n$  people in total (this much is true). I'm inviting  $n$  people to a conference, and to foster good communication, I want to write some seating plans so that each pair of people sit next to each other exactly once during dinner. Is this possible?

This question — the Oberwolfach problem — was posed by Ringel in 1967 and despite attracting a lot of attention among mathematicians, was unsolved until recently. Again we give each puzzle piece, in this case each required seating plan, a colour  $c_1, \dots, c_\ell$ . Since each person needs to sit next to  $n - 1$  other people, and they sit next to 2 of them at each meal, we need  $\ell = \frac{n-1}{2}$  colours, and  $n$  needs to be odd so that this is an integer. This means we want to colour the edges of  $K_n$  with  $\frac{n-1}{2}$  colours so that the edges

in each colour look like the layout of the tables. In the jigsaw analogy, the whole picture is  $K_n$  and the table graphs are the pieces.

Even if all the tables seat 3, this problem is different to our first, since not only do we want a decomposition into triangles, but the triangles need to come in groups of  $\frac{n}{3}$  which share no vertices. It is non-trivial to show that there are triangle decompositions where the triangles cannot be grouped in this way; that is, the decomposition is not *resolvable*.

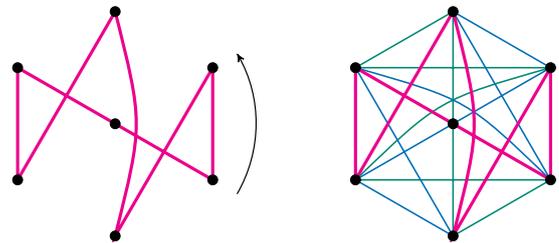


Figure 3. Walecki's theorem and proof for  $n = 7$ ; it is an exercise for the reader to generalise this for odd  $n$ .

If  $n$  is very small, a simple brute-force algorithm can solve the general problem on a computer. If there is a single table it was already solved by Walecki in 1892, predating the statement of the Oberwolfach problem (see Figure 3).

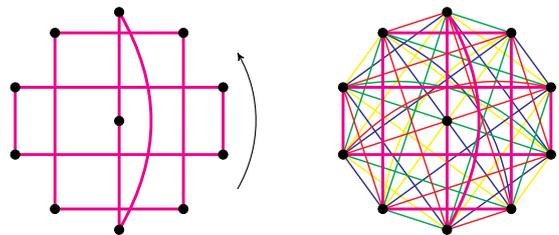


Figure 4. One table of size 3 and two of size 4.

Another special case is when all the tables are the same, i.e. the resolvable case. This is usually credited to Ray-Chaudhuri and Wilson from 1973, but actually their solution was predated by eight years by Lu Xia Xi. It was not until 2013 that Traetta solved the two-table problem.

In much of the work on the problem, researchers were able to describe seating plans explicitly, by finding one special seating plan and 'rotating' it to find the others (see Figures 3, 4). It turned out to be hard to apply this constructive approach to the general problem where the collection of tables is arbitrary. Eventually in 2018 Glock, Joos, Kim, Kühn

and Osthus were able to solve the problem for large  $n$  using a non-constructive method that makes random choices to find the seating plans. My recent work with Peter Keevash solved the ‘generalised Oberwolfach problem’ for large  $n$ , where every day the collection of tables is allowed to change (as long as they still seat  $n$ ). Both are examples of mathematical results which show that a solution exists without actually exhibiting it. Unfortunately neither of these two solutions would be any use for a real-life seating plan problem as the number  $n$  of people is far larger than any conference will ever be!

### Tree decomposition

Ringel had posed an earlier graph decomposition problem in 1963. He conjectured that  $K_n$  can be decomposed into identical puzzle pieces each of which look like a given tree  $T$  on  $\frac{n+1}{2}$  vertices, where here  $n$  is

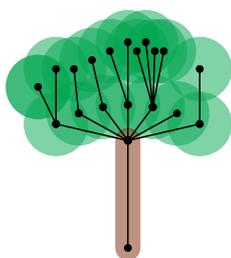


Figure 5. A tree

odd. A tree is any graph in which it is always possible to get between any two points by following edges, but you can only get back to where you started by backtracking (Figure 5 should explain its name).

This time, it turns out we need  $n$  pieces. Again, if  $T$  has a simple structure, or if  $n$  is small, the problem can be easy. Figure 6 shows how to find a decomposition when  $T$  has one vertex that lies in every edge (a *star*). But the structure of  $T$  could be very complicated; the challenge in solving Ringel’s conjecture is to find a proof that works for every  $T$ .

My recent work with Peter Keevash proves the conjecture, which has also been proved in independent work of Montgomery, Pokrovskiy and Sudakov.

### The power of randomness

A method for graph decomposition that has proved very powerful, and one which features heavily in our proof, is to run a random algorithm in which embedding decisions — where to put a piece or partial piece — are made by coin flips. Order the

pieces and the vertices within each piece. Decide where to embed one by one, in this order, by flipping a coin to decide between the available options. If the probability that this is successful for the first 1000 steps is positive, then we know it’s possible to complete the first 1000 steps. Such an algorithm is simple; the mathematical challenge is in its analysis, which involves showing that a large collection of random variables remain concentrated about their means.

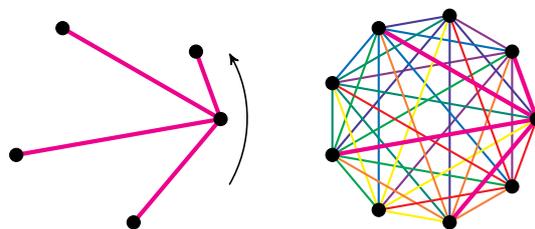


Figure 6. Ringel’s conjecture for the star. Montgomery, Pokrovskiy and Sudakov prove that every tree can be embedded cyclically like this.

Random embeddings do not work well when there are vertices of high degree (lying in many edges) because there is a high probability that too many edges will be placed at a single host vertex, more than the maximum of  $n - 1$ . To get around this issue while still harnessing the power of random embeddings, in our proof of Ringel’s conjecture we embed the high degree vertices first, in a balanced way. There are  $n$  copies of each high degree tree vertex  $v$  so we can arrange that each host vertex receives exactly one copy of  $v$ . This is balanced because we use the same number of edges at each host vertex. The crucial consideration is doing this in such a way that we can then embed the rest of the trees; to do this we want to ensure the unused part of the host graph is ‘randomlike’ so that the random embedding can proceed.

Even if there are no high degree vertices, such a random algorithm will not work all the way to the end: eventually we are left with very few edges and a small number of pieces to fit into them perfectly; it is highly unlikely that there is any choice left that works. For this reason, it’s the last 1% of the embedding where 99% of the difficulty lies, and for which new ideas have been developed.

## Counting and typical properties

Once we know there is a solution to the jigsaw puzzle, we may ask: how many solutions are there? Maybe it was so difficult to solve because there are only a few. This is actually very far from the truth. Answering a question of Wilson, results of Linial and Luria, and Keevash, combine to show that the number of Steiner triple systems of order  $n$  is

$$\left(\frac{n}{e^2} + o(n)\right)^{\frac{n^2}{6}}$$

when  $n$  is triangle divisible. This estimate comes from analysing a random triangle embedding algorithm.

Given that there are so many ways to piece together the puzzle, it is natural to ask what a typical solution looks like. In other words, if I pick a solution out of a hat, what properties do I expect it to have? Despite the fact that resolvable Steiner triple systems seem to ask for much more, Ferber and Kwan showed that almost all Steiner triple systems are almost resolvable, and conjecture that this second ‘almost’ can be removed.

## Open questions

There is still much more to understand regarding graph decompositions and there are a great many tantalising open problems.

**Conjecture 1** (Gyárfás, 1976). Can  $K_n$  be decomposed into any collection  $T_1, \dots, T_n$  of trees where  $T_i$  has  $i$  vertices?

A quick check reveals that the total size of the pieces and the completed puzzle are indeed equal. This conjecture looks similar to Ringel’s conjecture, but now that the trees are different, the high degree parts could be very different and there may be no balanced way to embed them.

**Conjecture 2** (Hadamard, 1848). There is an  $n \times n$  matrix  $H$  with entries  $\pm 1$  satisfying  $H^T H = nI_n$  whenever  $n = 1, 2$  or  $4|n$ .

At first sight this conjecture does not appear to be related to graph decomposition, but some linear algebra shows that it’s equivalent to asking for the existence of a  $K_{\frac{n}{2}, \frac{n}{2}}$ -decomposition of  $\frac{n}{2}K_n$ , the  $n$ -vertex multigraph where each pair has  $\frac{n}{2}$  edges

between them. The *complete bipartite graph*  $K_{\frac{n}{2}, \frac{n}{2}}$  has two equally-sized parts, every edge between the parts, and no others. It is known to be true for infinitely many  $n$ ; the smallest open case is  $n = 4 \times 167$ . Here, the puzzle pieces are extremely large.

A particularly interesting direction is to replace the complete graph with another host graph. A natural class of graphs to consider is those with some minimum degree condition, where each vertex lies in at least some number of edges.

Earlier I mentioned the result of Walecki from 1892 that  $K_n$  can be decomposed into  $n$ -cycles (or *Hamiltonian cycles*) when  $n$  is odd, which has an easy and appealing proof. In 1970 Nash-Williams asked whether any graph with minimum degree  $\geq \lfloor \frac{n}{2} \rfloor$  — examples show this number cannot be smaller — and in which every vertex has even degree — a necessary condition — also has this property. This was solved only in 2016 by Csaba, Kühn, Lo, Osthus and Treglown in a complex 170 page proof. The following version for triangles is still open.

**Conjecture 3** (Nash-Williams, 1970). Any graph with minimum degree  $\geq \frac{3n}{4}$  in which every vertex has even degree has a triangle decomposition.

This conjecture, like its Hamiltonian sibling, is difficult because there are a great many possible host graphs.

## Acknowledgements

A shorter version of this article originally appeared on the Oxford University Mathematical Institute website.

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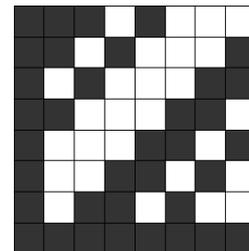


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### Katherine Staden

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in Cambridge and hasn’t gone a day without chocolate in over a decade.

## (Hyper)graph decompositions across time and across the globe

The full solution of Woolhouse’s problem for general  $p$  and  $q$  and large  $n$  was only solved quite recently by Keevash in 2014. This is equivalent to finding a suitable decomposition of a *hypergraph*, where an edge now contains exactly  $q$  vertices rather than 2. This is also known as a (*combinatorial*) *design*.

