The London Mathematical Society

UK government funding for mathematical sciences research

A briefing paper of the London Mathematical Society

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EXECUTIVE SUMMARY

1. **UK government funding for universities comes essentially in three parts: funds for teaching, baseline funds for research, and funds for specific people and projects.** The first two of these come from the **funding councils**, and the third from the **research councils**. The division between the funding and research councils is called the **dual support system for research**.

2. **The seven research councils have UK-wide remits, and are grouped together under Research Councils UK (RCUK).** They all give some funding to mathematical scientists, but it is hard to gather the relevant figures: funding is to individual projects and cannot easily be aggregated, and the research councils use different formats for any data they do display. The main overall funder is the Engineering and Physical Sciences Research Council (EPSRC), which has a specific Mathematical Sciences programme and funds research across the discipline.

3. **RCUK research grants have been funded on a full economic cost (fEC) basis since April 2006.** The RCUK budget was increased to allow this extra cost to be factored in.

4. **The division between teaching and baseline research funding was not explicit until the 1980s.** Since then, the baseline research funding has been allocated following a review and rating of all research in a process called the Research Assessment Exercise (RAE).

5. **The RAE is due to be replaced by the Research Excellence Framework (REF).** The biggest planned change is to include a measure of impact outside academia, i.e. political, social, economic and cultural impact. This has caused widespread concern because of the difficulty in ensuring that the methodology for assessing impact is robust enough to consistently identify excellence.

6. **Each country within the UK has its own funding council, except Northern Ireland where the task is undertaken by the Department for Employment and Learning (DEL).** These are the Higher Education Funding Council for England (HEFCE), the Scottish Further and Higher Education Funding Council (SFC), and the Higher Education Funding Council for Wales (HEFCW). Overall levels of funding are not identical, and each funding body has adopted a different scheme for division of their funds.

1 Introduction: the dual support system

This briefing paper outlines the main UK sources of funding for mathematical sciences research. UK government funding for universities comes essentially in three parts: funds for teaching, baseline funds for research, and funds for specific people and projects. The first two of these come from the **funding councils**, and the third from the **research councils**. Research funding is also available from other government departments, European Union agencies, industry, charities and so on, but the total amount of this in mathematical sciences is far smaller. The division between the funding and research councils is called the **dual support system for research**.

The costings for government research and teaching are based on the Transparent Approach to Costing (TRAC) methodology[1], used by all universities and the funding bodies to study the uses to which funds are put, and the appropriate amounts to charge or provide. Not everybody believes TRAC's results are meaningful (it involves annual data collection, often based on surveys of staff time and reporting from institutions), but it did in particular indicate that research funders did not (and still do not) cover the full cost of the work they support. As a consequence, applications to research councils now have to be in terms of the **full economic cost (fEC)**, although they only fund 80% of it (this is still a considerable increase on previous levels of research overheads). This system was introduced for new grants starting on or after 1 April 2006, and the RCUK budget has been increased ("uplifted") to allow this extra cost to be factored in.

Dual support is an important mechanism, although it complicates the explanation of government research funding: the remit of the seven research councils is UK-wide, whereas there are separate funding councils for England (Higher Education Funding Council for England, HEFCE), Scotland (Scottish Further and Higher Education Funding Council, SFC) and Wales (Higher Education Funding Council for Wales, HEFCW), and in Northern Ireland the task is undertaken by the Department for Employment and Learning (DEL). The Research Councils UK (RCUK) website[2] describes dual support as follows:

> *Under the dual support system, the Research Councils provide grants for specific projects and programmes, while the UK's Funding Councils provide block grant funding to support the research infrastructure and enable institutions to undertake ground-breaking research of their choosing. Such funding also provides the capacity to undertake research commissioned by the private sector, Government Departments, charities, the European Union and other international bodies.*

Funds for teaching are essentially per capita based on student numbers, and are very important for the stability of departments especially as undergraduate intake in mathematics has been buoyant in recent years. (Some departments, in universities which could expand numbers without exceeding the total numbers limits imposed by their funding council, have doubled their undergraduate intake in recent years.)

