International Review of Mathematical Sciences

Draft report to be presented at the Town Meeting

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Contents

Executive Summary

An International Review of Mathematical Sciences research in the United Kingdom (UK) was conducted in early December 2010 by a panel of sixteen experts whose research interests span the range of the Engineering and Physical Sciences Research Council (EPSRC) Mathematical Sciences Programme. The panel received extensive data from EPSRC before the review and gathered further information in six intense days that included site visits at universities throughout the country. Broadly framed, the panel's charge was to assess UK mathematical sciences research in comparison with the rest of the world, and to respond to a list of related questions.

An overall finding is that actions taken by EPSRC since the 2004 International Review of Mathematics and the 2004 Review of Operational Research have greatly contributed to invigoration of mathematical sciences research, including improved structures for PhD education.

The backdrop for all of the panel's judgements includes several principles: the mathematical sciences provide a universal language for expressing abstractions in science, engineering, industry and medicine; mathematical ideas, even the most theoretical, can be useful or enlightening in unexpected ways, sometimes several decades after their appearance; the mathematical sciences play a central role in solving problems from every imaginable application domain; and, because of the unity of the mathematical sciences, advances in every subarea enrich the entire field.

UK mathematical sciences research is world-leading in some fields, outstanding in many others and strong overall, although there are, of course, areas that could be strengthened. A major factor in the present excellence of the UK academic mathematical sciences enterprise is its diversity—in area, group size and size of institution—and its geographically distributed nature. To preserve and increase the present level of quality, it is essential for research funding structures to honour this diversity. In this time of great change, understanding between the mathematical sciences community and EPSRC is especially important.

Mathematical sciences researchers participate actively in multidisciplinary collaborations involving important and complex problems, and serve as valuable partners for industry in addressing its long-term challenges. Even so, better mechanisms are needed to encourage and strengthen the links between the mathematical sciences, other disciplines and industry.

The most worrying factor in education and training is that, for the most part, UK PhDs are not competitive for academic positions in today's global market. The UK has been able to attract outstanding research mathematicians from other countries, but the situation is fragile and needs continuing attention.

High-level recommendations of the panel include:

- Provision of a variety of funding programmes to enhance the diversity of the UK mathematical sciences research community;
- Creation of a new structure for enhancing communication between EPSRC and the mathematical sciences community;
- Actions and strategies focussed on individual subfields of the mathematical sciences;
- An array of mechanisms to encourage and strengthen links within the mathematical sciences, and between the mathematical sciences, other disciplines and industry; and
- Continued action and attention directed toward improving the quality of PhD training.

A complete list of findings and recommendations is given in Section 2.

Acknowledgements

Panel members are grateful to the Steering Committee for their excellent preparatory work that led to a well-structured, albeit busy, schedule. The panel also offers enormous thanks to the EPSRC staff involved in managing the review, and to the other Research Council staff who assisted and provided information throughout the review process. The panel is particularly grateful for the tireless efforts of EPSRC staff to ensure that members of the panel were in the right place at the right time; for their overall project management and preparation, including distribution of documentation for the review; and for their organisation of what would have been first-rate travel arrangements if the UK transport system and weather had cooperated. All of these elements contributed to a (pleasantly) memorable experience and the success of the review.

The panel is deeply appreciative of the time and effort expended by the participants in the site visits, especially those who travelled long distances under adverse weather conditions. The presentations made during the site visits were uniformly of high quality, obviously requiring substantial preparation, and the panel's frank and lively discussions with members of the mathematical sciences community were very enlightening. The panel expresses particular gratitude to collaborators and users who spent their valuable time informing the panel about their connections with the mathematical sciences. PhD students, postdocs, and junior faculty members are due special thanks for providing helpful insights about their experiences and perspectives. The panel also appreciates members of the mathematical sciences community, and a variety of organisations, who responded to EPSRC's call for consultation responses.

Foreword

This is a challenging time for science, when the UK seeks to extend both its international reputation for cutting edge fundamental research and its economic and social impact. This is the second International Review of Mathematical Sciences and reflects the important contribution of the area to the UK. This contribution, which arises from advances in fundamental knowledge and understanding of mathematics and the mathematical sciences, has a direct impact on areas as disparate as security, telecommunications, finance, industry, transport and medicine. Mathematical Sciences research underpins a wide range of activities that benefit society, including engineering, economics and computer science. Just as technology plays an increasingly vital part in virtually every aspect of modern society, mathematical sciences research continues to be indispensable.

The preparation for this review has been under way for over a year and we would like to thank our colleagues on the Steering Committee, which included representation from the Edinburgh Mathematical Society, Industrial Mathematics KTN, Institute of Mathematics and its Applications, Institute of Physics, London Mathematical Society, Operational Research Society, and the Royal Statistical Society. We also thank our colleagues from the Biotechnology and Biological Sciences Research Council (BBSRC), Economic and Social Research Council (ESRC), the Medical Research Council (MRC), the Natural Environment Research Council (NERC) and the Science and Technology Facilities Council (STFC) who also supported and helped to guide the review. Specific thanks must go to the EPSRC staff for their unwavering support and hard work which enabled the panel to do its work so effectively.

This report, the culmination of the whole review process, is entirely the work of the International Review Panel to whom we are very grateful—their expertise, team work, enthusiasm and capacity for sheer hard work impressed all those who came into contact with them. To Professor Margaret Wright, Chair of the International Review Panel, we are hugely indebted; her commitment and leadership were vital to both the review and the completion of this report.

We also warmly thank all those in the academic research community, together with their collaborators in industry and elsewhere who are so vital to the health of UK Mathematical Sciences research, for rising to the challenges and opportunities that this review presented. We are especially grateful to those who coordinated and participated in each visit, often travelling some distance to meet the panel in what were often difficult weather conditions, and being able quickly to organise video conferencing when the weather made it impossible to travel. The close working with the community meant that the panel was able to interact with a great many researchers and to witness a wider range of exciting advances than would otherwise have been possible in the short time available.

We hope that this report will stimulate further debate around the findings and recommendations highlighted and we genuinely welcome feedback on any issues raised. Comments should be sent to the Mathematical Sciences International Review team at `MathematicsIntReview@epsrc.ac.uk`.

Professor David Delpy	Professor Tim Pedley
Chief Executive, EPSRC	Steering Committee Chair

1 Introduction and Background

1.1 Purpose and scope of the review

The Engineering and Physical Sciences Research Council (EPSRC) holds regular international reviews to inform itself and the community (including stakeholders, industrial bodies, learned societies, academia and government departments) about the quality and impact of the science base in the United Kingdom (UK) compared to the rest of the world and to highlight any gaps or missed opportunities. Each international review provides a broad perspective on the research activity in a particular discipline in the UK, and is undertaken with the relevant learned institutions and other research councils as appropriate; it has been a rolling programme in which the research base in each discipline is reviewed approximately every five years. Twelve reviews have been conducted since 1999.

This report presents the conclusions of an international panel of sixteen experts convened by a Steering Committee on behalf of all UK Research Councils to review mathematical sciences research in the UK. The scope of the present review covers the entire mathematical sciences portfolio of the Research Councils, including all of mathematics, statistics, and the more mathematical aspects of operational research (OR). This collection of areas represents a broadening of the scope of the 2004 International Review of Mathematics [13] in that OR was the subject of a separate review [21] in 2004; the present panel therefore specifically includes researchers with expertise in operational research and related fields.

The broad tasks to be addressed by the panel—its Terms of Reference[1]—are:

1. Assess the quality of the UK research base in mathematical sciences in comparison with the rest of the world;

2. Assess the impact of the research base activities in mathematical sciences internationally and on other disciplines nationally, on wealth creation and quality of life;

3. Comment on progress since the 2004 Reviews of Mathematics and Operational Research (including comment on any changed factors affecting the recommendations); and

4. Present findings and recommendations to the UK research community and Research Councils.

The review focussed mainly on research and training in UK academic institutions, but also considered collaborations and users outside academia. Other EPSRC international reviews were addressed when relevant. Although most of the covered activity falls within EPSRC's remit, the review also encompassed areas such as mathematics embedded within environmental research (in NERC's remit); mathematical biology (in BBSRC's remit); biomedical informatics, statistical genetics and computational biomedicine (in MRC's remit); and mathematical physics (in STFC's remit).

1.2 Structure of the review

The review was commissioned and managed by EPSRC, and guided by a Steering Committee (listed in Section 19) chaired by Professor Tim Pedley, University of Cambridge. After receiving nominations from the community, this Steering Committee selected the panel chair, Professor Margaret Wright, Silver Professor of Computer Science and Mathematics in the Courant Institute of Mathematical Sciences, New York University, USA to take on this role for the review. In concert with the chair, the Steering Committee selected the other fifteen members, all based outside the UK. Panel members represented a balanced mix of international researchers with expertise spanning the mathematical sciences and relevant Research Council disciplines. The names and affiliations of panel members are given in Section 20, which also contains brief academic biographies. [Biographies are not included in the version of the report to be presented at the town meeting on 28 January 2011.]

In concert with the chair, the Steering Committee accomplished the following tasks: agreeing on the high-level questions to be addressed by the review panel (the Evidence Framework; see Section 18); agreeing on the institutions to be visited by the panel; advising on the background data to be provided to the panel; agreeing on the agenda for the review period; receiving a preliminary briefing from the panel

[1] See www.epsrc.ac.uk/research/intrevs/2010maths/Pages/scope.aspx.

at the end of the review period; and facilitating the disposition and use of this final report. Throughout, EPSRC provided support for the organisation, planning and logistics.

The eight areas of the Evidence Framework assessment (given in full, with annotations, in Section 18) address the following questions:

A. What is the standing on a global scale of the UK mathematical sciences research community both in terms of research quality and the profile of researchers?

B. What evidence is there to indicate the existence of creativity and adventure in UK mathematical sciences research?

C. To what extent are the best UK-based researchers in the mathematical sciences engaged in collaborations with world-leading researchers based in other countries?

D. Is the UK mathematical sciences community actively engaging in new research opportunities to address key technological/societal challenges?

E. Is the mathematical sciences research base interacting with other disciplines and participating in multidisciplinary research?

F. What is the level of interaction between the research base and industry?

G. How is the UK mathematical sciences research activity benefitting the UK economy and global competitiveness?

H. How successful is the UK in attracting and developing talented mathematical sciences researchers? How well are they nurtured and supported at each stage of their career?

The framework was used extensively by the panel in the review process, although these questions are not a template for organisation of the report. Responses to framework questions are embedded in the report. In addition, for the reader's convenience Section 18 contains the questions in the evidence framework, annotated with report section numbers that address each question. Section 2 summarises the panel's findings and recommendations, with cross-references to sections in the report.

1.3 Review panel activities

During the week starting Sunday, 5 December 2010, the full panel convened at Heathrow, England, for a preliminary assessment of the quality and impact of academic mathematical sciences research in the UK. After a briefing meeting on Sunday, the panel divided into three sub-panels for 'site visits', with each sub-panel visiting a separate set of venues during the ensuing Monday to Wednesday for an intense period of review. During the site visits, the sub-panels interacted with researchers from 42 institutions that receive mathematical sciences funding, holding separate meetings at each site with collaborators (academic and industrial) and with early-career researchers. During each site visit, EPSRC colleagues were not present during a block of time, typically 30 minutes, in which panel members continued their discussions with site visit participants; the panel and EPSRC representatives agreed, in advance, on this format.

The full panel reassembled in London on the morning of Thursday 9 December for a meeting with industrial 'users' of mathematical sciences research, and then moved outside London to discuss and develop preliminary findings and recommendations.

Throughout the week, panel members considered the eight areas of the evidence assessment framework and also added additional topics as appropriate. EPSRC provided an overview of research in mathematical sciences in the UK in terms of its people, funding, organisation and policy; further details about these data are given in Section 1.5.1.

In closed meetings on 9 and 10 December, the panel reached a common understanding about the context, vision and opportunities afforded by the Mathematical Sciences Programme; made judgements on strengths and weaknesses of the programme to date; and developed recommendations related to research excellence, knowledge exchange, people and research infrastructure, in accordance with the panel's terms of reference (see Section 1.1) and to 'help inform the development of future strategies by EPSRC and other key stakeholders'.[2]

This report is the panel's final product, developed from further analysis and refinement of findings following from the evidence available to the panel (Section 1.5).

1.4 Relevant external events

A report on research typically does not describe the political context in which it occurred, but a short discussion is warranted by the unusual circumstances during and shortly after this review. The relevant context is that a new UK coalition government, which took office on 12 May 2010, announced plans on October 21 for an 'austerity budget' in which most of the Research Councils escaped the deepest cuts.

On December 9, when the panel was in London to meet industrial users of the mathematical sciences, Parliament voted to adopt the recommendations of the 'Browne Review', officially titled 'Securing a Sustainable Future for Higher Education in England' [10]. Student demonstrations with some violence occurred in London to protest against a substantial increase to the upper limit on fees for undergraduate education. (The panel left London before the main protests began.) The panel is therefore aware of a high level of concern at many universities about how implementation of the Browne report's 'consumer preference' model, in which student willingness to pay is likely to guide the courses offered by universities, will affect undergraduate education in mathematical sciences and, consequently, research as well as preparation for graduate education.

The panel heard repeatedly that severe recent visa/work permit restrictions for non-EU citizens, imposed by the government in November 2010, are jeopardizing new research appointments, collaborations, visitors and PhD student intake.

It was apparent to panel members that these factors had caused great anxiety among the researchers, PhD students and postdocs whom we met during our site visits.

A final element of the external context is that the EPSRC Delivery Plan for 2011–2015 [6] was published on 20 December 2010. The chart on page 21 of that plan shows that EPSRC's planned expenditure in 2014/2015, measured in real terms, represents a 3% decrease in programme expenditure and a 50% decrease in capital expenditure compared to 2010/11. The immediate effect of the cuts in capital spending is likely to fall on the Science and Technology Facilities Council (STFC), which funds research in particle physics, astronomy and (as noted in Section 5.6) mathematical physics.

1.5 Evidence used by the panel

The evidence used by the panel in agreeing on its findings and recommendations (Section 2) includes three elements: the data described in Section 1.5.1; information gathered from presentations and discussions at the sites visited by the panel (see Sections 1.3 and 1.5.2); and the extensive and varied scientific and professional expertise of panel members (see Section 1.5.2).

1.5.1 Data used by the panel

The panel was provided with a hefty document, *Information for the Panel*, which contained information, data, and commentary to supplement the information acquired in site visits. Evidence in this document was compiled from the Research Councils' management records, the Higher Education Statistics Agency and Thomson Reuters. The document contained 85 pages of evidence prepared by EPSRC, 95 pages of 'Landscape Documents', and 206 pages of 'consultation responses' to EPSRC's call for public comment on the evidence framework.

Selected bibliometric data from Thomson Reuters were included in the evidence prepared by EPSRC, along with appropriate cautions as to the interpretation of this information. (The mathematical sciences community is, to a large degree, highly aware of the dangers in over-reliance on 'impact factors' and 'citation indices', based on a 2008 report [3] prepared by three international organisations in the mathematical sciences.)