These (hyper)graph decomposition problems appear in many guises and all over the world, from Latin squares constructed by Arabic mathematicians in the first millennium AD (see Figure 8) and later studied by Euler in the 18th century, to modern day Sudoku puzzles and in application to the design of experiments. Despite often appearing in recreational mathematics, there is some deep and transformative mathematics lurking beneath.

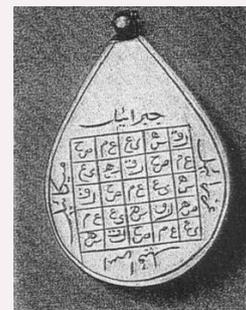


Figure 8. A silver amulet from Damascus with a Latin square inscribed, from *Latin Squares* by Lars Døvling Andersen in *Combinatorics: Ancient and Modern*.

# Jesse Douglas, Minimal Surfaces, and the First Fields Medal

JEREMY GRAY

Jesse Douglas received one of the first two Fields Medals in 1936 for his work on minimal surfaces, yet his work provoked long-running and painful battles with Tibor Radó and Richard Courant, and today it is not easy to find out what Douglas actually did, or much about his life.

(Everything in this Hirst lecture is joint work with Mario Micallef, University of Warwick).

## Introduction

Jesse Douglas received one of the first two Fields Medals in 1936 for his work on minimal surfaces: he was the first person to solve the Plateau problem for discs spanning an arbitrary contour, and to successfully generalise the problem to surfaces with two or more contours and, eventually, of arbitrary topological type. Yet his work provoked long-running and painful battles with Tibor Radó and Richard Courant, and today it is not easy to find out what Douglas actually did, or much about his life. Here I sketch some of the background, give an indication of what Douglas accomplished and Radó's response, and describe Douglas's later life.

## The 19th century background

There are several ways to think of a minimal surface. Lagrange, building on some earlier work by Euler, had defined it in 1761 as a surface with fixed boundary that minimises the area functional. This makes it the two-dimensional analogue of a geodesic: it is a surface  $\Sigma$  with the property that the piece of  $\Sigma$  enclosed by a closed curve  $\gamma$  on the surface has area less than or equal to that of any other surface having  $\gamma$  as a boundary. However, the only example Lagrange could find was the plane. In 1776, Meusnier sharpened some related ideas of Euler's to show that the catenoid and the helicoid were minimal surfaces, and despite some flaws in his work he succeeded in establishing that minimal surfaces have zero mean curvature. This says that their shapes

will be everywhere curved like saddles. His paper was eventually published as (Meusnier 1785) – savour the publication delay, exceptional even for the time. After that, a few other examples were found, each by ad hoc means.

So it was one of the successes of complex function theory in the 19th century to show that it was easy to produce pieces of minimal surfaces at will. Perhaps as early as 1860, in work published only after his death in 1866, Riemann reworked the whole theory of minimal surfaces. He defined a minimal surface as one for which the first variation of the area integral vanishes, and showed that this implies that the Gauss map of a minimal surface to the sphere is conformal. This allowed him to use the new methods of complex function theory that he was introducing to obtain arbitrary minimal surfaces, by giving parameterisations of them in a form equivalent to what are called today the Weierstrass–Enneper equations.

Riemann was particularly interested in the Plateau problem, which asks for a minimal surface spanning a given boundary. This is a much harder problem, and he was only able to indicate how the problem for a polygonal boundary could be reduced to a second order linear ordinary differential equation, in which the locations of the vertices indirectly determine the coefficients of the equation. The equation can be solved in terms of known functions only in the simplest cases, but Riemann did show that there was the right number of parameters for the problem always to admit a solution.

Independently, around 1865, Hermann Amandus Schwarz, relying on ideas introduced by Julius Weingarten, showed how to solve the hardest of Riemann's specific problems: To fit a minimal surface to a symmetrical quadrilateral in space. At the same time, Karl Weierstrass, who started from the idea of a minimal surface as a surface with zero mean

curvature, had also shown how the representation of a minimal surface is accomplished using complex function theory. Because this agrees with Enneper's representation, which relied on the methods of differential geometry, and because it is slightly different from Riemann's, there is some justice in the modern name for this representation: the Weierstrass–Enneper equations.

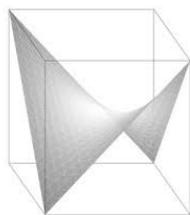


Figure 1. Schwarz's minimal surface spanning four edges of a regular tetrahedron

Also in these years the Belgian physicist Joseph Plateau published his extensive researches into the shape of soap films, which are generally minimal surfaces spanning a wire frame, and corresponded with Schwarz about the topic. Then Gaston Darboux, in the two editions of his book *Théorie des Surfaces* (1887, 1914), extended Riemann's programme for tackling the Plateau problem. He also drew on extra, if sometimes less than thorough, remarks by Weierstrass and Schwarz about the ordinary differential equations involved, and offered some more solutions, albeit for a contrived class of polygonal contours. But his book suggested that all had been done with existing methods, and new ones would be needed,

## Jesse Douglas



Figure 2. Jesse Douglas (1897–1965)

Jesse Douglas was born in New York City in 1897 and educated at public schools in New York City. He went to City College, and then to Columbia University, New York as a University scholar in 1916, where he became a fellow in 1917. He took his PhD in 1920 under Edward Kasner, who inspired in him his love of differential geometry, and became an instructor at Columbia from 1920 to 1926. Then he became a National Research Fellow and travelled widely in Europe for four years, financed in part

by the International Education Board (IEB), which had been established in 1923 by the Rockefeller Foundation, and where he was supported by Oswald Veblen.

Douglas was in Paris in January 1929 and again in December, when Jacques Hadamard wrote to the President of Cornell University:

His exposés on Plateau's problem are among the best I have had; the solution he has found is of a remarkable elegance (sic) and simplicity, almost unexpected for this difficult problem, one of the most beautiful (sic) in Mathematical Physics; moreover he presents it with a perfect clearness, a clever thing at my seminary (sic), where details must be left aside and only the general development of ideas brought in full light. Mr. Douglas has perfectly succeeded in that difficult achievement, and every auditor has carried from his lecture a perfectly clear understanding of the question.

He would be a first rate recruit for any mathematical staff ...

Others may have needed more persuading, or heard only earlier versions of his talk. Douglas had a hostile reception when he talked on this material in Göttingen in April 1929 (see Reid's *Courant*, p. 174), when his ideas were still developing. If the argument did not come together fully in Douglas's mind until the second Hadamard seminar in December 1929, then perhaps he was on the defensive at some points (possibly his account of degenerate parametrisations of the boundary curve).

However, on his return to America, Douglas took up a position in January 1930 as an Associate Professor at MIT, which never suited him, and sought unsuccessfully to be appointed at the Institute for Advanced Study in Princeton. Eventually, in July 1938 Douglas resigned from MIT. Dirk Struik, who was there, wrote in his (Struik 1989) that Douglas

had his own lifestyle which did not include coming to class on a regular schedule, so that Phillips (the Head of Department), who stuck to the Runkle discipline of conscientious teaching, had to let him go, to my and others' regret.

Much of what we know about how Douglas may have discovered the  $\mathcal{A}$ -functional is drawn from abstracts

of talks he gave to the AMS and published in its *Bulletin*. These show that by 1926 Douglas knew that any conformal parameterisation of a least area surface is also harmonic. He then reformulated the problem as the search for a conformal, harmonic parameterisation of a spanning surface, setting aside the issue of least area. This reduces to the search for the parameterisation of the boundary whose harmonic infill will be conformal.

Douglas knew that a monotone continuous map  $g$  of the boundary of the disc  $\mathbb{B}$  to a simple closed curve  $C$  in  $\mathbb{R}^n$  can be extended continuously by the Poisson integral formula to a harmonic map  $h_g$  of  $\mathbb{B}$ . So Douglas looked for a parameterisation  $g^*$  of  $C$  that would make  $h_{g^*}$  also conformal. He formulated this condition as an integral equation for  $g^*$ . He noticed that this integral equation was, in fact, the Euler–Lagrange equation of a functional. This is not, however, the one he later called the  $A$ -functional; it is not bounded below, and so was useless. But it seems to have given Douglas almost undue confidence in his approach, and he began to hint that he had solved the Plateau problem.

### Fréchet theory

Maurice Fréchet introduced the idea of metric spaces in 1906. They are spaces with a concept of distance between two points and a concept of limit.

Among the examples he gave of spaces with limits are spaces of curves or surfaces, and spaces of parameterisations of a given curve, where the notion of convergence is defined by a suitable definition of separation.

Under suitable circumstances, a lower semi-continuous functional on a such a space attains its lower bound.

Douglas wrote to Veblen in February 1928 to say that he had succeeded in establishing the existence of a solution of his integral equation, albeit under conditions that prevent it from applying to the Plateau problem, and he found this encouraging. In the July 1928 announcement in the *Bulletin of the AMS* he introduced Fréchet theory to his approach, having noticed that his integral equation is the Euler–Lagrange equation of the first version of his

later-to-be-famous  $A$ -functional:

$$A(\varphi) := - \int_0^{2\pi} \int_0^{2\pi} K(t, \tau) \log \sin \frac{1}{2} |\varphi(t) - \varphi(\tau)| dt d\tau. \quad (1)$$

Douglas took an arbitrary parameterisation of the boundary  $\mathbf{g} : \partial\mathbb{B} \rightarrow C$ , and varied it by means of a homeomorphism  $\phi : \partial\mathbb{B} \rightarrow \partial\mathbb{B}$  to obtain a new parameterisation  $\mathbf{g}\phi^{-1} : \partial\mathbb{B} \rightarrow C$ . Typically, he wrote  $t, \tau \in \partial\mathbb{B}$ . Fréchet theory implies the existence of a minimizer  $\varphi^*$  of the  $A$ -functional – at least in the case when  $K(t, \tau)$  is positive for all values of  $t$  and  $\tau$ . This makes the above integral inapplicable to the Plateau problem, for which the kernel  $K(t, \tau) = \mathbf{g}'(t) \cdot \mathbf{g}'(\tau)$ . The problem is that  $\mathbf{g}'(t) \cdot \mathbf{g}'(\tau)$  is not always positive, but this is not a serious obstacle, replacing it with  $\frac{1}{2} \|\mathbf{g}'(t) - \mathbf{g}'(\tau)\|^2$  in the functional does not affect its Euler–Lagrange equation, although it seems it took Douglas a year to notice this. A bigger problem is posed by the need to prevent a minimizing sequence from converging to an improper or degenerate representation of the contour. This was not mentioned in his announcement, and he solved it using a three-point condition in his (Douglas 1931a).