2 Research councils

The seven research councils are:

- Arts and Humanities Research Council (AHRC)
- Biotechnology and Biological Sciences Research Council (BBSRC)

[1] see e.g. `www.hefce.ac.uk/finance/fundinghe/trac/`

[2] `www.rcuk.ac.uk`

- Engineering and Physical Sciences Research Council (EPSRC)
- Economic and Social Research Council (ESRC)
- Medical Research Council (MRC)
- Natural Environment Research Council (NERC)
- Science and Technology Facilities Council (STFC)

Research Councils UK (RCUK) is a strategic partnership between them, established in 2002. According to the RCUK website, the research councils are all: *Non-Departmental Public Bodies (NDPBs), established by Royal Charter and are independent legal bodies outside of Government, accountable to Parliament.* It also states that: *Research Council funds are awarded on the basis of applications made by individual researchers, which are subject to independent, expert peer review. Awards are made on the basis of the research potential and are irrespective of geographical location.*

Mathematical sciences obtains some funding from all of the research councils (e.g. for philosophy and logic from AHRC, mathematical biology from BBSRC, statistics from ESRC, mathematical medicine from MRC, environmental mathematics from NERC, and mathematical physics from STFC). EPSRC has a **mathematical sciences programme** from which it funds research across the full breadth of the discipline, and mathematical scientists are also funded by other EPSRC programmes and mission themes.

It is unfortunately extremely difficult to gather information on how much funding goes to the mathematical sciences (or any other discipline) across RCUK: funding is to individual projects and cannot be easily aggregated, and the research councils use different formats for any data they do display. A common, searchable database structure across RCUK would make it feasible to determine whether funding was maintaining the "health of discipline", and also to compare total funding levels between disciplines.

2.1 EPSRC funding

EPSRC provides funding for PhD students, fellowships, and research. It manages its portfolio through **programmes**: some are discipline-based (such as *materials, mechanical & medical engineering* or *physical sciences*), and others are based around **mission themes**, such as *digital economy* or *energy*.

Mathematical scientists receive funding from many of EPSRC's programmes, and not only the mathematical sciences programme. The existence of a separate programme is extremely important for a "people-based" subject like mathematical sciences, which typically has different strategic priorities from lab- or equipment-based disciplines. Examples include funding for post-doctoral fellowships, allocation of (DTA) PhD studentships by peer review, rather than algorithmically, and funding for the Isaac Newton Institute (INI) and International Centre for Mathematical Sciences (ICMS). Both the INI and ICMS encourage and facilitate leading-edge national and international collaborative research, and also provide Knowledge Exchange (KE) and Knowledge Transfer (KT) support for the activities of UK-based mathematical scientists.

Mathematical sciences programme budget

The programme budget for 2010-11 is £11.4M for **training**, £9.3M for **fellowships** and £13.7M for **research**.

The main spend on **training** is for doctoral training accounts (DTAs) to fund PhD students. Although the mathematical sciences programme uses peer review to allocate DTA funding

(DTA funding for other disciplines is determined algorithmically on the basis of the value of EPSRC grants obtained), it is rolled into the total award at institutional level. This means that there is no guarantee that it will in fact be used to fund PhDs in mathematical sciences. More information on DTAs are given on the relevant sections of EPSRC's website[3] and in the LMS *Doctoral Training* paper.

All EPSRC funding for Masters courses in applied/industrial/computational mathematics (and most other disciplines) was withdrawn in 2008[4], and this has been profoundly damaging. Masters studentships in statistics and operational research were provided in 2010-11 (£800K was top-sliced from the mathematical sciences DTA), but it is not clear what will happen in future. If there is no further funding, then masters courses in these strategically important areas will be inaccessible to most UK students (the standard undergraduate student loans scheme does not apply to stand-alone masters courses). This is already the case for all other mathematics masters courses apart from Cambridge's Part III.

The three main types of **fellowship** funded are postdoctoral (for recent PhD graduates; up to 10 a year); career acceleration (for those with 3-10 years of postdoctoral experience) and leadership (for established academic staff). EPSRC is currently piloting a new *dream fellowship* scheme, whose stated aim is to "create a prestigious group of supporters and role models". This is a closed scheme – potential applicants are identified by EPSRC and invited to apply. It is apparently part of a "move to bespoke peer review".

"Responsive mode" (**research**) awards are also an important source of funding for postdoctoral positions, and provide a bridge between a PhD and permanent posts in academia or industry. There are concerns at the low number of these (compared to other subject areas), and the threat this poses to the health of the discipline. Only about half of the research budget in 2010-11 will be used for standard responsive mode grants, with the rest earmarked for various initiatives, including research projects in two of the **mission themes**. More controversially, £1.5M of the mathematical sciences research budget for 2010-11 has been earmarked for *mathematical sciences platform grants*. This was a closed call in which five departments were invited to apply based on the amount of funding received from the EPSRC mathematical sciences programme[5]. The stated rationale for these platform grants is to *provide underpinning funding to support the volume and range of research activity within a department*, and not to support a specific project.

EPSRC's annual reports contain details of total research grant expenditure per programme, and the figures for mathematical sciences are tabulated below.

