LONDON MATHEMATICAL SOCIETY
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NEWSLETTER

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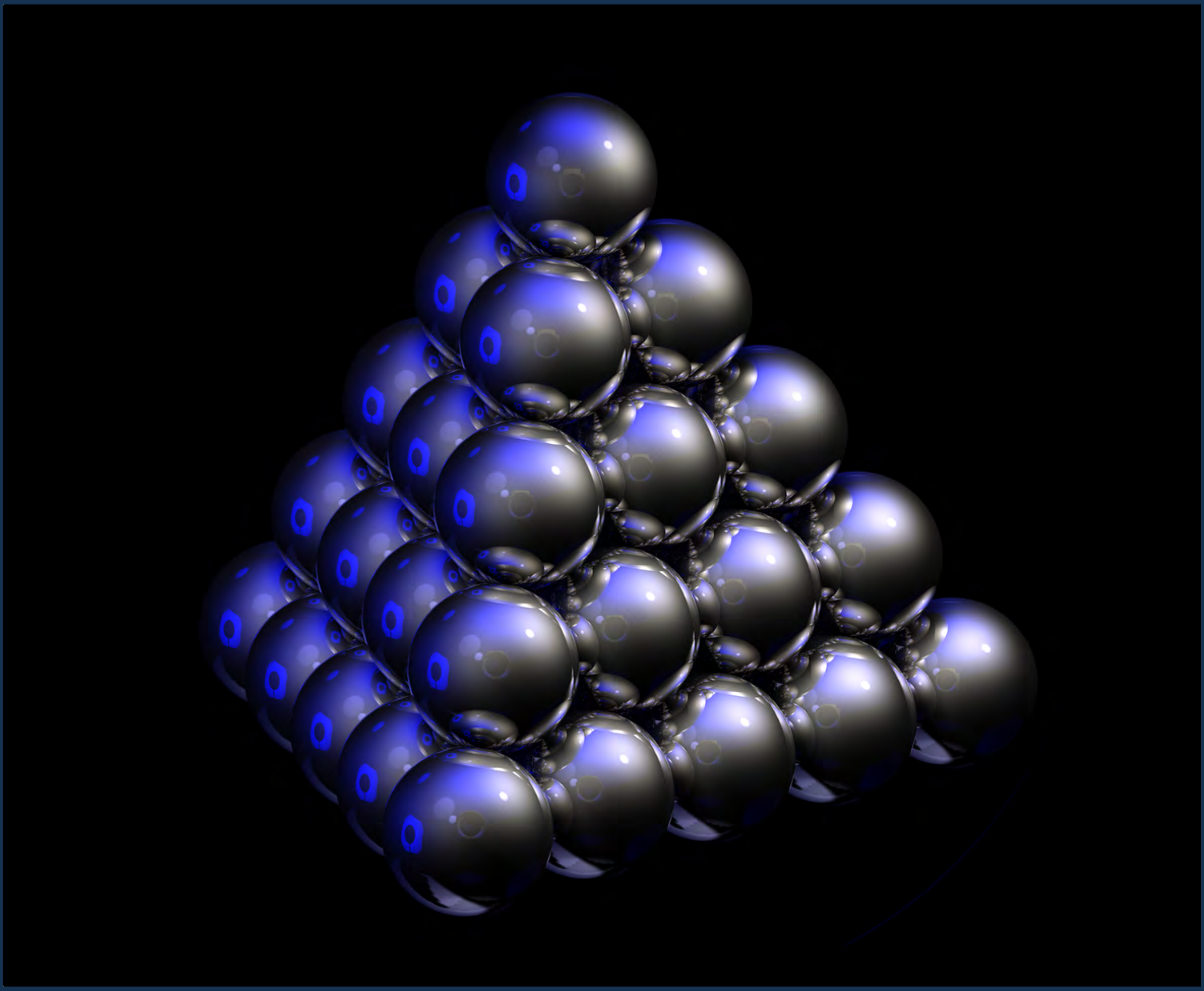

HELSINKI 2022: AN ICM LIKE NO OTHER

NOTES FROM A NUMERICAL ANALYST

MATHEMATICS NEWS FLASH

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COVER IMAGE

The face-centred cubic packing described in the interview with Fields Medal winner Maryna Viazovska, page 39. Image: Greg A L, CC BY-SA 3.0.

Do you have an image of mathematical interest that may be included on the front cover of a future issue? Email images@lms.ac.uk for details.

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LMS NEWS

De Morgan Portrait

Ulrike Tillmann, Janet Foster and Jane Vaucher (left), and the De Morgan Portrait (right)

In February this year the LMS received an email from Jane Vaucher (Thompson) in Texas, on behalf of herself and her four siblings all of whom live in the UK. She wondered whether the Society would be interested in having a photograph portrait of Augustus de Morgan whom they knew to be involved with the establishment of the LMS in 1865. The Thompson siblings are descended from Augustus De Morgan through their paternal grandfather, Reginald Campbell Thompson, the first son of Augustus De Morgan's daughter Anne Isabella De Morgan who married Reginald Edward Thompson. They are the children of R.C. Thompson's son John De Morgan Campbell Thompson.

An image of the portrait was attached to the email and from that it was possible to establish that it showed Augustus in profile giving a different aspect from the two portraits already displayed in De Morgan House. The gift of this new portrait was gratefully accepted and, since Jane was travelling to London in the summer, she would be able to deliver the portrait in person. Her visit took place on 7th June and she was welcomed to De Morgan House by the current LMS President, Ulrike Tillmann, myself and other LMS members of staff. Jane was shown the Members' Book signed by all members including Augustus De Morgan on the first page and given a tour of the building before the official handover, pictured here, took place. After refreshments and a chat about Augustus, Jane signed the Visitors' Book and was given a copy of the Presidents Book which has a biography of Augustus De Morgan as the first entry. Jane Vaucher later informed us that her siblings were delighted that she was able to share this before her return to the States. In a subsequent communication Jane wrote "Lovely to think Augustus will be cleaned up and found such a good resting place."" The plan is to have the portrait remounted and framed by a conservator before being hung in the Members' Room with a label acknowledging its donation by the Thompson family.

Janet Foster
LMS Archivist

LMS Honorary Members 2022

2022 Honorary Members Vladimir Drinfeld and Jennifer Chayes

The LMS has elected Professor Jennifer Tour Chayes, of the University of California, Berkeley, and Professor Vladimir Drinfeld, Harry Pratt Judson Distinguished Service Professor at the University of Chicago, Illinois, to Honorary Membership of the Society in 2022.

Jennifer Tour Chayes has made fundamental contributions to many of the most prominent topics in the mathematics, computation, and application of network science, data science, and allied areas. She co-invented the field of graphons, and she has also made foundational contributions to phase transitions in networks, social influence and other spreading processes, spin-glass models, ethical decision-making, climate change and machine learning, and many others. Professor Chayes's research has developed and utilized deep connections between mathematics and statistical physics. It also emphasises the importance of these ideas for practical computation and machine learning.

Starting with his proof of the geometric Langlands correspondence for GL2, Vladimir Drinfeld has in the course of a half-century solved difficult problems and introduced essential new structures that have unified and deepened our understanding of algebra, geometry, and mathematical physics. In recognition of his many contributions he was awarded the Fields Medal in 1990 and the Wolf Prize in 2018.

Full citations will appear in the LMS *Bulletin* at lms.ac.uk/publications/blms.

2022 LMS Prize Winners

The Society extends its congratulations to the following 2022 LMS Prize Winners and thanks to all the nominators, referees and members of the Prizes Committee for their contributions to the Committee's work this year.

Professor Sir John Ball FRS of Heriot-Watt University and the University of Oxford is awarded the De Morgan Medal for his multi-faceted and deep contributions to mathematical research and the mathematical community over many years.

Professors John Greenlees (Warwick University) and **Brooke Shipley** (University of Illinois Chicago) are awarded the Senior Berwick Prize for their paper 'An algebraic model for rational torus-equivariant spectra', published in the *Journal of Topology* in 2018.

Professor Andrew Lobb of Durham University is awarded the Shephard Prize in recognition of his remarkable paper 'The Rectangular Peg Problem', published in the *Annals of Mathematics*.

Professor Richard Thomas of Imperial College London is awarded the Fröhlich Prize for his extraordinary mastery and vision in creating and developing what has become known as the Donaldson–Thomas theory.

Dr Asma Hassannezhad of the University of Bristol is awarded an Anne Bennett Prize for her outstanding work in spectral geometry and her substantial contributions toward the advancement of women in mathematics.

Dr. Jessica Fintzen of the University of Cambridge, Duke University, and Universität Bonn is awarded a Whitehead Prize for her groundbreaking work in representation theory, in particular as it relates to number theory via the (local) Langlands program.

Professor Ian Griffiths of the University of Oxford is awarded a Whitehead Prize for his many contributions and insights to a wide range of challenging questions in applied and industrial mathematics, which he has achieved using a combination of asymptotic analysis and numerical simulations, supplemented by outstanding physical understanding.

Dr Dawid Kielak of the University of Oxford is awarded a Whitehead Prize for his striking, original and fundamental contributions to the fields of geometric group theory and low-dimensional topology, and in particular for his work on automorphism groups of discrete groups and fibrings of manifolds and groups.

Dr Chunyi Li of the University of Warwick is awarded a Whitehead Prize for his deep contributions to a wide range of questions in algebraic geometry, in particular in the theory of stability conditions and moduli spaces.

Professor Tadahiro Oh of the University of Edinburgh is awarded a Whitehead prize for his contributions to the theory of dispersive PDEs, in particular to the understanding of their interaction with random data.

Professor Euan Spence of the University of Bath is awarded a Whitehead Prize for his profound contributions to the theoretical understanding and design of numerical algorithms for wave propagation and scattering at high frequency, particularly through the development and application of methods from the world of semiclassical analysis.

Read the full citations for this year's prize winners at lms.ac.uk/lms-prize-winners-2022.

Forthcoming LMS Events

The following events will take place in forthcoming months:

LMS/IMA/BSHM: Women in Astronomy Meeting: 16 September, De Morgan House and online (bit.ly/3C7Ezqk)

Black Heroes of Mathematics: 4-5 October, online (bit.ly/3SPGHck)

LMS Popular Lecture 2022: 27 October, Birmingham (bit.ly/3QubdqE)

LMS Annual General Meeting & Naylor Lecture 2022: 18 November, Goodenough College, London and online (bit.ly/3C7H2B6)

LMS South West and South Wales Regional Meeting: 17 January 2023, Southampton (bit.ly/3QrQwvy).

A full listing of upcoming LMS events can be found on page 63.

2022 LMS Prize Winners


John Ball
De Morgan Medal


John Greenlees
Senior Berwick Prize


Brooke Shipley
Senior Berwick Prize


Andrew Lobb
Shephard Prize


Richard Thomas
Fröhlich Prize


Asma Hassannezhad
Anne Bennett Prize


Jessica Fintzen
Whitehead Prize


Ian Griffiths
Whitehead Prize


David Kielak
Whitehead Prize


Chunyi Li
Whitehead Prize


Tadahiro Oh
Whitehead Prize


Euan Spence
Whitehead Prize

2022 Christopher Zeeman Medal Winner

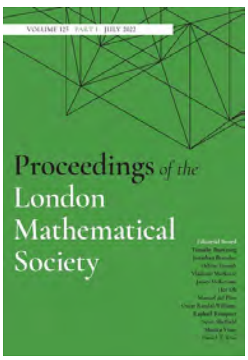
The Councils of the LMS and IMA are delighted to announce that the 2022 Christopher Zeeman Medal has been awarded to Simon Singh MBE (left). The Christopher Zeeman Medal is the UK award dedicated to recognising excellence in the communication of mathematics.

Simon Singh acts as an advocate of mathematics at the highest level, both in the media, through his many appearances, and also with policy makers, serving for example on the Advisory Council for the Campaign for Science and Engineering. Few people have inspired so many of the public to have a strong interest in mathematics.

The Society extends its congratulations to Simon on receipt of this award. An award ceremony and lecture will be held in spring 2023; details will be posted to the LMS website in due course.

Read the full citation for Simon Singh at tinyurl.com/zeeman-citation.

Proceedings of the LMS: Editors' Choice



Founded in 1865, the *Proceedings* is the flagship journal of the LMS. Over its long history it has published many significant articles including papers by Atiyah, Donaldson, Hardy, Ramanujan, Rayleigh and Turing.

Until 2019 the *Proceedings* shared an Editorial Board with the *Journal* and *Bulletin* of the LMS. In 2019 a new, independent Editorial Board of international experts was appointed to identify papers for the *Proceedings* of the highest quality and significance across a broad spectrum of mathematics. The aim of this change was to further elevate the position of the journal and strengthen its position with relation to other top-flight mathematics journals.

Over the last three years the *Proceedings* Editorial Board have accepted many papers of high quality and significance. The Managing Editors Tim Browning and Oscar Randal-Williams have chosen six such articles to highlight to readers, all of which are free to read online.

- **Symplectomorphisms with positive metric entropy**
  Artur Avila, Sylvain Crovisier and Amie Wilkinson
  doi.org/10.1112/plms.12437
  A general and deep result providing a complete answer to an old question of Mañé.
- **The density of polynomials of degree n over $\mathbb{Z}p$ having exactly r roots in $\mathbb{Q}p$**
  Manjul Bhargava, John Cremona, Tom Fisher and Stevan Gajovic
  doi/10.1112/plms.12438
  Some surprising and down to earth statements about roots of polynomials over $\mathbb{Z}p$.
- **Shift-invariance for vertex models and polymers**
  Alexei Borodin, Vadim Gorin and Michael Wheeler
  doi.org/10.1112/plms.12427
  This paper establishes some highly non-trivial invariance properties for a wide class of integrable models.
- **Complete Moduli of Cubic Threefolds and their intermediate Jacobians**
  Sebastian Casalaina-Martin, Samuel Grushevsky, Klaus Hulek and Radu Laza
  doi.org/10.1112/plms.12375
  This paper solves an important algebraic geometry question about the Torelli map from the space of cubic threefolds.
- **Motivic Chern classes and K-theoretic stable envelopes**
  László M. Fehér, Richárd Rimányi, Andrzej Weber
  doi.org/10.1112/plms.12374
  An influential paper relating invariants of singular spaces to quantum groups.
- **Independent sets in hypergraphs with a forbidden link**
  Jacob Fox and Xiaoyu He
  doi.org/10.1112/plms.12400
  This work provides a solution to a 50-year-old problem of Erdős and Hajnal.

To find out more about the *Proceedings* and how to submit a paper visit lms.ac.uk/PROC.

As a reminder, a benefit of LMS Membership is free online access to the *Proceedings* and a number of other LMS Publications; see tinyurl.com/2p823mee.

Survey of LMS Members on Proposals for a National Academy

Foreword

Since the Bond Report first put forward the idea of a National Academy for the Mathematical Sciences as its number one recommendation, it has been intensely discussed within the Society and wider mathematical science community. If implemented, such an academy will potentially have a great impact on mathematics in the UK. For me, it was therefore important to hear the views of our members before the LMS, as one of the stakeholders and a member of the Council for the Mathematical Sciences, helps to shape the new academy as we move to the implementation stage.

Your numerous and thoughtful responses have been interesting to read, and Council agreed that we should not just share the summary below but also individual comments, the raw data. While it will be some time until there is a fully developed proposal on which the Society can form a definitive view, the results of this survey will help guide the Society's Council in its deliberations relating to the Academy. It is also good to know what members value most highly in the LMS and we will strive to preserve those aspects also in future.

I want to extend my warm thanks to everyone who responded to the survey.

Ulrike Tillmann
President

Background

In December last year, the Council for Mathematical Sciences (CMS) published a Green Paper (bit.ly/3bT5xHB) outlining a proposal for a National Academy for Mathematical Sciences. The paper suggested that the Academy should embrace academia, education, industry and government agencies, and that its primary focus should be on external advocacy and enhancing connections across the broad mathematical sciences community. Though the CMS would be expected to cease to exist, individual membership based mathematical societies would continue.

The subsequent consultation of the mathematical sciences community by the CMS showed widespread support for such an Academy in principle. A working group set up by the CMS produced a Next Steps paper. This envisaged using time limited funds available through the Isaac Newton Institute (INI) and International Centre for Mathematical Sciences (ICMS), among others, to employ a senior administrator to serve as Executive Director and setting up a focused Executive Committee of a proto-Academy. A go/no go decision for the Academy would be taken at the end of the period (two and a half years). A paper summarising the feedback from the CMS consultation and the recommendations from the working group has been published on the CMS website at bit.ly/3dkfKgz.

The Society wanted to hear the views of its members on these proposals. A small working group consisting of the President, the Vice-Presidents and the Executive Secretary developed a brief set of questions about the National Academy proposals for Members to consider and the survey was launched in late April. The name of the respondent was requested as part of the survey, to understand whether the views expressed were those of members or non-members, and to ensure that the consultation was robust. Only the Society's Officers and staff who worked on the survey had access to this personal information and the results of the survey were anonymised in all other discussions.

The consultation was launched via an 'all member' email. Letters were also sent to those members with whom we communicate (at their request) by post, providing them with the link to the consultation. There were 127 responses from the 'all member' email and three from the letters to members. We would ideally have liked more responses. However, we noted that there were only about 70 responses from across the mathematical community to the CMS's consultation on the green paper so this level of engagement from the Society's membership alone seemed to us to be positive.

Council reviewed the results of the survey at its June meeting along with a draft summary of responses. Council decided that, in the interest of transparency, the summary of responses should be published and that the underlying data should also be published, including any 'free text' comments where respondents had opted into the publication of their anonymised data. This underlying anonymised data and free text comments (where respondents opted in) can be found at tinyurl.com/acad-survey-results. It should be borne in mind that the summary of responses below is based on all the responses received, not just those responses included in the file of anonymised data.

Summary of results

Almost all respondents were Members of the Society (more than 97%). Most respondents worked in academia or were retired. Nearly 45% of respondents were Professors.

Around 78% of respondents thought that the impact of the planned National Academy as described in the Green Paper would be positive or very positive, and fewer than 7% thought that it would be negative or very negative.

Amongst the main positive impacts mentioned were:

- Speaking with a unified voice, which should raise the profile of mathematics and make it more influential with government, funding agencies, the public and the media, which in turn will also have positive effects on the economy
- More influence internationally
- Facilitation of cross-fertilisation of ideas between different areas of mathematics and closer links between pure and applied mathematics
- Responding to the Government preference to engage with a small rather than large number of representative bodies
- Ability to attract top mathematicians into the Academy, who will be a model for others to follow
- A clear ambition to be inclusive.

However, concerns were also raised, including:

- The risk that yet another mathematical organisation will make the community less rather than more effective due to the proliferation of institutions
- A National Academy may not necessarily change mathematicians' behaviour or resolve the differences in view in the community
- The emphasis on 'impactful' mathematics — or an expectation of short-term benefits — is misplaced. This cannot be a requirement for supporting pure mathematics, which is important regardless of 'impact'. Pure mathematics could become drowned out and distinctive perspectives lost
- The same benefits could be achieved by improving the visibility of the Society rather than creating yet another institution. The Society's members will have little influence on an Academy
- Learned societies could be side-lined more generally
- The value of an Academy over and above the CMS is unclear
- It is not clear where the resources for an Academy are going to come from. The financial basis for the Academy needs to be established. There is a risk that funding for an Academy will reduce funding for other grants and research
- It is not clear how government control of an Academy will be prevented
- A larger organisation might move more slowly.

Respondents expressed a degree of uncertainty about the impact of the Academy on the Society, given that the design of the Academy was at an early stage. It was noted that the Society had a long history and it was hoped that the voice and independence of the Society would not be reduced. Some respondents felt that proper consultation of the Society by, and involvement of the Society in, the Academy, would be critical, and wondered if there was a risk that the Society would be absorbed into the Academy. It was felt that the Society would need to be careful not to become too inward looking and would need to invest effort in discussions and be willing to compromise.

It was noted that an Academy might free the Society's resources from having to respond to policy matters on its own and allow the Society to focus on its core activities and support for research mathematicians. More generally, there could be reduced duplication between the existing learned societies.

Any improvement in the visibility of mathematics was felt to be of benefit to the Society. On the other hand, the Academy could be a rival to the Society and there was felt to be a risk that Society membership could decline.

The survey asked about potential pitfalls during the set-up phase of the National Academy.

Respondents felt that there must be sufficient communication and consultation with the mathematical community during the Academy set-up phase, not least with the Society's Members. In addition, respondents commented that the vision for the Academy needed to be the right one, not too narrow, not neglecting education but not with a focus on teaching and applications to the detriment of pure mathematics. Balance across the mathematical community was considered to be vital, along with avoiding in-fighting.

Concern was expressed about choosing a founding President based on mathematical stature rather than political awareness — but also about doing the opposite. The concept of Fellowship was also met with caution — there was felt to be a risk it would be exclusive rather than inclusive and that the Fellowship would lack diversity. More broadly, there was concern that the Academy discussions so far do not prioritise diversity sufficiently.

Respondents noted that it would be critical how disagreements were dealt with and that more could be said on this than was currently included in the Next Steps paper, including around governance/decision-making to ensure that all views were heard and that there was a balanced outcome.

Funding also attracted a number of comments. It was felt that securing funding would be critical, as well as making sure that the Academy was not funded at the expense of other existing activities. It was noted that reliable medium-term funding would not necessarily be secured quickly.

Excessive bureaucracy within the Academy was also identified as a risk.

The survey asked what respondents valued the most about the Society and wanted to see preserved. The most common responses to this question included the following:

- The Society should remain a leading society for academic mathematicians and in particular it should retain its focus on pure mathematics.
- The Society's small grants are greatly valued, and the lack of bureaucracy in the grant application process.
- The Society's work to maintain the community of members, even after retirement, including through conferences, events and the Newsletter, should continue.
- The Society's independence from government and industry should be preserved.
- The Society's publications should be preserved.
- The Society's commitment to diversity should be preserved.

This summary was prepared by Caroline Wallace, Executive Secretary, and approved by Council for publication in the interests of transparency.