To sum up:

- This integral equation is the Euler–Lagrange equation of a functional (eventually his  $A$ -functional) defined on a space of parameterisations.
- This functional, by Fréchet theory, attains its minimum – its Fréchet derivative vanishes.
- For that parameterisation, the harmonic infill is also conformal.

Douglas’s first major published paper on the Plateau problem (1931a), began with an account of his  $A$ -functional.

$$A(\mathbf{g}) := \frac{1}{4\pi} \int_0^{2\pi} \int_0^{2\pi} \frac{\sum_{i=1}^n [g_i(\theta) - g_i(\varphi)]^2}{4 \sin^2 \frac{\theta - \varphi}{2}} d\theta d\varphi. \quad (2)$$

It is defined on the space of all monotonic parameterisations  $(g_1(\theta), \dots, g_n(\theta))$  of the boundary curve  $C$ , and has good continuity properties in terms of Fréchet’s theory of functionals. It is lower semi-continuous on that space, which made it easier to use at the time than the Dirichlet integral.

Douglas showed directly that the harmonic extension of a minimiser of the  $A$ -functional is conformal,

without any reference to the integral equation. He did this by varying the Fourier coefficients of a Fourier expansion of the boundary parameterisation. In this way Douglas showed that a conformal harmonic spanning surface  $\Sigma$  topologically equivalent to a disc could be found for any Jordan contour.

Douglas's approach yielded other striking successes. He noted that his proof applied to contours  $C$  in  $\mathbb{R}^n$  for any  $n \geq 2$ , not just  $n = 3$ , and so in the special case  $n = 2$ , which concerns Jordan curves in the plane, Douglas's proof then provided a proof of the Riemann–Carathéodory mapping theorem. Douglas was the first to connect the Plateau problem to this important result in the theory of conformal mapping.

In his paper (1931a) Douglas also dealt with contours where all spanning surfaces have infinite area, and the  $\mathcal{A}$ -functional itself becomes infinite. Each surface  $\Sigma$  of this kind is the limit of conformal harmonic surfaces, each of which has finite area, and the area of any region in  $\Sigma$  that does not include boundary points is finite. This was the first time a result of such generality had been obtained.

In papers that followed his (1931a), Douglas also published further extensions of the Plateau problem to surfaces other than topological discs; non-orientable surfaces; and surfaces with two contours. A surface with two contours is topologically an annulus and so its conformal structure belongs to a one-parameter moduli space, which affects the harmonic functions that extend the parameterisations of the boundary curves. Douglas was able to show how the correct modulus is determined under a suitable restriction on the boundary curves.

It is striking that Douglas's work made no use of the Riemann–Weierstrass–Enneper equations. Its one disadvantage was that, by taking a minimal surface to be a conformal harmonic image, Douglas could not assert that the minimal surface he produced had *least area* among all surfaces with the same boundary. A fairly simple argument shows that the desired conclusion follows if it can be shown that every surface can be conformally parameterised. However, the Korn–Lichtenstein theorem of 1916 on the existence of isothermal parameters for a non-degenerate metric on a surface provides a conformal parameterisation only for a surface *immersed* in  $\mathbb{R}^3$ , and it is not known a priori that an area-minimising surface that spans  $C$  is necessarily immersed. So the full generality of

Douglas's argument was unclear, and this was to lead to a dispute with Radó.

### Tibor Radó

Tibor Radó was born in 1895, and studied civil engineering at the Technical University in Budapest. After the first world war, he switched to mathematics, and a Rockefeller Foundation award enabled him to spend 1928–1929 in Munich with Constantin Carathéodory and in Leipzig with Paul Koebe and Leon Lichtenstein. In 1929 he visited the United States, lecturing at Harvard. Very likely he had been recommended by Carathéodory, who had enjoyed a very successful visit there the previous year. In 1930 he became a full professor of mathematics at Ohio State University, where he stayed until his retirement in 1965.

Radó defined a minimal surface spanning a simple closed curve  $C$  as a conformal harmonic map of a disc with appropriate boundary. To obtain a parameterisation of  $C$  whose harmonic extension is conformal he relied on papers by Schwarz and on some of Koebe's proofs of the uniformisation theorem. In a paper of 1927, Koebe had proved that a polyhedral surface can be conformally parameterised away from the vertices. In his (1930), Radó published his solution of the Plateau problem for a polygonal contour  $C$ , which runs along these lines: For  $\epsilon > 0$ , he chose a polyhedral spanning surface  $P_\epsilon$  whose area exceeded the infimum of areas of polyhedral surfaces spanning  $C$  by no more than  $\epsilon$ . He then parameterised  $P_\epsilon$  conformally using a recent theorem of Koebe's. This gave him a parameterisation of the boundary, from which he obtained, using the Poisson integral formula, a harmonic surface  $\Sigma_\epsilon$ . Crucially, the area of  $\Sigma_\epsilon$  is less than the area of  $P_\epsilon$ . Radó then showed that, as  $\epsilon \downarrow 0$ ,  $\Sigma_\epsilon$  converges to a conformal harmonic surface spanning  $C$ .

He had already proved in an earlier paper in 1930 that, if a simple closed curve  $C$  of finite length can be approximated by a sequence of simple closed curves each of which spans a conformal harmonic image of the disc of uniformly bounded area, then  $C$  can also be spanned by a conformal harmonic disc-type surface. He combined this result with his solution of the Plateau problem for polygonal contours, and so solved the Plateau problem for rectifiable contours.

It is most likely that Radó heard of Douglas for the first time when he was in Harvard. He pointed out that his approach provided a solution of the *minimum area* problem for disc-type surfaces, which Douglas had not been able to solve. Douglas soon promised to show that his methods allowed him to do that, but he never delivered on that promise, and had to fall back on a use of Koebe's uniformisation theorem in order to establish that his solution had least area among discs spanning  $C$ .

Radó then went on the attack. In a paper in 1931 he showed that his solution of the Plateau problem gives simpler proofs of all the results in Douglas's (1931a). He also remarked – not, we think, justifiably – that the existence of a minimiser for the  $A$ -functional was easy to establish, even without assuming his solution of the Plateau problem. He went on to show that the harmonic extension  $h_g$  of a monotone map  $g$  from the unit circle to a contour  $C$  is conformal if it is a stationary point for *Dirichlet's integral* with respect to composition of  $h_g$  with a diffeomorphism of the unit disc (that is, with respect to reparameterisation of the surface  $\Sigma_g$  parameterised by  $h_g$ ). This opened the way to what Courant was to do, after he emigrated to New York in 1936, which eventually led to the eclipse of the  $A$ -functional.

Douglas responded to Radó's attack in a paper in 1931, where he gave simpler proofs of his results in his (1931a) by making use of Koebe's uniformisation theorem from the start. He then commented that Koebe's uniformisation theorem and the Plateau problem were equally deep and so, in his opinion, what Radó had done was simply to make the Plateau problem a corollary of Koebe's uniformisation theorem, which is surely a rather unfair dismissal of Radó's ingenious solution of the Plateau problem.

We have no information on why the Fields Medals committee gave a medal to Douglas but not Radó — for what it's worth, Douglas was just under 40, Radó just over. The IMU has no records of the discussions leading up to the award of the first Fields Medals, which makes the fragments in the recent account by Hollings and Siegmund-Schultze in their book (2020) all the more interesting. But in any case Carathéodory's speech (1937) is odd. He praised Douglas for his originality: his methods made much less use of existing theory and so were of greater significance. But he then sketched an approach to the minimum area problem that was essentially Radó's, and only then noted that Douglas had solved the Plateau problem by looking among the class of harmonic spanning surfaces for a conformal

one. He then remarked that Douglas had solved the Plateau problem even when every spanning surface has infinite area, only to note that this problem should not really be called a *Plateau* problem. Finally, although he praised the great generality of Douglas's solution, he did not mention that this included the new proof of the Riemann–Carathéodory mapping theorem.

### Douglas's later life

In the later 1930s Douglas extended the Plateau problem to the problem of spanning arbitrarily many boundaries by surfaces of arbitrary, but specified, topological type. Teichmüller theory had yet to be developed, and Douglas was forced to use the indirect theory of theta functions to handle the moduli questions that arise. His calculations are difficult to follow and have probably never been fully checked. Nevertheless, Douglas was the first mathematician to provide a viable program for solving the Plateau problem at this level of topological complexity; his results have been generalised and made more accessible by modern methods.

Sadly, between 1936 to 1938 Douglas seems to have suffered problems with his mental health. Veblen wrote to Abraham Flexner (23 November 1938):

Douglas has been a member of the Mathematics Department of the Massachusetts Institute of Technology, but a couple of years ago he had to stop his work because of a nervous breakdown. Since then he has been on leave of absence recuperating at his home in Brooklyn. I was therefore much pleased to see him returning in a good state of health.

On his recovery, Douglas visited the IAS in Princeton in 1938–1939, and G.D. Birkhoff wrote to T.H. Hildebrandt at Michigan on 10 September 1939 to say that “He [Douglas] seems to have completely recovered his mental balance and is producing mathematics at a rapid rate.”

In 1940–1941 Douglas became a Guggenheim Fellow at Columbia, collaborating on a research project to do with gas masks. He hoped to be hired permanently at Columbia, and Marston Morse wrote unsuccessfully on his behalf, praising “the great talents of Douglas as a teacher”, but to no avail and Douglas moved to

Brooklyn College, where he taught from 1942 to 1946. His teaching during the War went well, and he was presented with a Distinguished Service award from Brooklyn College in 1944.

In 1943 the American Mathematical Society awarded him its Bôcher Prize in analysis for two of his papers on the Plateau problem and one on the inverse problem of the calculus of variations, and in 1946 Douglas was elected a Fellow of the National Academy of Sciences (NAS). But then his personal problems returned, and from 1946 to 1950 the record of his career is blank. George Garrison, the Department Head at City College, noted in February 1955 that: "During the period 1946–51, Doctor Douglas did not have a University or College position." In 1951 Douglas taught in the School of General Studies at Columbia; in 1953 and 1954 he was a Lecturer in Mathematics at Columbia. He also taught at the Institute of Mathematics at Yeshiva from 1952–1954, where, in the Fall of 1952, he taught a course in Modern Algebra.

In January, 1955, and not without further difficulty, Douglas returned to City College. There, he does seem to have been liked. When students went to the Head of the Mathematics Department of City College to complain about one of their teachers they were assured that whatever the man's failings as an instructor, he was a distinguished mathematician with a fine research record, and it was a privilege to be taught by him. The students were not impressed. "We don't want someone like that", they said, "we want a real mathematician, someone like Douglas!"