Year	**2004**	**2005**	**2006**	**2007**	**2008**
Research grant expenditure (£M) (mathematical sciences programme)	7.681	8.778	10.138	11.699	14.331

Much of the increase over this five-year period is due to the introduction of the full economic cost (fEC) funding model for new grants starting on or after 1 April 2006[6]. Unfortunately comparable figures from 2009 are not available – that annual report has the category heading of "mathematical sciences & public engagement" and does not give a separate figure for mathematical sciences expenditure.

Note that the EPSRC expenditure tabulated above is just that from the mathematical sciences programme, and does not include awards to mathematics or statistics departments from other

[3] `www.epsrc.ac.uk`

[4] when the "Collaborative Training Account" scheme was replaced by the "Knowledge Training Account"

[5] A selection based on ranking by EPSRC grant holding would have been different.

[6] Note that the entire "grant spend" will not be on an fEC basis until April 2011.

EPSRC programmes. This information is not easy to find from EPSRC documentation, but a very useful source of EPSRC funding information is the website belonging to Prof. Bill Lionheart (University of Manchester)[7], in particular the table of total value of EPSRC grants held by mathematical sciences departments[8]. This uses a Perl script to extract the information from the EPSRC website each month, and lists mathematical sciences departments alphabetically and also ranks them by total value of EPSRC grants currently held.

2.2 Other RCUK funding

As noted above, it is not easy to obtain overall funding information (or at least not of a type that is particularly meaningful) from any of the RCUK websites. Grant "spend" information is collected in the Research Assessment Exercise (RAE), and the most recent information available from the RAE 2008 website is for 2006-07. In this year the amount of RCUK grant spent by mathematical sciences (i.e. the sum of the figures for pure and applied mathematics and statistics and OR) from all institutions submitting in these areas is recorded as £28.3M, which is 68% of the total spend over all categories. Other significant categories of income are: UK-based charities (with grant spend of £2.4M, or 6% of the total), other UK government bodies (£2.5M, or 6%) and EU government bodies (£3.7M, or 9%). Between them these four categories comprise 89% of the mathematical sciences total. The total mathematical sciences research spend for 2006-07 over all 16 categories is £41.4M.

These RAE data probably underestimate the total research spend, especially for categories other than RCUK grants, but even so they provide some sort of comparison with the EPSRC expenditure tabulated in the previous subsection. Taking the average of the EPSRC mathematical sciences programme figures for 2006 and 2007, and comparing this with the RCUK and total spends for 2006-07, we see that the EPSRC mathematical sciences programme provided about 40% of research funding from RCUK, and about 25% of the total research funding in 2006-07. This is the best estimate we have, but it may not be very accurate.

3 Funding councils

The three funding councils and DEL use the results of the Research Assessment Exercise (RAE) to selectively distribute research funding to the universities in their jurisdiction. The RAE and its proposed successor REF are outlined below, and this is followed by a brief description of the research funding mechanism used by the different UK countries. More detailed information on funding is provided in the Appendix to this paper.

3.1 Research assessment exercise (RAE)

This is a periodic peer review exercise to evaluate the quality of research in UK higher education institutions. The most recent of these reported in late 2008, and it differed from previous RAEs in giving research quality as a graded profile rather than as a single number. Mathematical science was evaluated in three separate sub-panels: pure mathematics (20), applied mathematics (21), and statistics & operational research (22). The weightings used for the final quality profile were: 70% for research outputs, 20% for the research environment, and 10% for esteem indicators, each being measured on the five–point quality scale tabulated below.

[7] `www.maths.manchester.ac.uk/~bl/maths_league/2009/`

[8] `www.maths.manchester.ac.uk/~bl/maths_league/mathsgrants.html`

4*	world-leading in terms of originality, significance and rigour
3*	internationally excellent in terms of originality, significance and rigour but which nonetheless falls short of the highest standards of excellence
2*	recognised internationally in terms of originality, significance and rigour
1*	recognised nationally in terms of originality, significance and rigour
Unclassified	falls below the standard of nationally recognised work, or does not meet the published definition of research

Full information on RAE 2008, including details of submissions and results can be obtained from `www.rae.ac.uk`.

3.2 Research excellence framework (REF)

This is the name for the proposed successor to the RAE. The main differences are that REF will (controversially) also evaluate institutions on the "impact" of their research in each discipline (although the details of how this will be done are not yet finalised), and that the three mathematical sciences sub-panels will be combined into a single panel[9].

3.3 England (HEFCE): research funding methodology

The Higher Education Funding Council for England (HEFCE) refers to its funding as a "quality-related research" (QR) grant, where quality is as determined by the RAE. The main research funding method is known as "mainstream QR": in 2009-10 it totalled £1,074M (out of a total research fund of £1,572M). The allocation of mainstream QR is a two-stage process: the amount of research funding provided for different subject disciplines is decided first, and the total subject pot is then distributed amongst institutions. Both stages are purely algorithmic, but use different data. More information is given in Section A2 of the Appendix[10].