Each landscape document covers UK research in a specific subfield, providing an overview of activity in the area, relevant data collected during the most recent (2008) Research Assessment Exercise (RAE)

[2] `www.epsrc.ac.uk/research/intrevs/2010maths/Pages/more.aspx`.

[20], and (most usefully) commentary on strengths, weaknesses, and opportunities. The landscape documents were prepared by small groups of UK researchers and vetted with their colleagues. Because these documents will be made public after publication of the present panel's report, they have, perhaps inevitably, the flavour of 'laundry lists', including lists of names, locations and research areas, mentioning primarily generic rather than specific strengths, weaknesses and concerns.

There was a noticeable degree of agreement among the many opinions in the consultation responses to the Evidence Framework.

Each panel member received a folder of 'case studies'. These included a series of press releases, titled 'Mathematics Matters' and written for a general audience, that describe how the mathematical sciences have contributed to the solution of real-world problems.

The data provided for use by the International Review will be published on the EPSRC web site on completion of the review. (Documentation provided in confidence will not be published.) The panel requested some additional information from EPSRC during the review week.

The panel acknowledges three papers produced by the London Mathematical Society [16]: on the relationship between research and teaching, UK government funding for mathematical sciences research, and doctoral training in the UK.

The data listed above will not, for the most part, be cited explicitly in this report. For the record, the panel affirms that they formed an important foundation for our discussions and conclusions.

1.5.2 Evidence provided by site visits and panel members' expertise

During the site visits, panel members heard from a broad cross-section of the mathematical sciences community. (As noted in Section 1.3, each site visit contained sessions that included only panel members and site visit participants.) Hence panel members acquired, while in the UK for the review week, a substantial amount of information from mathematical sciences community members about their views and experiences related to research excellence, knowledge exchange, people and research infrastructure.

All panel members were chosen in part for their experience and knowledge of mathematical sciences research policy in their home countries, to provide the international perspective needed for this review. These experiences and knowledge appropriately informed the panel's discussions as well as its findings and recommendations.

1.6 Overview of the panel's report

The main body of the report begins with Section 2, which lists the panel's findings and recommendations along with a subfield-by-subfield list of the panel's views on challenges and opportunities in those subfields.

As background for the remainder of the report, Section 3 describes key elements that characterise the mathematical sciences. Section 4 explains why panel members view 'diversity' as a defining and essential feature of the UK mathematical sciences research community.

Section 5 reviews subfields of the mathematical sciences in some detail, providing the panel's assessment of strengths and points of concern; Section 6 does the same for application areas that are tightly connected to the mathematical sciences. Section 7 describes the UK's research capability in the imprecisely defined but crucial area of industrial mathematics.

Sections 8 and 9 comment on the importance of institutes and learned societies in the quality and visibility of UK mathematical sciences research.

Multidisciplinary collaborations constituted a major focus of the questions posed to the panel. At every site visit, the panel met research collaborators; Section 10 describes several multidisciplinary interactions that were presented at those meetings.

Section 11 reports on the many topics touched upon at the panel's session with industrial users, including comments about how to improve connections with industry.

Based on the site visits and the panel's own impressions, Section 12 makes a strong recommendation for a new mechanism to enhance communication between EPSRC and the mathematical sciences community.

Research excellence is necessarily related to funding and support. The high value placed by the panel on the community's diversity led to Section 13, which stresses the importance of diverse funding structures that honour diversity while remaining entirely consistent with support of excellence.

Section 14 discusses various ways for improving connections between the community, academic colleagues in other areas and industry.

Like the 2004 International Review of Mathematics [13], this panel devotes a section of its report (Section 15) to special concerns about structural fragility in UK statistics research.

Section 16 addresses issues concerning undergraduate students, PhD students, postdocs and early-career researchers. One of the questions posed to the panel involves gender diversity, which is explicitly discussed in Section 16.5.

Finally, Section 17 considers the extent of collaborations of UK mathematical scientists with leading international researchers.

1.7 On names of individuals and lists of institutions

This report does not intend, nor does it attempt, to assess or rank individual researchers or institutions. Individuals are named only when their standing or contribution is unique. Lists of institutions are alphabetical, with no implied ranking.

2 Findings and Recommendations

For convenience, all of the panel's findings and recommendations are contained in this section, with cross-references to the parts of the report where they are discussed.

2.1 Findings

F-1. Overall, mathematical sciences research in the UK is excellent on an international scale, with world-leading researchers in every subfield and closely connected application area considered by the panel. *All subsections of Sections 5 and 6.*

F-2. The high quality of UK mathematical sciences research depends critically on the diverse and distributed research community, where 'diverse' includes research area, group size and institution size, and 'distributed' refers to geographical location. *Section 4.*

F-3. Actions taken by EPSRC since the 2004 International Review of Mathematics and the 2004 Review of Operational Research have greatly contributed to invigoration of the mathematical sciences, including improved structures for PhD education. *Sections 5, 6 and 16.2.2.*

F-4. Newly established institutes and centres dedicated to furthering research in specific topics, interdisciplinary research and connections with industry have improved the UK's international visibility and standing in the associated areas. *Sections 5.2, 5.3, 5.4, 5.8, 5.9, 5.10, 5.11, 6.1, 6.3, 6.4, 7, 10 and 14.*

F-5. The institutes make a significant contribution to the visibility and quality of UK mathematical sciences research, as do activities of the learned societies. *Sections 8 and 9.*

F-6. Despite improvements, most UK-educated PhDs in the mathematical sciences are not adequately trained to be competitive on the international academic job market; hence a large proportion of postdocs and junior faculty consists of researchers trained outside the UK. *Sections 16.3 and 16.4.*

F-7. Action about gender diversity is not a sufficiently high priority for the UK mathematical sciences research community. *Section 16.5.*

2.2 Broad recommendations

R-1. **To research funders**: The panel strongly recommends that diversity and distributedness of the UK mathematical sciences (Finding F-2, above) should be enhanced by providing a variety of funding programmes designed so that the best mathematical sciences researchers can advance their activities in research and graduate education.

To provide maximal support for top-quality researchers in this context, it is highly desirable to have flexible funding models that permit geographically distributed researchers working in a broad scientific area to receive adequate long-term funding. *Sections 4, 13, 15.2 and 16.2.2.*

R-2. **To research funders, learned societies and the mathematical sciences community**: Open, frank and timely communication between EPSRC and the mathematical sciences community is extremely important. In addition to strengthening existing processes, the panel strongly recommends the establishment, as soon as possible, of a new structure for communication between EPSRC and the mathematical sciences community. A joint effort between EPSRC and leadership of the learned societies is an obvious way to begin to define such a structure. *Section 12.*

R-3. **To research funders, institutes and learned societies, and the mathematical sciences community:** Activities such as workshops, possibly arranged by the institutes or learned societies, should continue to be organised in order to encourage and enhance connections within the mathematical sciences, with academic colleagues in other areas and with industrial collaborators, and to involve the mathematical sciences community in proposed or contemplated major research initiatives. *Sections 3.2, 14.1.1 and 14.2.*

R-4. **To research funders, universities and the mathematical sciences community:** Strong efforts should be made to ensure that UK PhD training meets the highest international standards. This recommendation relies on the provision of PhD student funding during an adequate period of training. *Section 16.3.*

R-5. **To universities, funders of PhD research and the mathematical sciences community:** To complement existing EPSRC programmes that support PhD education (such as Taught Course Centres and Centres for Doctoral Training), UK universities should consider establishing, as a norm, a PhD programme that begins with a special one-year research Master's degree, followed by three years of PhD education and training. *Sections 16.2.2 and 16.2.4.*

R-6. **To research funders, industry, universities and the mathematical sciences community:** In addition to existing programmes that connect industry and mathematical sciences research, long-term collaborations focussing on basic research driven by industrial challenges should be explicitly encouraged. These collaborations should include multi-year, close relationships that involve academics and one or more financially committed industrial partners, as well as graduate education in the theory and methods needed to solve industrial problems.

Companies should explore establishing long-term strategic relationships with universities. Smaller companies should take full advantage of the Industrial Mathematics Knowledge Transfer Network.

Sections 11.2 and 11.3.

R-7. **To universities, industry, the mathematical sciences community and research funders:** Arrangements should be explored to create appropriate roles within academia for experts from industry so that universities can benefit from their technical knowledge and experience in collaboration. *Section 11.3.*

R-8. **To research funders:** The review panels for multidisciplinary proposals whose topics include the mathematical sciences should include members with expertise in both multidisciplinary research and relevant areas of the mathematical sciences. *Sections 14.1.1 and 14.1.2.*

A separate panel is desirable for reviewing proposals in operational research and statistics. *Sections 14.1.2, 5.9 and 15.2.*

R-9. **To universities and research funders:** In statistics, decisive action is needed to enhance the ability of small departments to compete on the international level for new faculty. A more flexible grant structure such as the one proposed in Recommendation R-1 would help to address this issue. *Section 15.1.*

R-10. **To universities:** Strong statistics research and teaching programmes should be supported at a large number of UK universities. *Section 15.2.*

R-11. **To research funders, the mathematical sciences community and universities:** Urgent action should be taken to improve participation of women in the mathematical sciences community, with special reference to EPSRC's strategy for developing leaders. *Section 16.5.*

R-12. **To EPSRC:** After enough time has passed, EPSRC should assess and analyse the outcomes of the variety of institutes and centres that are intended to further multidisciplinary and industrial collaborations involving the mathematical sciences. *Section 14.3.*

2.3 Field-specific recommendations—challenges and opportunities

These recommendations represent field-specific opportunities for universities, research funders and the mathematical sciences community to strengthen the quality of UK mathematical sciences research. The ordering of the topics follows that in the main body of the report.

C-1. **Algebra:** Several outstanding young appointments provide an opportunity to establish new vital research areas in an array of subdisciplines and to pursue synergies between them and beyond. *Section 5.1.*

C-2. **Analysis:** Despite progress since 2004, analysis in the UK is still underrepresented compared to the rest of the world, with a notable shortage of home-grown talent; efforts to strengthen existing excellent groups in analysis of all sizes should accordingly be continued. Opportunities should be exploited for fruitful collaborations with other fields of the mathematical sciences and with applications. *Section 5.2.*

C-3. **Combinatorics:** Further improvements should be made to build and strengthen connections among discrete mathematics, combinatorics, theoretical computer science and related areas such as graph theory; this applies as well to number theory (C-7, below). *Section 5.3.*

C-4. **Fluid mechanics:** Joint projects with engineering and physics departments would be beneficial, as would recruitment of outstanding computational fluid dynamicists and numerical analysts of partial differential equations. *Section 5.4.*

C-5. **Geometry and topology:** A big challenge in geometry is exploring the relations of quantum field theory to geometry and number theory. The part of geometry that most needs strengthening in the UK is the connection between geometric analysis and partial differential equations. *Section 5.5.*

C-6. **Mathematical physics:** Special attention should be paid to ensuring that excellent proposals in mathematical physics will be supported despite budget cuts in STFC. *Section 5.6.*

C-7. **Number theory:** More links should be developed with ergodic theory. There should be more analytic and algebraic expertise in all aspects of modular and automorphic forms, including the Langlands programme. UK research in number theory should look to build in important new directions, rather than reinforcing areas that are already well represented. Stronger connections are needed with computer science to develop work in cryptography (as for combinatorics; C-3, above). *Section 5.7.*

C-8. **Numerical analysis/scientific computing:** Numerical analysis should become more closely connected with computational science and engineering. Optimisation researchers should develop closer ties with analysis, operational research, probability and statistics. *Section 5.8.*

C-9. **Operational research:** Further strengthening would be beneficial for research in simulation, risk, forecasting, revenue management, supply chain problems, and applications in production, manufacturing, bioinformatics and data mining. Closer links are encouraged with optimisation researchers in numerical analysis. *Section 5.9.*

C-10. **Probability:** Because of likely retirements of prominent probabilists during the next few years, attention needs to be paid to preserving the UK's excellence in the areas they represent. Due to emerging applications, support is needed for in-depth engagement between highly trained probability researchers and first-class scientists. *Section 5.10.*

C-11. **Statistics:** For reasons discussed in detail in Section 15, measures are needed to avoid the loss of the UK's high international stature in statistics; see Recommendations R-9 and R-10, above.

C-12. **Computational science and engineering:** Additional support is needed for collaborative computational science and engineering research, with research teams that include mathematical scientists in key roles. UK programmes should be integrated as much as possible with related international networks. *Section 6.1.*

C-13. **Financial mathematics:** The UK research field should continue its expansion into study of commodity markets, including resources like oil, gas, electric power and agricultural products, and should continue developing more advanced methods. *Section 6.2.*

C-14. **Materials:** Mathematical research in materials science and solid mechanics remains underrepresented in the UK compared to the rest of the world. *Section 6.3.*

C-15. **Mathematical biology and medicine:** More systematic integration of the mathematical sciences in the established systems biology centres would increase UK strength. With respect to funding, new 'high-risk' funding programmes for mathematical biology are desirable. It is perceived that progress is sometimes impeded by differences between the funding structures of Research Councils. *Section 6.4.*

3 Nature of the Mathematical Sciences

The mathematical sciences form an essential part of science, engineering, medicine, industry and technology, and serve as one of the pillars of education at all levels. Major contributions to the health and prosperity of society arise from insights, results and algorithms created by the entire sweep of the mathematical sciences, ranging across the purest of the pure, theory inspired by applications, hands-on applications, statistics of every form and the blend of theory and practice embodied in operational research. Although the panel's charge focuses on research, an underlying message of this report is that mathematical sciences research is inextricably linked to both undergraduate and graduate education, including teaching university students in all disciplines the mathematical skills that are needed to function as informed citizens.

Every field of science and engineering is unique in some ways, and the mathematical sciences are no exception. Some distinctive properties of the mathematical sciences are sketched next as a foundation for the remainder of this report.

3.1 Unity

A longstanding practice has been to divide the mathematical sciences into categories that are, by implication, close to disjoint. Two of the most common distinctions are drawn between 'pure' and 'applied' mathematics, and between 'mathematics' and 'statistics'. These and other categories can be useful to convey real differences in style, culture and methodology, but, in the panel's view, they have produced an increasingly negative effect when the mathematical sciences are considered in the overall context of science and engineering, by stressing divisions rather than unifying principles. Furthermore, such distinctions can create unnecessary barriers and tensions within the mathematical sciences community by absorbing energy that might be expended more productively. In fact, there are increasing overlaps and beneficial interactions between different areas of the mathematical sciences, as discussed in several parts of Section 5.

Invoking the moral of Aesop's *Four Oxen and the Lion* [1], the panel urges the UK mathematical sciences community, relevant learned societies, and government funding bodies to adopt 'united we stand' as the most appropriate perspective on mathematical sciences research in the context of this review, where the features that unite the mathematical sciences dominate those that divide them. Our discussion (Section 5) of separate subfields for purposes of a more detailed assessment in no way contravenes this unified view.