OTHER NEWS

Ferran Sunyer i Balaguer Prize

Moritz Egert (left) and Pascal Auscher

Award of the 2022 Prize

The Ferran Sunyer i Balaguer prize is awarded annually by the Catalan Mathematical Society in memory of Catalan mathematician Ferran Sunyer i Balaguer (1912–1967). The 2022 prize was awarded to Pascal Auscher (Université Paris-Saclay) and Moritz Egert (TU Darmstad) for their monograph *Boundary Value Problems and Hardy Spaces for Elliptic Systems with Block Structure*.

Call for the 2023 Prize

The prize will be awarded for a mathematical monograph of an expository nature presenting the latest developments in an active area of research in mathematics. The monograph must be original, unpublished and not be subject to any previous copyright agreement. The prize consists of €15,000 and the winning monograph will be published in Birkhäuser series Progress in Mathematics.

Deadline for submission is 2 December 2022 at 1 pm (GMT+1). For further information visit the website at ffsb.espais.iec.cat.

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MATHEMATICS POLICY DIGEST

‘The Bedrock of all Sciences’: Letters in the *Times* and *Guardian*

Via the Protect Pure Maths campaign (protectpuremaths.uk), letters by LMS President Ulrike Tillmann reiterating the importance of investment in mathematics and calling on the Government to deliver on its £300 million committment in additional funding for the mathematical sciences have been published in the *Times* and *Guardian*.

The letters followed the publication of a House of Lords Science and Technology report (bit.ly/3STpdMb) on the government’s global ambitions for science and technology, which include making the UK a ‘science and technology superpower’.

You can read the *Guardian* letter (‘Lack of maths funding will hinder UK’s scientific progress’; 8 Aug) at bit.ly/3w7NhRO.

The *Times* letter was in response to an article by William Hague on the importance of the Conservative leadership candidates and new prime minister prioritising science investment (‘It’s time to do for science what we did for sport’; 2 Aug; bit.ly/3QvaiWQ). The letter, published on 6 Aug, can be read (behind a paywall) at bit.ly/3QPD4I4. The text of the letter is also available to read on the LMS website at lms.ac.uk/news/times-letter.

The Protect Pure Maths campaign later published a ’maths manifesto’ with six key ’asks’, aimed at the current Conservative leadership candidates. You can read more about the manifesto at tinyurl.com/maths-manifesto.

A-Level Results 2022

A-level results were released on Thursday 18 August as students across England, Wales and Northern Ireland received grades for the first formal exams since 2019. Scottish Higher results were released on 9 August.

The LMS extended its congratulations to all students receiving their results, particularly in light of the recent and ongoing difficulties caused by the Covid-19 pandemic and its impact on both students and teachers.

As expected, the percentage of students receiving A* and A in mathematics had decreased compared with 2021 figures, with 23.4% receiving A* (compared with 28.7% in 2021), and 48.2% receiving A (compared with 55.2% in 2021). This year’s percentages were higher than 2019 figures, however, when results were last based on exams, and the percentage of students receiving A* in further mathematics was significantly higher in 2022 than in 2019 (40.6% vs 24.7%, respectively).

Members of the mathematics community highlighted the fact that, although mathematics remains the most popular A-level choice, the number of students opting to study mathematics at university is decreasing and further government support is essential to boost the UK’s pipeline of mathematics talent. Dr Kevin Houston, LMS Education Secretary and spokesperson for the Protect Pure Maths campaign, said: “[w]ithout a healthy flow of mathematics undergraduates, the nation’s national security, healthcare services and future as a STEM superpower is put at risk. As a first step the candidates to be Prime Minister ought to endorse our six-point maths manifesto and pledge to honour the government’s promise of £300 million in extra funding for the mathematical sciences.”

Digest prepared by Katherine Wright
Society Business, Research &
Communications Officer

Note: items included in the Mathematics Policy Digest are not necessarily endorsed by the Editorial Board or the LMS.

Istanbul Center for Mathematical Sciences

The EMS is shocked and saddened to hear of the sudden closure of the Istanbul Center for Mathematical Sciences (IMBM). The EMS President Volker Mehrmann has written to the Rector of Boŏaziçi University, the President of the Turkish Research Council, the President of the Turkish Higher Education Council, and the President of the Turkish Republic to urge that this decision be reconsidered.

From Budapest to Tel Aviv, the IMBM is the only centre in the region catering to all areas of the mathematical sciences, including physics, engineering, economics and statistics. It has been serving the Turkish mathematical community for the last 16 years and has been visited by hundreds of colleagues from around the world. The need for such a place to discuss mathematical research is essential for the community, and since its inception IMBM has provided an ideal environment for local and international researchers. It would be an enormous loss to take this facility away from Turkish mathematicians.

The IMBM is requesting support from international colleagues through its 'Math or Not' campaign. You can declare a seminar at your institution to be an 'IMBM Seminar' and report this activity to the IMBM via their website imbm.org.tr. The IMBM can be followed on Twitter: @imbm_icms.

Predatory Journals

The EMS has launched a new awareness campaign on the problem of predatory publishing. For more information see euromathsoc.org/predatory-publishing which contains advice on identifying problematic journals and practices.

Resources for Ukrainian Colleagues

The EMS has begun compiling a list of resources, information, and opportunities for colleagues displaced by the Russian invasion of Ukraine. See: https://tinyurl.com/mteavzzm.

EMS News prepared by David Chillingworth
LMS/EMS Correspondent

Note: items included in the European Mathematical Society News represent news from the EMS and are not necessarily endorsed by the Editorial Board or the LMS.

OPPORTUNITIES

Atiyah UK–Lebanon Fellowships

The Atiyah UK–Lebanon Fellowships were set up in 2020 as a lasting memorial to Sir Michael Atiyah (1929–2019) in the form of a two-way visiting programme for mathematicians between the UK and Lebanon, where Sir Michael had strong ties. The scheme operates in partnership with the Centre for Advanced Mathematical Sciences (CAMS) at the American University of Beirut and its purpose is to foster exchange and interaction between mathematicians from the two countries.

Each year, the Atiyah Fellowship will either support an established UK-based mathematician to visit Lebanon for a period of between one week and six months, or alternatively a mathematician from Lebanon at the level of an advanced MSc or PhD student or above, to visit the UK for research and interaction with mathematicians in a host institution of their choice for a period of up to 12 months.

For mathematicians from the UK visiting Lebanon, the Fellowship will cover up to £2,000 towards actual travel and related expenses and will pay accommodation and subsistence of £1,000 per month pro rata for up to six months. In addition, CAMS will cover accommodation and provision of office space and logistical support. This will be independent of the host institution. There is the possibility of additional subsistence/payment for agreed teaching and some consideration may be given for additional support to Fellows travelling with a family.

For a mathematician from Lebanon visiting the UK, the Society will fund the Atiyah Fellow with up to £2,000 towards travel and related expenses and will pay accommodation and subsistence up to a maximum of £2,000 per month from an allocated maximum of £6,000. There is a possibility for Fellows from Lebanon to apply for further funding from the International Centre for Mathematical Sciences in Edinburgh. Please visit icms.org.uk/visiting-fellows for more information. Additional support will be available for PhD or MSc candidates in either mathematics or mathematical physics.

Applications for Atiyah UK–Lebanon Fellowships to be held the academic year 2023–24 will open on 1 September 2022, with a closing date of 31 January 2023.

For further information about the Fellowships and information on how to apply, please visit lms.ac.uk/grants/atiyah-uk-lebanon-fellowships.

LMS Grant Schemes

The next closing date for research grant applications (Schemes 1–6 and AMMSI) is 15 September 2022. Applicants for LMS Grants should be mathematicians based in the UK, the Isle of Man or the Channel Islands. For grants to support conferences/workshops, the event must be held in the UK, the Isle of Man or the Channel Islands. Applications are invited for the following grants to be considered by the Research Grants Committee at its October 2022 meeting:

Conferences (Scheme 1)

Grants of up to £5,500 are available to provide partial support for conferences. This includes travel, accommodation and subsistence expenses for principal speakers, UK-based research students, participants from Scheme 5 countries and Caring Costs for attendees who have dependents.

Visits to the UK (Scheme 2)

Grants of up to £1,500 are available to provide partial support for a visitor who will give lectures in at least three separate institutions. Awards are made to the host towards the travel, accommodation and subsistence costs of the visitor. Potential applicants should note that it is expected the host institutions will contribute to the costs of the visitor. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents.

Joint Research Groups in the UK (Scheme 3)
Application deadline 30 September 2022

Grants of up to £1,500 are available to support joint research meetings held by mathematicians who have a common research interest and who wish to engage in collaborative activities, working in at least three different locations (of which at least two must be in the UK).

Online Graduate Lecture Series (Scheme 3)
Application deadline 30 September 2022

Grants of up to £1,000 can be applied for by those who would like to organise an Online Graduate Lecture

Series. The introduction of the online lecture series element to the Joint Research Groups follows the successful Online Lecturer Series grant scheme, which was run in 2020 in response to the impact of the Covid-19 pandemic on the mathematical community. However, applications for this element of the Scheme 3 grants is open both to Joint Research Groups (new and current) and to mathematicians who are not part of a Joint Research Group.

Research in Pairs (Scheme 4)

For those mathematicians inviting a collaborator, grants of up to £1,200 are available to support a visit for collaborative research either by the grant holder to another institution abroad, or by a named mathematician from abroad to the home base of the grant holder. For those mathematicians collaborating with another UK-based mathematician, grants of up to £600 are available to support a visit for collaborative research either by the grant holder to another institution or by a named mathematician to the home base of the grant holder. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents.

Research Reboot (Scheme 4)

Grants of up to £500 for accommodation, subsistence and travel plus an additional £500 for Caring Costs are available to assist UK mathematicians who may have found themselves with very little time for research due to illness, caring responsibilities, increased teaching or administrative loads, or other factors. This scheme offers funding so that they can leave their usual environment to focus entirely on research for a period from two days to a week. For applications submitted by the next deadline (15 September 2022), the Reboot Retreats should take place between 1 November 2022 and 30 January 2023.

Collaborations with Developing Countries (Scheme 5)

For those mathematicians inviting a collaborator to the UK, grants of up to £3,000 are available to support a visit for collaborative research, by a named mathematician from a country in which mathematics could be considered to be in a disadvantaged position, to the home base of the grant holder. For those mathematicians going to their collaborator's institution, grants of up to £2,000 are available to support a visit for collaborative research by the grant holder to a country in which mathematics could be considered to be in a disadvantaged position. Applicants will be expected to explain in their application why the proposed country fits the circumstances considered eligible for Scheme 5 funding. In addition, the Society allows a further amount (of up to £200) to cover Caring Costs for those who have dependents. Contact the Grants team if you are unsure whether the proposed country is eligible, or check the IMU's Commission for Developing Countries definition of developing countries (tinyurl.com/y9dw364o).

Research Workshop Grants (Scheme 6)

Grants of up to £10,000 are available to provide support for Research Workshops. Research Workshops are intended to be an opportunity for a small group of active researchers to work together for a concentrated period on a specialised topic. Applications for Research Workshop Grants can be made at any time but should normally be submitted at least six months before the proposed workshop.

African Mathematics Millennium Science Initiative (AMMSI)

Grants of up to £2,000 are available to support the attendance of postgraduate students at conferences in Africa organised or supported by AMMSI. Application forms for LMS-AMMSI grants are available at mathematics.uonbi.ac.ke/.

The next closing date for early career research grant applications for ECR Travel Grants is 15 October 2022. Applications are invited for the following grant to be considered by the Early Career Research Committee at its June 2022 meeting:

Postgraduate Research Conferences (Scheme 8)

Grants of up to £2,500 are available to provide partial support for conferences, which are organised by and are for postgraduate research students. The grant award will be used to cover the costs of participants. In addition, the Society allows the use of the grant to cover Caring Costs for those who have dependents.

Celebrating new appointments (Scheme 9)

Grants of up to £400-£500 are available to provide partial support for meetings to celebrate the new appointment of a lecturer at a university. Potential applicants should note that it is expected that the grant holder will be one of the speakers at the conference. In addition, the Society allows the use of the grant to cover Caring Costs for attendees who have dependents.

ECR Travel Grants

Grants of up £500 are available to provide partial travel and/or accommodation support for UK-based Early Career Researchers to attend conferences or undertake research visits either in the UK or overseas.

For full details of these grant schemes, and to find information on how to submit application forms, visit the LMS website: lms.ac.uk/content/research-grants. Queries regarding applications can be addressed to the Grants Administrator Lucy Covington (020 7927 0807, grants@lms.ac.uk), who will be pleased to discuss proposals informally with potential applicants and give advice on the submission of an application.

Cecil King Travel Scholarship 2023

For many years the London Mathematical Society has been operating the Cecil King Travel Scholarship funded by the Cecil King Memorial Foundation. The scholarship is designed to support young people who display outstanding potential in mathematics in order to advance their educational and vocational training.

Every year two early career mathematicians are awarded £6,000 each to support their study or research abroad, typically for a period of three months. One scholarship is usually awarded to a mathematician in any area of mathematics and one to a mathematician whose research is applied in a discipline other than mathematics.

The awards are competitive and are based on written proposals describing the intended programme of study or research abroad, and the benefits to be gained from such a visit. The Early Career Committee of the London Mathematical Society has a panel of specialists representing a broad area of mathematics who are tasked with analysing the quality and standard of the applications, along with the scientific merit of the research project.

Who can apply and how?

Mathematicians in the United Kingdom or the Republic of Ireland who are registered for a doctoral degree or have completed one within 12 months of the closing date for applications are invited to apply for the Cecil King Travel Scholarship 2023 round. The Society encourages applications from women, disabled, Black, Asian and Minority Ethnic candidates, as these groups are under-represented in the field of mathematics in both the UK and the Republic of Ireland.

The application deadline for the 2023 programme is 15 November 2022. The online application form is available on our website. Shortlisted applicants will be invited to interview during which they will be expected to make a short presentation on their proposal. Interviews will take place in January 2023.

More information about the Scholarships is available on our website at tinyurl.com/2p92jzpz.

LMS–Bath Mathematical Symposia 2024: Call for Proposals

Deadline: 15 December 2022

The LMS is pleased to announce its Call for Proposals for the LMS–Bath Mathematical Symposia, to be held at the University of Bath in 2024. Core funding of approximately £40,000 is available to support up to two Symposia in 2024.

The LMS–Bath Mathematical Symposia are an established and recognised series of international research meetings, which since their foundation in 1974 have provided an excellent opportunity to explore an area of research in depth, learn of new developments and instigate links between different branches.

The format is designed to allow substantial time for interaction and research. The meetings are by invitation only and will be held in July/August, with up to 50 participants, roughly half of whom will come from the UK. A novel element of the Symposia is that they will be complemented by a summer school to prepare young researchers such as PhD students, or a "research incubator," where problems related to the topic of the conference are studied in groups. The entire event, summer school/incubator and workshop will typically last around two weeks.

Prospective organisers should send a formal proposal to the Grants team (Grants@lms.ac.uk) by 15 December 2022. Proposals are approved by the Society's Research Grants Committee after consideration of referees' reports.

Proposals should include:

- A full list of proposed participants, divided into specific categories:
  - Category A – Scientific Organisers
  - Category B – Key Overseas Participants
  - Category C – Key UK-based Participants

- Category D – Important Overseas Participants
- Category E – Important UK-based Participants

- Proposers are encouraged to actively seek to include women speakers and speakers from ethnic minorities or explain why this is not possible or appropriate.
- A detailed scientific case for the symposium, which shows the topic is active and gives reasons why UK mathematics would benefit from a symposium on the proposed dates.
- Details of additional support from other funding bodies, or proposed avenues of available funding.
- Indicative plans for the summer school or research incubator.
- Where appropriate, prospective organisers should consider the possibility of an 'industry day.'

For further details about the LMS Mathematical Symposia, please visit the Society's website: tinyurl.com/vz73dt93 or the LMS–Bath symposia's website: bathsymposium.ac.uk.

Before submitting: organisers are welcome to discuss informally their ideas with the Chair of the Research Grants Committee, Professor Andrew Dancer (Grants@lms.ac.uk).

Emmy Noether Fellowships 2022: Call for Applications

The LMS Committee for Women and Diversity in Mathematics encourages applications to the second call of the LMS Emmy Noether Fellowships, closing on Friday 7 October 2022.

Thanks to the continued generosity of the Liber Stiftung (tinyurl.com/2s3vbwvk), several Emmy Noether Fellowships with a value between £2,000 and £10,000 will be awarded in 2022, up to a total of £25,000. The amount awarded for each fellowship reflects the individual requirement of the applicant. The fellowships are designed to enhance the mathematical sciences research, broadly construed, of holders, either by re-establishing their research programme after return from a major break associated with caring responsibilities or supporting them to maintain their research programme while dealing with significant caring responsibilities.

VISITS

Visit of Bruno Lombard

Dr Bruno Lombard will be visiting the Department of Mathematical Sciences, Loughborough University from 7 to 12 November 2022. Dr Lombard is a Senior Researcher CNRS and Deputy Director of the Laboratory of Mechanics and Acoustics in Marseille, France. He is an internationally recognised expert in the areas of linear and nonlinear waves, homogenisation techniques and numerical modelling. During his visit Dr Lombard will give lectures at:

- Imperial College London, 7 November (contact Richard Craster: r.craster@imperial.ac.uk)
- Loughborough University, 9 November (contact Karima Khusnutdinova: K.Khusnutdinova@lboro.ac.uk)
- University of Manchester, 11 November (contact William Parnell: William.J.Parnell@manchester.ac.uk)

For further details contact Karima Khusnutdinova (K.Khusnutdinova@lboro.ac.uk). The visit is supported by an LMS Scheme 2 grant.

Visit of Professor Serge Cohen

Professor Serge Cohen will visit the Statistical Laboratory, University of Cambridge from 27 February to 18 March 2023. Professor Cohen is a member of the Laboratoire de Statistique et Probabilités at Université Paul Sabatier, Toulouse. His research encompasses many aspects of random fields and fractional processes. During his visit Professor Cohen will give lectures at:

- University of Cambridge, 28 February (contact James Norris: norris@maths.cam.ac.uk)
- University of Warwick, 1 March (contact Aleks Mijatovic: a.mijatovic@warwick.ac.uk)
- Imperial College, 7 March (contact Xue-Mei Li: xue-mei.li@imperial. ac.uk)

For further details contact James Norris. The visit is supported by an LMS Scheme 2 grant.

LMS Education Committee

The Education Committee of the LMS has a wide brief, from supporting mathematical education in schools, colleges and universities, to encouraging the public and young people to appreciate and engage with mathematics.

The difficulties facing the mathematics education community are many. There is a national shortage of highly trained mathematics teachers, a need for widening participation, and a long-term poor public perception of mathematics.

The Society has two significant activities to help tackle the lack of mathematics teachers. For many years the Committee has provided bursaries for maths teachers to attend continuing professional development events to improve their skills. In particular, the committee provided £4,000 in the past year for attendance at major teaching conferences. Our most important recent development is the creation of the 'Teaching Mathematics as a Career' (TeMaC) scheme. This scheme helps HE departments encourage their students to decide on a teaching career. Please see the LMS website for details on how to be involved: lms.ac.uk/policy/education/temac.

Another important issue is access to education. Thanks to a generous donation from Tony Hill, the Society is piloting the Levelling Up maths scheme: levellingupscheme.co.uk. This aims to widen participation from under-represented groups in mathematics degrees. A-level students are given tutorials and other help to raise grades and aspirations. This is being rolled out nationally. Please contact LevellingUp@lms.ac.uk to become involved.

The Education Committee has a number of outreach activities to raise the profile of mathematics. We return to in-person Popular Lectures, with the next to be held in Birmingham in October on the mathematics of Antarctica; see details at lms.ac.uk/events/popular-lectures. We work with Gresham College to produce an annual lecture, which this year was a well-received lecture by Hugh Hunt (Cambridge). You can watch the lecture at gresham.ac.uk/watch-now/gyroscopes-boomerangs.

As part of improving the maths outreach community, the Committee runs Mathematics Communication Workshops with beginners and advanced training for mathematicians who wish to participate or improve their skills in maths communication. In addition, we have the long-running Holgate Lectures and Workshops Scheme (tinyurl.com/holgate-scheme), which supports maths communicators (freelancers or lecturers), to give mathematics talks in schools. Both these activities have had very positive feedback.

To support mathematics teaching in higher education, the Committee recently introduced Grants for Teaching and Learning in HE. These provide financial support for workshops to disseminate good practice in this area. The Committee also awards Small Education Grants to support interesting ideas in education. See information on all Education Grants at lms.ac.uk/grants/education-grants.

In addition, the committee works with organisations such as the Institute of Mathematics & its Applications (IMA), Royal Statistical Society (RSS), The Operational Research Society (ORS), Heads of Departments of Mathematical Sciences (HoDoMS), Royal Society Advisory Committee on Mathematics Education (RS ACME), Joint Mathematical Council (JMC), British Science Association (BSA), and others, to improve mathematics and its perception.

There is still much work to be done and the Education Committee aims to be at the forefront of efforts to improve mathematics education and outreach in the UK.

Dr Kevin Houston
Education Secretary

Annual LMS Subscription 2022–23

Members are advised that renewal reminders about their annual subscription, including payment for additional subscriptions, for the period November 2022–October 2023, will be sent to them either by email or by post in late September/early October 2022. Annual subscriptions become due on 1 November 2022 and payment should be received by 1 December 2022. Note that payments received after this date may result in a delay in journal subscriptions being renewed.

Tiered Ordinary Membership Subscription Rates

Members who pay the Ordinary membership rate can choose which membership rate they wish to pay based on whether their annual professional income falls within the following ranges:

- Above £65,000 per annum: Ordinary (high) member rate.
- Between £35,000-£65,000 per annum: Ordinary (middle) member rate.
- Up to £35,000 per annum: Ordinary (low) member rate.

The Society will not collect any data on members' actual professional income nor require proof of earnings. Instead, Ordinary members are asked to advise either via their online member record or the subscription form which tier of Ordinary membership subscription they will be paying. For members who pay by direct debit, we encourage you to update this information by 14 October 2022.