On September 22, 1965 Douglas took sick leave from City College; he died in hospital from a heart attack on 7 October 1965. After his death the question arose of who should write his obituary for the NAS. The Secretary, Hugh L. Dryden, wrote to Courant to ask if he would do it, but Courant declined and suggested Hans Lewy, Kurt Friedrichs, or Charles Morrey. But Lewy and Morrey declined, and the new Home Secretary of the NAS, Merle Tuve, wrote again to Courant who now promised to try to collect some biographical information, but said that he could not promise to be successful. In the event he seems to have failed. Samuel Eilenberg was apparently amazed that Courant could be asked, and that he would accept. There is still no entry for Douglas in the *Biographical Memoirs of the National Academy of Sciences*.

## References

Full references will be found in the forthcoming book *Jesse Douglas, Minimal Surfaces, and the first Fields Medal* by Jeremy Gray and Mario Micalef.

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## Jeremy Gray

Jeremy Gray is an Emeritus Professor of the History of Mathematics at the Open University and an Honorary Professor at the University of Warwick. His research is largely on the history of

mathematics in the 19th century, and he gave the Hirst lecture of the LMS in 2019. He has recently and very happily become a grandparent for the first time.

## Notes of a Numerical Analyst

# Random Fibonacci sequences

NICK TREFETHEN FRS

Random Fibonacci sequences are generated by the recurrence  $x_{n+1} = x_n \pm x_{n-1}$ , where each  $\pm$  is an independent coin toss. Let's look at the "semi-Fibonacci" variant

$$x_{n+1} = x_n \pm \frac{1}{2}x_{n-1}. \quad (1)$$

If you run (1) for a few steps, you may get a perplexing result, as in Figure 1.

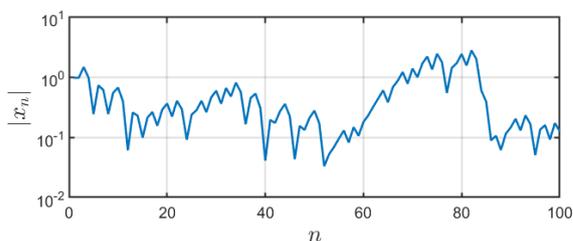


Figure 1. 100 steps of (1) with  $x_1 = x_2 = 1$ .

With more steps, however, as in Figure 2, the pattern becomes clear. *Random semi-Fibonacci sequences decrease exponentially.*

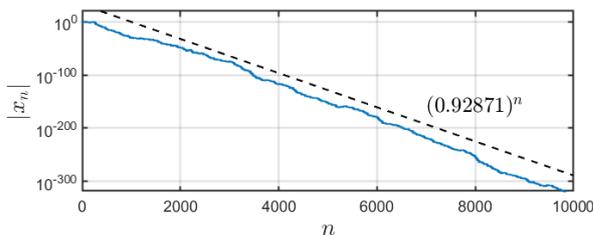


Figure 2. 10000 steps.

This effect is so elemental that it can illustrate many ideas of mathematics and science. (Warning! — all details below omitted.)

*Geometric means.* Some intuition for the decrease goes like this. Suppose  $\{x_n\}$  were of approximately constant magnitude. Then each step would multiply the magnitude at random by  $\frac{1}{2}$  or  $\frac{3}{2}$ . But the geometric mean of  $\frac{1}{2}$  and  $\frac{3}{2}$  is less than 1, suggesting decay on average after all.

*Almost sure behaviour of random processes.* In principle,  $\{x_n\}$  could decay or grow at any rate from

$(0.707)^n$  to  $(1.366)^n$ , but with probability 1, you'll see  $(0.929)^n$  as  $n \rightarrow \infty$ . It is effects like this that give meaning to quantities of physics starting with pressure, temperature, and entropy.

*Lyapunov constants.* The Lyapunov constant of this dynamical system is 0.92871. One could hardly devise a simpler illustration of this idea.

*Fractals.* Analysis of (1) reveals an invariant measure of fractal form, and if  $\frac{1}{2}$  is generalized to a parameter  $\beta$ , the dependence of the Lyapunov constant on  $\beta$  is also fractal.

*Products of random matrices*  $[1 \pm \frac{1}{2}; 1 \ 0]$ . This topic was made famous by Furstenberg and Kesten.

*Stochastic differential equations.* (1) is related to approximations to SDEs with multiplicative noise like the exponential martingale  $dX_t = \sigma X dW_t$ . Here, too, solutions decay exponentially.

*Heavy-tailed distributions.* Though  $x_n$  decreases exponentially, its variance and standard deviation grow exponentially! At the endpoint of Figure 2, the standard deviation is  $10^{677}$ .

*Some numbers can't be determined analytically.* I believe the digits 0.92871 are correct, but it would be astonishing if anyone found an exact formula.

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### Nick Trefethen



Trefethen is Professor of Numerical Analysis and head of the Numerical Analysis Group at the University of Oxford. His book *An Applied Mathematician's Apology* has just been published by SIAM.

## Mathematics News Flash

Jonathan Fraser reports on some recent breakthroughs in mathematics.

### A Proof of the Kahn-Kalai Conjecture

**AUTHORS:** Jinyoung Park, Huy Tuan Pham

**ACCESS:** <https://arxiv.org/abs/2203.17207>

The Kahn-Kalai Conjecture was posed in 2006 and is related to ‘thresholds’ for the appearance of certain structures inside a random object, for example Hamiltonian cycles inside a random graph. Consider a finite set of size  $N$  and construct a random graph by adding in an edge between each pair of distinct points independently with probability  $p \in (0,1)$ . Since a Hamiltonian cycle requires lots of edges the probability that there will be a Hamiltonian cycle will increase as  $p$  increases. The *threshold* is the value of  $p$  where the behaviour changes from it being unlikely (less than 50%) to likely (at least 50%) that there will be a Hamiltonian cycle. The threshold in this case turns out to be  $p = (\log N)/N$ .

One can ask similar questions about the ‘threshold’ for many different problems in a much more general setting than random graphs. It is convenient to estimate the threshold by an averaging method resulting in the *expectation threshold*. The Kahn-Kalai Conjecture is that the estimate provided by the expectation threshold will never be wrong by more than a logarithmic factor. This remarkable – and much discussed – paper proves the Kahn-Kalai Conjecture in just 6 pages!

### Measures of maximal entropy for surface diffeomorphisms

**AUTHORS:** Jérôme Buzzi, Sylvain Crovisier, Omri Sarig

**ACCESS:** <https://arxiv.org/abs/1811.02240>

An important result in smooth ergodic theory due to Newhouse is that a  $C^\infty$  surface diffeomorphism necessarily has an ergodic measure of maximal entropy. Newhouse asked if the number of ergodic measures of maximal entropy is finite provided the diffeomorphism has positive topological entropy. This paper, published in *Annals of Mathematics* in 2022, answers this question in the affirmative. In fact,

more is true. If the diffeomorphism is topologically transitive, then the measure of maximal entropy is unique, and if the diffeomorphism is topologically mixing, then the unique measure of maximal entropy is (isomorphic to) a Bernoulli scheme.

The proof is long and technical and involves an impressive generalisation of Smale’s spectral decomposition theorem to the non-uniformly hyperbolic setting as well as the use of irreducible countable state Markov shifts to model the system.

### Canonical Heights on Shimura Varieties and the André-Oort Conjecture

**AUTHORS:** Jonathan Pila, Ananth N. Shankar, Jacob Tsimerman, Hélène Esnault, Michael Groechenig

**ACCESS:** <https://arxiv.org/abs/2109.08788>

Mathematicians have long been intrigued by integer solutions to polynomial equations. For example, recall Fermat’s Last Theorem which states that there are no (positive) integer solutions to  $x^n + y^n = z^n$  for integers  $n \geq 3$ . Andrew Wiles’ proof of this theorem stands as one of the greatest mathematical achievements of the 20th Century.

The André-Oort Conjecture is in this direction and is stated in terms of *Shimura varieties*. This paper proves the conjecture by finding an appropriate bound for the heights of special points. An *algebraic variety* is the set of solutions to a system of polynomial equations and a *Shimura variety* is, roughly speaking, a family of complex algebraic varieties associated to a reductive algebraic group together with a certain conjugacy class of homomorphisms.

Interestingly, Klingler and Yafaev proved the conjecture in 2014 under the assumption that the Generalised Riemann Hypothesis was true.



**Jonathan Fraser** is a Professor at the University of St Andrews and an Editor of this Newsletter. He likes fractals and crawling with Reuben.

## Reciprocal societies: Indonesian Mathematical Society



The Indonesian Mathematical Society (IndoMS) was founded in 1976 in Bandung, Indonesia. Professor Muhammad Ansjar, Professor Slamet Dajono, Dr Bana Kartasasmita

and Professor A. Arifin were among its founding members, along with each and every mathematician who attended IndoMS's first national-level mathematics conference in Indonesia.

The Society aims to build networks for the development of mathematics research and education, provide input to relevant education agencies on mathematics curricula and teaching in accordance with national education goals, and apply the results of mathematical studies to increase community welfare. In order to achieve these goals, the society holds conferences, workshops, and outreach events for researchers, students, educators, and government bodies.

Since its founding, the Society has held the National Mathematics Conference (KNM) once every two years at Indonesian universities. The Conference is the largest gathering of mathematics researchers in Indonesia. Its objective is to provide researchers with

the opportunity to disseminate and discuss their work. The Society has also held seven instances of the IndoMS International Conference of Mathematics and Its Application. The first and second South East Asian Women Mathematicians Meetings were held in collaboration with IndoMS, as the Society is keen on supporting the endeavours of women in mathematics research.

Each year, IndoMS publishes three issues of the Journal of The Indonesian Mathematical Society, where members can submit their work for review from peers. This publication is Scopus-indexed and follows an open access publishing model.

As a large proportion of IndoMS members are also mathematics professors and educators, the Society is especially concerned with mathematics education. Over the last two years, the Society has held webinars, discussion sessions, and online workshops focusing on changes in educational policy and curriculum implementation. These activities were also held offline prior to 2020.

All publications of the Society are available at a discounted price for the personal use of all Society members. Annual memberships are available at half the annual membership fee for members of the London Mathematical Society.

Microtheses and Nanotheses provide space in the Newsletter for current and recent research students to communicate their research findings with the community. We welcome submissions for this section from current and recent research students. See [newsletter.lms.ac.uk](http://newsletter.lms.ac.uk) for preparation and submission guidance.

## Microthesis: Triangulations of Cyclic Polytopes in Representation Theory

NICHOLAS WILLIAMS

Cyclic polytopes are shapes with remarkable properties. Their triangulations have a rich combinatorial structure which appears in diverse areas of mathematics. In my thesis I prove a conjecture on triangulations of cyclic polytopes and show how this problem appears in representation theory of algebras.