3.2 Abstraction and long time scales

Much of the content of mathematical sciences research is expressed in terms of general relationships such as equations, formulae, and diagrams, which constitute a universal language for expressing abstractions in science, engineering, industry and medicine. Without the convenient forms provided by the mathematical sciences, other disciplines might develop their own specialised terminology for mathematical concepts, thereby inhibiting communication outside their field. Happily, the same (or closely related) abstractions can provide insights and intellectual tools for understanding and solving problems that seem disparate when represented in their 'natural' form. For example, the mathematical ideas in modelling the flow around swimming multicellular organisms might seem far removed from designing the Internet or controlling congested traffic because experts in each area use a different vocabulary; see Figure 1. But underlying similarities, as well as differences, are often revealed through mathematical abstractions, which also serve to translate fundamental concepts with different names from one field to another.

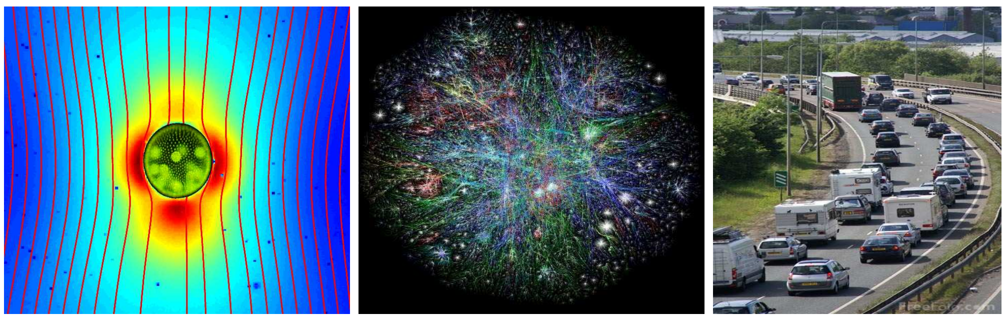

Figure 1: Three real-world settings in which the underlying mathematical models are closely related. In the first figure, streamlines (red) and magnitude of velocity field (colour) are shown around a colony of multicellular green alga *Volvox carteri* swimming freely upward. (Image courtesy of K. Drescher, M. Polin, I. Tuval, and R.E. Goldstein, DAMTP, University of Cambridge.) The second image depicts the structure of the Internet, and is a featured picture on the English language Wikipedia, shown under the Creative Commons Attribution 2.5 Generic License. The third picture, taken by Ian Britton, depicts congested traffic near Newcastle-upon-Tyne and is shown under the Creative Commons Attribution-Noncommercial-No Derivative Works 3.0 License, reference number 41-17-74.

Since mathematical results can be expressed, analyzed and verified in an abstract form independent of physical reality, the broad significance and multiple interpretations of some mathematical ideas are, in a real sense, everlasting, even when their importance is not fully recognised until decades after their publication. A mathematical theorem remains true and a counterexample remains false, regardless of subsequent advances in technology or computing power; the passage of time does not make mathematical ideas obsolete.

Examples abound where:

(i) A mathematical idea initially viewed as purely abstract turns out, much later, to be important in real-world applications. A quintessential instance arises in tomography, a ubiquitous non-invasive imaging technique that recovers the internal structures of an object, such as the heart, from external measurements. The Radon transform, published in 1917, provides the mathematical basis for recovering images from measured projection or scattering data [5].

Much more recently, algebraic topology is being applied to analyze complex data in a variety of fields (including biology, imaging, and neural science) that resist application of classical methods (see [4]).

(ii) In all areas of the mathematical sciences, ideas from one subfield serve as the inspiration and foundation for major results in a seemingly different subfield. For example, the 1960s 'Langlands programme' of conjectures in number theory led to the 'geometric Langlands programme' of conjectures in complex algebraic geometry, which were then shown to be related to quantum field theory (an important topic in mathematical physics). Very recently, as mentioned in Section 5.5, Ngô's Fields Medal-winning work, which established the 'Fundamental Lemma' of the Langlands programme, relied on the Hitchin fibration (developed in the context of geometry and mathematical physics).

(iii) Algorithms once dismissed as unsuitable or unstable are shown to be efficient following the development of new insights or the availability of new computer architectures. A well known instance in numerical linear algebra is the iteration proposed in 1950 by Lanczos for eigenvalue problems [14], which, despite its welcome novelty, provably suffered from a loss of orthogonality. Only in 1971 was it shown how this weakness could be overcome, and variants of the Lanczos method pervade today's state-of-the-art software for large-scale eigensystem computation.

It is essential to keep these unpredictable shifts in interpretation and applicability in mind when assessing the impact of the mathematical sciences on key technological/societal challenges (Question D in the Evidence Framework). The not-to-be-missed point is that the contributions of the mathematical sciences community should be *considered as a whole*. Although some researchers focus some of the time on addressing real-world challenges, other researchers devise remarkable insights and results that advance and strengthen the entire discipline by pursuing self-directed adventurous research.

3.3 Community structure and research needs

A significant fraction of excellent research in the mathematical sciences is carried out by individuals or small groups of collaborators. But this property does not hold uniformly: some mathematical scientists regularly work, and do their best work, as part of a large, focussed team. Using a description from the 2004 IRM [13], the mathematical sciences are 'people-intensive'. Attempts to generalise at a finer level about the conduct of research within the entire community are almost certainly misleading.

Special care is needed when placing the mathematical sciences within the context of research funding in science and engineering. Although many mathematical sciences researchers have 'modest', or even 'negligible', equipment needs, especially compared with areas such as high-energy physics that require expensive, high-maintenance shared facilities, there are mathematical scientists—for example, in fluid mechanics or mathematical biology—whose research requires complicated equipment and 'wet lab' space. Across the spectrum of the field, the research of many mathematical scientists relies on computing, which enables complex symbolic manipulation and formal verification of intricate hypotheses as well as number-crunching simulations, visualisations and data mining. Mathematical scientists working in computational science and engineering (Section 6.1) typically require access to high-end computing. Even mathematical scientists who work on their own and use computers only to process email and search the Web need financial support to attend conferences, visit colleagues and host visitors.

4 Excellence and Capability through Diversity

A metaphor used at the initial presentation (10 December 2010) of the panel's findings was that the mathematical sciences research strength of the UK can usefully be interpreted by analogy with the UK's world-renowned greatness in theatre. The UK has, without question, the best theatre in the world, and this is universally explained by its deliberately diverse, geographically distributed nature. For example, a lively article in *The Times* on 15 July 2010 [22] attributes the success of UK theatre to 'both culture and structure' and an 'instinct to keep taking risks' within an array of 'beacons of excellence, ... smaller fry punching well above their weight, ... and a host of provincial houses with local audiences', where 'every layer matters, bringing joy in its own right and feeding talent to the next'.

In the same way, the UK's internationally excellent mathematical sciences research community is both diverse and distributed. Here 'diverse' has three meanings: (i) including a broad expanse of research areas, (ii) belonging to groups of varying sizes, and (iii) representing institutions ranging from large to small. 'Distributed' means that outstanding mathematical sciences research is done at geographically spread institutions. *In the panel's view, it is essential to honour diversity in all three senses, as well as distributedness, when defining the structures available for support of UK mathematical sciences research* (Recommendation R-1, Section 2). In making this recommendation, the panel wishes to underscore its consistency with the principle (repeated at the beginning of Section 13) that only internationally excellent research should be supported by EPSRC and other Research Councils.

The subsections of Section 3 justify support for diversity; the remainder of this section explains why there should be adequate support for researchers at geographically distributed institutions.

The most recent (2008) UK Research Assessment Exercise [20] identified world-leading, internationally recognised mathematical scientists at a wide variety of institutions throughout the UK. An extensive official assessment thus confirms the existence of research excellence and capability at an array of geographically distributed universities.

During site visits, the three sub-panels consistently met outstanding researchers at institutions of varying sizes located around the country, and the subsections of Sections 5 and 6 clearly indicate the presence and productivity of excellent mathematical sciences researchers at geographically distributed locations. Given the competitive global academic job market, such researchers are likely to leave their current institutions if there is significantly reduced support for their research, with unfortunate consequences because those researchers and, indirectly, their institutions, play several essential roles that could not be maintained in their absence:

- In the past, graduate students, postdocs and junior faculty at geographically distributed institutions have become research leaders in the mathematical sciences. In particular, some of today's luminaries either did not receive their PhD from a university usually described as 'elite', or else did not begin their faculty career at such an institution. The strategy of 'Developing Leaders' in EPSRC's Delivery Plan [6] refers to 'world-leading individuals', but these individuals will not necessarily emerge from one of a small number of universities. Without support for excellent researchers at a broad range of institutions, the panel believes that there is a serious risk of irretrievably losing the talent of future world-leading researchers in the mathematical sciences.

- There appears to be an ample and growing number of undergraduates in the UK who are interested in studying the mathematical sciences (see Table 5 in the first LMS position paper [16]). If top mathematical sciences researchers were concentrated at only a small number of universities, undergraduates at other institutions would have less exposure to the interplay between high-level research and stimulating teaching that draws potential leaders into the field;

- There would be little opportunity for close interaction with local industry without outstanding researchers at nearby universities, thereby foreclosing a crucial element in the contribution of mathematical sciences research to the UK's economic prosperity and scientific/technological strength.

Based on evidence gathered by its panel, the 2004 IRM report [13] made the same point: the UK's capability and excellence in mathematical sciences research require a broad and deep field of excellent mathematical scientists, not only to produce outstanding research, but also to provide top-quality training across the UK for the next generation:

> The UK cannot afford to concentrate its advanced training in mathematics, which has to be maintained by the most up-to-date research, in a small number of highly competitive universities. [13, page 31]

A major ingredient in this panel's review is to make comparisons with the best research, and consequently the best research structures, in other countries. In this regard, one of the main conclusions in a 1997 US National Research Council report on international benchmarking of US mathematics is that the continued attractiveness of the US to talented people rests in large part on 'diversity and flexibility of the US research enterprise':

> Before World War II, only a handful of US research universities were distinguished in mathematics.... Rather than being dominated by a few institutions or individuals, ...[today's] diffuse structure allows a wide range of mathematicians from across the entire country, and their institutions, to excel. [11, page 30]

Valuing diversity and distributedness, and the concomitant excellence of the UK mathematical sciences research community, is thus in complete alignment with EPSRC's goal to 'create an environment that promotes excellence, encourages innovation, stimulates creativity and drives cultural, commercial and technological advances' [6, page 4], and leads to the panel's affirmation of two points, which should be interpreted as an integral part of Recommendation R-1 (Section 2):

The best mathematical sciences researchers, wherever they are located, should be provided with opportunities to advance their research activities and to create excellent training programmes for future research leaders;

To preserve and strengthen the diverse structure of the UK's mathematical sciences research capability, funding structures should be created that foster interactions and collaborations among excellent researchers at institutions of all sizes.

5 Subfields of the Mathematical Sciences

Broadly speaking, this panel found that mathematics in the UK has been invigorated by measures undertaken responding to earlier reviews [13, 21] and the most recent RAE [20]. In particular, the mathematical sciences research base has been strengthened by outstanding new research appointments, often from outside the UK, resulting in high international visibility and connectivity.

To answer Question A in the Evidence Framework, in this section and the next we discuss research in individual subfields, where each assessment is based on data provided for the review, site visits, and the panel's own knowledge and opinions.

This section covers subfields of the mathematical sciences, listed in alphabetical order. Fields in which it is difficult to separate the mathematical sciences and another area of science or engineering are discussed in Section 6. Recommendations, challenges and opportunities for all subfields are summarised in Section 2.

5.1 Algebra

Historically the UK has led the world in the important field of algebra, from the 19th-century work of giants like Cayley and Burnside to the 20th-century seminal algebraic applications by Fisher to statistics and the design of scientific experiments. The requirements of computational science and new communication technologies, as well as from within mathematics itself, have led to a resurgence in activity in algebra. Invitations to eight leading UK algebraists to lecture at the 2006 and 2010 International Congresses of Mathematicians constitute a widely accepted mark of prestige and high international standing.

UK researchers in geometric group theory occupy a leading role in the world, with large groups in Oxford and Southampton, as well as flourishing links with topologists and geometers in places like Warwick. The UK continues as world leader in the area of profinite groups, with, for example, proof of Serre's conjecture by Nikolov and Segal on open subgroups of finitely generated profinite groups. The work in representation theory is world-leading: since the 2004 IRM [13] there has been substantial growth in the strength and size of research groups in this area, especially at Scottish universities, with many new appointments of very high quality.

Outstanding world-class work is done in finite group theory, with strong research groups in Aberdeen, Birmingham, Imperial, Queen Mary and St Andrews, as well as high-quality individual researchers throughout the UK. Major research programmes exploit and advance the theory of finite simple groups and their associated geometries.

Excellent research at an international level is done also in asymptotic group theory, computational algebra and non-commutative algebra. The group in St Andrews coordinates the computational discrete algebra software system GAP[3] (Groups, Algorithms, Programming).

Strong cross-overs between algebra, combinatorics, number theory and algebraic geometry are evident, in terms of problem inspiration, theory development and applications. Impressive recent work, with perhaps unanticipated impact internationally, involves expansion in groups, spanning the boundaries between these subdisciplines. The UK is a leader in this area. The work has principal applications in analytic number theory and expander graphs, showing in particular that essentially all finite simple groups give rise to expanders. Numerous other notable UK advances have arisen from fruitful interactions: profinite groups and algebraic graph theory, group representations and zeta functions, geometric group theory and the structure of manifolds.

[3] `www.gap-system.org/`.

In addition to already established research groups in algebra, strong groups have grown recently (for example, at Aberdeen, East Anglia, Kent and Newcastle). Outstanding new appointments have been made, especially at Imperial, Oxford and Southampton.

A challenge for mathematicians working in algebra is to realise the opportunities created by several outstanding young appointments, to establish new vital research areas in the associated subdisciplines and pursue synergies between them and beyond.

5.2 Analysis

Analysis plays a key role in the mathematical sciences, first because of its deep connections to several other subfields. For example, Perelman's spectacular 2003 proof of the Poincaré conjecture emphasised the importance of analysis in topology and geometry, and the work of Green (Cambridge) and Tao on the existence of arbitrarily long arithmetic progressions is at the interface of number theory and harmonic analysis. In addition, analysis has long been, and remains, ubiquitous as an underpinning in the application of mathematics in science and engineering, in particular through the theory of ordinary and partial differential equations. Knowledge of recent developments in analysis and of new analytical concepts and methods is necessary for modeling real-world processes, characterising and computing solutions to systems, and describing fundamental mathematical structures.

Analysis has been always strong in the UK in classical subfields and is at the leading edge of research in dynamical systems and ergodic theory, operator algebras, and operator theory. However, the 2004 IRM [13] was concerned that UK research efforts were 'sparse and sporadic' in 'core areas of modern analysis and related areas of partial differential equations'. The previous IRM report also noted that, because analysis is regarded as a difficult subject for a PhD, few prospective students chose that area of research.

The situation has noticeably changed since 2004, thanks to several EPSRC initiatives. In 2006, two Science and Innovation Awards established research centres in analysis: at Oxford on 'Analysis of Nonlinear Partial Differential Equations', and at Edinburgh and Heriot-Watt on 'Analysis and Nonlinear Partial Differential Equations'. The new Cambridge Centre for Doctoral Training in analysis will also train more graduate students in this area.