3.4 Scotland (SFC): research funding methodology

The Scottish research funding methodology changed substantially in 2009-10, and is now very different from the two-stage processs used in England. There is a change in nomenclature to reflect the change in methodology: research funding is now called the Research Excellence Grant (REG) instead of QR, and is calculated from a single formula. One key difference from England is that the size of the "discipline pot" in Scotland depends heavily on the number of research associates, research students and grant income. This penalises grant–poor subjects like the mathematical sciences and led to a 22% cut in funding in Scottish mathematical sciences in 2009-10 compared to 2008-09, despite a good RAE performance and a large increase in academic staff submitted. Scottish mathematical sciences research funding is further penalised compared to that in the rest of the UK because SFC uses substantially lower "cost weights" in the funding formulae for Applied Mathematics and Statistics & Operational Research than those used elsewhere. More information on the Scottish allocation algorithm is given in Section A3 of the Appendix[11].

[9] Further information on what is proposed for the REF is available from `www.hefce.ac.uk/research/ref`.

[10] or for full details see the HEFCE document "Recurrent grants for 2009-10" available from `www.hefce.ac.uk`

[11] or for full details see the document "Grant Letter: General Fund" documents for 2008-09 and 2009-10 available from `www.sfc.ac.uk`

3.5 Wales (HEFCW): research funding methodology

HEFCW also refers to its funding a "quality-related research" (QR) grant, although aspects of the allocation appear more similar to SFC's REG allocation than HEFCE's QR. In 2010-11 HEFCW's QR allocation totalled £73.9 million, and a sum of £6.1m, which was added to QR from other sources in 2009-10, is allocated in proportion to 4* work only. HEFCW use a quality-volume threshold: submissions with fewer than 3 staff FTE of classified work (at 1* and above) are not eligible for QR funding. For further details see Section A4 of the Appendix.

3.6 Northern Ireland (DEL): research funding methodology

The Department for Employment and Learning (DEL) in Northern Ireland also uses the terminology "Quality-related research" (QR) funding, and the total amount of mainstream QR for 2009-10 was £38.1 million.

The allocation procedure for mainstream QR is similar to that in Wales. More information is given in Section A5 of the Appendix[12].

[12] full details of the allocation process can be found on the *Quality-related Research (QR) funding* page of the `www.delni.gov.uk` site

Appendix: Funding council allocation of research support

A1 Introduction and definitions

The UK funding councils (one each for England, Scotland and Wales) and the Department for Employment and Learning in Northern Ireland use the results of research assessment exercises (RAEs) to selectively distribute research funding to the higher education institutions (HEIs) within their jurisdiction.

The most recent RAE was held in 2008, and it assessed research by peer review in 67 separate subject discipline **units of assessment (UoAs)**, labelled 1–67. Mathematical science was evaluated in three of these: pure mathematics (20), applied mathematics (21), and statistics & operational research (22). The UoAs were grouped into 15 **main panels**, labelled A–O: the three mathematical sciences UoAs and computer science & informatics (23) comprised main panel F[13].

The term **submission** is used for the research submitted by one HEI to a single UoA. Key quantities for determining the funding attached to a single submission are:

V = full time equivalent number of academic staff in the submission

C = discipline-specific research cost weight multiplier (described below)

Q = quality rating (the various funding councils use different weightings to convert the grades in the research quality profile into a single number Q, as described below).

The 2008 RAE differed from previous ones in the way research quality was reported. Institutions received a *quality profile* for their research in each subject area, and this showed the proportion of research activity which met defined levels of quality on a five–point scale ranging from 4* down to unclassified (UC) as tabulated below.

4*	world-leading in terms of originality, significance and rigour
3*	internationally excellent in terms of originality, significance and rigour but which nonetheless falls short of the highest standards of excellence
2*	recognised internationally in terms of originality, significance and rigour
1*	recognised nationally in terms of originality, significance and rigour
UC	falls below the standard of nationally recognised work, or does not meet the published definition of research

Each of the four UK Funding Councils allocates its (RAE-based) research funding in a different way, and these are described below. However, since they all depend heavily on the quality profile, disciplines whose panels were more generous in their interpretation of the above definitions are favoured. That there was some inconsistency in grading even seems to have been noted officially. For example, the minutes of the final meeting of Panel F (29 Oct 2008) state that it was agreed:

- *That whereas the average overall profiles for the four sub-panels were comparable, the average proportion of 4* in the final profiles over those institutions which submitted to three or four of the sub-panels was notably higher for sub-panel F23* [computer science & informatics] *than for the other three* [mathematical sciences] *sub-panels.*

[13] Full information on panel membership and working methods is available from `www.rae.ac.uk/panels`.