### Cyclic polytopes

Cyclic polytopes are an incredibly special family of polytopes (shapes with straight sides). Out of all polytopes with the same dimension and number of vertices, cyclic polytopes have the largest possible number of faces of every given dimension. Furthermore, every sufficiently large collection of points in general position must contain the set of vertices of some cyclic polytope. What's more, any polytope which has this property must be a cyclic polytope.

The standard construction of cyclic polytopes is as follows. Consider the curve  $t \mapsto (t, t^2, \dots, t^\delta)$ , which is known as the *moment curve*. The convex hull of  $m$  points on this curve is a cyclic polytope  $C(m, \delta)$ . Here *convex hull* means the smallest convex shape containing a given set of points, where a shape is *convex* if it always contains the line segment connecting two points inside it.

### Triangulations of cyclic polytopes

A *triangulation* of a convex polytope is a subdivision of it into simplices. The set of triangulations of a cyclic polytope has a very interesting combinatorial structure. Triangulations of cyclic polytopes were used by Kapranov and Voevodsky to give examples of higher categories, and were used by Dyckerhoff and Kapranov to define higher Segal spaces. Dimakis and Müller-Hoissen showed how triangulations of cyclic polytopes described solutions to a differential equation describing solitary waves.

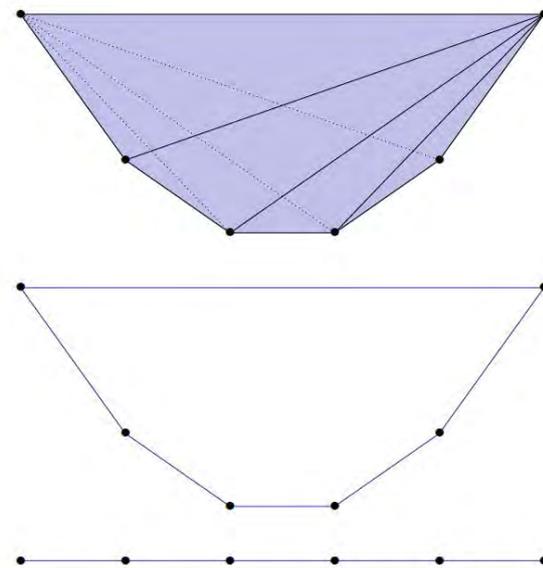


Figure 1. Cyclic polytopes in dimensions 1, 2, and 3

In the 1990s, partial orders were introduced on the set of triangulations of a cyclic polytope. Kapranov and Voevodsky first introduced a partial order known as the first higher Stasheff–Tamari order. This was followed by Edelman and Reiner, who introduced the second higher Stasheff–Tamari order in 1996. Edelman and Reiner conjectured that the two higher Stasheff–Tamari orders were the same. Proving this conjecture is the main result of my PhD thesis.

### Representation theory of algebras

An algebra is a mathematical structure in which one has addition and multiplication, but not

necessarily multiplicative inverses. One also has scalar multiplication from some field, such as the real numbers or the complex numbers.

A way of defining a large class of examples of algebras is to use path algebras of quivers. Here ‘quiver’ is a synonym for ‘directed graph’. The path algebra of a quiver is defined by taking the vector space which has the set of paths in the quiver as its basis. Multiplication is then defined by concatenating paths and extending linearly, with multiplication giving zero if the paths do not concatenate. By setting some linear combinations of paths equal to zero, one can also define path algebras of quivers *with relations*.

A *representation* of an algebra is given by a vector space along with a homomorphism from the algebra to the space of linear transformations of the vector space. This means that each element of the algebra corresponds to a certain matrix. Representations of algebras are also known as *modules*.

The algebras I consider in my thesis are the *higher Auslander algebras of type A*, which were defined by Iyama. Examples of quivers of these algebras are shown in Figure 2. The higher Auslander algebras of type  $A$  are defined as the path algebras of these quivers subject to certain relations.



Examples of the quivers of the higher Auslander algebras of type  $A$

### Triangulations and representation theory

The story of the interaction between triangulations of cyclic polytopes and representation theory starts with a remarkable paper of Oppermann and Thomas published in 2012. In this paper these authors show that triangulations of even-dimensional cyclic polytopes were in bijection with certain modules over the higher Auslander algebras of type  $A$  known as *tilting modules*.

The first question I asked myself is whether this bijection meant that there were natural interpretations of the two higher Stasheff–Tamari

orders in terms of modules. It turns out that there are. In fact, the two orders on tilting modules that one obtains were introduced in representation theory by Riedtmann and Schofield around the same time that Kapranov and Voevodsky introduced the first higher Stasheff–Tamari orders.

One of the great mysteries about the paper of Oppermann and Thomas paper was why it was only triangulations of even-dimensional cyclic polytopes that appeared in representation theory, with triangulations of odd-dimensional cyclic polytopes seemingly playing no role. This mystery was solved by seeing how the first higher Stasheff–Tamari order could be interpreted in representation theory. A beautiful fact about triangulations of cyclic polytopes is that triangulations of  $(\delta + 1)$ -dimensional cyclic polytopes can be built from maximal chains of  $\delta$ -dimensional triangulations in the first higher Stasheff–Tamari order.

The maximal chains one builds on the algebra side produce what are known as *maximal green sequences*, which, again, were already studied in representation theory. I obtain that triangulations of odd-dimensional cyclic polytopes are in bijection with natural equivalence classes of maximal green sequences, thereby solving the mystery. The higher Stasheff–Tamari orders may also be interpreted on these equivalence classes of maximal green sequences, giving interesting orders which have not previously been studied.

The algebraic upshot of my proof of the equivalence of the higher Stasheff–Tamari orders is then that these algebraic orders are equivalent for the higher Auslander algebras of type  $A$ , raising the question of whether this is true more generally in representation theory.



### Nicholas Williams

Nick is a PhD student at the University of Cologne. His research focuses on interactions between combinatorics and representation theory, and, in general, enjoys unexpected connections between different areas of mathematics. Nick grew up in London and sometimes finds it difficult to escape its gravitational pull.

# Heilbronn Annual Conference 2022

**8 – 9 September 2022**  
**University of Bristol**

The Heilbronn Institute for Mathematical Research welcomes eight distinguished speakers, to deliver lectures intended to be accessible to a general audience of mathematicians.

This year's invited speakers are:

**Viviane Baladi** (LPSM, Paris)

**Jennifer Balakrishnan** (Boston)

**Martin Bridson** (Oxford)

**Toby Cubitt** (UCL)

**Nicolas Curien** (Paris-Sud Orsay)

**Laure Saint-Raymond** (ENS-Lyon)

**Laura Schaposnik** (Illinois)

**Benny Sudakov** (ETH, Zurich)

Please contact [heilbronn-coordinator@bristol.ac.uk](mailto:heilbronn-coordinator@bristol.ac.uk) for further information or visit [heilbronn.ac.uk/events](https://heilbronn.ac.uk/events)



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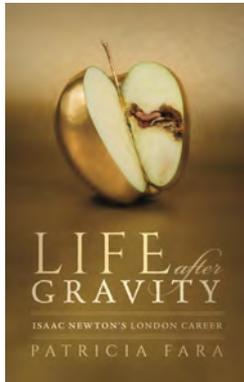


Funding is available to support a limited number of PhD students and Early Career Researchers. We also welcome applications for caring costs.

# Life after Gravity: Isaac Newton's London Career

by Patricia Fara, Oxford University Press, 2021, £25, ISBN: 9780198841029

Review by Niccolò Guicciardini



When I visit a major library, I am at first overwhelmed by the quantity of books that surround me, ordered and class-marked on the shelves. I know that many more are carefully kept in the deposits accessible only to the librarians. But then I realize that what is truly overwhelming is the absence of any mention of a vast multitude of human beings who were denied a memory written in ink and paper. In the end, what overwhelms me is a deafening silence making manifest the vastness not of what is preserved but of what has been lost. It is hard to find a woman authoring a book, especially a science book, prior to 1727, the year of Newton's death. It is even harder to find a black science writer in that period. I understand Fara's latest book as an attempt to find a place in our historical narrative of the Scientific Revolution for the forgotten ones, the men and women who, in Newton's times, made his fortune possible. The book is deliberately *engagé*: Fara posits herself in the historiographic tradition opened by Boris Hessen, the Soviet physicist of the seminal talk entitled 'The Social and Economic Roots of Newton's *Principia*' delivered in 1931 at the Second International Congress of History of Science held in London (see pp. 219–220). For some, it marks the founding moment of the 'externalist' approach to the history of science, an approach which is certainly followed in the book under review [1].

Fara's book can be read as a welcome reminder of the economic and social conditions that made Newton's fortune in London possible. The London that she vividly portrays is a town in which women were the property of their fathers and husbands and in which an affluent ruling class made a living out of human trafficking. Women feature prominently in

the book under review, from the poet Elizabeth Tollet (pp. 27–33) to the three great Leibnizian Hanoverians, Sophia, Sophia Charlotte, and Caroline of Ansbach, who, when Princess of Wales, played a pivotal role in the philosophical exchange between Newton, Samuel Clarke and Leibniz. The woman who deserves pride of place is, of course, Catherine Barton, Newton's brilliant half-niece (esp. pp. 47–50). Black African people appear in *Life after Gravity* as miserable subjects of the slave trade, and Fara proves in detail how Newton made a profit out of it. The data on tides he needed for his *Principia*, the gold employed in the casting of guineas at the Mint at which he presided as Warden and then Master, the money he foolishly invested and lost in the South Sea Bubble, would not have been possible without the mercantile and imperial community in which Newton participated (p. 182 *passim*). Fara guides her readers into Newton's unjust and ruthless cosmopolitan world. One can smell the stink of the elephants and lions and hear the noise of the nine deafening presses housed at the Tower of London, witness the details of the protocols that regulated the manners in the salons where Barton thrived, ponder the inventory of Newton's sumptuous furniture. Fara's attention to the material conditions in which Newton operated sheds new light on the character of the celebrated mathematician and natural philosopher. Fara's Newton is a man who sought power and money during his successful London career. Newton spent the latter part of his long life in London, from 1696 to 1727. In this period he was an influential high-ranking public servant. His long service at the Mint, the presidency of the Royal Society, the elections as member of the Parliament, can be cited to prove that the somewhat obscure Lucasian Professor of Mathematics at Cambridge morphed into Sir Isaac Newton, a public persona, and an influential actor in English politics.