Added strength in analysis is attributable to the UK's successful attraction of a number of excellent researchers at both senior and junior levels. Today, world-leading groups in analysis are located at Cambridge, Edinburgh, Oxford, Warwick and several smaller institutions. There is a cluster of excellent researchers in linear partial differential equations and spectral theory in the London area and excellent groups in ergodic theory in Bristol and Warwick.

Since analysis is such a central subject, exceptionally important breakthroughs in analysis have also been made by UK mathematicians whose main focus and motivation lies in other areas. Ever since Atiyah's work, the profound contributions of geometers to analysis have been recognised and valued. This great tradition continues with the work of Donaldson (Imperial) and his school. A much more recent phenomenon is that work in additive combinatorics, led by UK mathematicians, has seen deep and important common themes developing in many traditional analytic areas (harmonic and functional analysis, ergodic theory, analytic number theory, combinatorics and theoretical computer science), and strongly influencing algebra and group theory through expanders. Although such unification of areas has been the most outstanding feature of the development of algebraic geometry and topology in the last fifty years, this is new and exciting in analysis. There seems no obvious end in sight to the revolution that such methods are causing, and UK mathematicians should continue to be at the forefront of these developments.

Analysis is a huge and extremely active field worldwide, covering topics across the spectrum from pure to applied. The panel therefore believes that, despite progress since 2004, analysis in the UK is still underrepresented compared to the rest of the world, with a notable shortage of home-grown talent. The latter difficulty may require attention to undergraduate teaching; analysis is undoubtedly a demanding subject and may suffer if future undergraduate course offerings are directly based on a philosophy of students as 'course consumers' (see Section 1.4).

Efforts to strengthen existing excellent groups in analysis of all sizes should be continued, and opportunities should be exploited for fruitful collaborations between analysis and other fields of the mathematical sciences, most notably in geometry and topology, as well as in numerical analysis, mathematical biology and medicine, materials science and industrial mathematics.

5.3 Combinatorics and discrete mathematics

In combinatorics, areas of strength at the time of the 2004 IRM [13], such as algebraic and probabilistic combinatorics, continue to flourish in the UK. As well, many talented new appointments in this subdiscipline (often in computer science departments or business schools) have raised the international profile of UK research in combinatorics. In particular, there is new internationally strong research in the areas of hypergraphs, extremal combinatorics and quantum computation (Bristol); UK researchers remain world leaders in information security (Royal Holloway).

Discrete mathematics and combinatorics are inherently cross-disciplinary and are routinely invigorated by interactions with other areas. For example, the new UK-led theory of synchronisation[4] (Queen Mary, University of London), inspired by the 'celebrated' Černý conjecture from automata theory, impacts on graph theory and permutation group theory; and combinatorial methods have been crucial for algorithms in computational biology. Moreover, the new UK-led developments in additive combinatorics mentioned in Sections 5.1 and 5.5 also involve and energise combinatorics. A particularly interesting recent development was the solution of problems with a combinatorial flavour by a Web-based team, initiated by Gowers (Cambridge).

The fields of discrete mathematics and combinatorics have a strong tradition of interaction with industry, representatives of which regularly attend the key biennial British Combinatorics Conferences. Researchers in radio frequency allocation (City University, London School of Economics, Oxford) interact with the telecommunications industry; work on statistical design theory (Queen Mary, University of London) is essential to underpin the efficiency of scientific experiments.

The 2004 IRM [13] recommended that more attention should be paid to research involving discrete mathematics and the algorithmic side of theoretical computer science (which is not as strong in the UK as elsewhere in the world). Thanks to EPSRC initiatives, there has been improvement in connections between discrete mathematics and theoretical computer science. The Centre for Discrete Mathematics and Its Applications[5] (DIMAP) was established at Warwick in 2007 by an EPSRC Science and Innovation Award, founded on a collaboration between a group in the business school and the departments of computer science and mathematics. Nonetheless, the panel believes that further improvements could be made to build and strengthen connections among discrete mathematics, combinatorics, theoretical computer science and related areas such as graph theory.

Another issue noted by the 2004 IRM was the need to provide enough students with training in combinatorics and discrete mathematics to industrial and government employers. For employers connected with UK government intelligence, a supply of home-grown talent is obviously essential. The UK Government Communications Headquarters[6] (GCHQ) seeks to hire a significant number of PhDs in the mathematical sciences, but the panel did not hear directly from a representative of GCHQ whether the situation has improved since 2004. The creation in 2005 of the Heilbronn Institute of Mathematical Research (see Section 8), as well as comments by the representative of the Defence Science Technology Laboratory[7] at the users' meeting (Section 11), indicate that organisations with needs for highly trained people in combinatorics are working hard to build connections with UK universities.

5.4 Fluid mechanics

The UK has a prestigious tradition in fluid mechanics, going back to Newton, with many twentieth-century giants such as Taylor, Batchelor and Lighthill. The UK remains a world leader in this field, behind the US, ahead of Asian countries and of all other countries in Europe. This leadership rests not only on the excellence of the UK research in fluid dynamics but also on strong institutional vectors such as the Cambridge-based *Journal of Fluid Mechanics*. The Isaac Newton Institute and the International Centre for Mathematical Sciences (see Section 8) also provide world-renowned forums in the form of workshops for the international fluid dynamics community.

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[4] `www.maths.qmul.ac.uk/∼ pjc/LTCC-2010-intensive3`.
[5] `www2.warwick.ac.uk/fac/cross_fac/dimap`.
[6] `www.gchq.gov.uk`.
[7] `www.dstl.gov.uk`.

There are many centres of excellence, including Bristol, Cambridge, Imperial, Manchester, Nottingham, Oxford, University College and Warwick among others. The UK community is thus highly diversified with excellent research in most of the main subdisciplines of fluid dynamics. UK researchers in fluid dynamics are masters at devising elegant quasi-analytical models that effectively capture the dominant physical phenomena of a given flow configuration without excessive formalism. They bring to bear on the discipline the full range of sophisticated mathematical methods and are the undisputed leaders in this type of approach.

The UK is world-leading in hydrodynamic stability theory and transition, with recent landmark investigations on a fundamental problem which had been poorly understood for a long time: the transition from a laminar state to fully turbulent flow in pipes. The UK is very strong in many diverse areas including astrophysical fluid dynamics, low Reynolds number hydrodynamics, interfacial phenomena, waves, vortex flows, geophysical fluid dynamics, stratified and rotating flows, and magnetohydrodynamics.

It is significant that UK fluid dynamics research is not confined to mathematical sciences departments; departments of engineering are also actively engaged. The panel accordingly believes that UK research on both sides would benefit from joint research programmes in which (for example) mathematical scientists would draw inspiration from real configurations and validate predictions through laboratory and numerical experiments.

Colleagues from the physics community have invigorated areas such as the dynamics of complex fluids (granular materials and non-Newtonian flows) and interfacial phenomena (bubbles and drops) by relying on observations as the starting point for the derivation of deceptively simple scaling laws. Such a fresh point of view has proved to be particularly successful. In contrast with the US and continental Europe, relatively few UK physicists have been involved in fluid dynamics research. The Manchester Centre for Nonlinear Dynamics,[8] which gathers applied mathematicians and physicists around a variety of theoretical and experimental projects in nonlinear science, could serve as a model for enhancing these links.

The 2004 IRM [13] recommended more active development of computational fluid dynamics (CFD) as a central means of investigation, on a par with experiments and theory, because CFD has become essential to investigate large Reynolds number turbulent flows where wide ranges of length and time scales are dynamically relevant. Compared with the US, the UK contribution to CFD still appears to be relatively modest. This research area could be more aggressively pursued by mathematical scientists without setting aside elegant asymptotic methods but in conjunction with them. Hence the present panel repeats the recommendation of the previous IRM that the interface between CFD and numerical analysis (Section 5.8) should be strengthened. Other areas that could be strengthened include turbulence and complex media.

Fresh and insightful points of view at the interface with aerospace and mechanical engineering, and with the macroscopic physics community, should be promoted by fostering joint projects with engineering and physics departments. Likewise, recruitment of outstanding computational fluid dynamicists and numerical analysts of partial differential equations is to be encouraged. Such developments are needed if the UK is to keep its preeminence in fluid dynamics in Europe.

5.5 Geometry and topology

The UK has been a world leader in geometry and topology for many years, and it continues that tradition today. One indication of this status is that four UK geometers/topologists gave invited talks at recent International Congresses of Mathematicians, while Donaldson (Imperial) has been the recipient of several prestigious international prizes (King Faisal, Nemmers and Shaw).

The past seven years have seen exceptional activity and progress worldwide in geometry, with the solution of the Poincaré conjecture, finite generation of the canonical ring and solution of the Mumford and Kervaire invariant conjectures as particular highlights. UK mathematicians were essential contributors to the solutions of the second and third problems, which lie in the areas of algebraic geometry and algebraic topology respectively.

There are also world-leading UK groups in mirror symmetry, moduli spaces, string topology, differential geometry, symplectic geometry and geometric group theory. There are very strong groups in many other parts of the subject such as surgery theory, integrable systems and low-dimensional topology. A

[8] `www.mcnd.manchester.ac.uk`.

notable recent phenomenon has been the rise of an excellent group in geometry/topology distributed among the London institutions. Although many of the other leaders in this area work in main centres such as Cambridge, Oxford and Warwick, this is not true in all cases; there are several smaller departments (Aberdeen, for example) with world-class geometers/topologists.

One of the great strengths of UK geometry is its broad base. It combines ideas from numerous fields such as algebraic geometry, representation theory, string theory, topology and analysis and, in turn, its main results often have profound implications in these fields. As an example, the work by Hitchin (Oxford) on Higgs bundles and the geometry of the Hitchin fibration played an important role in Ngô's recent spectacular proof of the 'Fundamental Lemma' in the Langlands programme that won a 2010 Fields Medal. Similarly, Donaldson and other members of his school have made very significant contributions to the theory of partial differential equations. New areas of the subject, such as geometric group theory, are of increasing importance, building links to yet other areas such as dynamical systems.

Much of geometry and topology was initially developed to understand questions arising in physics and astronomy, and its most significant applications are still in those fields, for example string theory and the theory of relativity. However, now UK researchers are developing interesting applications in biology, for example the use of knot theory and three-dimensional topology to study DNA/protein interactions or the study of the symmetries of the surfaces of viruses. There are also some applications in robotics.

A big challenge for geometry is to explore the relations of quantum field theory to geometry and number theory (e.g., mirror symmetry in geometry, Witten-Kapustin in number theory). With its many brilliant and broadly trained geometers, the UK is very well placed to contribute here.

The part of geometry that most needs strengthening in the UK is the connection between geometric analysis and partial differential equations. Since Perelman's solution of the Poincaré conjecture using the Ricci flow, this approach to understanding topological questions has become much more important. Although the UK has one or two world-class mathematicians working in this area, it would certainly benefit by having more. On the positive side, in the past few years the UK has attracted some brilliant young geometers from abroad; they should be supported at a level that will encourage them to stay.

5.6 Mathematical physics

The UK continues to have world-class research programmes in several of the more mathematical areas of theoretical physics, including general relativity, cosmology, string theory and quantum chaos. There is a very healthy interaction between these areas of theoretical physics and other areas of mathematics. A few examples are: (1) physical arguments in string theory have motivated new mathematical conjectures which algebraic geometers have later proved; (2) physical questions in general relativity often require knowledge of the long time behavior of solutions to nonlinear partial differential equations which people working in geometric analysis can sometimes provide; (3) string theory has recently provided a surprising connection between general relativity and fluid dynamics; and (4) there are close interactions between researchers studying quantum chaos and number theory.

Since the 2004 IRM [13], there have been several appointments of outstanding young researchers (especially in Cambridge, Cardiff, Durham and Imperial) to continue the longstanding tradition of excellence in these areas. Some of these appointments were in response to recent retirements, but others represent growth. Some of the more senior researchers have won prestigious prizes.

One concern for the future is that recent severe budget cuts, especially in the Science and Technology Facilities Council (see Section 1.4), are making it difficult for many people in this area to obtain research support. It is inefficient as well disheartening for researchers to learn only after the fact that research in their area is not being funded. Because work in mathematical physics often falls between traditional areas of mathematics and physics, the panel recommends that special attention be paid to ensuring that excellent proposals in mathematical physics (which do not require capital expenditure) can still be supported. The UK's excellence will certainly be threatened if support disappears, even temporarily.

5.7 Number theory

The UK is the world leader in all aspects of the circle method, in analytic number theory, in harmonic analysis, and in quantitative arithmetic geometry, led by researchers at Cambridge, Bristol and Oxford. This includes some of the most exciting research in analytic number theory worldwide. (Three of the last six speakers at the International Congress of Mathematicians in analytic number theory come from this group in the UK.)

Outstanding number theory research is being done at quite a few UK universities. Dynamic and highly capable groups have emerged recently at Bristol, primarily younger researchers in analytic and computational number theory, quantitative arithmetic geometry and ergodic theory; at the London Universities, primarily younger researchers in a wide array of subjects in arithmetic geometry; at Oxford, where there has long been a small, high-quality group. This group has been greatly enhanced very recently by several strong hires, including Sir Andrew Wiles (an obviously world-leading figure).

Cambridge has had a tradition of strength in arithmetic geometry in the last twenty years, and many UK faculty received their training there. The internationally leading group in additive combinatorics at Cambridge has not only led a revolution in analytic number theory, but has been training excellent young people, with the potential for greatly strengthening UK combinatorics, analysis and number theory. However, both Cambridge groups should be broader.

Recent work (led by UK researchers) in additive combinatorics has had an important influence on group theory, ergodic theory and theoretical computer science; the panel hopes that future hiring in UK number theory might reflect these new directions.

Many smaller excellent groups in number theory are located at institutions throughout the UK, and several of those include participants who have had a large impact on the international stage. There are significant strengths in most important areas of number theory, particularly in analytic number theory, higher dimensional arithmetic geometry (both geometric, as in obstructions to local-global principles, and quantitative), arithmetic geometry and algebraic number theory, diophantine approximation, and computational and algorithmic number theory. The link with quantum chaos, via random matrix theory, continues to be strong, and links with logic have become even stronger with some excellent recruitment. (In both of these themes, UK researchers are world-leading.)

Some researchers are exploring the exciting boundary between ergodic theory and diophantine approximation (from the direction of Margulis/Lindenstrauss), and in modular and automorphic forms (both analytic and algebraic), and the panel would like to see more work along these lines. Indeed much recent important number theory has taken place on interdisciplinary boundaries, but, in the panel's view, hiring has tended to focus on areas that are already very well represented rather than these new directions.

Echoing a theme from Section 5.3 about discrete mathematics, there is too much separation between number-theoretic cryptographers in computer science departments and interested number theorists in mathematics departments. Efforts should be made by both communities to close this gap.

Although there is work in areas related to transcendental number theory, no specialists in the subject are working in the UK at present. With the proximity of the strength in Paris, it would be good to see more interaction that might lead to more development of this subject in the UK.