All LMS Membership Subscription Rates

The annual subscription rates to the London Mathematical Society for 2022–23 are:

LMS Membership Subscription Rates 2021-22		
Ordinary Member (high) rate	£125.00	US$250.00
Ordinary Member (middle) rate	£102.00	US$204.00
Ordinary Member (low) rate	£83.00	US$166.00
Reciprocity rate for members based outside the UK and a member of one of the LMS' Reciprocity partners	£51.00	US$102.00
*Associate (postdoc) rate for early career members whose PhD completion was more than 3 years ago and who are on a non-permanent contract	£51.00	US$102.00
Associate rate for PhD student members and members whose PhD was completed in the last three years	£25.50	US$51.00
*Concessionary rate for members working part-time, unemployed or otherwise in hardship	£25.50	US$51.00
Associate (Undergraduate) rate for undergraduate student members	£12.75	US$25.50
Senior rate for members who have paid fees for at least 35 years	£0.00	US$0.00

*These rates are by request and subject to agreement by the Treasurer.

LMS member benefits

Members are reminded that their annual subscription entitles them to the following range of benefits:

- Free online access to selected journals for personal use only; the *Bulletin, Journal* and *Proceedings of the London Mathematical Society*, and *Mathematika* (provided by Wiley) and to *Nonlinearity* (provided by the Institute of Physics)
- 50% of European Mathematical Society membership and discounted subscription to the Journal of the European Mathematical Society

- Voting in the LMS Elections
- Option for European Women in Mathematics members to pay their EWM fees via the LMS
- The bi-monthly Newsletter
- Use of the Society's library at UCL
- Use of the Verblunsky Members' Room at De Morgan House, London

For a full list of member benefits, see lms.ac.uk/membership/member-benefits.

Renewal and Payment

Online: Members can log on to their LMS user account (lms.ac.uk/user) to make changes to their contact details and journal subscriptions, and to make payment either by card via WorldPay or by setting up a direct debit via GoCardless, under the 'My LMS Membership' tab.

By subscription form: Members can also renew their subscription by completing the subscription form and including a cheque either in GBP or USD. We regret that we do not accept payment by cheques in Euros.

Please email any queries to the LMS membership team (membership@lms.ac.uk).

Elizabeth Fisher
Membership & Grants Manager

Council Diary — A Personal View

It was great to see a majority of Council members in person at De Morgan House for the last Council meeting of this academic year on 1 July 2022, despite the steady increase of Covid infections nationwide. The President herself was not spared but could nevertheless use an online channel to provide Council with a detailed report on her activities since the previous Council meeting and offer an informed opinion on several points of discussion, letting Vice-President Iain Gordon chair in person. What follows is a brief digest of the matters that occupied most of the session.

The in-house salient news was the confirmation of Jens Marklof FRS as successor of Ulrike Tillmann FRS as President-Elect in a year's time. It was also the last Council meeting for the current Executive Secretary, Caroline Wallace, who was warmly thanked for her excellent leadership during very challenging times. Fiona Nixon will step in for a short transitory period before Simon Edwards, the new Executive Secretary, joins in October.

Among the many events the President attended since April, let us mention a meeting in Queen's College Oxford in memory of former LMS Vice-President Peter Neumann O.B.E., a dinner at the Houses of Parliament organised by the Protect Pure Mathematics Campaign and a meeting with the Secretary of State for Education. This was an opportunity to argue for Core Mathematics to become more universally taught to 16–18 year olds, as well as make the case for enhanced Continuing Professional Development (CPD) for mathematics teachers.

On the international scene, the President reported news from the European Mathematical Society Council meeting recently held in Bled, Slovenia. More on this elsewhere in the Newsletter.

Congratulations to Simon Singh were in order, as he was announced as the recipient of this year's prestigious Zeeman Medal.

The concrete actions the LMS could take in order to help Ukrainian mathematicians in this time of war remained a topic of discussion at this Council meeting, with Vice-President Gordon announcing that the LMS would complement a solidarity scheme from the Isaac Newton Institute by offering a mentoring system for those remaining in Ukraine, a free LMS membership as well as the possibility to network through a Scheme 3 programme. The LMS continues to engage on this matter with the ICMS and the IMA which explores ways to complement efforts made elsewhere in the UK. Moreover the LMS has agreed, together with IOPP, to suspend for the time being the publication of three mathematics journals translated from Russian to English, with immediate effect. It was also reported that the Moscow and St Petersburg Mathematical Societies are not national institutions, and therefore the EMS does not intend to suspend their EMS membership.

Another societal concern of Council is sustainability at De Morgan House and more generally when travelling to London or when flying to international destinations for any LMS-related meetings. The Society endeavours to scrutinise the need for plane travel in particular and the necessity for meetings to be held in person while of course aiming to meet its core charitable aims of advancing and supporting mathematics.

The financial health of the LMS is constantly being challenged, and Niall MacKay, the new Publication Secretary, reported on the outcome of the 'publications strategic retreat' held in June, where the eternal dichotomy between commercial gain and the publication of excellent mathematics at any cost was discussed. Council heard from the Publication Secretary that there is no intention to launch new journals, but that it would pay to develop a strategy to lead rather than trail in the near future which will see big changes in the publishing world. In the meantime, Council agreed to an increase of 4% in the membership subscription, bringing the middle tier membership annual cost to £102. This increase (which is less than last year's 6%) was felt to be appropriate, the current rate of inflation notwithstanding.

On the other hand, it was agreed to keep publishing a paper copy of the LMS Newsletter in 2022–23 for members who elect to receive print, as there are a substantial number of members who value this medium.

The four-hour meeting concluded after a short discussion on whether the restriction on members of Council and the Prizes Committee being ineligible for LMS Prizes should be lifted. Council decided that this restriction should remain in place for the sake of transparency and fairness. On the other hand, the question was raised of whether the Prizes structure should be reviewed to ensure it enables the recognition of talent across the whole ED&I spectrum.

A number of Council members then walked to the British Medical Association building to attend the General Meeting and enjoy an enlightening double-act by Becky Armstrong and Lisa Orloff Clark, who delivered this year's Aitken Lecture.

Anne Taormina
Member-at-Large

Maximising your Membership Benefits: Voting in the LMS Elections 2022

The LMS has around 3,100 members who form a vibrant international mathematical community. The engagement of all its members is essential for the functioning of the Society. Nominating and voting in the annual Council and Nominating Committee elections, which will take place in October and November 2022, is an excellent way to get involved and influence the future of the Society. The deadline to submit a direct nomination is noon on 1 September 2022. Hearing members' voices through the ballots helps to keep the Society's governing body fresh, accountable, and credible while bringing a diversity of opinions and enabling members to connect with the Society's Council.

The preliminary slate of candidates for this year's LMS Elections for Council and Nominating Committee can be found at our website tinyurl.com/5x7pedhc and all members are invited to an online discussion forum which is available at discussions.lms.ac.uk/lmselections/.

Instructions on how to vote will be sent to members by e-mail or post by Civica Election Services, the organisation that administers the elections, on 14 October 2022, the day voting opens. Members are encouraged to check that their contact details are up to date at lms.ac.uk/user.

The results of the Council and Nominating Committee elections will be announced at the Annual General Meeting that will be held on Friday 18 November at 3.00pm.

The Society promotes a fair and transparent election that results in the general welfare of the mathematical community and portrays a positive public image of its members. Competitive elections allow the Society to find and form a strong governing body that will be responsible for the general control and management of its administration, strategies and plans and the charity's financial operations.

Valeriya Kolesnykova
Accounts and Membership Assistant
membership@lms.ac.uk

REPORTS OF THE LMS

Report: Operator Algebras: Subfactors, K-theory, Conformal Field Theory

The conference 'Operator Algebras: Subfactors, K-theory, Conformal Field Theory' was held in honour of the 70th birthday of David Evans (left) from Cardiff University. It was organised as an in-person event from 27th July to 2nd August 2022 at Gregynog Hall in Wales after it had to be postponed in 2021 due to the pandemic. The organisers gratefully acknowledge the financial support received from the LMS, EPSRC, the Learned Society of Wales, and the Clay Mathematics Institute.

The conference was focused on a wide range of interconnected themes, among them operator algebras and subfactor theory, fusion categories, topological and operator-algebraic K-theory and applications of all these areas in conformal field theory. This list of topics reflects the rich body of research David Evans has contributed to and cultivated over the years. The occasion brought together high-profile established speakers as well as early career researchers and featured talks by two Clay lecturers: Dan-Virgil Voiculescu (University of California, Berkeley) and Sorin Popa (UCLA). Voiculescu, who is an elected member of the National Academy of Sciences, spoke about the quasicentral modulus in functional analysis. This measure of how well n-tuples of operators commute with finite-rank approximate units leads to a non-commutative generalisation of the condenser capacity. Popa, who currently holds the Takesaki Endowed Chair in Operator Algebras, gave an account of W^*-representation theory for subfactors.

Stuart White (Oxford) reviewed the recent breakthroughs in the Elliott classification programme for simple separable nuclear Z-absorbing C^*-algebras, a topic that he also discussed as an invited speaker at the recent virtual ICM 2022. Another former ICM speaker (of which there were eight present in total) was Constantin Teleman (University of California, Berkeley) who presented his work on equivariant higher twists of K-theory and how it connects with Coulomb branches in gauge theory. Roberto Longo (Tor Vergata, Rome) spoke about his research programme on the role of Tomita-Takesaki theory in quantum field theory (QFT). The algebraic framework of QFT was also the topic of the talk by Kasia Rejzner (York), who outlined a C^*-algebraic approach to QFT with interactions. Yasuyuki Kawahigashi (University of Tokyo) reported on his work on producing subfactors of finite depth. Subfactor theory was developed by Vaughan Jones and its main object of study are inclusions of 'building block' von Neumann algebras, called factors. Surprisingly, finite-dimensional invariants that are called commuting squares already capture a lot of information about such inclusions and have recently also appeared in condensed matter physics in connection to topological orders.

The Thompson group, whose amenability has been a long-standing open problem, was the subject of two talks: Emily Peters (Loyola University Chicago) discussed Conway's rational tangles. Her talk was based on work of Vaughan Jones that has built a bridge between the theory of knots and links and that of the Thompson group. Alina Vdovina (Newcastle University) outlined its higher-dimensional analogues that can be described using buildings, i.e. geometric structures with a strong combinatorial flavour. The intricate connections between stable homotopy theory and topological quantum field theories were also a recurring theme: Apart from Constantin Teleman's presentation it featured in a talk by Mayuko Yamashita (RIMS, Kyoto) about invertible QFTs and differential cohomology. The topological view on quantum field theory was shared by Katrin Wendland (Trinity College Dublin) in her talk about elliptic genera for orbifolds with simple singularities. Terry Gannon (University of Alberta) spoke about modular invariants for rational conformal field theories. In two dimensions the restriction of a CFT to its chiral halves has enough symmetries to help classify them. In particular, those theories have a modular invariant partition function. This condition is quite restrictive and allowed Terry Gannon to find a solution to the problem up to rank 7. d

This diversity of topics and speakers reflects David Evans' widespread scientific interests and relations to colleagues around the world — including several former students and PostDocs — and the central role played by operator algebras in connecting these different branches of mathematics.

Providing an opportunity to discuss and connect with other researchers after the long 'communication drought' and 'Zoom fatigue' caused by the pandemic was one of the main aims of the conference. The venue, Gregynog Hall in Wales, was chosen with this goal in mind. Acquired by the Davies sisters in 1920 it first became a hub to bring art, music and creative skills to the people of Wales in the aftermath of the First World War. After its wartime use as a Red Cross convalescent home, it was bequeathed by the sisters to the University of Wales as a conference centre and in 2019 transferred to the Gregynog Trust. Their substantial collection of French impressionist and post-impressionist art is now in the National Museum in Cardiff. Gregynog has a history of serving as a location for mathematics colloquia, such as the annual meeting of mathematicians in Wales, and workshops including satellite meetings for the Isaac Newton Institute programmes, EPSRC networks and an LMS regional meeting and workshop. Greygnog has a comfortable atmosphere and all participants had accommodation on site during the event. We were happy to witness that this sparked many mathematical discussions among participants and hope that a lot of interesting future discoveries in the diverse research areas covered by the speakers will be traced back to a conversation that began over tea in the Blayney room or in the evening after the talks under the stars in the courtyard.

Masaki Izumi (Kyoto)
Gandalf Lechner (Erlangen)
Ulrich Pennig (Cardiff)
Mathew Pugh (Cardiff)

Call for Proposals
RIMS Joint Research Activities 2023-2024

Application deadline : November 30, 2022, 23:59 (JST)

Types of Joint Research Activities

*RIMS **Workshops(Type A)/Symposia** 2023
*RIMS **Workshops(Type B)** 2023

More Information : RIMS Int.JU/RC Website
https://www.kurims.kyoto-u.ac.jp/kyoten/en/

京都大学 KYOTO UNIVERSITY
Research Institute for Mathematical Sciences

Report: General Meeting of the Society & Aitken Lecture 2022

The General Meeting and Aitken Lecture were held on 1 July at BMA House in London, and online. The meeting room was filled with those attending in person, and many attended online. The meeting was opened by LMS Vice-President Professor Cathy Hobbs who relayed apologies from LMS President Professor Ulrike Tillmann who was indisposed. The minutes of the AGM held on 12 November 2021 were unanimously approved. The scrutineers for general elections to be held in November 2022 were proposed as Charles Goldie and Chris Lance, and this proposal was also approved. Honorary Membership 2022 was given to Professor Vladimir Drinfeld (University of Chicago), and to Professor Jennifer Tour Chayes (University of California at Berkeley). Professor Hobbs read the list of LMS Prize winners for 2022, and announced that the LMS President elect is Professor Jens Marklof FRS (University of Bristol). Finally, there had been many new members admitted to the LMS this year: three were present in person and duly signed the famous register.

The General Meeting was followed by two engrossing lectures. The first speaker was Dr Becky Armstrong (University of Münster) whose lecture was entitled *Groupoidology*, a term still hot from the forge. And Dr Lisa Orloff Clark (Victoria University of Wellington) gave the Aitken Lecture concerning C^*-algebras. These topics are interconnected and the speakers had collaborated together.

The first half of Becky's lecture was about the application of groups and groupoids on two well known puzzles. It is common knowledge that Rubik's cube can be modelled using group theory with the six operations of turning faces about an axis. But group theory cannot model the fifteen puzzle: the square with fifteen sliding tiles and a blank space. Why not, she asked? Well it was on the tip of everyone's tongue, honest, that although there are four operations moving the space around, one cannot always apply all of them since it depends upon the state of the tiles. This is where groupoids can be used. The theory can be couched algebraically or as a subset of category theory. Examples given were groupoids of an equivalence relation R over a set, and of a group G over X. Given a groupoid one can construct a group within it: an isotropy group. Using these, Becky demonstrated how the fifteen puzzle can be resolved methodically. The second half of the lecture concerned Becky's research about C^*-algebras, which originally arose in Quantum Mechanics. Examples are $n \times n$ matrices and continuous functions over a Hausdorff space. Becky expanded on her research on twisted groupoids with work on classification of C^*-algebras and the problem of producing a C^*-algebra from groupoids.

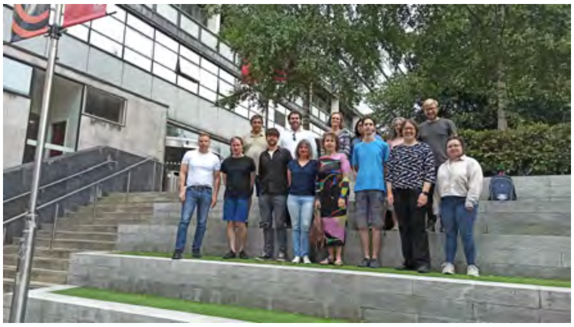
The Aitken lecture is a collaboration of the LMS and its NZMS equivalent. Lisa presented various classes of C^*-algebras. To help us recognise the key classes, she used amusing pictures, not of a Kiwi, but rather of a NZ flightless parrot, the Kakapo. The audience soon became conversant with the smiley and the diagonal Kakapo. Lisa listed a key theorem in classification of C^*-algebras, and explained that a goal, given a C^*-algebra, was to discover a class $M_n(\mathbb{C})$ of $n \times n$ matrices over complex numbers, using Raeburn's proposition to identify matrix units. This example of a C^*-algebra was the smiley Kakapo. Lisa showed that another example was to consider directed graphs, where if it was possible to construct an étale equivalence relation this would give rise to a C^*-algebra. Lisa explained that her area of research is principally about which purely infinite C^*-algebras can be realised in this way. And in closing she revealed that in Wellington she is known as the groupoid girl!

John R. Jones

Report: LMS Invited Lecture Series

The *Equations in Groups and Complexity* conference took place in the Curtis Auditorium in the Herschel Building at Newcastle University on 18–22 July 2022. It was centred around the LMS Invited Lecture Series 2022, given by Prof. Olga Kharlampovich (above) and funded by the LMS, Heilbronn Institute and Newcastle University.

The conference was a hybrid meeting, with 20 people attending in person and 40 taking part via Zoom. All early career mathematicians were given an opportunity to give a talk, and there was a good balance of talks given by well-established researchers and early career mathematicians.



The details of the conference, including the recordings, abstracts and slides of talks can be found at sites.google.com/view/nclcomplexity.

Dr Alina Vdovina
Senior Lecturer, Newcastle University

Report: EMS Council Meeting

The postponed European Mathematical Society (EMS) Council Meeting was held in Bled, Slovenia, on 29–30 June. The LMS delegation consisted of President Ulrike Tillmann, LMS Council Member-at-Large Frank Newmann and LMS Newsletter Editor-in-Chief Alina Vdovina. The event took place as a hybrid meeting, and many delegates participated via Zoom.

The results of the EMS elections were as follows:

- President: Jan Philip Solovej, University of Copenhagen, Mathematical Physics.
- Vice-President: Beatrice Pelloni, Herriot Watt University, Edinburgh.
- Treasurer: Samuli Siltanen, University of Helsinki.
- Member-at-Large: Victoria Gould, University of York.

We were pleased to see two LMS members elected to senior EMS roles.

Some other important topics discussed included the EMS Young Academy, the EMS Topical Activity Group, the EMS Publications House, the Nordic Congress of Mathematicians 2023, which will take place in Aalborg, and the next European Congress of Mathematicians (ECM9), which will take place in Seville in 2024.

Dr Alina Vdovina
Editor-in-Chief, LMS Newsletter

Records of Proceedings at LMS meetings
Ordinary Meeting: 7 July 2022

Held on 7 July at the Centre for Geometry and Topology, University of Copenhagen, as part of the Geometry and Topology Sectional Workshop during the virtual International Congress of Mathematicians 2022 (vICM 2022). Over 50 members and guests were present for all or part of the meeting.

The meeting began at 4.45pm with The Chair of the Society, Lectures and Meetings Committee, Professor Brita Nucinkis, in the Chair.

There were no members elected to Membership at this Society Meeting.

No members signed the Members' Book to be admitted to the Society.

The Chair, Professor Nucinkis, introduced the London Mathematical Society Lecture at the vICM 2022, given by Professor Karen Vogtmann FRS (University of Warwick) on *Spaces of Graphs*.

Professor Nucinkis thanked Professor Vogtmann for her excellent lecture and then expressed the thanks of the Society to the organisers for a wonderful meeting and workshop.

Afterwards, a reception was held at the Centre for Geometry and Topology, of the University of Copenhagen.

Records of Proceedings at LMS meetings
General Meeting & Aitken Lecture 2022

Held on 1 July 2022 at the BMA House, Tavistock Square, London, and online via MS Teams, along with the Aitken Lecture. Over 55 members and guests were present for all or part of the meeting.

The meeting began at 3.30pm (BST) with a Vice-President, Professor Cathy Hobbs, in the Chair.

It was agreed to ratify the minutes of the Annual General Meeting, which was held on 12 November 2021. The President invited members to vote, by a show of hands if present at BMA House or by voting in an electronic poll if attending via MS Teams, to ratify Council's recommendation. The minutes were ratified unanimously.

On a recommendation from Council, it was agreed to elect Professor Charles Goldie and Professor Chris Lance as scrutineers in the forthcoming Council elections. The President invited members to vote, by a show of hands if present at BMA House or by voting in an electronic poll if attending via MS Teams, to ratify Council's recommendation. The recommendation was ratified unanimously.

Vice-President Hobbs, on Council's behalf, proposed that following people be elected to Honorary Membership of the Society: Professor Vladimir Drinfeld of the University of Chicago, Illinois; and Professor Jennifer Tour Chayes of the University of California, Berkeley.

This was approved by acclaim. Vice-President Hobbs read a short version of the citations, which would be published in full in the Bulletin of the London Mathematical Society.

Vice-President Hobbs then announced the awards of the prizes for 2022: De Morgan Medal: Professor Sir John Ball FRS FRSE (Heriot-Watt University and the University of Oxford); Senior Berwick Prize: Professor John Greenlees (Warwick University) and Professor Brooke Shipley (University of Illinois, Chicago); Shephard Prize: Professor Andrew Lobb (Durham University); Fröhlich Prize: Professor Richard Thomas (Imperial College London); Anne Bennett Prize: Dr Asma Hassannezhad (University of Bristol); Whitehead Prizes: Dr Jessica Fintzen (University of Cambridge, Duke University, and Universität Bonn); Professor Ian Griffiths (University of Oxford); Dr Dawid Kielak (University of Oxford); Dr Chunyi Li (University of Warwick); Professor Tadahiro Oh (University of Edinburgh); Professor Euan Spence (University of Bath).

Vice-President Hobbs also announced the winner of the Bachelier Prize: Professor Beatrice Acciaio (ETH Zürich).

Vice-President Hobbs then announced that the President-Elect to be Professor Jens Marklof FRS.

Fifteen people were elected to Ordinary Membership: Ms Christodoulos Asiminidis, Dr Mahak Bhatia, Dr Jessica Crawshaw, Mr Martin Hansen, Dr Jingyu Huang, Dr Martino Lupini, Dr Volodymyr Pavlenkov, Dr Ana Ros Camacho, Dr Yuriy Semenov, Professor Y.D. Sharma, Mr Syed Muhammed Shavalliuddin, Dr Wahiba Toubal, Professor Natalya Vaysfeld, Dr Mats Vermeeren and Mr Jonathan Wallace.

Eight people were elected to Associate Membership: Mr Amlan Banaji, Dr Nilasis Chaudhuri, Dr Bryn Davies, Mr James Doran, Miss Marica Minucci, Mr David Murphy, Dr Xavier Pellet and Mr Louis Yudowitz.