Two episodes can be cited in order to alert the reader of this review to some inevitable lacunae in Fara's account of Newton's metropolitan life. In July 1726 Newton met Samuel Crell, a Socinian who

was in England with the purpose of publishing an anti-Trinitarian work. The Polish heretic was seeking Newton's patronage. The President of the Royal Society had already assisted him in his return to Germany some fifteen years before. We don't know the words they exchanged during this last encounter. What we know for sure is that Newton placed ten guineas in Crell's hand [2, p. 404]. Religion meant a great deal to the ageing money- and power-seeker described by Fara. At the time, he was also a member of the Parliamentary Committee for the Board of Longitude, which established that a prize would be awarded to those who could solve the problem of longitude at sea with the required precision. In his London period Newton worked hard on the mathematical theory of the Moon, a possible way to solve the longitude problem: his tentative answer was published in Latin and English in 1702.<sup>1</sup> After his move to London, Newton was still active in science and mathematics. He attended to the publication of the *Opticks* (1704, and several editions), of the second (1713) and third (1726) editions of the *Principia*, of the *Arithmetica universalis* (1707 and several editions). He carried out experiments on fluid resistance, electricity, and chemistry. He involved himself in debates, especially addressed against Leibniz, concerning, for example, the philosophical justification of gravitation theory, the nature of space and time, the foundations of calculus. Science and mathematics meant a great deal to the ageing man-about-town (p. xxi), and one might even cautiously surmise bisexual (p. 44), socialite and gourmand (pp. 15–22) described by Fara.

Yet, neither 'Crell' nor 'longitude' appear in the name and subject index of Fara's book. And the list of missing entries relevant for a biographical account of Newton's intellectual endeavours would be long. Science and religion are a real absence in this

book. And I am afraid that, without taking into due consideration these aspects of Newton's intellectual biography, we end up having a description of a typical representative of the Whig establishment, who, we should not forget, set himself with passion and determination, and sometimes with notable success, ambitious and deep-seated political, religious, and scientific agendas until his last days. Fara opens her innovative book with an epigraph (p. xiii) from John Ruskin's *The Crown of Wild Olive* concerning 'the first of all English games', a game 'absolutely without purpose': making money. Indeed, Newton made, and lost, money, and it is a shame that Newtonian scholarship has not devoted enough attention to his strategies of social climbing. Yet, those who have perused his manuscripts know that he devoted much of his time and energy to purposeful tasks.

#### FURTHER READING

[1] Boris Hessen, *The Social and Economic Roots of Newton's 'Principia'*, in: *Science at the Cross Roads*, Kniga, London, 1931, pp. 149–212.

[2] Stephen D. Snobelen, *Isaac Newton, Heretic: The Strategies of a Nicodemite*, *Brit. J. Hist. Sci.* 32 (1999), 381–419.



#### Niccolò Guicciardini

Niccolò Guicciardini teaches History of Science at the State University of Milan. He has devoted many years to studying Newton's mathematics and its

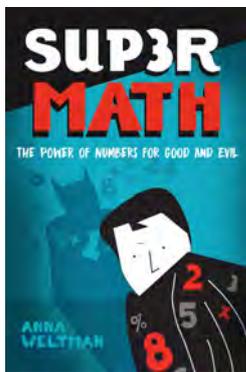
reception. His latest book is *Isaac Newton and natural philosophy* (Reaktion Books, 2018).

<sup>1</sup>The Latin 'Theoria Lunae' appeared in Gregory's *Astronomiae physicae et geometricae elementa* (Oxford, 1702). The English version was published as a separate pamphlet.

# Supermath: The Power of Numbers for Good and Evil

by Anna Weltman, Johns Hopkins University Press, 2020, US\$ 24.95,  
ISBN: 978-1421438191

Review by Tomoko L. Kitagawa



Anna Weltman's *Supermath* follows her art activity books, *This Is Not a Math Book* (2015) and *This is Not Another Math Book* (2018). *Supermath* is written for students and educators of mathematics and provides good examples of how mathematics plays out in the larger

framework of STEM education. The book is not aimed at professional mathematicians, but it is an enjoyable read for those interested in sociological development associated with mathematics, especially in the United States.

Weltman poses five intriguing questions that make maths good *and* evil. The questions are: Is maths the universal language? Can maths eliminate bias? Can maths predict the next move? Can maths open doors to greater opportunities? What is genuine beauty? If you have been looking for answers to these questions, the book will provide food for thought.

In Chapter 1, Weltman introduces her first question — is maths the universal language? The opening story is about two mathematicians' endeavour to communicate with aliens. In 2003, Stephane Dumas and Yan Dutil sent a message into space. Weltman develops the story of Dumas and Dutil's project for SETI (Search for Extraterrestrial Intelligence), which was funded by NASA, to show the various ways of expressing mathematical logics from ancient civilisations (on earth!).

Chapter 2 deals with the practical usefulness of maths by asking whether it can help make

important decisions. The chapter begins with the 1914 Christmas truce. Six months into World War I, the shooting suddenly stopped on Christmas Eve. British and German soldiers exchanged greetings, sang Christmas carols together and shook each other's hands. Weltman depicts this historical moment in the language of game theory, including the prisoner's dilemma and Nash's equilibrium. Could maths make you win (or at least, not lose)? The chapter pinpoints the good and bad attributes of mathematical modelling, which show the way people behave in the real world.

Chapter 3 asks another important question: Can maths help eliminate societal bias? As Weltman's primary audience is US readers, she starts this chapter with a courtroom in New Jersey, where a new bail-setting method for nonviolent offences was introduced in 2016. The new system was meant to eliminate possible prejudice at trial, given that the old system disproportionately affected defendants from minority groups, especially black people. Weltman investigates cases in other US states and tells us about the improvements made in the algorithm for social and political justice. This is a timely topic as we are all called upon to tackle the discriminations highlighted, for example, by the Black Lives Matter movement.

In Chapter 4, Weltman discusses the under-representation of women and individuals from minority groups in the field of mathematics. She is eager to find a way of giving more opportunities to those who wish to study mathematics and become professional mathematicians. However, she brings up the stories of Terry Tao and Don Zagier (not Zager, as the surname is spelt in the book). Here, professional mathematicians, especially the readers of the *LMS Newsletter*, may feel that both the anecdotes and

the current mathematical developments introduced in this chapter need more rigorous fact-checking.

The book ends with Chapter 5, which asks a big question: What is genuine beauty? To answer it, Weltman quotes ideas from G. H. Hardy's *A Mathematician's Apology*. Hardy's book is certainly a great point of reference here. Weltman goes further with the final episode about Marjorie Rice, a housewife who discovered new types of pentagons — a “most unlikely mathematician”. Rice's story is a valuable addition to our endeavour of finding the beauty of mathematics; it also links back to the previous chapter, which discusses the need for more female scientists to shine in the field.

The themes Weltman touches upon are inspiring, and you may find yourself looking for more books to read. If you become interested in the historical roots of modern mathematics, e.g., algebra, geometry, calculus, and statistics, Michael Brooks' *The Art of More: How Mathematics Created Civilisation* (2021) would be a good read. If you are looking for more books about mathematics for college students, another book with illustrations, *Man vs. Maths* (2016) by Timothy Revell, would be an excellent choice. For female scientists, *I Died for Beauty: Dorothy Wrinch and the Culture of Science* (2012) by Marjorie Senechal would be an exciting read. For gender and equality issues, *Programmed Inequality: How Britain Discarded Women Technologists and Lost its Edge in Computing*

(2017) by Mar Hicks gives a more concrete historical understanding for British readers.

By reading *Supermath*, I learnt about the sociological conditions of the US and felt an urgent need to understand how effectively (or ineffectively) mathematics and its applications are used in other parts of the world. As Weltman points out, we must face up to some of mathematics' negative aspects — the “wrongs” the field has done. Numbers have enormous power over humans; we always need to check their use and correct biases whenever we find them. Weltman rightly focuses on gender and racial inequality for a general audience to think about and act upon these problems. The four other questions she poses in her book should also be investigated in different cultural contexts. From this perspective, the book provides good themes for classroom discussion and is useful anywhere in the world.



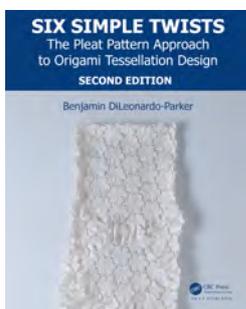
### Tomoko L. Kitagawa

Tomoko L. Kitagawa is a historian, specialising in mathematical cultures of Europe, East Asia, and South Africa. Her research often focuses on minority groups. She is a Happiness Coordinator of mathematicians' social groups Prime Powers (Oxford) and Riemann Rockers (Bonn).

# Six Simple Twists: The Pleat Pattern Approach to Origami Tessellation Design (Second Edition)

by Benjamin DiLeonardo-Parker, CRC Press, 2021, £27.99,  
ISBN: 978-1-138-31188-6

Review by Dorothy Leddy



As soon as we open the pages of this book Benjamin DiLeonardo-Parker's enthusiasm for the design and creation of origami tessellations is clear. The author reveals the patterns that can be formed by paper pleats

and their combinations and demonstrates how by folding paper in different configurations the beautiful structures known as origami tessellations can be constructed. The author explains to his readers how to fold tessellations, and encourages them to apply their learning to explore the subject to generate their own folding techniques and create their own designs, offering case studies to experiment with. The book is generously supplied with photographs and diagrams illustrating each step of the techniques, and a comprehensive glossary is provided. The author's creativity and attention to detail are a delight.

This second edition is a greatly expanded version of its predecessor [1], benefiting from more instruction in basic skills and pleat manipulation methods, enhanced illustration of the processes, a greater range of demonstrated patterns, an improved notation system, and further mathematical analysis of the behaviour of twists.

The author begins with first principles, teaching the skills required to create tessellations and explaining their components. The use of diagrams and the "Benet notation" for documenting folds is explained, along with crease patterns — drawings that show every crease in a folded form after folding and then unfolding a work of origami. We are shown how to fold uniform parallel creases, and to create a grid of

desired density by repeated folding. Triangle grids are formed from three creases along the diagonals of a hexagon, each grid consisting of an intersecting set of parallel creases. The folds that are created are subsequently unfolded, leaving the final grid consisting of a structured set of creases to be used later for pleat patterns.

Instructions are given for using a grid for folding a pleat, which is at the core of the pleat patterns that are the subject of this book. Pleats may have a single pair of mountain (up) and valley (down) folds, or may have more pairs of folds. They may be parallel and therefore non-convergent, or non-parallel and so convergent unless intersected. Pleats may be manipulated by moving the pleat along the grid or by splitting a pleat to multiply it.

The Six Simple Twists of the title are described and their construction explained. Twists are formed where a layer of the paper rotates as multiple pleats intersect, in order to lie flat. The author documents pleat intersections by drawing a modified crease pattern, called the "pleat schematic".