5.8 Numerical analysis/scientific computing

As described in the 2004 IRM [13], the UK was a pioneer in numerical analysis during the 1950s, when it was recognised that rigorous mathematical techniques were needed to understand the properties of algorithms implemented on computers. The dual name of this section arises because, in many parts of the world, 'numerical analysis' is called 'scientific computing'.

Numerical analysis in many areas, including but not limited to those that were strong in 2004—linear algebra, multiscale and adaptive algorithms, ordinary and partial differential equations (especially stochastic), and numerical optimisation—remains at a very high international level in the UK, with world-leading individuals in several categories. UK researchers in numerical analysis have received significant international recognition, as confirmed by their list of honours, editorships and prestigious invited talks.

The selection of research topics in UK numerical analysis reflects the challenges seen as difficult in this area around the world, including problems involving multiple scales, complex systems, stochastic systems, high nonlinearity and sparse, high-dimensional data. A welcome and longstanding feature of UK numerical analysis research is its standard practice of producing general-purpose numerical software, which provides a highly efficient mechanism for knowledge transfer from research to practice.

The landscape document for numerical analysis provided to the panel (Section 1.5.1) notes that, based on the 2008 RAE data [20], the number of active researchers in numerical analysis has noticeably increased since 2001. In addition, there has been a welcome growth in scientific breadth and connections with other fields, especially science and engineering. Nonetheless, the panel believes that there should be a greater effort to encourage numerical analysis to become part of approaches labelled as 'computational science and engineering' (Section 6.1). Such a push, if successful, would enhance the computational capabilities available to numerical analysts, as well as create added ties to key societal/technological challenges. A related research theme, well-funded and actively pursued in several other countries, is the development of innovative numerical methods that are provably reliable and efficient on changing computer architectures (as exemplified by the current widespread interest in multicore systems). Some steps along these lines have been taken in the UK, but more would be desirable.

Optimisation, which also appears in this report under the heading of 'operational research' (Section 5.9), is seen internationally as a crucial part of problems at the frontiers of science and engineering, which involve (for example) emerging connections between optimisation and partial differential equations, probability, statistics and discrete mathematics. For both analysis (Section 5.2) and fluid mechanics (Section 5.4), the panel has recommended a greater focus on computation, which implies a complementary recommendation that UK numerical analysts should develop closer links with those areas.

5.9 Operational research

In 2004, operational research (OR) received its own review [21]. The present review includes the mathematical aspects of OR, which limits consideration of research activity to a relatively small number of programs of varying size and strength. The OR landscape document provided to the panel (Section 1.5.1) indicates clearly that other researchers in OR may well classify themselves in non-mathematical areas.

OR is an interdisciplinary subject that arose after World War II and was originally concerned with solving real-world decision problems from military and industrial contexts. The scope of OR today is much more general, extending to numerous problems in science, engineering, and medicine, and much of the subject overlaps, sometimes confusingly, with other areas. OR methodologies range from conceptual problem structuring and formulation to mathematically and/or computationally based methods, deterministic or stochastic. OR has always maintained a particularly strong practitioner base and a focus on applications; the 2004 review of operational research commented '[I]n no other country in the world is the orientation of OR towards applications as strong as in the UK' [21, page 2].

While traditional OR methods frequently have a mathematical foundation, for historical reasons the subject is located today in departments of management, business, industrial engineering, computer science and statistics, as well as mathematics. In the UK the subject is most frequently represented in management and business departments, in contrast to the US, where OR features strongly in industrial engineering. Several OR-related topics, such as probability, statistics and optimisation (both continuous and discrete), are considered in Sections 5.8, 5.10 and 5.11, emphasising the already-mentioned difficulty in assigning OR research to tidy disjoint categories.

As reported in the 2004 review of OR [21], at that time researchers in the field had achieved substantial progress in helping to solve important real-world problems, but much of the community was pessimistic about the prospects for funding success. Since then, EPSRC has made significant investments to support and encourage foundational OR:

- NATCOR[9] (National Taught Course Centre in Operational Research; see Section 16.2.2) was established in 2006 as a collaboration among ten universities;
- LANCS,[10] a collaboration among Lancaster, Nottingham, Cardiff and Southampton, was created by a Science and Innovation Award designed to strengthen foundational operational research;
- STOR-i[11] is a Centre for Doctoral Training (see Section 16.2.2) in statistics and operational research, based at Lancaster University, with additional funding from industry.

Today UK researchers represent international excellence in continuous, stochastic, and combinatorial optimisation; some of this work overlaps with research in numerical analysis and probability. Especially notable research includes creation of novel heuristic approaches in search and optimisation. In particular,

[9] `www.natcor.ac.uk`.
[10] `www.lancs-initiative.ac.uk`.
[11] `www.stor-i.lancs.ac.uk`.

UK work on hyper-heuristic methods is world-leading and has had a significant international impact; these methods address important mathematical problems from applications that cannot be solved today by conventional exact techniques, and are closely connected with computer science.

Much of the optimisation research in OR involves applications to finance, data envelopment analysis, scheduling, multi-criteria decision-making and decision-making under uncertainty, whereas optimisation on the international scene tends to be viewed more broadly as an important approach to problems in the physical and biological sciences as well as engineering. An obvious suggestion, in the spirit of Section 3.1, is for optimisation researchers in OR and numerical analysis to examine, together, the possible connections between their separately developed approaches.

Contributions in stochastic OR include simulation, risk, forecasting and revenue management, but these are not as strong and would benefit from further investment. In some cases the UK presence in these areas depends on a small number of researchers who are approaching retirement. Strengthening is needed in research on supply chain problems and applications in production and manufacturing; revenue management, auctions pricing, and market regulation; and contributions to bioinformatics and data mining.

Since OR is interdisciplinary by definition, researchers expressed concerns to the panel about proposal evaluation. (See Section 14.1.2.)

5.10 Probability

Probability is a diverse subject with research pursued on both intrinsic foundations and applications. There are many connections to other fields, as sophisticated probabilistic methods are now pervasive and influential through much of science and engineering, as well as other areas of mathematics. Research in the fundamentals of probability and its applications is conducted at the highest international level in the UK. This is a historical UK strength and continues to have a vigorous community that includes a number of senior international leaders and a strong group of junior researchers. There are extensive collaborations with the world-wide probability community.

The UK is one of the world leaders in stochastic analysis, including the topics of coagulation and fragmentation, rough path theory, stochastic differential geometry, stochastic partial differential equations, nonlinear filtering, Levy and Levy-type processes, and numerical methods for stochastic equations. The UK is also an international leader in the study of probabilistic models related to mathematical physics and discrete probability including percolation, Markov chains, random graphs and random matrices. In applications of probability, the UK is an international leader in the analysis of probabilistic models for genetics, bioinformatics, communication and transportation networks, finance and epidemics.

While the UK probability community is strong in significant areas, a few very active areas worldwide, such as stochastic Loewner evolution, random planar maps, Gaussian free fields, combinatorial optimisation and energy applications, are thinly represented.

Major examples of interdisciplinary activity occur in the interactions of probability, statistics and biology in genetics; probability, operational research and computer science in the study of engineered networks; and probability, statistics, partial differential equations and numerical analysis in finance. There are many potential opportunities for further highly productive cross-disciplinary activities involving probability. In this domain, there is a need for support of in-depth engagement between highly trained probability researchers and first-class scientists.

There is a strong demand for PhDs with knowledge of probability from areas of application and industry. However, apparently (according to a rough estimate in the landscape documents provided to the panel) only about one-fourth of probability PhD students are from the UK, which is of potential concern. The probability-focussed residential weeks offered by the Taught Course Centre NATCOR[12] and by the probability research centres at Bath, the Swansea node of the Wales Institute of Mathematical and Computational Sciences, and Warwick are important for enlarging the base of PhDs with knowledge of probability and for exposing those specialising in probability to a wide spectrum of probability topics that include those not well represented in the UK.

Two Centres for Doctoral Training[13]—Mathematics and Statistics Centre for Doctoral Training[14]

[12]See Sections 5.9 and 16.2.2.
[13]See Section 16.2.2.
[14]`www2.warwick.ac.uk/fac/sci/masdoc/`.

at Warwick and Cambridge Centre for Analysis[15]—have significant probability components. These are relatively new, but should be beneficial for bringing probability PhD students in contact with researchers in related fields of analysis, computational analysis and statistics.

One area of concern is that the UK probability community is already experiencing retirements of some prominent probabilists and could experience more before too long. Some excellent junior researchers have been appointed in recent years; however, quite a few of these are from outside the UK, and there are signs that some of these, as well as others in mid-career, are being tempted by the availability of attractive 'chairs' in other countries.

5.11 Statistics

Statistics as a discipline, wherever it is found, is rooted in research and ideas from the UK in the late nineteenth and early twentieth centuries. In the first part of the twentieth century the field was transformed when challenges from biology and agriculture stimulated insightful ideas with profound implications from mathematicians at Rothamsted and University College, London. Many innovations in statistics still follow this pattern, whereby concrete questions arising in domains such as social science or medicine lead to the development of new statistical methodology that can then be deployed much more widely. Major UK contributions continue to the present day; in the last forty years, new directions in both frequentist and Bayesian statistics have been pioneered by Cox and Lindley, figures of world stature. It is on these foundations, both early and more recent, that many of the UK strengths today are built.

Areas in which UK statisticians are world-leading include Bayesian modelling, computational statistics, spatial statistics, time series, the statistical study of shape, statistical methodology related to bioinformatics, multivariate and longitudinal data analysis, functional data analysis and extremal modelling. Research in these areas is an eclectic blend of theory, methods, computation and applications. For example, recent contributions to Bayesian modelling include novel computational algorithms, deep work on convergence properties, formulation of new models for generic problems especially those involving hierarchical structure, and applications to fields ranging from engineering to economics, from medicine to machine learning, and from graphics to genomics.

A major feature of modern UK statistics is its strong motivation from, and enduring connection to, an extraordinary array of important applications in the sciences, industry, business and government. A sizeable part of this work, although not the whole, lies outside the area conventionally defined as the mathematical sciences. There are deep connections, however, between UK applied statistics and relatively theoretical contributions, particularly in areas such as methodology for dependent data, risk analysis and techniques for feature selection in high-dimensional data.

At the same time, the connections of modern statistics to mathematics remain strong and are growing. For example, both the theory and the methodology of functional data analysis rely on ideas and results from mathematical analysis. Even algebraic geometry, through its capacity for reducing complex relationships to properties of families of equations, and number theory, which in some statistical problems determines convergence rates of estimators, have important uses in statistics. Modern statistics has an ethos and foundation of its own, however, and statistical theory is today a large discipline in its own right, reaching out to other disciplines and finding new challenges from them.

The 2004 IRM [13] gave separate consideration to the field of statistics because the UK's international position in research was seriously threatened then by structural and personnel issues (rather than by gaps in research content *per se*). In response to these concerns, EPSRC took several major actions: three Science and Innovation Awards were made in statistics, at Bristol, Cambridge and Warwick; and two Taught Course Centres and two Centres for Doctoral Training[16] including elements of statistics were established, involving researchers from subsets of Aberdeen, Bath, Bristol, Cambridge, Dundee, Edinburgh, Glasgow, Heriot-Watt, Lancaster, Nottingham, Oxford, Southampton, St Andrews, Strathclyde and Warwick.

Unfortunately, today the panel finds once again that the UK statistics research enterprise is in a fragile and weakened condition, despite all its areas of excellence and the welcome measures taken during the

[15] `www.maths.cam.ac.uk/postgrad/cca/`.
[16] See Section 16.2.2.

intervening years to strengthen it. This worrying situation has arisen for several reasons, primarily structural, which are discussed in Section 15.

6 Applications Connected with the Mathematical Sciences

In this section we discuss, in alphabetical order, subfields in which the mathematical sciences are very closely connected with one or more disciplines in science, engineering, or business. These topics are highly mathematical, but their nature depends strongly on the application area. Industrial mathematics is considered in Section 7. The discussion in every subsection addresses Question E (multidisciplinary collaborations) and, for some areas, Question D (key societal/technological challenges).

6.1 Computational science and engineering

Throughout the world, 'computational science and engineering' is interpreted as a tightly coupled multidisciplinary approach that combines research-level contributions from the mathematical sciences, computer science, and one or more scientific and engineering disciplines. Unfortunate confusion about this terminology arises for two reasons: its similarity with 'scientific computing', which is taken in many countries (including the US) as a synonym for 'numerical analysis' (see Section 5.8), and increasing usage of 'computational X', where X can be any other field. For example, the subject of a 2010 workshop[17] in the Netherlands is 'Computational Humanities'.

A widely used definition of computational science and engineering includes all aspects of modelling (formulation, analysis, calibration and validation) along with computational methods, data analysis and numerical/symbolic simulation. Many researchers (but not all) regard high-performance computing (HPC) as an essential part of computational science and engineering.

The 2004 IRM noted a concern that, after pioneering UK activities in the 1960s, 'the scale of activity [in computational science and engineering] is small compared to its scientific importance' [13, page 25]. Thanks to EPSRC, progress has been made in recent years toward supporting research in mathematical sciences focussed on advancing computational science and engineering. In particular, a large Science and Innovation Award, 'Numerical Algorithms and Software for the Evolving HPC Platform', was made in mid-2009 to a consortium of Edinburgh, Heriot-Watt, and Strathclyde. EPSRC has also supported development of a 'roadmap' in high-performance numerical algorithms (involving Manchester, Oxford, Rutherford Appleton Laboratory, and University College, London), to identify research areas for the next five years; further related programmes are ongoing or planned.

Because these activities are very recent, it is too early to assess their national or international impact. Nonetheless, the present panel, like our 2004 predecessor, believes that EPSRC should provide additional support and encouragement for collaborative computational science and engineering research, subject to the requirement that research teams must include mathematical scientists, preferably as co-principal investigators. (See Section 14.1.1.) In addition, UK programmes in this area should be integrated as much as possible with related international networks.

6.2 Financial mathematics

As a consequence of financial and economic crises in recent years, quantitative methods for assisting decision processes in these fields are in great demand. The relatively new label of 'financial mathematics' applies to an inherently interdisciplinary subject that contains extremely challenging problems and interfaces with econometrics, probability, statistics, partial differential equations and numerical analysis. Because of the generality of the mathematical sciences (see Section 3.2), progress in financial mathematics is likely to lead to tools that can be applied to problems from other domains, such as systems of stochastic differential equations.

The UK is among the world leaders in financial mathematics, with particular strength in applications of stochastic analysis. A welcome development is that the UK research field is being enlarged from purely financial products and markets to commodity markets, including resources like oil, gas, electric power and agricultural products. In actuarial mathematics, which applies insights and techniques from

[17] `staff.science.uva.nl/~rens/CompHum2010.htm`.

mathematical sciences research to assess risk in insurance, finance and medicine (among other areas), the UK continues as a world leader. This subject is becoming increasingly intertwined with financial mathematics through the topics of insurance and risk management.