- *That this discrepancy should not be taken to be an indication of the relative strengths of the subfields in the institutions where comparisons are possible.*
- *The panel expressed the hope that, in the process of determining the relationship between RAE results and QR funding, or any other use to which the results of the exercise may be put, the areas covered by subpanels F20, F21 and F22 would not be adversely affected as a consequence of the choice of structure for Panel F.*

A2 England (HEFCE): research funding methodology

The Higher Education Funding Council for England (HEFCE) refers to its funding as a "quality-related research" (QR) grant, where quality is as determined by the RAE. The main research funding method is known as "mainstream QR": in 2009-10 it totalled £1,074M out of a total research fund of £1,572M. The allocation of mainstream QR is a two-stage process: the amount of research funding provided for different subject disciplines is decided first, and the total subject pot is then distributed amongst institutions. Both stages are purely algorithmic, but involve different quantities. They are outlined below, and more details are given in the HEFCE document "Recurrent grants for 2009-10" available from `www.hefce.ac.uk` and the spreadsheet `www.hefce.ac.uk/research/funding/qrfunding/2009/resdata0910.xls` (the contents of the various columns of this spreadsheet are described below). Details of funding for 2010-11 have also recently been released, and these are available from `www.hefce.ac.uk/research/funding/QRFunding`.

Definitions:

- Research cost weight multiplier (C). The three values of C used by HEFCE are 1.0, 1.3 and 1.6. The UoAs applied mathematics and statistics & OR both have $C = 1.6$, whilst for pure mathematics $C = 1.3$.
- Quality rating (Q). For each UoA this is a weighted average of the RAE quality profile. The weightings were made more selective in 2010-11 and those for both years are tabulated below.

Description	4*	3*	2*	1*	UC
Q weights for 2009-10	7	3	1	0	0
Q weights for 2010-11	9	3	1	0	0

Example: a submission with RAE 2008 quality profile of 30% at 4*, 35% at 3*, 25% at 2*, 10% at 1* and 0% UC, has

$$\begin{aligned} Q_{2009-10} &= (30 \times 7 + 35 \times 3 + 25 \times 1 + 10 \times 0 + 0 \times 0)/100 = 3.4 \\ Q_{2010-11} &= (30 \times 9 + 35 \times 3 + 25 \times 1 + 10 \times 0 + 0 \times 0)/100 = 4.0 \end{aligned}$$

- Volume above HEFCE's minimal quality threshold (denoted here by V^*).

  V^* = the volume of research in a submission that is rated at the 2* quality level or higher in the RAE.

  So if the example submission above had V academic staff FTE, then $V^* = 0.9\,V$.

A subscript on each of the quantities above (e.g. C_i or Q_i) refers to the value of that quantity for a single submission. Sums can be taken over all submissions in all subject areas (typically written as $i \in$ all), all submissions in one of the 15 main panels (e.g. $i \in$ panel F), or all submissions in a specific UoA (e.g. $i \in$ UoA 20).

Distribution of mainstream QR between submissions

Throughout Section A2, "Col" refers to a column of HEFCE's `resdata0910.xls` spreadsheet file described above. Columns A and B contain the code and name of the institution, and C–D are the UoA number and name. The entry in Col_E is the letter Z unless the institution has made more than one submission to that UoE. The percentage of the submission rated at each research grade (from 4* to UC) is given in Cols F–J, and Col_K is titled "2009-10 Model volume measure". Note that this is **V* rather than V**, and the conversion formula is:

$$V = \mathrm{Col}_K \times 100/(\mathrm{Col}_F + \mathrm{Col}_G + \mathrm{Col}_H)\,.$$

The quality rating Q of the UoAs is not listed explicitly, but Col_O (called "Model total quality-weighted volume") contains the product $Q\,V$ (note: it is $Q\,V$ and not $Q\,V^*$). Column P contains the sum of the entries in Col_O over the UoA, i.e.

$$\mathrm{Col}_P = \sum_{i \in \, UoA} Q_i\, V_i\,,$$

and so it is constant for the UoA.