Five people were elected to Associate (Undergraduate) Membership: Miss Veronica Bitonti, Mr Trystan Hooper, Mr Mark Lucas, Mr Anthony Ruiz and Mr Hugo Simoes Veloso.

Two people were elected to Reciprocity Membership: Dr Lesia Baranovska and Mrs Chinenye Nworah.

Three members signed the Members' Book and were admitted to the Society.

Vice-President Hobbs announced the dates of the next two Society Meetings. These are to be held on 7 July 2022 in Copenhagen as part of the virtual International Congress of Mathematicians and on 16 September at De Morgan House in London as part of the Joint Meeting with the Institute of Mathematics and its Applications and the British Society for the History of Mathematics on the topic of *Women in Astronomy*.

Vice-President Hobbs introduced the first lecture given by Dr Becky Armstrong (University of Münster) on *Groupoidology*.

After tea, Vice-President Hobbs introduced the Aitken Lecture 2022 by Professor Lisa Orloff Clark (Victoria University of Wellington) on *Equivalence Relations, Topology and C^*-algebras*.

Professor Hobbs thanked the speakers for their excellent lectures.

Afterwards, a wine reception was held at De Morgan House, Russell Square, London. The Society dinner was held at Trattoria Verdi, Southampton Row, London.

Records of Proceedings at LMS meetings
Ordinary Meeting: Society Lecture for the 73rd British Mathematical Colloquium

This meeting was held on Tuesday 7 June 2022, in-person at the Bush House of King's College London. Over 50 members and attendees were present in the auditorium, for all or part of the meeting.

The meeting began at 11.30am, with the President, Professor Ulrike Tillmann FRS, in the Chair.

No members were elected to membership.

Eight members signed the Members' Book and were admitted to the Society.

The Chair introduced the lecture given in-person by Isabelle Gallagher (École Normale Supérieure), On the dynamics of dilute gases.

Professor Tillmann thanked the speaker for her excellent lecture and then expressed the thanks of the Society to the organisers, the SLAM Committee and the BMC, for a wonderful meeting and workshop.

Afterwards, a lunch was held at Bush House. The BMC was held later that evening, at the Great Hall of King's College London.

Records of Proceedings at LMS meetings
Ordinary Meeting: 24 May 2022

This meeting was held in person and virtually on Zoom on 24 May 2022, hosted by the University of Leeds, as part of the Northern Regional Meeting and Workshop. Up to 81 members and visitors were present for all or part of the Society meeting session. This one day meeting took place as part of the LMS Workshop on *Bridges between Representation Theory and Algebraic Geometry: Singularities, Friezes and Cluster Categories* from 24 to 27 May 2022.

The meeting began at 2.00pm with the LMS Education Secretary, Dr Kevin Houston, in the Chair.

Three members signed the Members' Book and were admitted to the Society.

Professor Karin Baur (University of Leeds) introduced the first lecture given by Professor Sibylle Schroll (University of Köln) on *Full exceptional sequences in the derived category of gentle algebras*.

After tea, Dr Eleonore Faber (University of Leeds) introduced the second lecture by Professor Bethany Marsh (University of Leeds) on *An introduction to tau-exceptional sequences*.

Dr Matthew Pressland (University of Glasgow) introduced the third lecture by Professor Peter Jørgensen (Aarhus University) on *The index with respect to a rigid subcategory of a triangulated category* (joint work with Amit Shah).

Dr Houston thanked the speakers for their excellent lectures and then expressed the thanks of the Society for a wonderful meeting to the organisers, Professor Karin Baur and Dr Eleonore Faber of the University of Leeds and Dr Matthew Pressland of the University of Glasgow.

Records of Proceedings at LMS meetings
Ordinary Meeting: Diverse Perspectives on Alan Turing

held on Friday 4 March 2022 at De Morgan House, London and online via Zoom. Over 40 members and guests were present in the Hardy Room and over 70 members and guests were present on Zoom, for all or part of the meeting.

The meeting began at 1.30pm with The President, Professor Ulrike Tillmann FRS, in the Chair.

There were 31 members elected to Membership at this Society Meeting:

Associate membership: Mrs Andreea Dumitru, Miss Anastasia Hadjievangelou, Mr Gandhar Joshi, Mr Cameron Michie, Mr Diego Millan Berdasco, Dr Larissa Richards and Mr Thomas Smith.

Associate (Undergraduate) membership: Mr Scott Aitken, Mr Oliver Brown, Mr Sk Hassan Mahathi, Mr Alex McSweeney-Davis, Mr Spencer Rogers and Mr Girish P Vigrish.

Ordinary membership: Dr Demi Allen, Dr Zexun Chen, Mr Chris Daniels, Dr Viveka Erlandsson, Dr Zaniar Ghadernezhad, Dr Adam Jones, Dr Jean Lagacé, Dr Paula Macedo Lins de Araujo, Dr Gueorgui Mihaylov, Dr Daniel Nichol, Professor Markus Szymik, Dr Vaidehee Thatte and Mr Thomas Wilkinson.

Reciprocity membership: Dr Neranga Fernando, Mr Quinn Jackson, Dr Roa Makki, Professor Bimal Mishra and Professor Oswaldo Portobanco.

Seven members signed the Member's Book and were admitted to the Society.

The Chair introduced the first talk given in-person by Andrew Hodges (University of Oxford): *A double enigma: what did Alan Turing do in 1945 and in 1954?*

The Chair introduced the second talk given in-person by Debbie Marriott (Bank of England): *The Creative Journey of the New £50 Banknote Design*.

The Chair introduced the third talk given online by Janet Foster (LMS Archivist): *Turing and the LMS*.

After tea, the Chair introduced the fourth talk given online by Stephen H. Muggleton (Imperial College London): *Alan Turing and the Development of Artificial Intelligence*.

The Chair introduced the fifth talk given in-person by Jonathan Dawes (University of Bath): *Unpublished Patterns of Thought: Alan Turing's Later Work on Morphogenesis*.

After the break, the Chair introduced the final talk given in-person by Sir Dermot Turing: *Alan Turing's 'Wasted years': the Mathematics of World War II*.

Professor Tillmann thanked the speakers for their excellent lectures and then expressed the thanks of the Society to the organisers, Professor Eugenie Hunsicker, Dr Mark McCartney and Professor Brita Nucinkis, for a wonderful meeting and workshop.

Afterwards, a wine reception was held at De Morgan House. The Society dinner was held at the Blue Door Bistro, Montague on the Gardens Hotel.

Helsinki 2022: An ICM Like No Other

RACHEL THOMAS AND MARIANNE FREIBERGER

The 2022 International Congress of Mathematicians (ICM) was due to take place in July in St Petersburg, Russia. However, with Russia's invasion of Ukraine, the ICM couldn't take place as planned. Despite this challenge the international mathematics community has come together to create an ICM in 2022 like no other.

Every four years, thousands of mathematicians usually gather from around the world for the ICM. "The Congress plays a unique role in mathematics, both by recognizing exemplary recent research achievement in all of the various subfields of mathematics, and by giving the broader community of mathematicians the opportunity to learn about the latest mathematical research developments, and to interact with leaders in the field," said the committee of mathematicians overseeing the scientific programme of the 2022 ICM, in their 2019 report (see bit.ly/3zF2Y3U).

This exciting highlight of the mathematical calendar is held in a different location around the world each time, featuring invited lectures, panel discussions and events alongside the awarding of some of the most prestigious prizes in mathematics, including the famous Fields Medals. The ICM — particularly planning the scientific programme and selecting the recipients of the prizes — is overseen by the International Mathematical Union (IMU). The IMU is made up of over 80 member countries, each represented by a national organisation such as a learned society or research council. The UK is represented by the London Mathematical Society.

Every four years, alongside the ICM, the IMU holds its General Assembly where over 200 representatives of the member countries come together to discuss and decide on issues important to the wider mathematical community, and to decide on where the next ICM will take place. In 2018 the IMU General Assembly decided that St Petersburg, Russia, would hold the 2022 ICM.

St Petersburg ICM 2022

After this decision the local organisers of the ICM take over planning the logistics of the in-person event, and the host country's government provides financial assistance to enable the ICM to take place. The 2018 ICM was held in Rio de Janeiro, the first time it had been held in the Southern Hemisphere, and became a focus for the rapidly developing mathematical community in Brazil, and more widely supporting mathematics in other developing countries. Marcelo Viana, co-organiser of that ICM, spoke to us at last year's IMU centennial and said: "I saw it as an opportunity for advertising maths. From the very beginning we thought of the Congress as an excuse to have people talk about maths, and we framed it as part of a much broader initiative, which we called the Biennial of Mathematics."

At the IMU centennial, Stanislav Smirnov, the co-organiser of the planned 2022 ICM in St Petersburg, said that he hoped holding the ICM in Russia would not only showcase the Russian mathematical tradition, but also to help restart mathematics in Russia.

Preparations for the 2022 ICM initially went as planned, Helge Holden, Secretary General for the IMU, told us when we met him at the beginning of the 2022 ICM: "Our collaboration with our Russian colleagues had been smooth." But then came the global Covid-19 pandemic. "There was a lot of uncertainty, we had to prepare for all sorts of measures." The IMU continued working with the local organising committee in Russia for different eventualities including a fully virtual or hybrid ICM. But by December 2021, as vaccines made their impact and countries began to move out of pandemic measures, the plans had reverted to a fully in-person event in St Petersburg.

Everything changed

Then, on 24 February 2022, Russia invaded Ukraine. "The invasion changed everything overnight," said Holden. The IMU executive committee was in a long-planned virtual meeting that day, and they quickly came to a unanimous decision that the ICM and General Assembly could not go ahead in St Petersburg as planned. As they continued to meet virtually, scattered across 17 timezones over the next four days, the executive committee agreed that an in-person General Assembly and awards ceremony would take place outside of Russia, and the ICM would be held virtually, open to all mathematicians around the world. The committee said after the meeting: "An ICM is a unique meeting place for mathematicians from all over the world to assemble, put political and cultural differences aside, and discuss mathematics" (see the statement at bit.ly/3vNjCwY).

"We knew we wanted the General Assembly and awards to take place on the scheduled dates and in person. And we would not accept any support from the Russian government, neither money nor staff or anything," said Holden. "And that left us in a very difficult situation. We didn't know where we would hold this, or how we were going to pay for this."

The IMU was soon approached by several member countries offering support, and decided to accept the generous offer to hold these events in Helsinki, Finland. "We were very happy to come to Helsinki in Finland, for many reasons" said Holden. "We have a long tradition with Finland — they held the ICM in 1978 and they had the IMU archives — and it is located between the East and the West. So it was the right place to go."

The General Assembly was held on 3–4 July along more or less along traditional lines with over 200 delegates from member countries of the IMU meeting in person in Helsinki. Among other important business, the General Assembly decided that the next ICM would be held in Philadelphia, USA, in 2026. The delegates also unanimously voted to establish a new reserve fund to temporarily cover the membership costs of member countries that are facing extraordinary hardship, such as that currently faced by Ukraine.

An ICM like no other

Next came an unprecedented event. "Welcome to the IMU awards ceremony — the first to be held in over 100 years of the IMU," said Carlos Koenig on 5 July 2022. Instead of occurring in front of thousands of mathematicians at a full in-person Congress, the IMU awards ceremony took place in front of only 600 mathematicians, with thousands more watching live online from around the world. Koenig celebrated the work of the mathematical community, both for making the event happen in such unusual circumstances, but also thanking them for their involvement in championing the work of mathematicians in the decision process for the IMU awards: "This generosity of spirit is heartwarming and should make us all proud to be a part of this community."

The winners of the Fields Medals were Hugo Duminil-Copin, June Huh, James Maynard and Maryna Viazovska, and Mark Braverman won the Abacus Medal (previously known as the Rolf Nevanlinna Prize). Barry Mazur received the Chern Medal, Elliott Lieb the Gauss Prize and Nikolai Andreev the Leelavati Prize. You can see summaries of some of their work elsewhere in this newsletter.

Some aspects of the event felt particularly of our time, with appearances of some prize winners and laudators over zoom, with the usual accompanying technical hiccups. But it was a joyous event, despite the difficult and unprecedented circumstances. The laudations and short films about the winners gave a sense of their work, their motivations and their personal stories. The Fields Medal being awarded to Viazovska felt particularly significant, as she is only the second woman to be awarded this prize, after Maryam Mirzakhani in 2014. And as a Ukrainian mathematician Viazovska's film was particularly moving, describing the impact the war has had on her family and all their lives, alongside her passion and commitment to her mathematics.

The ceremony was followed by a day of lectures from the winners of the Fields and Abacus medals, watched in-person by several hundred excited mathematicians at Aalto University, with thousands more joining virtually from around the world. "Normally the ICM has three or four thousand participants," said Holden. "We had a ceiling of 7,000 participants for our online platform, which 'sold out' quickly. The largest number of participants at any ICM." The virtual ICM had no registration costs, and obviously no travel costs, enabling a much broader distribution of participants to take part than ever before from more countries all over the world.

"This ICM is totally different, we have never done this before," said Holden. Thanks to the work already done by the scientific committee of the ICM, all the speakers and lecture programme were already settled. But Holden and his colleagues at the IMU had to find an online platform to create this virtual ICM before they even had the funding in place. "We had to sign the contracts without having the money, which made me uneasy! But it worked out." The programme followed Central European Summer time, which meant some participants, and some speakers, would be joining in the middle of their night. All speakers had the chance to pre-record their lectures, partly as a backup in case of any last minute technical difficulties.

The virtual ICM has been a huge success with registered participants attending virtually live, and absolutely anyone able to watch the lectures afterwards on the IMU YouTube channel (see bit.ly/3PcBnwt; to decide what to watch, you can find the details of the programme at bit.ly/3BRwOFa). This open access to the lectures has been such a success that the IMU plans for all lectures at future ICMs to be recorded and made freely available online.

Another important lesson Holden said he had learned from this year's unprecedented ICM was: "We don't want a virtual ICM ever again! And important part of an ICM is to meet, to go for a beer and discuss what is happening in your field and in your university and with your colleagues. That is more important in some sense than the lectures."

This social aspect of mathematics came across in the work of so many of the mathematicians, and the importance of their close collaborations was clear in the way many of the prize winners described their mathematical lives. And despite a global pandemic, and a barbaric war, the mathematical community managed to come together to share and celebrate their mathematical research.

Mark Braverman, winner of the Abacus Medal, captured the spirit of the community well in the opening remarks of his lecture: "Putin's action over last years is profound and deep nihilism. Nihilism is like a nilpotent operator — if you try to invert it it will explode. You can't fight it with drones and rockets. The way to fight it is with important activities that celebrate life and the human spirit. And research and mathematics is definitely one of those. This event, [the ICM], had to go on not despite the war, but because of it."

As Editors of plus.maths.org we had the pleasure to talk to laureates in the run-up to the ICM 2022. Find out what they told us about their work in the following articles. This content was produced as part of the collaboration between plus.maths.org and the London Mathematical Society. You can find all our content on the 2022 International Congress of Mathematicians at bit.ly/3QqqXLk.

Marianne Freiberger and Rachel Thomas

Rachel and Marianne hard at work at the opening day of the ICM

Marianne Freiberger and Rachel Thomas, Editors of plus.maths.org, work with researchers to communicate their mathematics to a variety of audiences. They work as part of the Millennium Mathematics Project, based at the University of Cambridge.

Apart from editing plus.maths.org, Marianne and Rachel have written and edited several popular science books, worked with local and national radio, TV and streaming media and various newspapers and magazines. Between them they have over 35 years of experience writing about mathematics for a general audience. After studying semigroup theory at the University of Western Australia, Rachel was a maths consultant working for government and industry. Marianne did her PhD in complex dynamics at Queen Mary, University of London.

THE FIELDS MEDALS 2022: HUGO DUMINIL-COPIN

By Rachel Thomas, produced as part of the ICM coverage on plus.maths.org

Hugo Dumini-Copin (Photo Matteo Fieni)

Hugo Duminil-Copin, of the University of Geneva, has won one of this year's Fields Medals for "solving long standing problems in the probabilistic theory of phase transitions in statistical physics, especially in dimensions three and four".

Phase transitions and universality

When we spoke to Duminil-Copin in the run up to the International Congress of Mathematicians, he told us that he, like many kids, had wanted to understand how the world works and why it works that way. But he was also strongly drawn to the reassuring purity and finality offered by mathematical proofs, where you could build a "castle that wouldn't collapse, that wasn't made of cards. I always had this balance of these two loves and not really knowing how to reconcile them." Happily, while studying mathematics, Duminil-Copin discovered the area of statistical physics: "For me it was 'Oh my god what just happened!' Something I was looking for for years was just in front of me, an area at the crossroads of the two things I wanted!"

One application of statistical mechanics is the Ising model, originally developed in the 1920s to describe phase transitions in magnets. It represents the magnet as a set of pointlike dipoles arranged on a regular lattice. Ising-type models extend this approach to a wider range of situations. Duminil-Copin has been recognised for his work transforming the mathematical theory of phase transitions in statistical physics, and in particular for his results for Ising-type models in dimensions three and four.

Although Ising-type models are deeply unrealistic 'caricatures' they allow you to explain a wide range of physical phenomena, Duminil-Copin explained: "It's something related to a very deep phenomenon — *universality* — that I try to understand as a mathematician."

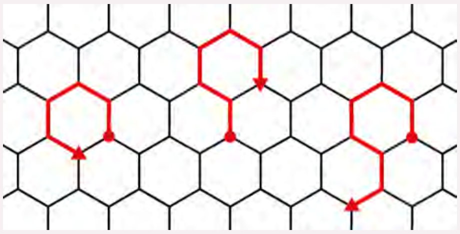
Universality is almost like wishful thinking come true: in certain situations the fine details of a mathematical model don't affect its global behaviour. The reason is that if a system involves many different random processes, such as many water molecules moving around, then the details of the underlying mechanisms should not matter. In the example of water freezing to ice, you can choose whatever arrangement of molecules you like — assume they are positioned regularly in a lattice of your choosing — and the phase transition you are studying will have the same properties regardless of your choice of lattice.

"This is very reassuring for mathematicians and physicists because it tells you that many systems in the end have the same behaviour, and you can choose the simplest example of such systems, which is those that lie on a lattice."

Beautiful problems

Another motivation for Duminil-Copin is that statistical physics offers problems that he is particularly drawn to: ones that are deceptively simple to state but require new mathematics to solve. He told us about one example that was one of the first conjectures he learned about when he was a postdoc, concerning self-avoiding walks (SAW) on a hexagonal lattice. How many SAWs are there of a given length?

As Duminil-Copin says, the rules are so straightforward a child can attempt the problem, but the complexity of the problem quickly appears. It's clear the number of possible SAWs grows exponentially with the number of steps taken, but as you take more steps it's increasingly hard to keep track as you try to make sure you never retread your steps. "You realise very quickly that you cannot compute exactly this number, it's a very difficult number to grasp."



Examples of self-avoiding walks of length 5, 6 and 7 (image from plus.maths.org)

This question isn't just of interest to mathematicians. In the 1940s chemists Paul Flory (who won the Nobel Prize for Chemistry in 1974) and W.J.C. Orr introduced SAWs as a way to study long chain-like molecules — *polymers* — and understand how they behave. "It's very related to physical phenomena, for example trying to understand what polymers, such as DNA molecules, are doing. These polymers are self-avoiding [walks] for obvious reasons: they are long sequences of molecules that can't be at the same place."

There's no exact answer known for how fast the number of SAWs grow on any lattice, but for the hexagonal lattice, we can get very close to an answer. In 1980 the statistical physicist Bernard Nienhuis conjectured that the rate of growth of the number of SAWs on a hexagonal lattice is almost $(\sqrt{2+\sqrt{2}})^n$ for a large number of steps n.

"I found it truly fantastic that there is an answer, and it's a really cool number!" Duminil-Copin says. "This was a conjecture I first learned about in my masterclass. It's funny because at the time it looked like there was no hope the conjecture could be proved — I discussed it with my PhD supervisor and we both agreed it was a terrible idea to try to prove it."

But prove it he did, using techniques from an unexpected area: complex analysis. Duminil-Copin was working on problems in complex analysis, seemingly disconnected with self-avoiding walks, when he started to understand some of the ideas that arose in the conjecture. "At some point it just started snowballing and we got this cool proof," he says. (You can get into some of the details of this proof in the lovely article by Duminil-Copin at bit.ly/3SCUDGp.) "This is a typical example of problems that we have in our field where you get inspired by many other fields of mathematics and physics. It puts you at the crossroads of many places [and] that is something I like very much."

Conformal invariance

Turning again to understanding phase transitions, initially it was physicists who made most progress, says Duminil-Copin, but "now it's almost like the other way around, where the mathematical [contribution] is very strong." One example is the recent progress by Duminil-Copin and colleagues in understanding *conformal invariance:* the rich set of symmetries that comes from invariance under any conformal (angle-preserving) map. "The understanding of conformal invariance in two dimensions for mathematicians has progressed so much that it is now shedding new light on the physical theory," he says.

Proving conformal invariance has been a very active research area, but since 2000 only a handful of specific models (for example, for just a few particular types of lattices) have been rigorously proved to be conformally invariant. "There was a list that could be counted on one hand of models that were conformally invariant," says Duminil-Copin.

To make things easier Duminil-Copin and colleagues focussed on rotational symmetries in two-dimensional models. Consider our example of SAWs on a hexagonal lattice, and suppose you are interested in the number of SAWs between a starting and finishing point on your lattice. Then it's clear that this number will be the same if you rotated your finishing point by $2\pi/3$ around the starting point — that's just a symmetry inherent in the lattice itself. "One marvellous property of systems at criticality, when phase transitions occur, is that the system gains more symmetries," says Duminil-Copin. As a consequence it was believed that a system undergoing a phase transition would become rotationally invariant under any angle, not just the obvious one you see far away from criticality.

Duminil-Copin and colleagues were able to provide a rigorous mathematical proof of this rotational symmetry for a much larger group of models. What's more, their methods could provide the missing ingredient that could lead to proofs of full conformal invariance, and all the mathematical power this would bring. (You can find out more about Duminil-Copin's work, read the articles provided on the website of the International Mathematical Union at bit.ly/3vUtvJl.)