Two recently-founded innovative institutes in financial mathematics are discussed elsewhere in this report. The Oxford-Man Institute (see Section 11.2) is entirely funded by industry; the Scottish Financial Risk Academy (see Section 10.4) is a consortium of academia and industry. The programmes of both institutes will, the panel believes, provide valuable information about 'best practices' in multidisciplinary work and industrial collaborations (Section 14.3).

Several companies with interests in financial mathematics were represented at the users' meeting; their views related to Question F are discussed in Section 11.

6.3 Materials science and engineering

New materials are often the driver of new technologies. In the past, better materials have mostly been found by experimentation, characterisation and incremental improvement, sometimes guided by empirical laws or simple continuum theories. Dramatically increased computational capabilities and added insights about the fundamental mathematical equations on all levels offer the prospect of a much more systematic route to the discovery of new materials. Key ingredients are a better understanding of N-body quantum mechanics and its upscaling to relevant length scales, and of the role of microstructures (on a large variety of scales from atomistic to macroscopic) and their impact on the overall material behaviour.

In the application of new mathematical ideas to materials science, the UK has a world-class group in Oxford, with very significant contributions from a number of other institutions including Bath, Bristol, Cambridge, Glasgow, Manchester, Nottingham and Warwick.

The 2004 IRM [13] described the UK as 'not a player' in the key area of multi-scale modelling. This situation has noticeably improved, partly due to EPSRC's awards of Multidisciplinary Critical Mass Awards[18] to the Bath Institute for Complex Systems (a collaboration between the departments of mathematical sciences and mechanical engineering) and the Oxford research programme in New Frontiers in the Mathematics of Solids. Nonetheless, mathematical research in materials science and solid mechanics is still largely underrepresented in the UK, especially compared to the traditionally strong representation of fluid mechanics (Section 5.4).

6.4 Mathematical biology and medicine

Connections between the mathematical sciences and life sciences arise in multiple ways, and are rapidly expanding worldwide, with spectacular growth since the 2004 IRM [13]. Mathematical methods, computational simulation and statistical/quantitative analysis are essential components in life sciences research. Biomedicine has become internationally recognised as an area of research with potential for strong growth, driven by national and industrial priorities.

It is impractical to make an exhaustive list of associated roles of the mathematical sciences, especially since these are expected to continue increasing into the foreseeable future. A few examples (among many more) are: to improve understanding of cell processes, cell interactions, and formation of biological structures; to automate and improve medical diagnosis; and to design and control therapeutic measures. In general, processes in biological systems are complex, which means that the derived models are typically nonlinear, multi-scale, often hybrid, discrete-continuous, involving high-dimensional dynamical systems.

UK excellence in mathematical biology (in which we include mathematical medicine) is based on a long tradition, pursuing aims and concepts that now define systems biology and synthetic biology. The Centre for Mathematical Biology[19] in Oxford, founded in 1983, acted as an important catalyst for a remarkable development of this discipline. Excellent research groups are distributed across the UK, located in a variety of universities and departments.

The research focus in the UK is broad, ranging from medical applications (e.g., electrophysiology for heart modelling) and strong biometry up to 'hot' areas as biomarkers, genomics, proteomics and stem cell research. This position in established areas of computational biology, including statistics and metabolic modelling, is complemented by advanced approaches in network biology and virtual physiology

[18] `www.epsrc.ac.uk/about/progs/maths/Pages/multi.aspx`.
[19] `www.maths.ox.ac.uk/groups/mathematical-biology`.

modelling. Mathematical research groups cooperate effectively with biology and medicine, and the UK publication track record in high-ranked journals is excellent.

UK research in biomechanics, in particular biological fluid mechanics, is very strong and benefits from a long tradition of seminal work. Several major centres have recently been established, outstanding researchers have been newly appointed and, as a result, the field has undergone rapid growth. This discipline is certainly more active than in countries of similar size in continental Europe. Although the influx of researchers at the interface between fluid dynamics and the biological sciences is not as massive as in the US, UK biomedical fluid mechanics is in a healthy and vibrant state. A wide variety of topics are covered, over a large range of scales: blood flow in vessels and arteries, numerical simulations of flow in the heart, pulmonary flows, cell biomechanics, collective dynamics of cells, mixing and transport in bacterial populations, and bioconvection phenomena.

Interdisciplinary biomathematics projects are often led by mathematical sciences researchers, who are supported in various ways from pharmaceutical and biotechnology companies (AstraZeneca, Glaxo Smith Kline, Pfizer) and agrochemical companies (Syngenta), which have made significant, long-term investments in UK universities, including mathematics and statistics.

Although pharmaceutical companies tend to outsource off-the-shelf work to countries with lower salary levels, there are increasing needs for PhDs with substantial knowledge of the mathematical sciences. As we note in Section 16, because students produced by the UK's traditional 3 + 3 model often have unacceptably narrow training, these companies tend to recruit outside the UK.

The UK strength in mathematical biology would be increased by more systematic integration of the mathematical sciences in the established systems biology centres. Separate Research Councils administer grants and funding in biology, and the panel's site visits indicate a widespread perception that, in medicine and the mathematical sciences, funding structures are often hampered by conflicting remits and criteria of EPSRC and BBSRC, leading to workarounds by researchers that consume time and energy better devoted to research. It may be helpful to have specific 'high-risk' funding programmes for mathematical biology.

7 Industrial Mathematics

The primary focus of this Section is industrial mathematics as an area of research capability. (Additional commentary on the interactions of mathematical sciences researchers with industrial users is given in Sections 11 and 14.2.) Industrial mathematics is treated separately from the topics of Section 6 because industrial applications can, and do, include literally every area of the mathematical sciences. In this regard, the panel chooses to follow the definition of 'industry' from a 2008 OECD report [19]:

> The term *industry* ... [is] broadly interpreted as any activity of economic or social value, regardless of whether it is in the public or private sector.

Industrial mathematics could be described in a lighthearted vein as 'multidisciplinary squared', in that it typically includes more than one area of the mathematical sciences as well as more than one area of science and engineering, plus complex constraints (often imprecise and shifting) imposed by the business environment.

The UK is widely recognised to have had, for a long time, pioneering and innovative activities in industrial mathematics, based on a consistent view of industry as an important source of inspiration, reputation, and funding. Institutions whose purpose is to further industrial mathematics have been founded by universities, industry, and government; special activities have been introduced to enhance the transfer of knowledge and personnel. Oxford Study Group workshops, for example, began more than 30 years ago to bring together academic and industrial researchers to work on problems defined by participating industries. Many UK activities related to industrial mathematics today are imbedded in international cooperation, e.g. within the European Consortium in Industrial Mathematics. Here the UK is providing important nodes in an international network in industrial mathematics, influencing global development in this field.

An especially notable instance of an organisation that links mathematical sciences research and industry is the Industrial Mathematics Knowledge Transfer Network[20] (KTN), principally funded by the

[20] A Knowledge Transfer Network is a national network in a field of technology or business application, intended to stimulate innovation through knowledge transfer. The Industrial Mathematics KTN, managed by the Smith Institute for

Technology Strategy Board with additional contributions from EPSRC, which has been highly effective in fostering industrial-academic partnerships.

EPSRC has created several valuable programmes to connect mathematical sciences research and industry.[21] EPSRC also makes industrial Collaborative Awards in Science and Engineering (CASE) awards to fund PhD studentships in which projects are arranged between industry and an academic partner. The CASE awards, which serve as enabling tools for students to learn about industrial workflows and approaches to problems, were highly praised at the panel's meeting with industrial users (Section 11) and by PhD students who had received these awards (Section 16.3).

The panel is impressed by these important activities. However, based on the panel's meeting with industrial users, it is clear that the two-way transfer of knowledge could be further improved; see Sections 11 and 14.2.

8 Institutes

The name of this section may be ambiguous because the term 'Mathematical Institute' is applied at some UK universities to denote what might elsewhere be called the 'Mathematics Department'. Adding to the confusion, there are 'institutes' and 'centres' of varying sizes and research directions in the mathematical sciences throughout the UK.

In this section we briefly discuss four mathematical sciences institutes that provided information during the panel's site visits: the International Centre for Mathematical Sciences (ICMS), Edinburgh, created in 1990; the Isaac Newton Institute (INI), Cambridge, founded in 1992; the Wales Institute of Mathematical and Computational Sciences (WIMCS), established in 2006; and the Heilbronn Institute for Mathematical Research, established at Bristol in 2005. Both ICMS and INI are funded in (large) part by EPSRC. WIMCS was set up by the Welsh Assembly Government through the Higher Education Funding Council for Wales, and the Heilbronn Institute is a partnership between Bristol University and the UK Government Communications Headquarters (mentioned in Section 5.3).

Panel members were impressed by the range and excellence of activities organised and supported by these institutes. Several panel members had previously attended programmes at one or more of these institutes (most often, at INI or ICMS) and hence had first-hand knowledge of their quality.

Top international researchers routinely form a part of institute workshops and programmes, which means that the institutes make a substantial contribution to the international visibility of UK mathematical sciences research. Because distinguished international visitors very often spend time at other UK universities when they attend institute programmes, the benefits of these programmes include enhanced opportunities for UK collaborations with world-leading researchers from other countries.

A further admirable property of the institutes is that their structures are sufficiently flexible to include activities related to fast-breaking 'hot' topics at short notice.

The panel makes two recommendations (R-3 and R-11, Section 2) for new roles for the institutes: creating connections within and outside the mathematical sciences (Section 14.2), and encouraging women to participate in mathematical sciences research (Section 16.5).

9 Learned Society Activities

The panel would be remiss if this report failed to mention the contributions to quality and capability of UK mathematical sciences research made by the six learned societies who nominated members of the Steering Committee—the Edinburgh Mathematical Society, the Institute of Mathematics and its Applications, the Institute of Physics, the London Mathematical Society (LMS), the Operational Research Society and the Royal Statistical Society (see Section 19).

Each of these organisations conducts a full programme of activities for researchers in its particular area of specialisation, including conferences, symposia, workshops and public lectures. For example, the EPSRC/LMS Durham Symposia, established in 1974, provide a venue that allows invited researchers to

Industrial Mathematics and System Engineering, identifies and develops collaborative projects through programme delivery and partnership engagements that create strategic collaborations. See `https://ktn.innovateuk.org/web/mathsktn`.

[21] `www.epsrc.ac.uk/funding/business/schemes`.

carry out an in-depth investigation of a research topic and to learn about new and evolving links between different subfields of the mathematical sciences and other areas.

In the panel's opinion, these activities of the learned societies strengthen the excellence of UK mathematical sciences research. As with the institutes (Section 8), because eminent international researchers participate in some of these events, opportunities for international and multidisciplinary collaborations, as well as the UK's visibility worldwide, are greatly enhanced. The panel's recommendations R-3 and R-11 (Section 2) involve potential roles for the learned societies to increase connections within and outside the mathematical sciences (Section 14.2), and to improve participation of women (Section 16.5).

10 Collaborations

Several questions in the Evidence Framework—Questions D (key challenges), E (multidisciplinary research), F (interactions with industry) and G (economic benefits)—involve, in various ways, connections between the mathematical sciences, other science and engineering disciplines, medicine and industry, as well as connections among subdisciplines of the mathematical sciences. Unfortunately, the answers do not separate neatly according to the labelling of the questions. The panel's responses accordingly appear in several sections of the report, starting with this one, which focuses on the outcomes of the panel's meetings with 'collaborators' at site visits.

As part of each site visit, a selection of collaborators, typically a mixture of academics and people from industry, had been invited to participate in a session about multidisciplinary interactions. The format of these meetings was that each collaborator made brief remarks about the nature of his/her collaboration (the fields involved, the role of the mathematical sciences, the nature of the results) and then answered questions from the sub-panel. A general discussion followed with the sub-panel and all collaborators.

As part of the panel's response to Questions D (especially D-1) and E-1 in the Evidence Framework, the remainder of this section highlights collaborations described during the site visits that are explicitly addressing key challenges mentioned in the EPSRC Delivery Plan [6], as well as improving the efficiency of the UK economy and global competitiveness. Only a small selection of these collaborations is presented here, each representing a concrete instance of the kinds of innovation that result from mathematical sciences research. A much more extended list could be provided if additional evidence (requested as part of Question E-1) is desirable. Some of the broad challenges in increasing and enhancing multidisciplinary collaborations, including those with industry, are addressed in Section 14 to address Question D-3.

The EPSRC staff who were present during the sessions with collaborators could confirm the panel's comments relevant to Question E-1, including the requested evidence. As described in Section 1.5.1, EPSRC provided the panel with additional evidence in the form of interesting 'case studies', each of which illustrates a multidisciplinary collaboration.

10.1 Manufacturing

The Inkjet Research Centre[22] carries out research into the foundational science for inkjet technology, with collaborating academics from Aberystwyth, Cambridge, Durham, Leeds and Newcastle as well as industry partners CDT, Domino, FFEI, Inca, Linx, Sericol, Sunjet and Sericol. The collaborator who described this work stated that new mathematical modelling was essential for understanding fluid flow at high rates and drop/surface interaction, and that the prestige of the affiliated mathematical sciences researchers was a key ingredient in attracting collaborators. This centre, in the panel's view, is an excellent example of how mathematical sciences research benefits the UK economy (Question G-1), as is the spin-out company mentioned in Section 10.4.

10.2 Energy

The Grantham Institute for Climate Change (Imperial) embodies interdisciplinary research that drives 'the fundamental technological transformation needed to tackle climate change'. A key element in one of its research themes, sustainable futures,[23] involves, among other topics, mathematical analysis of

[22] www.ifm.eng.cam.ac.uk/pp/inkjet/default.html.
[23] www3.imperial.ac.uk/climatechange/research/sustainablefutures.

different energy-generating and energy-saving technologies, along with the role of network technologies. An associated project uses a mathematical model based on fluid mechanics to predict the characteristics of a 'night purge' (driving out accumulated warm air) that is intended to reduce the energy needed for daytime cooling of buildings.

10.3 Health and healthcare

Researchers in the Infectious Diseases Consortium[24] in the Cambridge Veterinary School are studying the dynamics at the heart of infectious diseases, which involve multiscale mathematical modelling as well as more traditional epidemiological approaches. While presenting this work, one of the collaborators stated that this area has been 'completely transformed' because of the insights gained from the mathematical sciences, including a 'culture of precision'. This comment was echoed by the representative of a large biomedical research company during the Thursday morning meeting with industrial users (Section 11).

In Wales, an ambitious initiative[25] has recently been announced to create a health modeling centre whose goal is to improve the quality and efficiency of healthcare by bringing together research in medicine, computational modelling, operations research, stochastic analysis, nonlinear partial differential equations, image processing and high-performance computing.

10.4 Economic efficiency

The Scottish Financial Risk Academy[26] (SFRA), established in 2010, is a consortium of academia and industry, where the latter includes Aberdeen Asset Management, Barrie and Hibbert, and Lloyds Banking Group. Creation of SFRA was motivated by a recognition of the complexity of financial risks, and the consequent importance to Scotland of connecting financial services with advanced mathematical modelling. A major part of SFRA's activities involves knowledge exchange activities to improve interactions between mathematical sciences research and the financial services industry, such as a series of invitational 'Risk Colloquia'.