In the second chapter we are shown techniques for arranging the twists in evenly-spaced multiples on the paper, combined as building blocks to form the full pleat patterns that are the origami tessellations. The construction of 35 tessellations of increasing complexity and beauty is carefully set out and their underlying geometry explained. Every step is described in detail, each with pleat schematic diagrams and photographs. The significance of symmetry and regular polygons, only some of which can tile a flat plane indefinitely with no holes or overlaps, is discussed; origami tessellations are not true tessellations as they have holes and overlaps.

Chapter 3 considers pleat patterns as artwork, with a photo gallery of beautifully finished tessellations

of many kinds by various origami artists. Instruction on alternative twists follows, with an in-depth mathematical analysis of twist creation and of what makes a twist actually twist and fold flat. The mathematical implications of the Benet notation introduced in Chapter 1 are explored more fully here. Proposed as a systematic way to document pleat intersections and hence describe precisely the differences between twists, it is based on a gridded hexagon with its six axes drawn out on a coordinate grid. The location of each of the pleat folds is given as the number of grid spaces away from its parallel axis. To describe a pleat intersection an ordered hextuple shows the location of pleats for each of the six axes. The pleat intersection, where the pleats converge and the twist is formed, is termed a molecule. Pleat to Molecule analysis and Molecule to Pleat analysis together with model development investigate thoroughly the relationships between molecules and the pleat intersections that form them.

Seven molecule formats, called archetype sets, are defined based on the axes that contribute to the creation of molecules. A comprehensive molecule database based on archetype provides, for each molecule, the pleat schematic and photographs showing stages of construction. The Six Simple Twists are a mere sample of these numerous molecules.

Composition of pairs of archetypes creates archetype changes, the resultant archetypes remaining a member of the group of seven archetypes. Pleat schematic diagrams and photographs show the results of the compositions, and all possible pleat intersection compositions are tabulated. The effects of other actions on pleats, namely reorientation, drifting, inverting and splitting, are analysed in detail using the Benet notation, and all possible splitting combinations are tabulated.

A perfect twist is a molecule where the only creases are the mountains and valleys of the contributing pleats. Methods of perfect twist design are described, with especial attention to sectioning and notching, and comprehensive analysis of Brocard points.

In Chapter 4 the author highlights the roles of tessellations in textiles, interior design, artistic expression, mathematical analysis, modelling of galactic rotations and more. Matthew Benet shares his thoughts on the pleat notation he and the author developed, giving a technical breakdown of the reasoning behind it. This numerical system enables clear description of pleat patterns, and the mathematically sound operations precisely show changes to a pleat, enabling analysis of pleat behaviour and interactions. Benet considers that the operations described on pleats satisfy the axioms of a group.

The thorough, detailed, methodical treatment of this subject will surely inspire readers of this book to try out the techniques for themselves, and to embark upon creating many of the tessellations described here, and, with experience gained, even attempting their own tessellation designs. I certainly feel encouraged to reach for some paper and get folding.

#### FURTHER READING

[1] Benjamin DiLeonardo-Parker, *Six Simple Twists: The Pleat Pattern Approach to Origami Tessellation Design*, First Edition, CRC Press, 2015.



#### Dorothy Leddy

Dorothy's career has been in agricultural research and development, most recently as Senior Project Manager for R&D information systems in a global agriculture company. Her earlier research into symbiotic microflora of hill and upland pastoral systems enabled her to indulge in her favourite pastime of trekking in those regions. She also enjoys creating music with guitar, handbells and hammered dulcimer.

## Obituaries of Members

### Roger W. Carter: 1934 – 2022



Professor Roger Carter, who was elected a member of the London Mathematical Society on 18 June 1959, died on 21 February 2022, aged 87.

*George Lusztig writes:* Roger was born on 25 August 1934 in Shipley, West Yorkshire. His father was an income tax

collector for the Inland Revenue who, because of his job, had to move every few years. He had a younger sister, Audrey. His junior school was in Chesterfield and later he attended the Barnsley Boys Grammar School where he excelled in mathematics and won an Open Scholarship to Cambridge University. He went to Cambridge University in 1952. At Sidney Sussex College he achieved first class honours. He received his PhD in 1960 at Cambridge under the supervision of Derek Taunt with a thesis entitled *Some Contributions to the Theory of Finite Soluble Groups*. In 1959/60 he spent a year at Tübingen University on a Humboldt Stiftung research scholarship; during that year he became fluent in German. In 1961 he published a paper in which he introduced certain very important subgroups of a finite group (later called the ‘Carter subgroups’). These are the nilpotent self-normalizing subgroups. Roger proved that a finite solvable group has Carter subgroups and any two of these are conjugate. (For example, in the group of upper triangular matrices over a finite field, the diagonal matrices form a Carter subgroup.) After the year in Tübingen, Roger joined King’s College in Newcastle upon Tyne and in 1965 he moved to the University of Warwick where he became a Professor in 1970. He was Head of Department there in 1976–79 and retired in 2001.

Roger is famous for his research in the theory of reductive algebraic groups. In a 1972 paper he obtained some key results on the classification of conjugacy classes in Weyl groups in terms of what he called ‘admissible graphs’; these results are still the best reference in this area. In his 1974 paper (with his PhD student Bala) a new classification of unipotent classes in complex reductive groups was given in

terms of ‘distinguished’ parabolic subgroups. This is a beautiful result which has subsequently been widely used. Roger wrote several very influential expository books, for example *Simple Groups of Lie Type* (1972) and *Finite Groups of Lie Type: Conjugacy Classes and Complex Characters* (1985).

Roger was a very inspiring lecturer and PhD supervisor; one of his PhD students (Jian-Yi Shi) wrote a PhD thesis which is a big breakthrough related to the cells in affine Weyl groups of type A.

I was very fortunate to learn the theory of algebraic groups from Roger during the years that I was at Warwick (1971–77); we also wrote two joint papers, one on the representations of  $GL_n$  and the symmetric group in positive characteristic and one on the modular representations of finite Chevalley groups.

Roger very much liked hiking; I remember hiking with him for a week in the Zillertal Alps in Austria (around 1973) where we discussed much mathematics. As I learned from his sister, Roger rowed for his Cambridge college in the famous Bumps races; he also sang in the Cambridge Musical Society and enjoyed playing piano.

Many thanks to Audrey Edwards, Helen Smith and Anthony Manning for supplying information which allowed me to write this.

### Death Notices

We regret to announce the following deaths:

- Homer F. Bechtell, formerly of the University of New Hampshire, USA, who was elected a member of the London Mathematical Society on 21 November 1963, died on 22 May 2022, aged 94.
- John H. Coates FRS, formerly of the University of Cambridge, who was elected a member of the London Mathematical Society on 20 June 1986 and was LMS President 1988–90, died on 9 May 2022, aged 77.
- Dean Ives, formerly of University College London, who was elected a member of the London Mathematical Society on 19 November 1993, died on 22 April 2022, aged 62.
- Gerrit van Dijk, formerly of the University of Leiden, who was elected a reciprocity member of the London Mathematical Society on 19 June 1992, died on 16 April 2022, aged 82.

 LMS Meeting

## LMS Meeting at the vInternational Congress of Mathematicians 2022

7 July 2022, 16:45 (CEST); University of Copenhagen and online

Website: [lms.ac.uk/events/ICM-SocMtg](https://lms.ac.uk/events/ICM-SocMtg)

Speaker: Karen Vogtmann, *Spaces of Graphs*

The lecture will be aimed at a general mathematical audience. All interested, whether LMS members or not, are most welcome to attend this event.

Members can sign the Members' Book, which dates from 1865 when the Society was founded and

contains signatures of members throughout the years, including Augustus De Morgan, Henri Poincaré, G.H. Hardy, and Mary Cartwright. The meeting will be followed by a reception, which will be held at the University of Copenhagen

For further details about the meeting and to register for a place, see the website.

 LMS Meeting

## LMS Invited Lecture Series 2021 (postponed to 2022)

18–22 July, Newcastle University

Website: [tinyurl.com/invited22](https://tinyurl.com/invited22)

The Invited Lecturer is Olga Kharlampovich (CUNY Graduate Center and Hunter College), who will speak on *Equations in Groups and Complexity*.

Accompanying lectures will be held by: Martin Bridson (Oxford), Montse Casals-Ruiz (Basque Country), Laura Ciobanu (Heriot-Watt), Ilya Kazachkov (Basque Country), Alexei Miasnikov (Stevens Institute), Rizos Sklinos (Chinese Academy of Sciences), Riikka Kangaslampi (Tampere) and Vladimir Shpilrain (CUNY).

The annual Invited Lecturers Series aim to bring a distinguished overseas mathematician to the United Kingdom to present a small course of about ten lectures spread over a week. Each course of Invited Lectures is on a major field of current mathematical research, and is instructional in nature, being directed both at graduate students beginning research and at established mathematicians who wish to learn about a field outside their own research specialism.

For further details about the LMS Invited Lecture Series and to register for a place, see [sites.google.com/view/nclcomplexity](https://sites.google.com/view/nclcomplexity).



## vInternational Congress Of Mathematicians: Public Lectures 2022

8, 11 and 13 July, online via Zoom

The LMS is pleased to support the vInternational Congress of Mathematics by hosting three vICM Public Lectures as satellite events online via Zoom during the vICM in July 2022.

The lectures will be aimed at a general mathematical audience. All interested, whether LMS members or not, are most welcome to attend these events.

### Friday 8 July: Public Lecture by Geordie Williamson (Sydney)

Programme (timings in CEST):

12:00	Opening of the event
12:05	vICM Satellite Event: Public Lecture by Geordie Williamson (Sydney): <i>Machine learning as a tool for the mathematician</i>
12.55	Q&A
13.00	End of event

Further details, including on how to register, can be found at [bit.ly/3xg8V6f](https://bit.ly/3xg8V6f).

### Monday 11 July: Public Lecture by Elena Giorgi (Colombia)

Programme (timings in CEST):

18:00	Opening of the event
18:05	vICM Satellite Event: Public Lecture by Elena Giorgi (Colombia): <i>Black Holes: A Mathematical Enlightenment</i>
18.55	Q&A
19.00	End of event

Further details, including on how to register, can be found at [bit.ly/3NyFwLv](https://bit.ly/3NyFwLv)

### Wednesday 13 July: Public Lecture by Tadashi Tokieda (Stanford)

Programme (timings in CEST):

19:30	Opening of the event
19:35	vICM Satellite Event: Public Lecture by Tadashi Tokieda (Stanford): <i>A world from a sheet of paper.</i>
20.25	Q&A
20.30	End of event

Further details, including on how to register, can be found at [bit.ly/3NZiOfh](https://bit.ly/3NZiOfh).