The total mainstream QR funding (F_{tot} = £1,074M in 2009-10) is first allocated between the subject fields of the 15 RAE main panels in proportion to V^* weighted by C. That is, the total funding awarded to all the submissions covered by main panel X (where X is one of A–O) is:

$$F_X = F_{tot} \sum_{i \, \in \, panel \; X} C_i\, V_i^* \Big/ \sum_{i \, \in \, all} C_i\, V_i^*\,.$$

This figure was adjusted in 2009-10 to ensure that the proportion of mainstream QR allocated to each main panel group in science, technology, engineering, medicine and mathematics (STEMM; i.e. panels A–G) was at least what it was in 2008-09. The totals F_X are not given in the spreadsheet (either before or after adjustment), but are tabulated below. The quantity F_X (calc) is that given by the above formula, and F_X (actual) is the sum of the RAE submission amounts (which are listed in Col_R) over the main panel. "Ratio" is F_X (actual) / F_X (calc) as a percentage – note that the adjustment to some of the STEMM panels means that the amount given to each of the non-STEMM panels (H–O) is scaled by 0.89. The quantity α_X (which is the funding unit of resource for the panel) is described below.

Panel	F_X (calc, £)	F_X (actual, £)	Ratio (%)	α_X
A	97,806,364	97,807,091	100	6,791.16
B	53,998,114	53,998,512	100	7,345.54
C	58,342,590	58,343,023	100	7,540.84
D	94,826,466	108,831,324	115	8,976.07
E	104,232,340	118,598,923	114	8,077.74
F	95,034,400	95,035,104	100	6,992.37
G	117,631,292	139,497,268	119	8,297.32
H	53,680,922	47,716,412	89	6,200.28
I	73,611,967	65,432,924	89	6,172.56
J	85,184,127	75,719,293	89	6,514.10
K	61,068,701	54,283,342	89	7,062.27
L	12,356,965	10,983,980	89	6,791.11
M	50,180,805	44,598,149	89	6,178.61
N	50,817,603	45,171,244	89	6,036.64
O	65,195,888	57,951,955	89	6,058.00

The main panel total F_X is allocated between UoAs in proportion to the product of QVC, i.e. the research funding F_j awarded to submission j in panel X is:

$$F_j = F_X \, Q_j \, V_j \, C_j \Big/ \sum_{i \in \, panel \; X} Q_i \, V_i \, C_i \, .$$

(Note that although V^* is the volume measure used to distribute funding between main panels, ***V* is used when allocating money to individual submissions**.) It is easier to make comparisons between disciplines by giving F_j in terms of the funding unit of resource α_X for panel X, which is

$$\alpha_X = F_X \Big/ \sum_{i \in X} Q_i \, V_i \, C_i \, ,$$

its value is given for each panel in the final column of the table above. Then

$$F_j = \alpha_X \, Q_j \, V_j \, C_j \, .$$

If two submissions in two different disciplines have the same staff volume (V) and quality rating (Q), then their funding levels will be in proportion to the product of their research cost weight (C) and value of α_X. Most STEMM subjects (including applied mathematics and statistics & OR, but not pure mathematics) have $C = 1.6$ in England, and funding differences between these disciplines are solely due to variations in α_X. Neither α_X nor F_X are given explicitly in the spreadsheet, although α_X can be obtained from the cost-weight C and columns P and Q:

$$\alpha_X = \mathrm{Col}_Q / (C \, \mathrm{Col}_P) \, .$$

Note that within a UoA (for which C is constant), the quantity $F_i/(Q_i \, V_i)$ is constant, and equal to $\mathrm{Col}_R/\mathrm{Col}_O$.

The mainstream QR allocation for each submission, F_j, is listed in Col_R. In addition, Col_V contains a list of transitional QR funding (£12M in total) which is provided "in order to limit some of the more significant changes in rates of funding per funded volume for 21 UoAs" (not including any of the mathematical sciences ones). The transitional funding is said to be "provided for 2009-10 only", with no guarantee that it will continue in future years.

Allocation of other elements of QR

The previous subsection has described the HEFCE calculation of mainstream QR for 2009-10. The other QR elements which are allocated on a submission level are outlined below.

- £32M for London weighting for mainstream QR: this is given in Col_S, and appears to be equal to 12% of Col_R for London institutions.
- £203M for research student supervision: this is given in Col_U, and appears to be equal to the number of home research students (with London weighting where appropriate) from Col_M, multiplied by 3878. It does not appear to depend on Q or C.
- £194M charity-related funding: this is given in Col_V and appears to be equal to the charity income (from Col_N) divided by about 3.17 for an institution in central London, 3.29 for an institution in greater London, and 3.55 for an institution outwith London. It also does not appear to depend on Q or C.

There is also

- £63M business research (awarded at institutional level);
- £6M for national research libraries (awarded at institutional level).