At the operational level, efficient techniques for cutting and laying out materials such as glass, fabric, and sheet or precious metal are important in many industries. Operational research experts at Nottingham who investigate cutting and packing have produced automatic algorithms that increase manufacturing efficiency by devising greatly improved layouts, thereby reducing cost and waste. Keeping Question G-2 in mind, the panel notes that this research led to creation of a spin-out company, Aptia Solutions,[27] that has developed associated software products and services.

11 Industrial Users

Because of the importance of connections between the mathematical sciences and industry, the panel spent the morning of 9 December 2010 with sixteen industrial (non-academic) 'users' from a variety of companies and agencies. These users individually described the technical nature of connections between mathematical sciences research and their companies, and then offered commentary about how to improve relationships between industry and the mathematical sciences. The presenters were questioned by the panel during and immediately after their presentations. Following a general discussion among all participants, there were further opportunities at a sandwich lunch for small-group conversations and follow-up.

The thoughts of the assembled users about the relationship between industry and mathematical sciences research are covered in two places: the remainder of this section and in Section 14 (for ideas that apply in general to multidisciplinary collaborations).

For completeness and to respond to Question G-1, the following varied and impressive list, in alphabetical order, is included of the mathematical sciences topics that were mentioned as important to industry: applied mathematics, combinatorics, computational mathematics, control, cryptography,

[24] www.vet.cam.ac.uk/cidc/.
[25] www.walesinternationalconsortium.com/news/N10_Model_healthcareu.shtml.
[26] www.sfra.ac.uk.
[27] www.aptiasolutions.com/People.aspx.

data analysis, fluid mechanics, geometry, mathematical modelling, multiscale and risk modelling, networks, number theory, numerical analysis, operational research, optimisation, probability theory, risk assessment, statistics, stochastic analysis and uncertainty quantification.

In addition to considering collaborations with industry, several participants mentioned the vital contribution of the mathematical sciences at UK universities to educating numerate citizens in all fields who can intelligently cope with the technological challenges of modern society. This point is not directly related to the panel's charge, but it confirms the panel's view (in relation to Question G-1 of the Evidence Framework) that education is a large, albeit indirect, benefit of mathematical sciences researchers to the UK economy.

Among the users who met with the panel, there seemed to be no doubt about the importance, real and potential, of the mathematical sciences as a key element in industrial applications, and the panel heard numerous success stories from a wide variety of areas. Of course these particular users were, in some sense, 'self-selected' in that they were willing to spend company time to meet with the panel, but even so the panel came away with a strong feeling that mathematical sciences research is appreciated by and relevant to a significant cross-section of UK industry. One participant stated forcefully that 'A big mathematics push needs to happen in the long term'; another, representing a company that has traditionally emphasised engineering, said, 'Moving forward, mathematics will be more important than engineering'.

11.1 The range of interactions with industry

It is commonplace to assert that there is an inherent mismatch between the long time scales of academic research (see Section 3.2) and the immediate needs of industry. Based on the variations in length and content of interactions described at the users' meeting, this view may be exaggerated. Although mathematical scientists seldom offer an immediate solution to industrial problems, a few users mentioned that this kind of quick interaction can indeed happen, and is extremely gratifying when it does. For instance, one industrial participant cited, with a smile, an EPSRC/LMS Durham workshop at which a mathematician in the audience exclaimed, during an industry presentation, 'You really don't want to do it like that'. An obvious implication is that industry can receive high value from structured short-term interactions with mathematical scientists; hence it is reasonable to ask industry to make an appropriate financial commitment to support such activities, especially given the very large fees typically charged by external consultants.

In the most interesting cases, however, participants consistently observed that industry does not, and should not, expect its day-to-day needs to be addressed by a short conversation with a mathematical scientist. As one participant flatly stated, 'It is unreasonable for industry to expect academics to do industry's job'.

A strong message from these users was that, almost by definition, any healthy industrial sector has long-term needs for transformative technologies and paradigm-changing processes. To address these needs, long-term collaborations with mathematical sciences research should be encouraged, preferably with involvement of mathematical scientists from the beginning. The policy of early inclusion of mathematical scientists was a major benefit from existence in the past of industrial and government research laboratories. It is widely felt that 'many of these labs have decayed into near oblivion, and are becoming relics of the past' [17], and that opportunities for fundamental, adventurous research at the few remaining labs are severely constrained. Industry and government must therefore turn to universities for genuine partnerships, not as consumers who want to 'buy mathematics' on order.

11.2 Strategies to enhance industrial connections

As already mentioned in Section 7, several structures exist for encouraging and creating collaborations between industry and mathematical sciences research. The panel hopes that the mathematical sciences community and industrial collaborators will develop an increased awareness of these opportunities, for example through workshops of the kind described in Section 14.2.

Based on the informative session with users, the panel concluded that even more encouragement is needed for establishing fundamental research collaborations driven by broad classes of industrial problems rather than by the more frequent case studies of the past. Recognising that CASE awards (Section 7) are likely to be too short and tightly focussed for joint development of transformative mathematical sciences research, it would be desirable for EPSRC to create mechanisms for multi-year research collaborations as well as pre-competitive academic-industry consortia. For example, projects could focus on 'quantum leap' innovations requiring new mathematical sciences research, not application of known techniques to a new problem. These efforts could involve one or more industrial partners that should be required to demonstrate commitment by investing their own resources, plus public funding for at least one academic partner.

A different approach is provided by the Oxford-Man Institute of Quantitative Finance,[28] established in 2007 and funded entirely by the Man Group.[29] The institute is academically independent, with associated faculty from economics, engineering science, statistics, mathematics, computing, business and law. Executives from the Man Group are seconded to a co-located laboratory in order to pursue their own research projects in quantitative finance. Examples of other structures requiring a financial commitment from industry that have been mentioned elsewhere in this report are the Inkjet Research Centre (Section 10.1) and the Scottish Financial Risk Academy (Section 10.4). Certainly the track records of these organisations will be helpful in any future EPSRC evaluation of 'best practices' in academic-industry interactions (Section 14.3).

11.3 Issues in industrial connections

Despite the positive views of industrial users who met with the panel about connections with the mathematical sciences, several impediments, some more serious than others, were discussed.

With respect to graduate education, numerous users expressed a concern noted by the 2004 IRM: UK PhD students are too narrowly educated, and this issue is likely to become more pressing as globalization increases. (See Section 16.2.) Deep knowledge of a research area in the mathematical sciences is essential but, since the problems faced by industry are constantly changing, flexibility and adaptability are equally important in the long run.

The panel also heard about the inevitable difficulties in communication between experts in different fields, in particular the need to learn 'new disciplinary languages and culture' [9, page 65]. One participant from industry commented that, in addition to misunderstandings caused by incompatible terminology, some researchers in the mathematical sciences, at all levels, struggle when faced with a poorly posed problem.

To reduce these difficulties for mathematical sciences researchers who are interested in involvement in multidisciplinary or industrial research, elsewhere in this report the panel recommends (see Recommendation R-3 and the discussion in Section 14.2) institute or learned society workshops specifically designed to familiarise participants with multidisciplinary modes of operation. For PhD students who wish to work in industry (or to participate at some point in multidisciplinary research), training and experience in problem-solving should play a key role in the offerings of Taught Course Centres and Centres for Doctoral Training (Section 16.2.2).

Participants from several larger companies described their established and growing programmes to build long-term strategic relationships with universities, including formal mechanisms for sponsoring PhD students, hiring interns, supporting faculty chairs and consulting with academics. Since programmes of this magnitude are not feasible for smaller companies, the Industrial Mathematics Knowledge Transfer Network was mentioned as having helped to make a variety of connections (see Section 7).

Question F-1 asks about the movement of trained people between industry and the research base, and vice versa. The panel heard that successful PhD or postdoctoral internships often lead to a 'real job' in industry, and there are visible examples of the movement of researchers from academia into industry or government positions. In the other direction, in much of the world (including the UK) the reduced number of industrial and government research laboratories means that people from industry may well be unable to meet today's academic standards for publication. A positive development about which the panel heard was that, in some instances, highly placed executives, engineers and scientists from industry have joined universities as 'honorary faculty' to teach courses, advise students and lead workshops. Such arrangements, which appear to be mutually beneficial, deserve further consideration on a national scale; see Recommendation R-7.

[28] www.oxford-man.ox.ac.uk.
[29] www.mangroupplc.com.

12 The Need for Increased EPSRC–Community Connections

This panel consists of researchers, all of whom work outside the UK, who came together for one week (and have since exchanged many email messages) to do an arguably impossible task. Panel members are grateful to EPSRC for placing confidence in their opinions, but it is far more important—indeed, it is essential—for EPSRC and the UK mathematical sciences community to have a high level of trust and confidence in each other.

The panel is aware of the existence and valuable contributions of the EPSRC Mathematical Sciences Strategic Advisory Team,[30] as well as the regular meetings between the EPSRC Mathematical Sciences Programme Team and the Council for Mathematical Sciences (CMS).[31] Unfortunately, numerous strongly worded comments made during the site visits clearly indicated to the panel that, despite these structures, there is, in the words of the 2009 International Review of Chemistry, a 'tense environment for communication', and that

> It is precisely under such circumstances ... that government and academia need to redouble their efforts to sustain open and clear communication. This communication is not one way but instead must engage both sides. [2, page 33]

The present panel similarly believes that an additional structure for regular conversations between EPSRC and the mathematical sciences community would be helpful in several ways—in particular, by complementing existing channels of communication and allowing a broader dialogue to inform the community and increase its comfort level. The panel therefore strongly recommends (Recommendation F-2, Section 2) creation of a new structure, designed together by EPSRC and the mathematical sciences research community, that will allow communication, in advance rather than after the fact, about the most effective structures for funding, the nature of initiatives and other key issues. We regard this as an *essential* recommendation.

This panel is reluctant to suggest too many details, but urges consideration of the following general principles:

- There should be some continuity to provide institutional memory, allowing members to 'learn the ropes' while avoiding a self-perpetuating and potentially self-serving body;

- There should be explicit opportunities for discussion of a wide range of topics affecting the mathematical sciences community. Ideally, EPSRC could 'float' ideas that are being considered, and representatives of the community could comment on these ideas before firm decisions are made. Examples of such decisions are the nature of new calls for proposals and selection of proposal review panels. The panel understands that, because certain government information is embargoed, EPSRC might be unable to reveal exact plans to the community group.

13 The Need for Diverse Funding Structures

This section presents the panel's recommendations about funding structures and strategies, which rest on three consistent premises:

(1) Only internationally excellent research should be supported by EPSRC and other Research Councils;

(2) It is reasonable and desirable for EPSRC and other Research Councils to devise programmes that encourage research in the mathematical sciences associated with new areas and initiatives; and

(3) It is essential to preserve the diversity and distributedness of the UK mathematical sciences research community, as explained in Section 4.

[30] `www.epsrc.ac.uk/about/governance/sats`.

[31] The CMS contains representatives and observers from the mathematical sciences community and from the Edinburgh Mathematical Society, the Institute for Mathematics and its Applications, the London Mathematical Society, the Operational Research Society and the Royal Statistical Society. See `www.cms.ac.uk/activities.html`.

Given these principles, the panel recommends (Recommendation R-1) that EPSRC funding for the mathematical sciences should contain a *variety of programmes* designed to address the most important demands and challenges. Despite inevitable uncertainties about how future EPSRC budgets will be allocated, the panel wishes to emphasise this recommendation now in the hope of influencing EPSRC's definition of mathematical sciences funding structures during the next five years.

On the first day of the review, the panel was informed about EPSRC's admirable wish to support adventurous research with high impact. In this regard, a statement was made similar to one on EPSRC's Programme Grant website:[32]

> The funding of longer, larger research grants is key to achieving EPSRC's strategy to deliver greater impact than ever before.

It emerged from the ensuing discussion that the intended strategy is to support teams of researchers who collaborate closely on a single research theme. Panel members immediately pointed out that a strategy of supporting large, close-knit groups was inappropriate for many leading researchers in the mathematical sciences, and was definitely *not* a recipe for encouraging creative and adventurous research. The reminder of this section expands on that point.

The enormous benefits to UK mathematical sciences research of diversity in research area, location, and institution have already been explained in Section 4. The EPSRC policy suggested above, if interpreted so that large segments of the mathematical sciences research community are likely to receive less funding, will be damaging to both excellence and capability. In addition, increasing the proportion of project-like grants would tend to produce an unhealthy concentration of funding (and leading researchers) in an ever-smaller number of institutions if (as is likely to be the case) those grants enable significant numbers of new faculty to be hired to the detriment of other universities. (This situation has already arisen in statistics; see Section 15.)

An implementable solution, which the panel recommends, is for EPSRC to create a flexible model that permits geographically distributed researchers working in a broad research area to qualify for funding at levels analogous to a 'platform grant' or 'programme grant'. The panel heard repeatedly during the site visits that distributed groups of researchers working on a single broad topic are not eligible in practice for either platform or programme grants as they are presently defined. Based on available information about these grants, the panel agrees that neither type of grant matches the panel's suggestion.

The EPSRC website describing platform grants[33] states that they are intended to provide 'underpinning funding to well established, world leading research groups', and that '[applicants] should hold a substantial portfolio . . . of current EPSRC funding, typically over £2 million in value. . . '.

Online information about the assessment criteria for Programme Grants includes the following four statements—(1) '[Programme Grants] are intended to support a suite of related activities focussing on one strategic research theme', (2) 'it is expected that most proposals will be interdisciplinary and collaborative in nature', (3) 'the principal investigator should have brought together a world-class team with complementary expertise so as to enhance the potential to achieve the vision', and (4) 'the proposal must demonstrate that there is a clear management plan'.[34] These are highly unlikely to apply to a collection of leading researchers working independently on a major research theme (the kind of 'programme' needed in certain areas of the mathematical sciences).

In the panel's view, flexible structures that provide long-term support for distributed excellence would be entirely in line with EPSRC's goal of supporting excellent research in the mathematical sciences. The recommended new community group suggested in Section 12 should be able to provide useful thoughts about how to define and manage grants of this nature.

Question B in the evidence framework asks about high-risk, high-impact research. Almost by definition, all the areas in Sections 5 and 6 in which there is world-leading or world-class research include high-risk, high-impact research. As for Questions B-2 and B-3 regarding more adventurous research, the panel believes that, as of today, research base funding (formerly 'responsive mode') offers the best chance for optimally creative research in a wide array of subareas of the mathematical sciences.

Similar views about the high importance of research base funding in encouraging adventurous research were expressed by the two most recent international reviews conducted under the auspices of EPSRC.

[32] `www.epsrc.ac.uk/funding/grants/capacity/programme/Pages/scopeofscheme.aspx`.
[33] `www.epsrc.ac.uk/funding/grants/capacity/platform`.
[34] `www.epsrc.ac.uk/funding/grants/capacity/programme`.

For both fields under review (materials and chemistry), the panels were unhappy, more than a year ago, with strategies that might reduce community-initiated proposals in favour of top-down management of research.