Further details about the vICM itself can be found on the IMU website at [bit.ly/3xdhAX6](https://bit.ly/3xdhAX6).

## Quantum Chaos and Nodal Sets of Random Waves

Location: King's College London  
 Date: 25–29 July 2022  
 Website: [tinyurl.com/2p8rtwur](https://tinyurl.com/2p8rtwur)

In the recent years there has been a remarkable progress in understanding the high energy behaviour of quantum dynamical systems, from a number of perspectives: classical, spectral, and microlocal analysis; probability theory including percolation theory, and number theory. The conference will unite all of these disciplines, and will include research lectures from specialists in these rather different subjects, ranging from early stage researchers to the leading experts of their fields. Supported by an LMS Conference grant.

## 15th Algorithmic Number Theory Symposium

Location: University of Bristol  
 Date: 8–12 August 2022  
 Website: [antsmath.org](https://antsmath.org)

The ANTS meetings, held biennially since 1994, are the premier international forum for the presentation of new research in computational number theory and its applications. There are five planned plenary talks (Kevin Buzzard, Fredrik Johansson, Tanya Lange, Alina Ostafe, Peter Sarnak) and there will be about 25 contributed talks based on papers submitted to the ANTS Proceedings. Funding is available, especially for junior participants, thanks to sponsorship from HIMR, UKRI, NSF, LMS, and PQShield.

## One Day Function Theory Meeting

Location: De Morgan House/online  
 Date: 5 September 2022  
 Website: [tinyurl.com/4m3pdxj4](https://tinyurl.com/4m3pdxj4)

The annual ODFTM brings together established and early career researchers from the UK and beyond, working in the areas of Complex Analysis and Function Theory. It covers a broad range of topics including potential theory, holomorphic dynamics and complex differential equations. Talks this year include ones originally planned for the postponed 2020 meeting, plus an additional talk related to the work of Professor Walter Hayman FRS, a founder of the ODFTM, who died in early 2020. Supported by an LMS Conference grant.

## Computational Mathematics for Quantum Technologies

Location: University of Bath  
 Date: 1–5 August 2022  
 Website: [tinyurl.com/yee245hy](https://tinyurl.com/yee245hy)

This is an interdisciplinary event, with invited speakers from Numerical Analysis & Scientific Computing, Computational Quantum Chemistry and Physics, and Quantum Information Theory. The objective is to bring together computational mathematicians with diverse and complimentary expertise in Chemistry, Physics and Quantum Information Theory, and accelerating the development of computational techniques for quantum technologies. Further details are available on the webpage. Supported by an LMS Conference grant.

## Young Researchers in Algebraic Number Theory (Y-RANT) IV

Location: University of Glasgow  
 Date: 23–25 August 2022  
 Website: [y-rant.github.io/](https://y-rant.github.io/)

Y-RANT is an annual conference primarily aimed at PhD students and postdocs in algebraic and analytic number theory and related topics. All participants are strongly encouraged to submit a 20 minute talk, either on their own research or on a topic of interest. Email [y-rantconference@gmail.com](mailto:y-rantconference@gmail.com) to ask about registration. Y-RANT is funded by an LMS Scheme 8 grant and a HIMR grant through the UKRI/EPSCRC Additional Funding Programme for Mathematical Sciences.

## Scaling Limits: From Statistical Mechanics to Manifolds

Location: Cambridge  
 Date: 5–7 September 2022  
 Website: [statslab.cam.ac.uk/james60](https://statslab.cam.ac.uk/james60)

This 3-day workshop, originally planned for 2020, will (somewhat belatedly!) bring together mathematicians from around the world to celebrate James Norris' 60th birthday. In recent years, tremendous progress has been made in understanding the limiting behaviour of stochastic processes which serve as natural models for many physical phenomena. The techniques used are both probabilistic and analytic in nature, and the aim of this workshop is to emphasise this beautiful interplay between analysis and probability. This meeting is supported by an LMS Conference Grant.

### The Charm of Integrability: Honouring Alexander Its

Location: University of Bristol  
 Date: 12–16 September 2022  
 Website: [tinyurl.com/rh26up68](https://tinyurl.com/rh26up68)

The goal of this conference is to bring together experts and young researchers, mathematicians and physicists, scientists with different backgrounds and different takes on integrable systems theory, to discuss the latest achievements and to point at future research directions. Limited financial support is available and participants need to register on the website by 29 August 2022. Supported by an LMS Conference grant.

### Mathematical Challenges of Big Data: IMA Conference

Location: University of Oxford, hybrid  
 Date: 19–20 September 2022  
 Website: [tinyurl.com/2a45u2rn](https://tinyurl.com/2a45u2rn)

Mathematical foundations of data science and its ongoing challenges are rapidly growing fields, encompassing areas such as: network science, machine learning, modelling, information theory, deep and reinforcement learning, applied probability and random matrix theory. Applying deeper mathematics to data is changing the way we understand the environment, health, technology, quantitative humanities, the natural sciences, and beyond, with increasing roles in society and industry.

### Induction Course for New Lecturers in the Mathematical Sciences 2022

Location: Isaac Newton Institute, Cambridge  
 Date: 14–15 September 2022  
 Website: [tinyurl.com/3bbjpw9](https://tinyurl.com/3bbjpw9)

This IMA Induction Course has been designed by the mathematics community so that it is ideally suited for anyone who is new to or has limited experience teaching mathematics or statistics within UK higher education. It will be delivered by individuals with significant experience of teaching in the mathematical sciences and will focus upon the specific details and issues that arise in mathematics and statistics teaching and learning within higher education. The course is endorsed by the LMS.

### Clay Research Conference and Workshops

Location: Mathematical Institute, Oxford  
 Date: 26–30 September 2022  
 Website: [claymath.org/](https://claymath.org/)

Workshops on *p-adic Hodge Theory and Applications*, *O-minimality and Diophantine Geometry*, *Stability and Instability in General Relativity* and *Physics from the Point of View of Geometry* will be held throughout the week, with the conference held on 28 September. Conference speakers: Bhargav Bhatt, Kevin Costello, Igor Rodnianski, Jacob Tsimerman. To register for the conference and to register interest in a workshop e-mail Naomi Kraker at [admin@claymath.org](mailto:admin@claymath.org).

# Society Meetings and Events

## July 2022

- 1 Society Meeting and Aitken Lecture, BMA House, London
- 7 LMS Meeting at the International Congress of Mathematicians 2022, University of Copenhagen/Hybrid

- 18-22 LMS-IMI-Bath Symposium: K-Theory and Representation Theory, University of Bath
- 18-22 LMS Invited Lectures, Equations in Groups and Complexity, Newcastle University

## Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website ([www.lms.ac.uk/content/calendar](http://www.lms.ac.uk/content/calendar)). Please send updates and corrections to [calendar@lms.ac.uk](mailto:calendar@lms.ac.uk).

## July 2022

- 6-14 ICM 2022, online (500)
- 11-15 British Combinatorial Conference, Lancaster University (499)
- 13-15 Maths in Music Conference, Royal College of Music, London (498)
- 18-22 New Challenges in Operator Semigroups, St John's College, Oxford (498)
- 18-22 Rigidity, Flexibility and Applications LMS Research School, Lancaster (497)
- 20-22 Microlocal Analysis and PDEs, University College London (498)
- 24-26 7th IMA Conference on Numerical Linear Algebra and Optimization, Birmingham (487)
- 25-29 New Trends in Moduli Spaces and Vector Bundles, University of Warwick (499)
- 25-29 Quantum Chaos and Nodal Sets of Random Waves, King's College London (501)
- 27-2 Aug Operator Algebras: Subfactors, K-theory, Conformal Field Theory, Gregynog Hall, Wales (500)
- 30-7 Aug Groups St Andrews 2022 in Newcastle (500)

## August 2022

- 1-5 Computational Mathematics for Quantum Technologies, University of Bath (501)

- 8-12 Fifteenth Algorithmic Number Theory Symposium, University of Bristol (501)
- 22-26 Unlikely Intersections in Diophantine Geometry, University of Oxford (500)
- 23-25 Young Researchers in Algebraic Number Theory (Y-RANT) IV, University of Glasgow (501)

## September 2022

- 1-2 Applied Mathematical Challenges and Recent Advances in the Micro-Mechanics of Matter 2022, University of Bristol (500)
- 5 One Day Function Theory Meeting, De Morgan House/online (501)
- 5-7 Scaling Limits, Cambridge (501)
- 5-9 COMB in CAMB: Combinatorial Methods in Algebraic Geometry in Cambridge (500)
- 12-16 The Charm of Integrability, University of Bristol (501)
- 14-15 Induction Course for New Lecturers 2022, Isaac Newton Institute, Cambridge (501)
- 19-20 Mathematical Challenges of Big Data IMA Conference, University of Oxford/Hybrid (501)
- 26-30 Clay Research Conference and Workshops, Mathematical Institute, Oxford (501)

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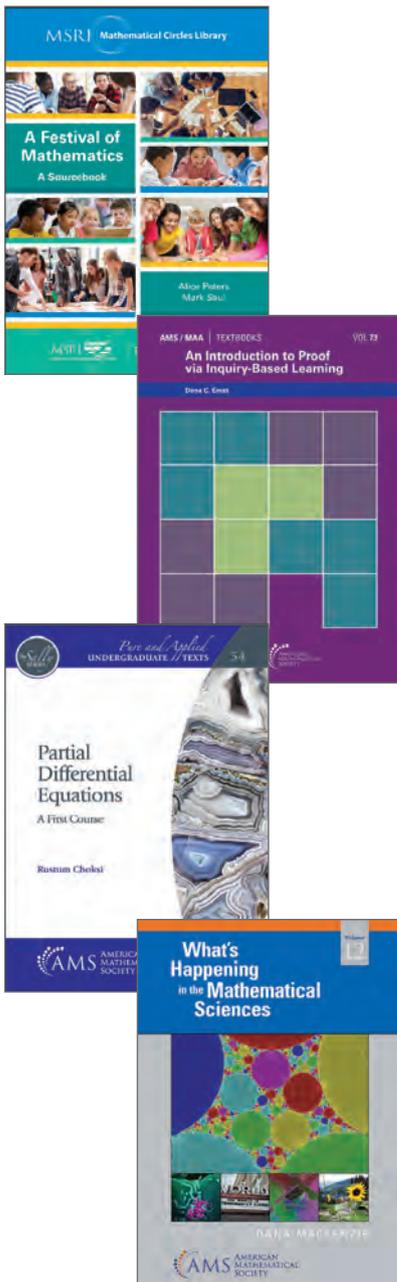
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Dana Mackenzie

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