A3 Scotland (SFC): research funding methodology

The Scottish research funding methodology changed substantially in 2009-10, and is now very different from the two-stage processs used in England. There is a change in nomenclature to reflect the change in methodology: research funding is now called the Research Excellence Grant (REG) instead of QR. In Scotland the main research grant (£203M out of a total of £208M in 2009-10) is distributed in proportion to $Q\,(V\,C + X)$, where X is a weighted combination of "other activity indicators" (postdoctoral research associates, research students and grant income). The key difference from England is that the size of the "discipline pot" in Scotland depends heavily on X. Grant–poor subjects like the mathematical sciences are thus penalised, and this resulted in a 22% cut in funding in Scottish mathematical sciences in 2009-10 compared to 2008–09 (despite a good RAE performance and a large increase in V). The allocation process is described briefly below, and full details are given in the "Grant Letter: General Fund" documents for 2009-10 and 2010-11 available from `www.sfc.ac.uk`.

Definitions:

- Research cost weight (C). The three values of C used by SFC are 1.0, 1.2 and 1.6 (UoA 4 had an intermediate rating of 1.4 in 2009-10, which was increased to 1.6 in 2010-11). The three mathematical sciences UoAs (pure mathematics, applied mathematics and statistics & OR) all have $C = 1.2$ (and are thus **rated as proportionally cheaper subjects in Scotland than in the rest of the UK**).
- Quality rating (Q). For each UoA this is a weighted average of the RAE quality profile. The Scottish weightings (which were unchanged in 2010-11) are based on the cube of the RAE rating and are tabulated below.

Description	4*	3*	2*	1*	UC
Q weights	8	3.375	1	0.125	0

Allocation of REG to submissions

There are two separate funding pots in REG, the main one being $R^{[1]} =$ £203.354M in 2009-10 (and increased to £208M in 2010-11), which is allocated amongst all subject disciplines. STEMM subjects (those in panels A–G) also receive a share of the smaller pot $R^{[2]} =$ £5M (unchanged in 2010-11).

The formula for allocating $F_j^{[1]}$, the share of $R^{[1]}$ allocated to submission j is:

$$F_j^{[1]} = \beta^{[1]}\, Q_j\ (V_j\, C_j + X_j)\ ,$$

where the unit of resource is

$$\beta^{[1]} = R^{[1]} \Big/ \sum_{i\,\in\, all} Q_i\ (V_i\, C_i + X_i)\ .$$

SFC helpfully list the total $\sum_{i\,\in\, all} Q_i\ (V_i\, C_i + X_i)$; it is called the "departmental rating" and given as 39,045.86 in 2009-10. There was a significant increase in the departmental rating (to 43,230.21) in 2010-11, caused mainly by a large increase in the total of "other activity indicators", X, which are measured every two years. Thus

$$\beta^{[1]}_{2009-10} = \frac{203.354 \times 10^6}{39,045.86} = 5208.08 \quad \text{and} \quad \beta^{[1]}_{2010-11} = \frac{208 \times 10^6}{43,230.21} = 4811.45\,.$$

Note that because values of Q, V and C are available (or can be calculated) for all submissions, this formula also allows X to be calculated for each submission. In total, 38% of $R^{[1]}$ was distributed due to other activity indicators in 2009-10, and this rose to 44% of the total in 2010-11.

The £5M pot $R^{[2]}$ is distributed amongst STEMM submissions in proportion to $Q\,V\,C$. That is, the formula for allocating $F_j^{[2]}$, the share of $R^{[2]}$ allocated to the STEMM submission j is:

$$F_j^{[2]} = \beta^{[2]}\, Q_j\, V_j\, C_j\,,$$

where

$$\beta^{[2]} = R^{[2]} \Big/ \sum_{i\,\in\, panels\ A\text{-}G} Q_i\, V_i\, C_i\,.$$

The sum in the above expression is also given (it is confusingly called the total of departmental ratings for STEMM, despite being given by a different formula). In 2009-10 it was 14,674.89, and it increased to 14,872.61 in 2010-11 (because of the cost weight change for UoA 4). This gives $\beta^{[2]}_{2009-10} = 340.72$ and $\beta^{[2]}_{2010-11} = 336.19$.

The total funding F_j allocated to submission j is then:

$$\text{STEMM:}\quad F_j = Q_j\,\left(\beta\, V_j\, C_j + \beta^{[1]}\, X_j\right)$$

$$\text{Non-STEMM:}\quad F_j = \beta^{[1]}\, Q_j\,\left(V_j\, C_j + X_j\right)\,,$$

where $\beta = \beta^{[1]} + \beta^{[2]}$ (so $\beta_{2009-10} = 5548.80$ and $\beta_{2010-11} = 5147.64$).

There was also £2.8M of transition funding in 2009-10 in order to smooth reductions in funding levels for two institutions.