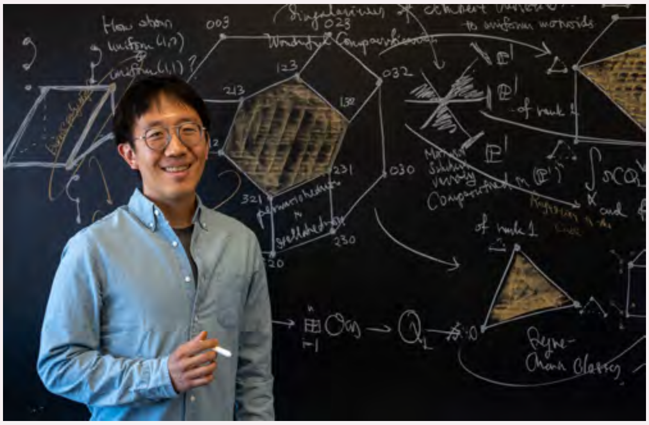
Sharing mathematics

Duminil-Copin sees his Fields Medal as recognition for all of those working in his area and the work they are developing together, and was looking forward to finally sharing this recognition with his collaborators once the prizes were announced at the ICM. "Mathematics is a very social activity, much more than people believe," he says. "There is this image of the mathematician as the lonely hero, but in my case it's not a vision of my mathematics or my way of doing mathematics. My work would not happen without this interaction with others."

Interview conducted May 2022

THE FIELDS MEDALS 2022: JUNE HUH

By Marianne Freiberger, produced as part of the ICM coverage on plus.maths.org



June Huh (Photo: Lance Murphey)

June Huh of Princeton University has won one of this year's Fields Medals "for bringing the ideas of Hodge theory to combinatorics, the proof of the Dowling–Wilson conjecture for geometric lattices, the proof of the Heron–Rota–Welsh conjecture for matroids, the development of the theory of Lorentzian polynomials, and the proof of the strong Mason conjecture."

Not your average mathematician

It's tempting to think of all brilliant mathematicians as prodigies. They excelled at the subject early on, won all the maths prizes at school, and had their path to greatness plotted out before them.

June Huh is different. At elementary school his grades in maths weren't great, and high school got him so bored, he chose to write poetry instead. When he finally did find his way to maths, it wasn't the topic itself that drew him, but a person. Around the time Huh was about to graduate from the Physics and Astronomy course he had ended up taking at Seoul National University, he noticed that the famous mathematician Heisuke Hironaka was giving a lecture course on algebraic geometry. "I knew nothing about algebraic geometry, but I had read Hironaka's autobiography. He's an interesting person, so I enrolled," he told us when we spoke to him in the run-up to this year's ICM.

The course turned out to be a real-time reflection on Hironaka's research, reporting on insights he had literally had the day before. "This was the first time I actually saw someone doing maths," says Huh. "It was my first exposure to mathematics as a human activity." The thrill of that human activity is what got Huh hooked.

Counting and seeing

Huh's way of talking about his mathematics is beautifully intuitive. Because he didn't have much maths training, he started his career focussing on objects "visible to the naked eye," as he puts it. "A lot of areas in modern mathematics are so developed that just to understand the central questions in that area you have to invest several years," says Huh. "It's like studying the kind of astronomy you can only do when you have a million dollar telescope."

An area of maths that's not like this is combinatorics, one of Huh's first loves. "All the objects are tangible — you can almost see them and touch them," he says. "Combinatorics was the part of mathematics that was visible to *my* eyes."

Geometry, Huh says, is equally attractive to a newcomer. "Geometry is intrinsically interesting. We are visual creatures, vision is our primary sense organ. We understand the world around us through geometry as opposed to sound, taste or smell." But because geometry generally is quite hard to formalise (just think of the effort it takes to define a manifold) Huh felt drawn to algebraic geometry. "In algebraic geometry, in order to specify a space, all you have to do is write down a [polynomial] equation. You can write it down in your notepad and look at it — it's something you can touch. These are the only kinds of spaces I could really put my hands on, at least in the beginning. That's why algebraic geometry also appealed to me."

From continuous to discrete and back

Many of Huh's results have bridged the gap between the combinatorial and the geometric, the discrete and the continuous. When asked about a result he is particularly proud of he points to his work with Petter Brändén on *Lorentizian polynomials*.

"There's a well-developed subfield of analysis called *convex analysis*, which studies convex functions or bodies," he explains. "It is used to understand the optima of various different types of problems, even [problems] with practical applications."

The reason why convex analysis is useful in optimisation theory is that convex functions have useful properties as far as their minima are concerned. For example, strictly convex functions over an open set have at most one minimum. When finding this minimum isn't straightforward, there are efficient algorithms to do the trick. *Gradient descent algorithms*, for example, inch their way towards a minimum from a given starting point just like a hiker might inch their way down from a mountain in a fog: they feel the region around them to see which direction is down, head a few steps in that direction, then feel again. If a function has several minima then this method might not work because it might land you in a local dip, rather than at a global minimum. But the existence of efficient algorithms means that many convex optimisation problems can be solved in polynomial time.

By its very nature, convex optimisation works when continuous functions are involved. "But of course our world is becoming more and more digital, often we want to optimise discrete problems," says Huh. "So several decades ago there was an attempt to develop the analogous theory in a discrete setting. For this you need a completely different technique." Discrete convex analysis is an analogy of the traditional theory for the discrete setting, but until Huh and Brändén came along it was just that: an analogy that didn't exhibit any direct connections to classical convex analysis.

The theory of Lorentzian polynomials that Huh and Brändén developed explicitly links the two subfields. A polynomial of n variables is a continuous function in the traditional sense, but you can also view it in a discrete way: as a function that takes a point in the discrete space of all monomials of n variables to the coefficient that this particular *monomial* has in the original polynomial. (A monomial here is a product of powers of the variable involved, such as x^4y^{17}.)

"What is amazing is that if you look at things in the framework of Lorentzian polynomials, the usual notion of continuous convexity and the notion of discrete convexity are naturally linked together in exactly the same way as you can view the polynomials in two different ways," explains Huh. "Finding this formal bridge was very satisfying. What was even more pleasant for us is that once you have this bridge you can approach problems that were considered very technical and difficult in a very natural and easy way."

For more on Huh's work, see the articles on the IMU website (bit.ly/3w11cJc).

Mathematics as mirror

Great advances in maths often happen when someone makes a connection between seemingly different areas. In some sense, though, Huh doesn't think we should be surprised by those connections. "It's not surprising because the subdivision of [mathematical] areas, or the subdivision of human intuition — combinatorial, geometric, and analytic — are just a result of millions of years of our experience as a species and the kind of sense organs we have. If we were a different kind of creature with a different kind of sense organs and a different kind of environment, we'd presumably have [developed] completely different areas of mathematics."

If the boundaries between the areas of maths are accidental, then it's not surprising that some of the deepest questions in maths transcend them. In that sense, the maths we develop is a mirror of our humanity. "It shows you who we are and how we think."

Interview conducted June 2022

THE FIELDS MEDALS 2022: JAMES MAYNARD

By Rachel Thomas, produced as part of the ICM coverage on plus.maths.org

James Maynard (Photo: Ryan Cowan)

James Maynard, of the University of Oxford, has won one of this year's Fields Medals, for "contributions to analytic number theory, which have led to major advances in the understanding of the structure of prime numbers and in Diophantine approximation."

The twin prime conjecture

"Number theory always had a slightly special status in my mind, even before I formally learned about it," said Maynard when we spoke to him in the run-up to this year's ICM. Maynard's main interest concerns the distribution of prime numbers and he has obtained many remarkable results in this direction.

Understanding the gaps between prime numbers is fundamental to understanding the distribution of the primes. Typically as you go down the number line the gaps between prime numbers get bigger, but the twin prime conjecture says there are infinitely many pairs of prime numbers that are separated by a gap of two. "The twin prime conjecture is very much one of these simple to state fundamental questions about numbers that has fascinated mathematicians for hundreds of years but remained slightly out of reach," says Maynard. "The twin prime conjecture is one of the first steps in trying to understand the distribution of prime numbers – looking at how close together prime numbers can possibly be."

After centuries of effort on the conjecture there was a big breakthrough in 2013, when Yitan Zhang proved that there were infinitely many pairs of primes that are separated by 70 million. "For mathematicians this was absolutely a huge breakthrough as this was the first time we had [a proof of gaps of] any finite number," says Maynard. "70 million is much bigger than 2 but it's a lot smaller than infinity."

Zhang's breakthrough was based on important earlier work from 2005 involving a set of techniques known as *sieve methods*. "Sieve methods are a mathematical tool for translating some information you understand about numbers to create some information you'd like to know," says Maynard. "[For example] if you understand certain technical questions about primes in arithmetic progressions, then you can translate that to information about small gaps between primes." The caveat was that to prove these results about bounded gaps between primes you needed very strong results to feed into the method. Zhang's breakthrough involved proving strong results about primes that could provide that input.

Maynard's approach was different: "Rather than improving the input to the method I changed the method itself. It became more efficient in turning one type of information into another and it meant that we needed much weaker inputs to get a result about boundary gaps between primes." With this new method he dramatically reduced the gap from 70 million to just 600. And after a flurry of collaborative work by mathematicians on the Polymath project, we now know there are infinitely many pairs of primes separated by a gap of just 246.

Even after such dramatic progress a proof of the twin prime conjecture still remains elusive. Work continues, but often involves taking new approaches. An example is Maynard's work proving there are infinitely many primes without certain digits, a test case for new techniques for counting primes with particular properties. It's hard to know at this stage when the twin prime conjecture will finally be proved in full, but we came away from our discussion with Maynard feeling optimistic. "We're still one big idea away from proving the twin prime conjecture, but maybe we're only one big idea away."

The Duffin-Schaeffer conjecture

Maynard has also produced fundamental work in Diophantine approximation, having solved the Duffin-Schaeffer conjecture with Dimitris Koukoulopoulos. This conjecture, first posed in 1941, describes how well a typical real number can be approximated by rational ones.

Suppose you want to guarantee your approximations will have a certain accuracy, and this accuracy can vary with the denominator q used in your approximations, p/q. That is, given a real number x then there are infinitely many pairs of p and q that satisfy $\left|x - \frac{p}{q}\right| < \frac{f(q)}{q}$ for some specified function $f : \mathbb{N} \to \mathbb{R}^+$.

Then the Duffin-Schaeffer conjecture says that the above statement is true for almost all real numbers (with respect to Lebesgue measure) if and only if $\sum_{q=1}^{\infty} f(q)\frac{\varphi(q)}{q} = \infty$ where $\varphi(q)$ is Euler's totient function.

"The Duffin-Schaeffer conjecture says either you only have a rare set of exceptions to this statement about [Diophantine] approximations, or it is basically never ever true," says Maynard. "And [the conjecture] gives you a very simple step to know whether you are ok or not."

The Duffin-Schaeffer conjecture might not sound so useful at first, but Maynard and Koukoulopoulos' proof of it gives mathematicians a powerful tool. "There's lots of mathematical statements that you might wish were true for absolutely everything but it turns out there's a few annoying exceptions," says Maynard. "But provided those exceptions are suitably rare, the result tells us they don't actually matter."

Traditionally, approaches to proving the conjecture had focussed on proving an underlying question about prime factors of the specified denominators. Maynard and Koukoulopoulos reframed this as a question in graph theory, where the nodes of the graph were the denominators and the edges connected denominators that shared many prime factors. They were able to use results from graph theory to then deduce the information they needed to prove the conjecture. To find out more about Maynard's work, see the articles on the IMU website (bit.ly/3w11cJc).

Prizes for playing

Maynard's work is described in the citation for his prize as "highly ingenious, often leading to surprising breakthroughs on important problems that seemed to be inaccessible by current techniques." But despite his many successes, the Fields Medal still comes as a surprise. "You don't think of yourself as getting a huge mathematical honour when you're sitting at your desk playing around with mathematical bobbins!"

And while receiving one of the biggest prizes in mathematics is a huge honour, Maynard also finds it daunting, and slightly surreal. "In some ways it's intimidating thinking of my name on this list of legends of mathematics from the past. People I looked up to, when I was a kid and thinking about mathematicians," he says. "It's certainly quite surreal in that way."

Interview conducted June 2022

THE FIELDS MEDALS 2022: MARYNA VIAZOVSKA

By Marianne Freiberger, produced as part of the ICM coverage on plus.maths.org

Maryna Viazovska (Photo: Matteo Fieni)

Maryna Viazovska, of the EPFL in Switzerland, has won one of this year's Fields Medals for "the proof that the E_8 lattice provides the densest packing of identical spheres in eight dimensions, and further contributions to related extremal problems and interpolation problems in Fourier analysis."

The problem

How should you arrange equally-sized $(n-1)$-spheres in $\mathbb{R}^n$ so that as much space as possible is filled by the spheres and their interiors? The *density* of a sphere packing in a finite region of space is the proportion of the volume of the region filled by spheres and their interiors. The *upper density* of a sphere packing is what you get when you consider $\mathbb{R}^n$ as a whole: it is defined as the limit superior of the densities of the packings in balls of radius r, as r tends to infinity. The *sphere packing density* of $\mathbb{R}^n$ is the supremum of the upper densities of all sphere packings in $\mathbb{R}^n$.

A result by Helmut Groemer shows that in all dimensions n there exists a sphere packing which achieves the sphere packing density. The question is, what is this densest packing and what is its density?

It's a natural question for geometers to ask, and also one that is linked to practical applications. Johannes Kepler, who studied the problem in the 17th century, was inspired by stacks of cannonballs, while greengrocers through the ages have faced the finite version of the problem with oranges and apples.

But, perhaps surprisingly, higher-dimensional sphere packings have applications too. If you want to construct a code to transmit messages over a noisy channel, you're best off choosing codewords that aren't too similar — that way you can still tell them apart even if they're slightly scrambled in transit. If your code words are points in n–dimensional space, you'd therefore like them to lie at centres of non-overlapping error balls of a particular size. The more codewords you have, the easier the communication, which is why you'd like to pack as many error balls into n-dimensional space as possible. That's why sphere packings play an important role in the theory of *error correcting codes*. And as we will see, this theory has in turn inspired the methods underlying Viazovska's proof.

Low dimensions

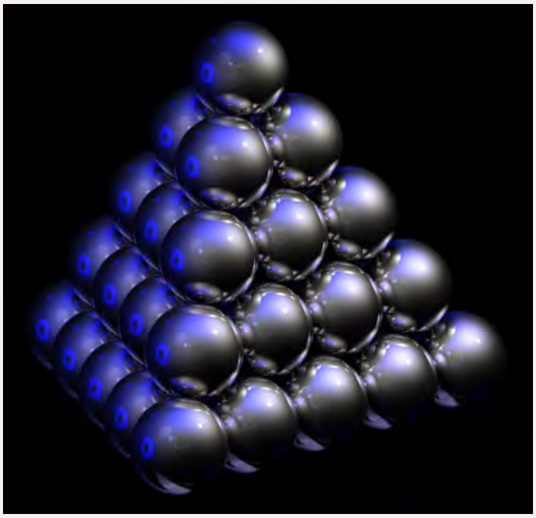
In dimension two the answer to the sphere packing problem comes from centring discs at the points of a hexagonal lattice, chosen so that neighbouring discs are exactly tangent. The honeycomb-like packing that arises comes with a density of

$$\frac{\pi}{\sqrt{12}} = 0.9068...$$

"In dimension three [the problem] was known as Kepler's conjecture and remained open for [nearly 400] years," Maryna Viazovska told us in an interview in 2018. "[Here] we don't have only one best packing, we have many equally good packings. One of them you can see on the market, where oranges are stacked in pyramids. The density [of this packing] is about 74%."

The arrangement Viazovska is talking about is known as the *face-centred cubic packing*. The precise value of the sphere packing density in three dimensions is

$$\frac{\pi}{\sqrt{18}} = 0.7404...$$



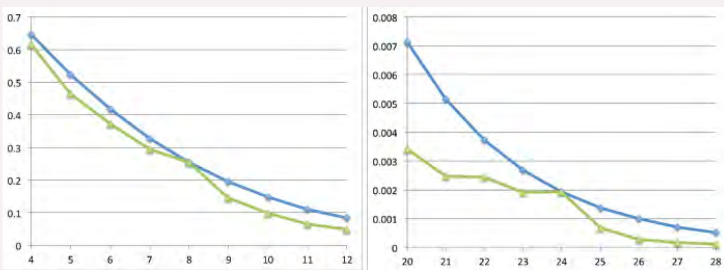
The face-centred cubic packing, one of the packings with maximal density in dimension three. (Image: Greg A L, CC BY-SA 3.0)

The proof that you really can't do any better in three dimensions famously arrived in 1998 when Thomas Hales produced 250 pages of traditional proof supplemented by over 3 gigabytes of computer code and data for making necessary calculations. It was humanly impossible to check the entirety of this exhaustive proof, but referees eventually decided they were 99% sure it was correct. Since then the proof has been verified using formal computer logic.

Finding bounds

Viazovska's proof for dimension 8 is an entirely different beast from the one produced by Hales for dimension 3, involving no computers and stretching to a mere 22 pages. It is based on the work of Henry Cohn and Noam Elkies who in 2003 came up with a theoretical method for finding upper bounds for sphere packing densities in any dimension. The method builds on a body of work started by P. Delsarte, who in 1972 developed a technique for finding bounds on the size of error-correcting codes using linear programming.

Unfortunately, Cohn and Elkies method is so hard to put into practice that they were only able to approximate the upper bounds, and they did this for dimensions up to 36. Results are shown in the graphs below (the blue line), together with the best-known values for the sphere packing density (the green line), for dimensions 4 to 12 and 20 to 28.



The Cohn-Elkies upper bound (blue) and the density of the best-known packing (green) for dimensions 4 to 12 and 20 to 28

While the two graphs appear to coincide for dimensions 8 and 24, Cohn and Elkies weren't able to prove this: there remained the annoying possibility that the true value of the sphere packing constant in these dimensions lay within the invisible gap between the graphs. "That's not a plausible scenario for anyone with faith in the beauty of mathematics, but faith does not amount to a proof," Cohn wrote in a beautiful article in the *Notices of the American Mathematical Society* in 2017 (bit.ly/3QmEIe3).

Closing the gap

Viazovska's work, which closed the gap for dimension 8 and was later extended with the help of Cohn, Abhinav Kumar, Stephen D. Miller, and Danylo Radchenko to dimension 24, builds on the centrepiece of Cohn and Elkies' work.

The existence of the upper bound suggested by Cohn and Elkies hinges on the existence of a radial Schwartz function which, together with its Fourier transform, must satisfy certain properties. If such a Schwartz function exists, then the value of the upper bound is the volume of a ball of radius $r/2$, where r is a real number related to the Schwartz function.

If, in addition, the Schwartz function vanishes at the non-zero points of a lattice with minimal vector length r, while its Fourier transform vanishes at the non-zero points of a corresponding dual lattice, then the upper bound for the sphere packing density is sharp and achieved by the sphere packing arising from the lattice. A starring role in the proof of this result is the *Poisson summation formula*, which relates the sum of a function over a lattice to the sum of the function's Fourier transform over a dual lattice.

A lot hinges on the 'magic' Schwartz function, then, but this is where the problem lies: controlling a function and its Fourier transform simultaneously is so difficult, the magic function tends to remain elusive. "Our inability to control [the magic function] without losing control of [its Fourier transform] is at the root of the Heisenberg uncertainty principle, and it's a truly fundamental obstacle," Cohn wrote in the *Notices of the AMS*.

> **The E_8 lattice sphere packing**
>
> The spheres in this eight-dimensional packing are centred on points whose coordinates are either all integers or all lie half way between two integers, and whose coordinates sum to an even number. The radius of the spheres is $1/\sqrt{2}$.
>
> The E_8 lattice is related to the exceptional Lie group E_8. As the name suggests the group is an exceptional object in mathematics, so it's perhaps not surprising that it is connected to an exceptional sphere packing.

It is this task Viazovska achieved for dimension eight in a 'bold construction' with the E_8 lattice (see box) providing the optimal packing. The E_8 lattice is "magnificently symmetric", it is its own dual lattice, and had long been conjectured to provide the densest sphere packing in dimension 8. Viazovska's method, however, was completely new, using the theory of modular forms to construct a magic function with just the right properties. Her result proves that the sphere packing density of $\mathbb{R}^8$ equals the density of the E_8 lattice, namely

$$\frac{\pi^4}{\sqrt{384}} = 0.2536\ldots$$

Together with Cohn, Kumar, Miller, and Radchenko, Viazovska then applied a similar technique to find the sphere packing density for $\mathbb{R}^{24}$. The underlying lattice here was discovered by John Leech in the 1960s, when he realised that a sphere packing related to a specific error correcting code could be made denser. Incidentally, the code in question was later used to transmit images of Saturn and Jupiter taken by the Voyager probes.

Viazovska and her colleagues showed that the Leech lattice provides the densest packing in dimension 24, with the sphere packing density equal to

$$\frac{\pi^{12}}{\sqrt{12!}} = 0.001929\ldots$$

Mysterious dimensions

So what makes dimensions 8 and 24 so special? "Everybody asks what is special about dimensions 8 and 24 — I don't know, it's a mystery," Viazovska told us. "In these dimensions we have these two extremely great configurations, which we don't have in other dimensions. They are so good that methods which fail in all other dimensions, in these dimensions give a sharp estimate. If you ask me why, I don't know."

Perhaps the reason for this will become clear in the future, but for now Viasovska's proof for dimension 8, and the follow-up for dimension 24, was spectacular enough to win Viazovska one of the highest honours in mathematics. She is only the second woman to receive a Fields Medal, following on from Maryam Mirzakhani who won it in 2014. Whatever the solution for the other dimensions will turn out to be, it will probably earn its discoverer another prestigious prize.

Interview conducted in 2018

THE ABACUS MEDAL 2022: MARK BRAVERMAN

By Rachel Thomas, produced as part of the ICM coverage on plus.maths.org

Mark Braverman (Photo Lance Murphey)

Mark Braverman of Princeton University has been awarded the 2022 Abacus Medal for "outstanding contributions in Mathematical Aspects of Information Science". The Medal used to be called the Rolf Nevanlinna prize, and is awarded every four years at the International Congress of Mathematicians.