Calculation of X

The 'other activity indicator' component of a submission's funding, X, depends on the following quantities.

- V_{RA}, the FTE number of postdoctoral research assistants;
- V_{RS}, the FTE number of research students;
- V_{RI}, the total of non-charitable research income divided by £35,000; and
- V_{CI}, the total of charitable income divided by £35,000.

The formula for X is

$$X = 0.15\,\left(V_{RA} + V_{RS} + \mu_R\, V_{RI} + \mu_C\, V_{CI}\right)\,,$$

where μ_R and μ_C are multipliers which are chosen to fill the 'gap' in funding the full economic cost, and are tabulated below.

Year	μ_R	μ_C
2009-10	1.599759	2.4870231
2010-11	1.732755	2.6598898

Allocation of other elements of research funding

The previous subsection has described how SFC calculates its REG. Other elements are outlined below.

- **Research postgraduate grant.** This is allocated in proportion to research postgraduate enrolments weighted by teaching subject. Its total was £29.4M in 2009-10 and £29.7M in 2010-11.
- **Knowledge transfer grant.** Each institution is awarded £70,000 for knowledge transfer, with an additional amount to be allocated from SFC's Horizon Fund.

A4 Wales (HEFCW): research funding methodology

HEFCW also refers to its funding a "quality-related research" (QR) grant, although aspects of the allocation appear more similar to SFC's REG allocation than HEFCE's QR. In 2010-11 HEFCW's QR allocation totalled £73.9 million, and a sum of £6.1m, which was added to QR from other sources in 2009-10, is allocated in proportion to 4* work only. HEFCW use a quality-volume threshold: submissions with fewer than 3 staff FTE of classified work (at 1* and above) are not eligible for QR funding.

Definitions:

- Research cost weight (C). The three values of C used by HEFCW are the same as HEFCE's, namely 1.0, 1.3 and 1.6, and the three mathematical sciences UoAs have the same cost weights as in England, namely $C = 1.6$ for applied mathematics and statistics & OR and $C = 1.3$ for pure mathematics. (There were no Welsh submissions to UoA 22, with all relevant staff being returned to another UoA. Most applied mathematicians were also returned to UoAs other than 21.)
- Quality rating (Q). For each UoA this is a weighted average of the RAE quality profile. The Welsh weightings for 2010-11 are the values which HEFCE used in 2009-10, as tabulated below:

Description	4*	3*	2*	1*	UC
Q weights	7	3	1	0	0

- Volume. As in Scotland, the effective volume measure includes "other activity indicators". It is here denoted by V_T and is equal to $V + V_O$, where V (as previously) is the number of FTE staff in the submission, and V_O is the sum of research students and research associates/fellows (weighted by 0.15 and 0.1 respectively) plus an amount proportional to charitable grant income.

Allocation of QR to submissions

Funding allocations are in proportion to the product $Q\,V_T\,C$. Any submissions which do not meet the volume threshold of at least 3 staff FTE of classified work (i.e. at 1* or above) are excluded from the calculation. The overall outcomes of the formula calculation are scaled to the total amount of funding available for QR (£73.9 million in 2010-11). Two elements of the total QR budget are allocated on a different basis:

- The additional £6.1 million added to QR in 2009-10 is allocated in proportion to 4* quality only. (The standard volume and subject weightings are also applied to this element.)
- £3.1 million is provided to help institutions meet the full economic cost (FEC) of research funded by charities. This funding is allocated in proportion to research income from UK charities to submissions which meet the QR volume threshold.

A5 Northern Ireland (DEL): research funding methodology

The Department for Employment and Learning (DEL) in Northern Ireland also uses the terminology "Quality-related research" (QR) funding, and the total amount of mainstream QR for 2009-10 was £38.1 million.

The allocation procedure for mainstream QR is very similar to that in Wales. The possible values of the research cost weight (C) are again 1.6, 1.3 and 1.0, and pure mathematics has $C = 1.3$ (there were no N Irish submissions to UoAs 21 or 22). The Quality rating (Q) is also the same as that used by HEFCW, and a submission's "volume" V_T is again the sum of V (the FTE number of staff submitted to the RAE) and the volume V_O of "other activity" indicators. The N Irish formula for V_O is the sum of the number of research fellows and research assistants, weighted by 0.067 and 0.06 respectively.

There are also separate allocations for postgraduate supervision, charities support, and two special funds: the Sustainability Research Fund and Research Capability Fund. Full details of the allocation process can be found on the *Quality-related Research (QR) funding* page of the `www.delni.gov.uk` site, and in the associated *University Recurrent Research Grant Summary Tables 2009-10*.