Braverman won the prize for his "path-breaking research developing the theory of information complexity." When we spoke to him in the run-up to this year's ICM, he told us his overall goal is to understand computation, both because it is a fundamental intellectual pursuit, just like studying black holes or prime numbers, but also because computation is now embedded in our daily lives. "Computers and communication are now so cheap they're part of many devices — even a toaster is a computer!"

From a mathematical theory of communication...

"Shannon's classical information theory is a beautiful area that explains communication," says Braverman. "There are still open problems, but communication is mathematically extremely well understood."

Braverman is referring to the mathematician Claude Shannon, who developed many of the key ideas used in digital communication today in his groundbreaking 1948 paper, *A mathematical theory of communication*. Shannon realised that binary digits, better known as bits, lay at the heart of information technology. Any type of information, be it pictures, music or words, can be encoded in strings of these 0s and 1s. Shannon worked out the minimum number of bits you need to encode the symbols from any alphabet, be it the 26 letters we use to write with, or the numbers that encode the colours in a picture.

Given an alphabet of n symbols and a probability distribution telling you the probability p_i with which a symbol i occurs in a text made out of those symbols, the number

$$H = p_1 \log(1/p_1) + p_2 \log(1/p_2) + p_3 \log(1/p_3) + \ldots + p_n \log(1/p_n)$$

is called the *entropy* of the distribution. Shannon proved that the average number of bits needed per symbol cannot be smaller than the entropy, no matter how cleverly you encode them.

If you're in the business of sending messages long-distance, then Shannon's entropy is a useful number to know. If you know you can transmit some number, say C, bits per second, and that the symbols in your message require around H bits per symbol on average, then you'd guess that you can transmit around C/H symbols per second on average. Shannon showed that this is indeed correct: by picking a clever way of encoding your symbols you can guarantee an average transmission rate that's as close to C/H per second as you like. Conversely, no matter how you encode your symbols, it's impossible to transmit at a faster rate than C/H. This result is known as Shannon's *source coding theorem*.

...to a mathematical theory of computation

The clear mathematical framework developed in information theory has led to communication being very well understood, with clear mathematical bounds on things like how much information you can transmit.

"You can use Shannon's entropy to define *channel capacity*," says Braverman. This is the maximum rate at which information can be communicated over some medium, such as via email, over the radio, or downloading to your phone. "Mathematically it makes it easier and you get very precise answers to questions like 'How long would it take me to transmit a billion bits over this medium?' You divide a billion by your channel capacity and you get your answer."

In contrast, a similar mathematical framework is not nearly so well developed for studying computation. Braverman compared the precise answers available for communication to the open fundamental questions in computation, such as *computational complexity:* how long a computational task, such as factoring a number, will take. Not only do we not have any algorithms that can reliably factor numbers in polynomial time, we don't even know if such an algorithm exists.

Braverman's groundbreaking idea was to bring communication into the picture. He is being awarded the Abacus Medal for his development of the theory of *information complexity*, the interactive analog of Shannon's information theory. "In hindsight it's almost obvious that information theory should be a core tool, and hopefully now it's a little more central to our understanding," says Braverman.

Communication complexity

One example of this approach is studying the *communication complexity* of a task, where instead of asking how long the task will take to compute (the computational complexity), you instead ask how much communication you need to compute the task.

"Communication is an important part of computation," says Braverman. There are many scenarios where a task is distributed, say between a number of servers holding a distributed data set, and you need some sort of coordination between the different servers working on the task. "In practice it's often not the local processing that is the bottleneck [in a task], but moving the data back and forth between the worker and manager computers."

A simple example is one where two parties, say you and Braverman, are trying to do some task together. Perhaps you have a file X, and Braverman has a file Y, and you both need to know if these files are identical. You could always just send your whole file to Braverman to compare with his, but communication complexity asks if there is a way of achieving this task that requires fewer bits of information to be communicated between you, for example by creating a *hash* of the file.

To define things more formally, the exchange of information between parties needed to complete a task is called a *protocol*, the *communication cost* of a protocol is the number of bits communicated during the completion of the task, and the *communication complexity* of a task is the smallest possible communication cost of a protocol that completes that particular task.

Information complexity

There are practical reasons, such as bandwidth limitations, to limit the amount of communication between parties. But in some settings the most important concern is parties being able to share aspects of their data without revealing too much information to each other. "It's a major objective to be able to do computation without revealing information," says Braverman. For example, regulators might require access to data from private organisations to prevent or detect crime, but should not be given access to all information a company holds. This led Braverman to think about revealing information from a theoretical standpoint. The information cost of a protocol is the amount of information the two parties learn about each other's inputs in order to successfully complete a task. And the information complexity of a task is the smallest possible information cost of a protocol for completing that task.

"In information complexity the goal is for the parties to teach each other as little as possible about their files," Braverman says, referring to our file comparison example above. "And it turns out, at least for problems involving two parties, you get a nice picture in the sense that the information complexity behaves in a very similar way to Shannon's information entropy."

Shannon's information theory applied to one-way communication, whereas the framework built by Braverman describes interactive communication in a way that provides a more fundamental understanding of computation. Braverman has proved important results in information complexity, understanding its links to communication complexity, and showing that it can be thought of as the interactive analog to Shannon's information theory. This work is relevant to other settings which depend on interactive communication including applications that are important in real life such as memory requirements in streaming algorithms, distributed error correction and information security.

Fundamental understanding

A prestigious prize such as the Abacus Medal, Braverman feels, also brings with it the responsibility to help set the direction of his field. “Theoretical computer science is very exciting because it’s between the deepest maths and applications that are developing at breakneck speed,” says Braverman. “But it’s kind of tricky — how do you maintain long term focus while staying relevant, while also not getting pulled into the latest trend?”

This dual nature of the field is something that Braverman feels might be unique: “Some fields have this feeling of a Buddhist monastery — where it is looking to the very long term. And some fields exist in the present, driven by immediate needs such as the high commercial demand for applied machine learning research, or important needs like developing COVID vaccines or cutting edge cancer therapies. Here it’s kind of both. Theoretical computer science might be unique in that it is simultaneously very slow and very fast”

For those of us outside of computer science we might think the biggest motivation would always be the applications, but it is the field’s deep questions that motivate Braverman. “My main goal is a mathematical tool for understanding computation,” he says. And this understanding could shed some fundamental insights.

“We’ve seen the stars for hundreds of thousands of years, we’ve had the natural numbers for 5000 years, and we’ve only been thinking of computation for about 100 years. But somehow, if we met aliens, they’d have looked at black holes, they’d have thought about prime numbers, and they probably have thought about computation. It’s hard to imagine being advanced and not realising that [computation] is this basic process that you can abstract. Computation is basic, and it’s important to understand its properties in the same way it is important to care about if the Universe is expanding or whether the Riemann hypothesis is true.”

Interview conducted June 2022

THE GAUSS PRIZE 2022: ELLIOTT LIEB

By Marianne Freiberger, produced as part of the ICM coverage on plus.maths.org

Elliott Lieb (Photo: Lance Murphey)

Elliott Lieb of Princeton University has been awarded the 2022 Gauss Prize at the International Congress of Mathematicians. The Prize is awarded every four years at the Congress to honour scientists whose mathematical research has had an impact outside mathematics.

Lieb won the prize for "outstanding contributions to physics, chemistry, and pure mathematics." One thing that links these disparate areas is something we all know even though we can't see it: that all matter in the world, including ourselves, is made up of a multitude of individual atoms and molecules. Everything we could care to explore in the physical world can be viewed as a system of many particles.

Lieb was interested in many particle systems "from the get go," he told us in the run-up to the Congress. "Whatever talents I have, they seem to fit in with this area. I didn't choose it, it chose me."

A magnetic attraction?

Although Lieb's career started with a PhD in mathematical physics from the University of Birmingham in 1956, the true power of mathematics only became clear to him when he took a position as a staff theoretical physicist at IBM in 1960. "I met two physicists here, Ted Schultz and Dan Mattis, who were young guys like me," he recalls. "We did mathematical things together, and all seemed to have an interest in proving things."

One thing Lieb and his new colleagues looked at was Werner Heisenberg's theory of *ferromagnetism*, developed in the 1920s. As you would expect, the theory worked in three dimensions, but Lieb and his colleagues considered the one dimensional set-up. "From a theoretical point of view, the effect [Heisenberg described] should be more pronounced in one dimension than in three dimensions," he explains. What Lieb and Mattis were able to prove, though, was that this was not only false, but that the reverse was true: rather than ferromagnetic, a one-dimensional system becomes *antiferromagnetic*. "So the Heisenberg theory, though there may be a germ of truth in it, by itself did not sufficiently explain ferromagnetism."

The result upset Lieb's teachers, which included Nobel-laureate-to-be Hans Bethe and the eminent Rudolf Peierls. "They didn't believe it at first, though later they came around to it," Lieb recalls. "This was the first time I truly applied mathematical thinking to physics. It made us think, 'this is good, we should keep going'."

The stability of matter

From then on Lieb applied his mathematical prowess to a vast range of problems in a career spanning seven decades and still going strong. He is modest about his name being attached to an astonishing number of results. "I am sorry to call it that, but it's how it's become known," is what he says when talking about them.

When it comes to atoms and molecules, the force of gravity is negligible, but there are intermolecular forces at play. “The question is, if I have an enormous number of atoms banging around, for example in a box full of gas, why doesn’t this stuff just stop behaving like atoms and condense?” he says.

The Lieb–Thirring and Lieb–Oxford inequalities are both of importance in this question. Derived from Schrödinger’s equation, both inequalities tell us about forms of energy involved in a system. They enabled Lieb and Walter Thirring to show that the stability of matter is not unexplained, but a consequence of accepted theory, improving on work that had previously been done by Freeman Dyson and Andrew Lenard.

Into the future

Proving the stability of matter is important from a fundamental view point, but many of Lieb’s results have also been useful in a practical sense. The Lieb–Oxford inequality is an example. For industrial purposes, for example in material science and chemistry, people often need to simulate the behaviour of many particle systems. The Lieb–Oxford inequality serves as a reality check here, says Lieb. “If you make a calculation and find that you have [broken the inequality] then you know you need to start over again. So the [inequality] is useful to people. There is lots of money attached to this.”

Another area Lieb has contributed to, which may find real-world applications with quantum computers, is *quantum information theory*. The analogue of the *Shannon entropy* (which measures the amount of information contained in a classical system) in quantum information is *Von Neumann entropy*. An important result Lieb was able to prove together with Mary Beth Ruskai, using some beautiful mathematical theory Lieb had developed previously, describes how the Von Neumann entropies of quantum systems that are all part of a larger system relate to each other. The result, known as strong *subadditivity of quantum entropy*, underlies many of the important results in quantum coding theory that people have come up with since.

Hard to rival

While this article barely scrapes the surface of Lieb’s contributions, one thing that’s clear is that mathematics has been a driving force. “I consider myself a mathematical physicist,” he says. “I believe that mathematics, especially pure mathematics, has a meaning and use in physics and should be encouraged.” The conversation also goes the other way around: some of the tools that Lieb has developed to study physics have found important uses in other areas of mathematics — functional analysis, knot theory, and matrix analysis for example. You can find out more about Lieb’s on the IMU website (bit.ly/3vUtvJl).

The Gauss prize citation puts it most succinctly. “Elliott H. Lieb’s work has a truly outstanding combination: as mathematical work, his contributions have a hard-to-rival impact on other sciences, and as applied work, his contributions have a hard-to-rival mathematical depth. It continues the tradition of a dialogue at the highest level between mathematics and physics, and beautifully demonstrates the power of mathematics as a theoretical and practical tool to understand nature.”

Interview conducted in May 2022

THE CHERN MEDAL 2022: BARRY MAZUR

By Marianne Freiberger, produced as part of the ICM coverage on plus.maths.org

Barry Mazur (Photo: Lance Murphey)

This year's Chern Medal has been awarded to Barry Mazur of Harvard University for his "profound discoveries in topology, arithmetic geometry and number theory, and his leadership and generosity in forming the next generation of Mathematicians."

From radio to knots

Mazur got into mathematics because he loved radio. "The physics of radio, the mystery and magic of that physics, got me hooked when I was 12 years old," he says. At high school he became an amateur radio operator, beaming messages in Morse code to other operators around the world. "I was fascinated by how this 'action at a distance' could possibly happen, and was dubious about any explanation of it. I thought of myself as a 'philosopher of electronics', and enrolled at MIT hoping to follow a path that had such a description. But the minute I got there, I discovered that what I thought was 'philosophy of electronics', everybody else called mathematics."

Once that realisation had sunk in, Mazur headed straight for pure mathematics, falling in love with knot theory specifically and topology more generally. "Knot theory is so wonderful, if you're into it you get a sense that your geometrical intuition is ever expanding," he says. His PhD thesis focused on a higher-dimensional version of the Jordan Curve theorem, often referred to as the Schoenflies Problem, and produced one of many mathematical notions that now carry his name: the Mazur swindle. It's called that way because it evokes the famous "illegal" summations of Grandi's series which purport to show that 0 equals 1. Used in the appropriate setting, Mazur showed, an analogue of the argument turns out to be perfectly legitimate and helps you reduce a supposed countable infinity of objects to just one.

Mazur's PhD work led to a foray into dynamical systems theory with Michael Artin, which involved the algebraic geometry of the real numbers. "Once I had become interested in real algebraic geometry, of course it [was] natural to try to understand algebraic geometry in the large, and that's what I did," he says. "Then the whole Groethendiek era emerged and so algebraic geometry began to combine with aspects of arithmetic and number theory — so it [was] natural to follow that route as well."

From knots to numbers

In a beautiful article from 1991 Mazur describes number theory as producing "without effort, innumerable problems which have a sweet, innocent air about them, [like] tempting flowers." You're probably familiar with many of those flowers — indeed one of them, the twin prime conjecture, is the subject of one of this year's Fields Medals, awarded to James Maynard for making progress towards its proof.

The flower that tempted Mazur, however, wasn't a simple statement about the primes or such like, but an analogy he spotted with knot theory. "What really got me into the number theoretical aspects of arithmetic geometry is that prime numbers in number theory play an analogous role to knots in knot theory," he explains. "Since I had real experience in knots, the idea of transposing the vocabulary on knots to the vocabulary on numbers was intriguing. It was not only enormous fun but also a good in-road into the intuition on number theory."

When asked about pieces of work he is particularly proud of, he points to two such eponymous results: the Lieb–Thirring inequality and the Lieb–Oxford inequality. Both are ingredients in the resolution of a worrying problem that, for a long time, lay at the heart of physics: it wasn't clear why matter doesn't just collapse into ever smaller heaps, perhaps producing violent explosions along the way. After all, this is exactly what happens when gravitational collapse gives rise to bodies like our sun, or even black holes.

Loosely speaking, on the knot side the analogy involves "Galois coverings" of the three-dimensional sphere containing the knot, ramified at the knot, and on the number theory side it involves cyclic coverings of the integers ramified at the prime. On the knot theory side, the ideas lead to knot invariants such as the Alexander polynomial. On the number theory side they lead to the Iwasawa characteristic polynomial and Kubota Leopoldt L-function.

Showing that the latter two objects are in fact the same was a prominent problem known as the Iwasawa Main Conjecture, which Mazur was able to prove together with Andrew Wiles in the early 1980s. The proof is one of Mazur's most celebrated results, a direct consequence of the knot analogy, and can be viewed as the analogue of the Riemann hypothesis for the p-adic zeta functions attached to Dirichlet characters. It also led Mazur to other important results and, crucially, set new directions in number theory.

A famous flower

It was of course Andrew Wiles who proved one of the most tempting flowers in number theory, Fermat's last theorem, in 1994. In this context too, Mazur's work has been instrumental. Wiles' proof of the theorem (to be precise of the Taniyama Shimura conjecture) involves elliptic curves (given by equations of the form $y^2 = x^3 + ax + b$ where a and b are integers), which are extraordinarily neat. In 1922, Louis Mordell proved that the rational points of an elliptic curve defined over $\mathbb{Q}$ form a finitely generated Abelian group. This group is isomorphic to $\mathbb{Z}^r \times F$, where F is a finite Abelian group called the torsion subgroup, while r is known as the rank of the curve.

An immediate question is whether you can classify what can happen both in terms of rank and torsion. And while the question is still open as far as the rank is concerned, Mazur essentially settled it in 1972 for torsion. His proof of the torsion conjecture for elliptic curves gives an explicit list of possibilities for torsion subgroups F. Apart from being a result of "supreme elegance and beauty", as the Chern Prize citation puts it, the proof initiated new areas of study that helped pave the way for a proof of Fermat's last theorem.

Maths is personal

The Chern Medal is awarded for lifetime achievement in mathematics. What we have touched upon in this article is only a small proportion of Mazur's work. For more information, see the information on the IMU website (bit.ly/3vUtvJl).

Mazur's prize citation describes him as having a "pluralistic view" of mathematics. "Often people make the distinction between 'problem-solvers' and 'theorem-provers' and, of course, between applied and pure mathematics," he says when asked what he thought this meant. "But any of these classifications are gross simplifications: all human beings have some way of approaching the world with mathematical, or near-mathematical sensibilities, intuitions, experience. And these mathematical approaches and predilections are, in the end, personal… hardly classified by the gross labels I mentioned."

Mazur is also being honoured for the leadership and generosity he showed to people just starting out in their careers, including the nearly 60 PhD students he supervised. When asked what he enjoyed about working with the next generation, his answer was simple: "Most of the time they teach me more than I teach them."

Notes of a Numerical Analyst

What's the degree of x^n?

NICK TREFETHEN FRS

The degree of x^n, of course, is n. But computational mathematicians keep running up against the fact that its effective degree on a real interval, as defined by approximations, is only $O(\sqrt{n})$ as $n \to \infty$. This effect was made precise in a 1976 paper by Newman and Rivlin [3], and another treatment has been sent to me by Nicholas Marshall and Vladimir Rokhlin (unpublished).

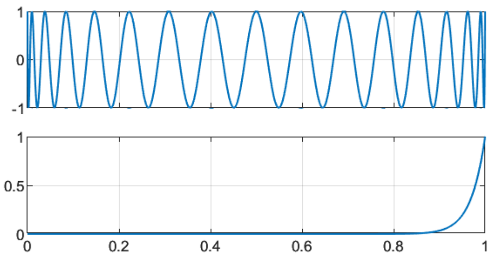


Figure 1. The Chebyshev polynomial $T_n(2x-1)$ and the monomial x^n for $n = 32$, both considered on $[0,1]$. The first has degree n by any measure, but x^n has effective degree only $O(\sqrt{n})$.

For example, in the Chebfun system for numerical computing with functions, a function f is approximated to about 15-digit accuracy by polynomials expressed as Chebyshev series (after transplantation of the interval of interest to $[-1,1]$). The function e^x on $[0,1]$ becomes a polynomial of degree 12. For x^n, the Chebfun degree is equal to n up to $n = 26$, but after this it is smaller, approximately $5\sqrt{n}$. For $n = 64$, 256 and 1024, the degrees are 43, 90 and 177.

Intuitively, what's going on is that on $[0,1]$, all high powers of x look the same, so that in the set $\{1, x, \ldots, x^n\}$, the higher powers can be well approximated by lower ones. The flip side of this observation is the phenomenon that to expand a more general degree n polynomial in this basis, you may need huge coefficients, potentially of size $O(C^n)$ with C as large as $3 + 2\sqrt{2} \approx 5.8$. In particular this is true for the transplanted Chebyshev polynomial $T_n(2x-1)$ shown in Figure 1 for $n = 32$. It is of degree n by any measure; it cannot be approximated by lower degree polynomials. However, its leading coefficient when expanded in the basis $\{1, x, \ldots, x^n\}$ is $\frac{1}{2}4^n$, and its largest coefficient in this expansion is even bigger.

To cook up even worse bases, we need look no further than the *Müntz approximation theorem* [1]. This theorem asserts that a necessary and sufficient condition for an infinite set of monomials x^{α_k} with unbounded exponents $0 = a_1 < a_2 < a_3 < \cdots$ to be dense in $C([0,1])$ is

$$\sum_{k=1}^{\infty} \frac{1}{\alpha_k} = \infty.$$

For example, the set $\{1, x^2, x^4, \ldots\}$ is dense in $C([0,1])$. Now, suppose you want to approximate the function $f(x) = x$ on $[0,1]$ in this basis to 6-digit accuracy. This is equivalent to the classic problem of polynomial approximation of $|x|$ on $[-1,1]$. It turns out you'll need $140{,}000$ terms in the series, with coefficients as large as $10^{100{,}000}$.

The effective *rational* as opposed to polynomial degree of x^n is much less than $O(\sqrt{n})$: just $O(1)$, for the best rational approximants converge exponentially. But that is another story [2].

FURTHER READING

[1] J. M. Almira, Müntz type theorems. I, *Surv. Approx. Theory* 3 (2007), 152–194.
[2] Y. Nakatsukasa and L. N. Trefethen, Rational approximation of x^n, *Proc. AMS,* 146 (2018), 5219–5224.
[3] D. J. Newman and T. J. Rivlin, Approximation of monomials by lower degree polynomials, *Aequationes Math.* 14 (1976), 451–455.

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Mathematics News Flash

Jonathan Fraser reports on some recent breakthroughs in mathematics.

A negative answer to Ulam's Problem 19 from the Scottish Book

AUTHORS: Dmitry Ryabogin
ACCESS: https://arxiv.org/abs/2102.01787

Consider a solid sphere of uniform density less than that of water. It has the pleasant property that it will float in water in every possible orientation. Last night my son and I tried to get a toy egg to do the same — it didn't work. The egg favoured certain orientations. One could reasonably conjecture that a solid sphere is the *only* solid convex body of uniform density which can float in water in all possible orientations. Indeed, Ulam explicitly asked if this was the case in Problem 19 of the *Scottish Book*. This remarkable paper, published in *Annals of Mathematics* in 2022, answers this question in the negative: there exists a strictly convex non-centrally symmetric body of revolution in $\mathbb{R}^3$ that floats in equilibrium in every orientation.

The *Scottish Book* is a famous notebook used by the Lwów School of Mathematics in the 1930s for recording open questions. It took its name from the *Kawiarnia Szkocka* (Scottish Cafe) in Lwów (now Lviv, Ukraine) where it was kept and where mathematicians such as Banach, Steinhaus, Kuratowski and Ulam often met at the time.

Seifert surfaces in the 4-ball

AUTHORS: Kyle Hayden, Seungwon Kim, Maggie Miller, JungHwan Park, Isaac Sundberg
ACCESS: https://arxiv.org/abs/2205.15283

A knot is a closed loop in 3-dimensional space — picture an embedding of S^1 into 3-dimensional space, such as a circle or a trefoil. Two knots are equivalent if one can be continuously deformed into the other within the ambient space. It is clear enough that the circle and the trefoil are not equivalent because — well — the trefoil has a knot and the circle does not! However, if we embed these knots in 4-dimensional space, then they *are* equivalent because there is more space to do the unknotting. This phenomenon of 'adding a spatial dimension and reducing complexity' is commonplace in geometry and topology. Here is a related problem replacing knots by certain surfaces. Roughly speaking, a *Seifert surface* is a surface in 3-dimensional space whose boundary is a knot. Similar to knots, many Seifert surfaces which are not equivalent in 3-dimensional space become equivalent once embedded in 4-dimensional space. Livingston asked in 1982 if, given two Seifert surfaces of the same genus and the same boundary, they are necessarily equivalent when embedded in 4-dimensional space. This paper, posted in May 2022, answers this question in the negative.

A phase transition for repeated averages

AUTHORS: Sourav Chatterjee, Persi Diaconis, Allan Sly, Lingfu Zhang
ACCESS: https://arxiv.org/abs/1911.02756

Fix an initial vector of real numbers $(x_1, \ldots, x_n)$ and iteratively modify the vector by choosing two indices $1 \leq i, j \leq n$ uniformly at random and replacing both the coordinates x_i and x_j with $(x_i + x_j)/2$. Almost surely all of the coordinates will converge to the arithmetic mean $(x_1 + \cdots + x_n)/n$. This paper, published in the *Annals of Probability* in 2022, conducts a detailed study of the rate of convergence of this random process. In doing so they answer a question posed by Bourgain related to 'cut-off transitions'. This type of problem appears in many contexts, including quantum computing (part of the motivation for this work) and game theory.

Jonathan Fraser is a Professor at the University of St Andrews and an Editor of this Newsletter. He likes fractals and floating eggs.

Microtheses and Nanotheses provide space in the Newsletter for current and recent research students to communicate their research findings with the community. We welcome submissions for this section from current and recent research students. See newsletter.lms.ac.uk for preparation and submission guidance.

Microthesis: Sheaves Over the Spectrum of a Tensor Triangulated Category

JAMES ROWE

The spectrum of a tensor triangulated category is a topological space which is vital in classifying different types of subcategories. We investigate sheaves built from data contained in the original tensor triangulated category defined over this spectrum and see how their behaviour relates to various categorical or algebraic properties.

We work with rigidly-compactly generated tensor triangulated categories (tt-categories). This is a triangulated structure, equipped with a well-behaved closed symmetric monoidal product such that the compact objects generate the category and play nicely with internal homs. Given a tt-category T, the full subcategory of compact objects T^c is itself a tt-category. We denote the tensor unit by $\mathbf{1}$ and require that T is generated by $\mathbf{1}$. The most well-known example is $\mathsf{D}(R)$, the derived category of a commutative ring R. The category of compact objects is $\mathsf{D}^{\mathsf{perf}}(R)$, the category of perfect complexes. These objects are those chain complexes quasi-isomorphic to a bounded complex of finitely-generated projective R-modules. In a more topological setting, the stable homotopy category of topological spectra also forms a tt-category under the smash product.

The spectrum of a tt-category

Given a tt-category T, we now consider the spectrum of prime ideals. We require our candidate primes $\mathcal{P}$ to be proper thick subcategories of T^c, i.e. triangulated subcategories of T^c closed under summands. As in commutative algebra, we say $\mathcal{P}$ is ideal if $\mathsf{T}^c \otimes \mathcal{P} \subseteq \mathcal{P}$ and prime if $X \otimes Y \in \mathcal{P}$ implies $X \in \mathcal{P}$ or $Y \in \mathcal{P}$. The collection of all prime ideals is the spectrum $\mathrm{Spc}(\mathsf{T}^c)$. For a compact object $X \in \mathsf{T}^c$, define the support of X by $\mathrm{supp}(X) = \{\mathcal{P} \in \mathrm{Spc}(\mathsf{T}^c) \mid X \notin \mathcal{P}\}$. The spectrum can be equipped with a topology by taking the collection $\{\mathrm{supp}(X) \mid X \in \mathrm{Spc}(\mathsf{T}^c)\}$ as a basis of closed subsets. By Balmer's classification theorem, the spectrum and support together classify all (radical) thick ideals of T^c. Every thick ideal $\mathcal{J}$ determines a Thomason subset $\bigcup_{X \in \mathcal{J}} \mathrm{supp}(X) \subseteq \mathrm{Spc}(\mathsf{T}^c)$, while every Thomason subset $Y \subseteq \mathrm{Spc}(\mathsf{T}^c)$ determines a thick ideal $\mathsf{T}^c_Y = \{X \in \mathsf{T}^c \mid \mathrm{supp}(X) \subseteq Y\}$ [1, 4.10].

Locally ringed space structures

Let $U \subseteq \mathrm{Spc}(\mathsf{T}^c)$ be a quasi-compact basic open subset with complement Z. Define the category T^c on U as the Verdier quotient $\mathsf{T}^c(U) = \mathsf{T}^c/\mathsf{T}^c_Z$. Fixing an invertible object u, define a presheaf defined over all basic opens by

$$_p\mathcal{O}^\bullet_{\mathsf{T}}(U) = \bigoplus_{i \in \mathbb{Z}} \mathrm{Hom}_{\mathsf{T}^c(U)}(\mathbf{1}, u^{\otimes i} \otimes \mathbf{1}).$$

Applying the sheafification functor we obtain a sheaf $\mathcal{O}^\bullet_{\mathsf{T}}$ on $\mathrm{Spc}(\mathsf{T}^c)$. This defines a graded locally ringed space

$$\mathrm{Spec}^\bullet(\mathsf{T}^c) = (\mathrm{Spc}(\mathsf{T}^c), \mathcal{O}^\bullet_{\mathsf{T}}).$$

By considering degree zero only, we obtain a locally ringed space

$$\mathrm{Spec}(\mathsf{T}^c) = (\mathrm{Spc}(\mathsf{T}^c), \mathcal{O}_{\mathsf{T}}).$$

Setting $R^\bullet_{\mathsf{T}} = \bigoplus_{i \in \mathbb{Z}} \mathrm{Hom}^c_{\mathsf{T}}(\mathbf{1}, u^{\otimes i} \otimes \mathbf{1})$, there are natural maps of locally ringed spaces

$$\rho^\bullet : \mathrm{Spec}^\bullet(\mathsf{T}^c) \to \mathrm{Spec}^h(R^\bullet_{\mathsf{T}})$$

and in degree zero

$$\rho : \mathrm{Spec}(\mathsf{T}^c) \to \mathrm{Spec}(R_{\mathsf{T}}).$$

The more algebraically flavoured your category, the closer these maps are to being isomorphisms.

For example, the map ρ is an isomorphism when considering $\mathsf{D}^{\mathrm{perf}}(R)$, but is not when T is the stable homotopy category of finite spectra, as it fails to be injective [2, 9.8].

Building sheaves associated to objects

With an invariant in the form of a locally ringed space, we investigate how certain sheaves behave over this structure. Given objects $A \in \mathsf{T}^c$ and $B \in \mathsf{T}$, and an invertible object u, define a presheaf via ${}_p[A,B](U) = \bigoplus_{i\in\mathbb{Z}} \mathrm{Hom}_{\mathsf{T}(U)}(A, u^{\otimes i} \otimes B)$ where U is a basic open subset and $\mathsf{T}(U) = \mathsf{T}/\mathrm{loc}(\mathsf{T}^c_Z)$. Applying sheafification we obtain an $\mathcal{O}^{\bullet}_{\mathsf{T}}$-module $[A,B]^{\#}$. This sheaf encodes the relationship between A and B at both the global and local level. In principle, studying the properties of these sheaves should tell us information about A and B at multiple different localisations, all at once. In my thesis we perform this construction in the generality of a tensor triangulated category *acting* on a triangulated category. One should think of this as the natural extension of the comparisons with commutative algebra so far. Just as we have the tt-category versions of prime ideals and spectra, so too can we bring the ideas of modules into the arena. For now, we will focus on one category at a time, and simply consider T as a module over itself.

Comparing support theories

We have already seen one definition of support for compact objects in a tt-category. Extending the notion of support to non-compact objects is a non-trivial task, and has lead to a variety of approaches involving ring actions, categorical actions, and residue functors. In the more familiar setting of algebraic geometry, sheaves themselves carry a notion of support. Applying this to our associated sheaves we can define a new support:

$$\mathrm{supp}^{\bullet}(B,A) = \{\mathcal{P} \in \mathrm{Spc}(\mathsf{T}^c) \mid [A,B]^{\#}_{\mathcal{P}} \neq 0.\}$$

When the spectrum $\mathrm{Spc}(\mathsf{T}^c)$ is a noetherian topological space, the tensor unit $\mathbf{1}$ generates T, and the invertible object u is equal to $\Sigma\mathbf{1}$. Then

$$\mathrm{supp}(B) \subseteq \mathrm{supp}^{\bullet}(B,A)$$

where the support on the left is defined by the residue functors alluded to earlier. If B is a compact object, this inclusion of the usual support upgrades to an equality. In fact, this sheaf based notion of support captures precisely those prime ideals $\mathcal{P}$ such that B is not contained in the localising ideal $\mathrm{loc}^{\otimes}(\mathcal{P})$. The case $u = \Sigma\mathbf{1}$ is the maximal case, and altering u changes the detective capability of the support.

Affine and schematic categories

We say a tt-category is *affine* if the comparison map is an isomorphism, and *schematic* if the comparison map is *locally* an isomorphism. As an example, given a quasi-compact quasi-separated scheme X, the category $\mathsf{D}^{\mathrm{perf}}(X)$ is schematic. Assuming that our category is affine or schematic, every sheaf we have constructed of the form $[\mathbf{1},X]^{\#}$ are quasi-coherent. These affine categories also interact well with certain t-structures. We say the $\mathbf{1}$ object is *connective* if $\mathrm{Hom}_{\mathsf{T}^c}(\mathbf{1},\Sigma^n\mathbf{1}) = 0$ for all $n > 0$. In this case we can equip our category T with a t-structure via the hom-functors $\mathrm{Hom}_{\mathsf{T}}(\mathbf{1},\Sigma^n-)$ [3]. Equipping an affine category with this t-structure provides even more information about the associated sheaf functors. In this scenario, the heart of the t-structure captures all quasi-coherent sheaves over the spectrum. More specifically there is an equivalence of categories $\mathsf{T}^{\heartsuit} \cong \mathrm{QCoh}(\mathrm{Spec}(R_{\mathsf{T}}))$. and moreover the associated sheaf of an object X is isomorphic to the sheaf associated to the zeroth cohomology H^0X induced by the t-structure. That is $[\mathbf{1},X]^{\#} \cong [\mathbf{1},H^0X]^{\#}$.

FURTHER READING

[1] Paul Balmer. The spectrum of prime ideals in tensor triangulated categories. In: J. Reine Angew. Math. 588 (2005), pp. 149–169.
[2] Paul Balmer. Spectra, spectra, spectra — tensor triangulated spectra versus Zariski spectra of endomorphism rings. In: Algebraic and Geometric Topology 10.3 (2010), pp. 1521–1563.
[3] Mitsuo Hoshino, Yoshiaki Kato, and Jun-Ichi Miyachi. On t-structures and torsion theories induced by compact objects. In: J. Pure Appl. Algebra 167.1 (2002), pp. 15–35.

James Rowe

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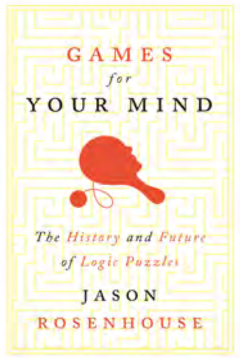
Games for Your Mind

by Jason Rosenhouse, Princeton Press, 2020 £25.00, US$ 29.95, ISBN: 978-0691174075

Review by Holly A. J. Middleton-Spencer



Jason Rosenhouse's *Games for Your Mind* is an engaging popular mathematics book written to enlighten the reader on the mathematics and logic behind popular puzzles. Immediately apparent is that the puzzles have not simply been added to a book about logic, but the content of the book is based and woven entirely around the puzzles within. Although the book focusses on the history and progression of logic and logical puzzles, Rosenhouse does not stick to the usual 'Aristotle, Stoicism, . . . Mills, Boole, Russell' that other history books may keep to. This bouncing between multiple topics guarantees the reader does not tire of one topic. Rosenhouse introduces many logicians throughout his text, however he puts special emphasis on two in particular — Charles Dodgson (better known as Lewis Carroll) and Raymond Smullyan. Of note is his dedication to explaining Dodgson/Carroll's contributions to the field, both in formal mathematics as well as his logic puzzles.

We open this book with 'The Pain and Pleasure of Logic' and the age-old debate on the level of tediousness of logic. Providing examples related to his cat, he briefs the reader on the fundamental rules of logic. Following this, Rosenhouse then begins to ease into the discussion of the logic behind ubiquitously popular puzzles and games. It is apt that Rosenhouse's first puzzle in the book is a Sudoku grid; this reflects on his earlier successful popular mathematics book *Taking Sudoku Seriously*, cowritten with Laura Taalman.

The second section starts with an introduction of Aristotelian logic. Here, the reader is briefed on syllogisms (an argument of two statements followed by a conclusion) and is walked through given examples. Carroll's *The Game of Logic*, considered the first text on logic for the general reader, is introduced and explained. Carroll's aim in his work was to entertain young children while also teaching them the basics of logic. Inventing a grid resembling a complicated Venn diagram, he taught children how to solve convoluted syllogisms with the placement of counters onto this aforementioned grid. This reviewer questions whether a child would indeed enjoy such a game, however she was unable to source a willing child in which to test this 'game' on! Also introduced is Carroll's notation for logical statements. Rosenhouse quips (albeit in a different chapter) that his lecturer had once told him 'we care about notions, not notations', however the reviewer cannot not exclaim how cumbersome Carroll's notation truly is.

The third section, 'Raymond Smullyan and Mathematical Logic' introduces the infamous 'Knights and Knaves' problem. Here, the player is placed on an island with two types of inhabitants; the player is tasked to work how who is who. The problem is that one type (the Knights) will always tell the truth whereas the other (the Knaves) will always lie. Chapter 8 takes a different tone to the rest of the section and provides an interesting chronological history of logic as a subject, and notes its distinctiveness from the rest of mathematics.

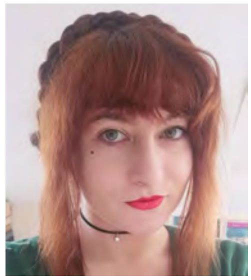
All mathematics and puzzles in the first three sections followed the 'classical' logic originally derived by Aristotle. In Section 4, Rosenhouse begins by highlighting the philosophical debate on whether this is the one 'true' logic, or whether multiple exist and highlights other logics studied in the last century. Interesting examples, including 'fuzzy' logic are introduced. In fuzzy logic, a truth is no longer 0 or 1, but can hold any infinite value in between. Smullyan's Knights and Knaves problem is brought back and viewed through our new nonclassical eyes: first from a three-valued perspective of truth and then a fuzzy one. These non-Aristotelian logics brings a variety of puzzling complications onto Smullyan's original puzzle.

The fifth and final section has chapters on miscellaneous topics not otherwise covered in the book. Rosenhose starts with George Boolos' 'Hardest Logic Puzzle Ever'; the bamboozling tale of three gods; one who speaks the truth, one who speaks falsely and one whose answers are entirely random. To make the system even more elaborate, these gods do not respond in

English but with 'ja' or 'da', of which the questioner does not know the meaning. Other topics include a discourse on the difference of puzzles, metapuzzles and paradoxes. This section, and book, concludes with a discussion of fictional logic and puzzles appearing in literature – covering Star Trek, Arthur Conan Doyle, and the mysteriously named 'Thinking Machine', otherwise known as Professor S. F. X. Van Dusen.

Formal logic can be a confusing and frustrating topic to master. In contrast to the dry textbooks this reviewer had to tackle during her undergraduate studies, Rosenhouse presents potentially different topics with humour and ease. His shear eagerness for his topic is catching and the personal touches give a liveliness to the text. The reviewer must state that she found a few parts of the text challenging to understand; some puzzles may be out of reach to mathematical laymen. Numerous times the reviewer found herself putting the book down and mulling over possible solutions to a puzzle before returning to see if she was correct. The reviewer must also confess that she was unable to complete many of the given puzzles without assistance. Thankfully, Rosenhouse makes sure to explain the steps to tackle each puzzle. Rosenhouse's detailed worked solutions mean that all readers can see how the answers are obtained and can easily follow along. In later chapters whole sections are dedicated to singular questions with thorough discussion.

There is a wide range of further reading listed at the end of each chapter. In giving such a broad range of technicality, from popular mathematical books to undergraduate-level textbooks and academic papers, Rosenhouse's recommendations means that any reader can expand their knowledge each according to his or her own mathematical background. Overall, the reviewer would recommend this book to all people who want a puzzling challenge. Although the puzzles towards the end of the book feel impossible, the thrill of that 'ah!' moment when you work through Rosenhouse's solution is surely a high for any mathematician out there.



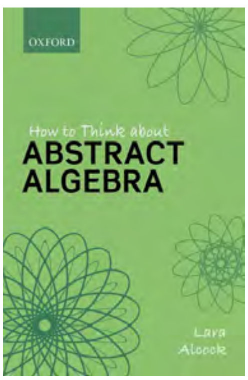
Holly A. J. Middleton-Spencer

Holly Middleton-Spencer is a PhD student in Applied Mathematics at Newcastle University. Her research focuses on the theory of and computational modelling of turbulence with in Bose–Einstein Condensates. She also has a keen interest in ancient mathematics and spends her free time trying to master Akkadian, Middle Egyptian, and the bassoon.

How to Think About Abstract Algebra

by Lara Alcock, Oxford University Press, 2021, £17.99, ISBN 978-0-19-884338-2

Review by Tony Mann



New undergraduates at UK universities often find abstract mathematics difficult. Their A-level training has not prepared them for the kind of thinking required when studying topics like group theory. University lecturers, for whom this abstract way of thinking mathematically has become second nature, sometimes struggle to communicate this mode of thinking to students who feel out of their depth.

Lara Alcock, who was recently awarded the inaugural John Blake University Teaching Medal, has written several highly-regarded books including *How to study for a mathematics degree* and *How to think about analysis*. The book under review is not a textbook in abstract algebra. Rather it is intended to supplement an undergraduate's first lecture courses in group theory or ring theory, providing background not so much in the technical content but rather in the ways of mathematical thinking and learning which we are hoping students will develop. In the preface Alcock describes it as "pre-reading or concurrent reading", and she emphasises the importance of *reading* mathematics productively.

The book itself will certainly develop the student's mathematical reading skills.

The writing is clear and engaging, with plenty of humour, and directly addresses the reader throughout. Alcock is excellent in identifying likely difficulties and misconceptions, and her friendly approach will reassure students who might worry that because they fail to grasp an idea, or have frequent misunderstandings, that means they are not a good mathematician. She spends time clarifying points which often puzzle students (such as the different meanings of "symmetry group" and "symmetric group"), but I don't feel that she is ever patronising. Alcock covers technical points in more detail than a lecturer might be able to do in the classroom, and throughout she signposts to the student the different approaches that lecture courses may take to the material, so that this book will support lecturers presenting the material from different angles.

She gives lots of examples for each concept, developing the student's understanding of the formal definitions they will come across in lectures, and frequently asking the reader to do some thinking. (There are no "Exercise" sections as such in this book because the exercises are fully integrated into the discussion.) This is not just a guide to algebra, but will lead the reader to understand how to read mathematics in general, and how to study the subject effectively. Alcock emphasises tricks like making sure one understands the statement of a theorem before starting to follow the proof. (I wish I'd had that advice: I recall as a student working through proofs in the hope that they would make the statement of the result clear!)

Indeed the first four chapters are about how to study, not about the specific content. These chapters are particularly valuable — I would like all my undergraduates to read them! I especially liked Chapter 4, which has sections "Who are you as a student?", "Myths about learning" and "Effective learning". The remaining chapters cover binary operations, groups and subgroups, quotient groups, isomorphisms and homomorphisms, and rings, presenting the material in a logical and helpful order.

The author is realistic enough to realise that many students will turn to this book when they get stuck, and will dive straight into the relevant technical section. Helpfully, she includes for such readers pointers to the earlier material that will help them with their particular difficulty. (There is one reference in a footnote to a non-existent Section 5.8, but that is the only slip that I noticed.)

The final paragraph of the book re-emphasises the point that understanding mathematics the right way saves you work! Alcock's comment that the students who get the very top marks typically work less hard than those who get slightly lower marks, because they have developed ways of studying mathematics more effectively, is one which I wish more students would appreciate: too many spend too much time memorising rather than understanding!

Many students will find this a valuable resource, perhaps picking and choosing the sections that will help them at a particular point rather than reading systematically from beginning to end (although I hope they all read Chapter 4). While the reference to "pre-reading" is perhaps optimistic, any student who uses this book will certainly learn a lot about how to study mathematics and how to think mathematically. I wish it been available when I was a fresh undergraduate!

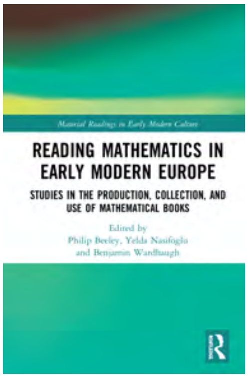
Tony Mann

Tony Mann teaches mathematics and organises university-wide mathematics and statistics support at the University of Greenwich. His academic interests include mathematics teaching and the history of mathematics. He is one of the editors of the journal *MSOR Connections* which covers mathematics teaching in higher education. He blogs occasionally at tonysmaths.blogspot.com and tweets as @Tony_Mann.

Reading Mathematics in Early Modern Europe: Studies in the Production, Collection, and Use of Mathematical Books

by Philip Beeley, Yelda Nasifoglu, Benjamin Wardhaugh (eds.), Routledge, 2020, £120, ISBN: 978-0367609252

Review by Jane Wess



Reading Mathematics in Early modern Europe, edited by Philip Beeley, Yelda Nasifoglu and Benjamin Wardhaugh constitutes an important and timely addition to the well-established history of reading. While the 'history of the book' thrives, little attention has been given to mathematical works within this genre.

The introduction sets out the laudable aim of 'closing the gap between the history of mathematics as a history of texts and the history of mathematics as a part of human culture'. The work certainly extends the range of focus to study the reception of mathematics in society using manuscripts, correspondence, and marginalia, so that texts become part of a wider whole. The different strands could be described as: a) the reception of major works, particularly Euclid, by able mathematicians, b) the role and use of mathematical diagrams and illustrative material, c) institutional mathematics including attitudes to its teaching, the politics of publication, and the foundation of libraries, and d) the use of material books, in particular marginalia, to discern a broad reception based on use. The introduction could have done more to pull together the various strands. The chapters by Vincenzo Risi, Nasifoglu, and Beeley are excellent but would have benefitted from conclusions.

Vincenzo de Risi starts by examining the reception to Euclid's *Elements* I.1, which he uses as a 'vantage point' for observing several aspects of the transformation of ancient Greek geometry into modern geometry. A rich and profound study of the concept of continuity follows, when initially space became the underlying premise rather than figures, to be replaced by more abstract principles.

Robert Goulding considers the addressing of Euclidean blemishes by Henry Savile and his circle, illuminating debates between lesser-known figures. Goulding studies the compounding of ratios which appeared in the fifth definition of Book VI, together with inconsistencies in the definitions of proportion elsewhere. He notes that in sixteenth-century Italy proportionality was a more pressing issue than the parallel postulate. Goulding describes a 'thicket of proportion theory' which continued as a controversy during the following century.

Nasifoglu notes that practical drawing was considered a representation of reality in the early seventeenth century, while classical diagrams were 'geometric thinking in action'. However, she argues the situation was changing by the end of the century, reflecting the increasing use of mathematics in many aspects of everyday life. In more advanced mathematics, however, algebra was taking the place not only of the text but the diagrams themselves.

Renee Raphael returns to the first strand by investigating responses of elite mathematicians to blemishes, this time in Tycho Brahe's work. By studying the responses of two well-known and three less well-known readers to a faulty diagram, she draws out common practices in mathematical reading in the late sixteenth century. A diverse mix of responses illuminates various approaches which involve personal style, publishing issues, religious belief, and pragmatism.

Mordechai Feingold begins by describing the Humanist approach to learning. While they established a place within the curriculum for mathematics at Cambridge and elsewhere, their aim was to prepare the social elite for public life, therefore many writers advised 'gentlemanly moderation'. Nevertheless, Feingold goes on to dispute the idea of Oxford being 'wretched' in its provision of mathematics teaching. He uses Samuel Ward's diary over many years to argue that more mathematics was being done than at first appears.

Richard Oosterhof's chapter constitutes a detailed study of manuscript notes by Brian Twyne, who is best known as an Oxford antiquary. Two sets of notes reveal a wide interest which included mathematics. Twyne's principal text was Billingsley's 1570 edition of Euclid. Oosterhof points out that this reliance on ancient texts erodes the idea of a clean break between ancient and modern mathematics.

William Poole gives a meticulous account of the foundation of the Savilian Library in Oxford, its development, and eventual demise. Savile considered a library, equipped with observing instruments, to be an essential adjunct of his newly-instated professors, stipulating that they deposited all their teaching material there. In 1884 the Bodleian Library took it over formerly as it had become a historical collection.

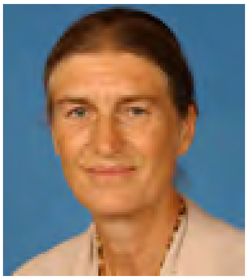
Beeley investigates behind the scenes of the production of the 'Gregory' edition of Euclid's *Elements* of 1703. Describing the 'maelstrom of [Oxford] University politics' he traces an earlier project which failed but contributed considerably to the eventual publication. The vastly ambitious project, steered by Edward Bernard from 1672, stalled due to lack of subscribers. Bernard tried to resurrect it in 1684 and 1694, allowing us to see changes in footnotes and the placing of diagrams. Only a year after his death in 1697 preparations began for 'Gregory's' edition. It was politic to present this as an entirely new project, not drawing on the previous failed one.

Wardhaugh has undertaken a meticulous empirical study of marks in 366 mathematical books from the eighteenth and early nineteenth centuries in libraries in Oxford and London. Mathematical books are much more likely to be deliberately marked than works on other subjects. Elementary books are marked more than advanced texts or practical works. The questions at the end point to further study.

Boris Jardine puts the problems of the study of marginalia in a nutshell. He then goes on to propose a system of classification of types of reading. This is a bold step, as it relies almost exclusively on the marginalia from one book. Nevertheless, it is a thought-provoking and fresh approach. From four types of comments Jardine identifies four types of reading. This chapter should stimulate further debate.

Kevin Tracey also studies one book: *Seller's Pocket Book* of 1685 in the London Science Museum's collection. Tracey identifies three 'lives' of this work: as a product of John Sellers, as the property of two students at Cambridge, and lastly as a repository of knowledge in a teacher's private library.

To summarise: the book is well worth reading and contains some excellent scholarship and interesting ideas. Each chapter is well-researched using a wide selection of original material in an extension of the traditional methodology of the history of mathematics.



Jane Wess

Dr Jane Wess was curator of the mathematics collection at the Science Museum in London for over twenty years. Her interests include the application of mathematics in the eighteenth century, experimental philosophy, and mathematics education. More recently she has ventured into the nineteenth century with a PhD on the role of mathematical instruments in exploration. She is not planning on retiring any time soon.

Obituaries of Members

John H. Coates: 1945–2022

Professor John H. Coates FRS, who was elected a member of the London Mathematical Society on 20 June 1986, died on 9 May 2022, aged 77. Professor Coates was LMS Vice-President 1986–88, LMS President 1988–90 and was awarded the LMS Senior Whitehead Prize in 1997.

Richard Taylor writes: John Henry Coates was born in the village of Possum Brush in rural Australia in 1945 and grew up on his family's farm. His mother died when he was seven and his father suffered from periodic illnesses. He won a scholarship that allowed him to study physics at the Australian National University, but during his first year there, partly influenced by an introductory course in number theory taught by Kurt Mahler, John switched to mathematics. He went on to complete an honours project in number theory under Mahler. As a student at the ANU, John met and married Julie Turner.

On completing his undergraduate degree John went for graduate study to the Ecole Normale Supérieure in Paris. However, he found himself ill prepared for the abstraction of Grothendieck's school and so after a year moved to Cambridge University, where he worked with Alan Baker on p-adic analogues of Baker's method, receiving his PhD in 1969.

After completing his PhD, influenced by the Birch–Swinnerton-Dyer conjecture, John decided to switch his interests back to algebraic number theory. He moved to a post-doctoral position at Harvard University, where he benefitted from the advice of Tate and, remotely from Princeton, of Iwasawa.

Algebraic number theory, and in particular Iwasawa theory, would remain his principal scientific interest for the rest of his career. In 1972 John took an associate professorship at Stanford University, in 1975 a lectureship in Cambridge, in 1977 a professorship at ANU, and in 1978 a professorship at Orsay, before returning to Cambridge in 1985 as Sadleirian Professor, where he remained until his retirement in 2012. After his retirement from Cambridge, he took a part time post in Postech in Korea and continued to work jointly with some young Asian mathematicians.

In the mid-1970's John with his student Andrew Wiles established the first general result towards the Birch–Swinnerton-Dyer conjecture. They showed that if an elliptic curve over $\mathbb{Q}$ with CM by the ring of integers of an imaginary quadratic field of class number one has infinitely many rational points, then its L-function vanishes. This work was based on the development of an analogue of Iwasawa's theory of cyclotomic fields for abelian extensions of imaginary quadratic fields. It introduced several very influential new ideas. In the mid-1980's John initiated the study of the Iwasawa theory for the symmetric square L-function of a modular form, a topic that turned to gold in Andrew Wiles' work on Fermat's Last Theorem. In the mid-1990s John turned to non-commutative Iwasawa theory, a topic that really took off under his influence.

As well as his own research work, John had an enormous influence on mathematics through his mentorship of young mathematicians and his administrative work. According to the Mathematics Genealogy Project he advised 35 PhD students including Andrew Wiles, Bernadette Perrin-Riou and Pierre Colmez, and he has 446 mathematical descendants. He was a powerful advocate for those young mathematicians he felt to be deserving.

John served as an editor of *Inventiones Mathematicae* for 19 years. He also served as President of the LMS, Vice-President of the IMU, and a member of the Council of the Royal Society. He left a big mark as an energetic and reforming head of the Department of Pure Mathematics and Mathematical Statistics in Cambridge in the 1990s.

John's mathematical achievements were recognized by his election to the Royal Society and the Academia Europaea, and by the award of the LMS Senior Whitehead Prize. He had wide ranging interests outside mathematics including Japanese literature and ceramics, and Asian culture more generally.

Biographical Memoirs and LMS Obituaries

Memoirs for the following people have been published in Biographical Memoirs of Fellows of the Royal Society:

- Graham Higman (1917–2008), tinyurl.com/547a9v42
- Sir Vaughan Jones (1952–2020), tinyurl.com/2s3mchru
- Freeman John Dyson (1923–2020), tinyurl.com/2c6s5pn5
- John Charlton Polkinghorne (1930–2021), tinyurl.com/5y9sh9zf

Obituaries for the following people have recently been published in the *LMS Bulletin*:

- Peter Michael Neumann (1940–2020), tinyurl.com/29773wts
- Peter Vámos (1940–2020), tinyurl.com/dcus9cvr

The obituary of David W. Lewis (1944–2021) has recently been published in the Irish Mathematical Society Bulletin: tinyurl.com/3bdc6uss.

All obituaries (both recent and historical) published in the LMS *Bulletin* are free to read and can be accessed at tinyurl.com/bduxhkhe.

Death Notices

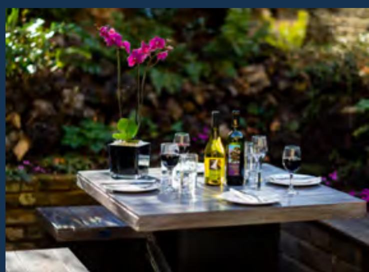
We regret to announce the following deaths:

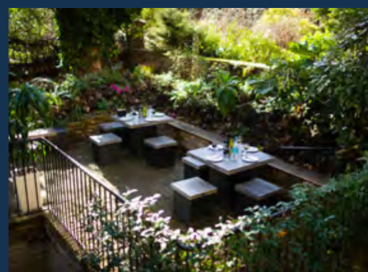
- Derek Goldrei, of The Open University, who was elected a member of the London Mathematical Society on 20 May 1983, died on 2 July 2022, aged 74.
- David W. Lewis, formerly of University College, Dublin, died on 20 August 2021, aged 77.
- John McKay, formerly of Concordia University, Montreal, who was elected a member of the London Mathematical Society on 16 November 1967, died on 19 April 2022, aged 82.
- Stewart A. Robertson, formerly of Southampton University, who was elected a member of the London Mathematical Society on 15 June 1961, died on 8 July 2022, aged 89.
- David Tipple, formerly of University College, Dublin, who was elected a member of the London Mathematical Society on 19 December 1968, died on 5 June, aged 79.

LONDON MATHEMATICAL SOCIETY
EST. 1865

CONFERENCE FACILITIES

De Morgan House offers a 40% discount on room hire to all mathematical charities and 20% to all not-for-profit organisations. Call 0207 927 0800 or email roombookings@demorganhouse.co.uk to check availability, receive a quote or arrange a visit to our venue.





ADVERTISE IN THE LMS NEWSLETTER

The LMS Newsletter appears six times a year (September, November, January, March, May and July).

The Newsletter is distributed to just under 3,000 individual members, as well as reciprocal societies and other academic bodies such as the British Library, and is published on the LMS website at lms.ac.uk/publications/lms-newsletter.

Information on advertising rates, formats and deadlines are at: lms.ac.uk/publications/advertise-in-the-lms-newsletter.

Examples in this issue can be found on pages 11 and 23, and on the back page.

To advertise contact Susan Oakes (susan.oakes@lms.ac.uk).

Persistent Homology and Applications

Location: University of Plymouth
Date: 12 September 2022
Website: tinyurl.com/3h8kd5ab

This one-day conference will focus on persistent homology and its applications. Persistent homology is a fairly new method of data analysis which uses techniques from topology, thus allowing a mathematically rigorous study of the shape of the data. The talks will be aimed both experts in the field and those interested in it, as well as early career researchers. The meeting is supported by an LMS Celebrating New Appointments Scheme 9 grant awarded to Mariam Pirashvili.

Ergodic Theory and Fractal Geometry

Location: University of Birmingham
Date: 14–15 September 2022
Website: tinyurl.com/6chjzexd

This 2-day meeting is to celebrate the recent appointments of Dr S. Baker and Dr S. Kombrink. The focus will be on recent advances in ergodic theory and fractal geometry. To register visit the conference website. Limited funds are available to support accommodation and travel expenses. Applications for funding can be made when registering. The meeting is supported by an LMS Celebrating New Appointments grant.

Cohomological Methods in Group Theory

Location: London
Date: 23 September 2022
Website: lancaster.ac.uk/maths/fcg/

The meeting will take place in person at Royal Holloway, University of London (Central London Campus). To register, contact Brita Nucinkis (Brita.Nucinkis@rhul.ac.uk). In this meeting, the three speakers will discuss cohomological methods and their applications to matrix groups and pro-p groups. Speakers: Paula Macedo Lins de Araujo, Yuri Santos Rego and Pavel Zalesski. The meeting is organised by the Joint Research Group *Functor Categories for Groups* supported by an LMS Grant Scheme 3 Research grant. Some funding is available for PhD students.

Geometry, Stochastics and Dynamics

Location: Imperial College, London
Date: 12–16 September 2022
Website: xuemei.org/UK-Japan-Winter-School

This special workshop is to celebrate the first 20 years of annual UK-Japan Winter Schools. The topics chosen reflect several regularly occurring themes of the School. On Friday 16 September the workshop will be followed by two public lectures at the Japanese Embassy in London: the speakers will be Sir Martin Hairer and Professor Hirosi Ooguri. All participants, from PhD students through to established researchers, are invited to attend as few or as many days of the workshop as they like.

LMS Women in Mathematics Day

Location: University of Stirling
Date: 21 September 2022
Website: tinyurl.com/5bwaybry

This event will provide an opportunity to meet and hold discussions with women scientists from a range of maths-related disciplines and at various stages in their careers. It will feature talks from four eminent female mathematicians from home and abroad: Tara Brendle (University of Glasgow), Annie Cuyt (University of Antwerp, Belgium), Gerlind Plonka (University of Göttingen, Germany), Yulia Timofeeva (University of Warwick). The meeting is supported by an LMS Women in Mathematics Grant.

LMS Popular Lecture

Location: University of Birmingham
Date: 27 October 2022, 5–8pm
Website: lms.ac.uk/events/popular-lectures

The Society's Popular Lectures present exciting topics in mathematics to a wide audience. The lectures are aimed at those who have an interest in mathematics as well as being useful for A-level students. This year's lecture will be given by Dr Samantha Buzzard (Cardiff) and is titled *Lakes, rivers... and waterfalls? The surprising things maths can help us to understand about Antarctica.*

LMS Meeting

Black Heroes of Mathematics

4–5 October 2022, online (hosted by ICMS)

Website: lms.ac.uk/events/black-heroes-mathematics

Speakers: Sophie Dabo-Niang, Toluope Fadina, Tosin Babasola, John Urschel, Aris Winger, Nandi Leslie, Ejay Nsugbe, Franck Kalala Mutombo.

The vision of this conference is 'to celebrate the inspirational contributions of black role models to the field of mathematics and mathematics education'. There will be a balance of technical talks by internationally renowned black speakers that include some detail of career paths and experience to provide a testimonial dimension. We plan to achieve a balance of career stage and gender.

LMS Meeting

Annual General Meeting

18 November 2022; 3.00–6.00pm; Goodenough College, London

Website: lms.ac.uk/events/meeting/agm

The meeting will start with Society business, which will include the presentation of certificates to all 2022 LMS prize winners and the announcement of the annual LMS Election results. This will be followed by the Naylor Lecture by Endre Suli (Oxford). Members can sign the Members' Book, which dates from 1865. A reception and the Society's Annual Dinner, to be held at Goodenough College, will follow the meeting. The cost of the dinner will be £60.00, including drinks. To reserve a place at the dinner, email Clare Ralphs (AnnualDinner_RSVP@lms.ac.uk). The talk will be aimed at a general audience and all interested, whether LMS members or not, may attend this event.

LMS Meeting

South West/South Wales Regional Meeting & Workshop

17 January 2023, Southampton

Website: https://www.lms.ac.uk/events/meeting/South-West-and-South-Wales

Regional Meeting Speakers (17 January): Peter Kropholler (Southampton), Karen Vogtmann (Warwick) and Ian Leary (Southampton).

Workshop speakers: (18–19 January): Tara Brendle (Glasgow), Jack Button (Cambridge), Mark Hagen (Bristol), Dawid Kielak (Oxford), Nadia Mazza (Lancaster), Brita Nucinkis (Royal Holloway) and Sarah Rees (Newcastle).

The meeting forms part of the South West and South Wales Workshop *Geometric Group Theory*. Members attending may sign the Members' Book and a Society Dinner will be held after the meeting; see the website for details and to register. Partial travel support for LMS members and research students is available.

Society Meetings and Events

September 2022

5 LMS Women in Mathematics Day, University of Stirling (502)

16 LMS/IMA/BSHM: Women in Astronomy Meeting, De Morgan House and online

October 2022

4-5 Black Heroes of Mathematics Meeting, online

27 LMS Popular Lecture 2022, Birmingham

November 2022

18 LMS Annual General Meeting and Naylor Lecture, London

January 2023

17-19 LMS South West and South Wales Meeting and Workshop, University of Southampton

Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website (www.lms.ac.uk/content/calendar). Please send updates and corrections to calendar@lms.ac.uk.

September 2022

1-2 Applied Mathematical Challenges and Recent Advances in the Micro-Mechanics of Matter 2022, University of Bristol (500)

5 One Day Function Theory Meeting, De Morgan House/online (501)

5-7 Scaling Limits, Cambridge (501)

5-9 COMB in CAMB: Combinatorial Methods in Algebraic Geometry in Cambridge (500)

12 Persistent Homology and Applications, University of Plymouth (502)

12-16 Geometry, Stochastics and Dynamics, Imperial College London (502)

12-16 The Charm of Integrability, University of Bristol (501)

14-15 Ergodic Theory and Fractal Geometry Meeting, University of Birmingham (502)

14-15 Induction Course for New Lecturers 2022, Isaac Newton Institute, Cambridge (501)

19-20 Mathematical Challenges of Big Data IMA Conference, University of Oxford/Hybrid (501)

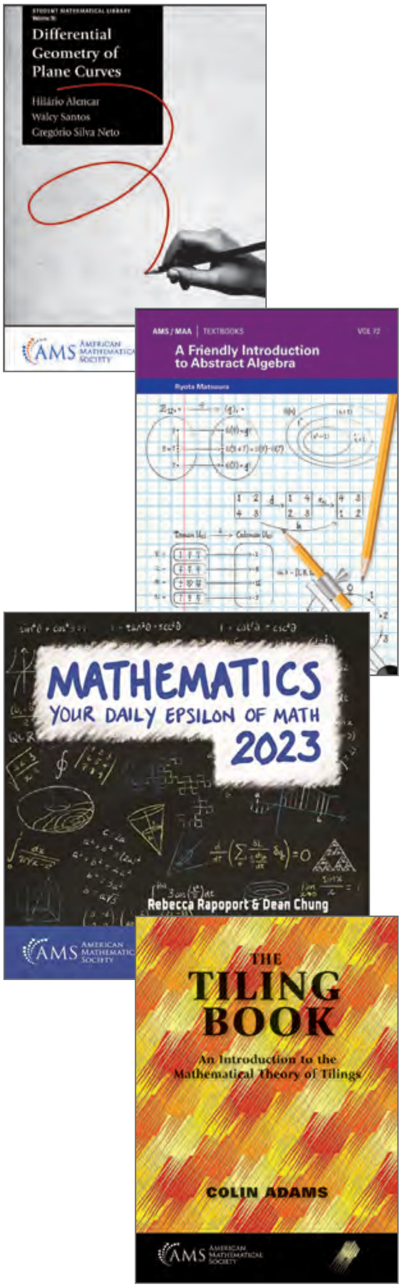
23 Cohomological Methods in Group Theory, Royal Holloway, University of London (502)

26-30 Clay Research Conference and Workshops, Mathematical Institute, Oxford (501)

December 2022

16-17 Random Matrix Theory, Brunel University, London (502)

AMERICAN MATHEMATICAL SOCIETY



DIFFERENTIAL GEOMETRY OF PLANE CURVES

Hilário Alencar, Federal University of Alagoas, Walcy Santos, Federal University of Rio de Janeiro & Gregório Silva Neto, Federal University of Alagoas

Student Mathematical Library, Vol. 96

Features plane curves to illustrate many deep and inspiring results in the field in an elementary and accessible way. After an introduction to the basic properties of plane curves, the authors introduce a number of complex and beautiful topics, including the rotation number, rotation index, Jordan curve theorem, and isoperimetric inequality.

Jul 2022 416pp 9781470469597 Paperback £54.95

A FRIENDLY INTRODUCTION TO ABSTRACT ALGEBRA

Ryota Matsuura, St. Olaf College

AMS/MAA Textbooks, Vol. 72

Offers a new approach to laying a foundation for abstract mathematics. Prior experience with proofs is not assumed, and the book takes time to build proof-writing skills in ways that will serve students through a lifetime of learning and creating mathematics.

Aug 2022 367pp 9781470468811 Paperback £63.95

MAA Press

MATHEMATICS 2023: YOUR DAILY EPSILON OF MATH

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