- The 2008 International Review of Materials panel stated, in response to a question in its evidence framework, that

> More adventurous research will often require longer-term planning on the part of EPSRC. There is concern that projects that do not mature in a 3–5 year time horizon will not be provided for in the future with responsive funding that emphasises new adventures.... There is a concern among a number of Panel members and researchers that excessive EPSRC top-down direction of research restricts innovation.... [12, page 36]

- Similarly, the 2009 International Review of UK Chemistry Research stated 'Current funding levels and implementation mechanisms are a significant barrier to adventurous research' and formally recommended that EPSRC should

> Increase the number of single PI-initiated grants to stimulate more adventurous research. ... Importantly, *such grants should not be in competition with very large grants* [emphasis added]. [2, page 25]

14 Making Connections for the Mathematical Sciences

A defining role of mathematical sciences research is its array of connections with other fields and industry as well as within itself. This section considers several issues related to creating and strengthening multidisciplinary and industrial connections.

14.1 Multidisciplinary connections

EPSRC has actively supported funding programmes to encourage multidisciplinary research, such as Multidisciplinary Critical Mass Centres,[35] Programme Grants[36] (where funding was expected to be for multidisciplinary research) and Interdisciplinary Research Collaborations.[37] Based on what the panel learned during its site visits, these programmes have encouraged formation of large groups with a focus on multidisciplinary topics.

Even so, multidisciplinary collaborations, including collaborations with industry, suffer from some well known difficulties, identified in other countries, that also apply to the UK. An extensive 2004 US National Research Council study *Facilitating Interdisciplinary Research* states

> Despite the apparent benefits of interdisciplinary research, researchers interested in pursuing it often face daunting obstacles and disincentives. [9, page 1]

14.1.1 Ensuring a substantive role for the mathematical sciences

The present panel consists of mathematical scientists, who (naturally) regard our common field as important, and this view extends to the role of mathematical scientists in research teams who tackle multidisciplinary and industrial problems. Because scientists and engineers have necessarily received training in the mathematical sciences, they may relegate the mathematical sciences to a 'service' role, genuinely but wrongly believing that 'We've already done the mathematics'. Given that the mathematical sciences have a clear and consistent history of delivering widely useful, often transformative, theory and methods, they should not be treated as an 'optional extra' within initiatives involving scientific, engineering, societal and technological challenges. (See Section 3.2.)

Based on the panel's own experience and the comments from collaborators and industrial users about the essential role of the mathematical sciences (Sections 10 and 11), major progress on 'grand challenge' problems will almost certainly require substantial involvement, from the beginning, by mathematical

[35] www.epsrc.ac.uk/about/progs/maths/Pages/multi.aspx.
[36] www.epsrc.ac.uk/funding/grants/capacity/programme/Pages/default.aspx.
[37] www.epsrc.ac.uk/funding/grants/capacity/Pages/ircs.aspx.

sciences researchers. Otherwise, there is a danger that the proposed research may boil down to applying standard mathematical sciences techniques to a new problem rather than seeking a fresh look from the perspective of an expert. Thoughtful advice from mathematical sciences researchers should be sought to ensure that proposed teams for multidisciplinary initiatives represent an appropriate scientific balance. The panel's recommendation (see Recommendation R-3 and Section 14.2) for workshops intended to improve connections with other areas should accordingly be activated whenever new major initiatives or 'grand challenge' focus areas are contemplated or announced, to encourage mathematical scientists to think creatively about their role in potential collaborative proposals.

14.1.2 Proposal review

The issue of appropriate proposal review came up several times during the site visits and the panel's own discussion. Of particular relevance to EPSRC and the other Research Councils, there is a strong feeling in the community, which the panel regards as serious because it was expressed by very distinguished researchers, that the peer review process for multidisciplinary proposals is not working well. Similar concerns about proposal review were expressed on behalf of mathematical biology (Section 6.4), operational research (Section 5.9) and statistics (Section 5.11), fields that combine a wide range of approaches and methodology. On this topic, the US National Research Council report on multidisciplinary research mentioned above notes:

> Reliable methods for prospective and retrospective evaluation of interdisciplinary research and education programmes will require modification of the peer-review process to include researchers with interdisciplinary expertise in addition to researchers with expertise in the relevant disciplines. [9, page 3]

Along these lines, the panel recommends that review panels for multidisciplinary proposals with substantial mathematical sciences content should include members with expertise in both multidisciplinary research and relevant areas of the mathematical sciences. For example, a proposal about future energy options could well involve comparisons of technologies that are not yet fully developed or understood; in such a context, judgements about the most promising approaches can be made properly only with a subtle and advanced mathematical understanding of the implications of uncertainty.

Because proposals in operational research and statistics typically involve an array of methodologies as well as application areas, the panel recommends that they should be reviewed by a separate panel covering both areas.

14.2 Broadening connections via workshops

During the period of the panel's review, panel members asked collaborators from other disciplines and industry, how they met their mathematical sciences partners. To no one's surprise, the almost-universal answer was a random meeting, for example at a seminar or social gathering where a conversation led to recognition of common scientific interests. Even within the mathematical sciences, connections often result from chance meetings.

In response to Questions E-3, F-2 and F-3, the panel believes that connections are so important for the mathematical sciences that they should not be left primarily to chance. During the site visits, panelists were repeatedly told about an eagerness to connect and collaborate that was frustrated by not knowing how to begin. A similar theme was repeated at the panel's meeting with users (see Section 11.3).

In this direction, EPSRC has arranged 'scoping' workshops designed to engage UK researchers in societal themes such as energy or the digital economy. The panel urges an extension and generalisation of this idea to provide information and guidance for mathematical sciences researchers interested in making new connections. In particular, short workshops are a promising mechanism to provide guidance for members of the community as well as academic colleagues from other fields and from industry. These gatherings could be arranged at one of the institutes (Section 8) or by a learned society (Section 9). With a well-chosen set of topics and sufficient funding, we believe that distinguished mathematical scientists would agree to attend; their prestige would in turn attract junior people in the field.

Because of the language disconnect mentioned in Section 11.3, great care would be needed in choosing those invited to present multidisciplinary or industrial problems, but the present panel's positive experience with the collaborators who spoke at site visits and with the Industrial Mathematics KTN (which invited attendees to the users' meeting; see Section 11) provides confidence that these potential workshops could be similarly successful.

14.3 Assessing best practices

The panel recognises and applauds the fact that UK funding agencies, including EPSRC, have made determined and successful efforts to encourage multidisciplinary research and industrial collaborations, as have individual academic institutions (including, but by no means limited to, those mentioned in Section 10). Support has also come from industry, as mentioned elsewhere in the report, and from other sources. For example, the Oxford Centre for Collaborative Applied Mathematics[38] is supported by the King Abdullah University of Science and Technology Global Research Partnership.

Given this variety of centres and institutes within the UK using different strategies to initiate and encourage multidisciplinarity and industrial connections, the panel recommends that, after sufficient time has passed to make a reasonable judgement, EPSRC should assess and analyse the associated 'best practices', as well as any less successful practices.

15 Structural Issues Specific to Statistics

This section follows up the mention in Section 5.11 that the situation in the UK statistics research community is of serious concern, even though its members are doing outstanding research. The reasons for this are primarily structural, involving the age profile of statistics researchers, the unanticipated effects of large Science and Innovation Awards in statistics, and the position of statistics in school curricula. Evidence to support the statements in this section was brought strongly to the panel's attention during the site visits by individual comments (which are not appropriate for inclusion in this report).

15.1 Weakening of small statistics departments

At many universities where excellence is represented by a relatively small number of researchers, research strength in statistics has been diminished by retirements, job moves by prominent statisticians and locally decided closures of statistics departments, which tend (on average) to be small. (In particular, there is now no free-standing statistics department in Scotland or Wales.)

Although the science and innovation centres in statistics have had highly positive consequences for the institutions that received them, an unforeseen result is that their ability to offer attractive salaries and reduced teaching inhibits the capacity of small departments, even those of high quality, to compete for new hires. Thus far this decline of small departments has been partially offset by junior appointments of young foreigners, but they may well leave if offers from competitor countries become more attractive.

In the fairly near future a generation of internationally leading statistical researchers will retire; their potential successors are not obvious, owing to a relative dearth of people in their 40s. The absence of these renowned senior figures will deny the UK access to internationally outstanding statistical scientists who could train the next generation. The landscape document in statistics sounds the alarm about the difficulties encountered in filling vacancies, warning that the situation will worsen after the expected retirements. Without decisive action to enhance the ability of small departments to compete on the international level for new faculty, a major decline seems likely, especially in the coming constrained funding environment.

15.2 Diminution of statistics as a separately recognised field

This panel has taken a consistent view that the mathematical sciences should be treated as a unity, but this does not mean that we advocate ignoring real differences in culture and methodology. Some of the distinctive properties of statistics are described in Section 5.11, and the panel believes that it is highly desirable for the UK to retain its international presence in statistics. In addition, the panel favours having strong statistics research and teaching programmes at a large number of UK universities, for two reasons:

[38] www.maths.ox.ac.uk/occam/.

- The study of statistics, as a science in its own right, is largely absent from school curricula (immediately preceding entrance to university). Usually a student encounters statistics only at university, often accidentally and in many instances through its use in other fields. Because of this lack of awareness by entering undergraduates, a small number of high-profile centres, such as those funded by the science and innovation awards, are unlikely to attract many of these students into statistics and to train them first at the undergraduate level and then in research. Some researchers argue that the omission of statistics at the school level is reasonable because a solid basis in mathematics is essential to study undergraduate statistics; hence schools should offer mathematics first, followed by statistics. However, this strategy works countrywide only if students at most universities have access to stimulating modern statistics courses given by experts in the field. The increasing concentration of statistics in a few major departments means that this approach is less and less viable.
- Beyond the role of statistics as a discipline, it is a collaborative science whose presence is enormously beneficial to scientists and engineers, within universities of all sizes, who need statistical tools to advance their own research activities and to ensure their global competitiveness.

This potential marginalisation of statistics in the UK, with a diminished or non-existent recognisable presence in small universities, is in contrast to the situation in the US, where undergraduate statistics majors are increasing, especially in multidisciplinary applications.

15.3 Strategies for strengthening statistics

To address the looming prospective fragility in small statistics department, one idea (as suggested in Section 13) might be a flexible grant scheme that could provide support for excellence in statistics across a range of universities; see Recommendation R-1 (Section 2). Advice specifically about statistics should also be part of the community involvement described in Section 12 and included in Recommendation R-2. A third possibility is an in-depth study of structural issues in statistics.

Since postdoctoral training programmes in the statistics centres have been particularly successful, ideally they should be continued for a period beyond the point at which the centres would otherwise cease to function. Postdoctoral training is especially important to the profession in a number of ways, for example by giving young researchers stronger foundations for their early careers and by providing employment opportunities when regular statistics appointments are not available. Maintaining a strong cohort of excellent postdocs is crucial for UK statistics at the moment because of the factors mentioned earlier.

Finally, there are sound reasons for special procedures to assess research base funding (formerly 'responsive mode') grant proposals in statistics. Such proposals often include many different aspects, for example methodology, theory, applications and computational algorithms, all of which need examination by knowledgeable reviewers. This diversity of focus is less common in several of the fields with which statistics currently shares a review panel. (See Section 14.1.2.)

16 Training and Nurturing the Next Generation

At every site visit, panel members spent an extended lunch period meeting early-career researchers (a mixture of PhD students, postdocs and junior faculty). Panel members typically spread out, 'working the room' in which lunch was served, each speaking at some length to one or more small groups. This format worked very well in providing the panel with perspectives on the many sub-questions of Question H in the Evidence Framework. Of course, any conclusions drawn from these conversations are based on a very small sample of data.

16.1 Students seeking undergraduate degrees in mathematics

Question H-1 asks about the demand for UK undergraduates to become engaged in mathematical sciences research and requests a comparison with other countries.

In the US and Canada, undergraduate research is regarded as extremely important, to the point that there is tremendous pressure on universities to establish programmes that encourage undergraduates across all disciplines to participate in research. Funding agencies in those countries are deeply involved as well, offering support to undergraduates for summer internships either as part of a faculty research grant or at a national laboratory.

Although the panel did not meet specifically with undergraduate students, at several site visits the panel was told that there is an ample supply, even an over-supply, of students applying for admission to undergraduate programmes in the mathematical sciences. The data in Section 4 of the London Mathematical Society position paper *UK Mathematical Sciences – Research and Teaching in Symbiosis* [16] confirm that the number of undergraduates at UK Higher Education Institutions studying the mathematical sciences has significantly increased in the past five years. But this trend does not necessarily mean that there is sufficient demand from undergraduates to become engaged in mathematical sciences research, and the panel had no opportunity to learn about undergraduate research programmes in the UK. Nonetheless, if (as suggested by the presence of Question H-1 in the Evidence Framework) there is a wish for more undergraduates to become involved in research, the panel believes that there could well be a potentially damaging effect from the recently enacted changes in funding of undergraduate education (see Section 1.4); left to their own devices, undergraduates may be unlikely to regard 'research' as helpful to their future earning power.

16.2 Education and support of PhD students

The 2004 IRM [13] expressed serious concern about the short and narrow training received by UK PhDs under the traditional $3+3$ model, which reduced their competitiveness for both academic and industrial jobs. The next three subsections discuss several related issues.

16.2.1 Doctoral Training Accounts

The information provided by EPSRC (Section 1.5.1) explains that most PhD assistantships in the mathematical sciences are funded by Doctoral Training Accounts (DTAs), which derive from awards made to institutions based on the results of a peer review panel convened by EPSRC. However, the panel's understanding, derived from the second LMS position paper [16], is that, although EPSRC informs institutions about the earnings through research grants originating in mathematical sciences departments, institutions are not obliged to follow the associated division of funds, but rather are encouraged to think 'strategically' about the internal allocation of DTA funds. As a result, researchers in a mathematical sciences department cannot count on receiving what might be considered their 'fair share' of DTA money, since this is decided by the local institution without review by EPSRC. In the panel's view, this policy about distribution of DTAs is likely to cause continuing harm to the pipeline of PhD students in the mathematical sciences, especially since, as announced on 6 January 2011, EPSRC will no longer support project studentships on research grants.[39]

16.2.2 Taught Course Centres and Centres for Doctoral Training

With the intent of improving the breadth of PhD education, EPSRC has funded two different programmes since 2004: Taught Course Centres (TCCs) in the Mathematical Sciences and Centres for Doctoral Training (CDTs), whose scientific content must be aligned with EPSRC's remit, but is not limited to the mathematical sciences.

The six Taught Course Centres were designed to encourage collaborations of universities, allowing each department to offer a broader education to its PhD students than would be possible with only its own faculty. Based on what the panel heard during its site visits, TCCs seem to be fulfilling the purpose of increasing students' knowledge of mathematical sciences areas outside their PhD topics. The secondary effects of TCCs have also been beneficial. For example, a consequence of creating the Scottish Mathematical Training Centre is that the faculty members from Aberdeen, Dundee, Edinburgh, Glasgow, Heriot–Watt, St Andrews and Strathclyde started to work together, at first in preparing the lectures and then more generally on research. This result suggests an effective collaboration mode that could be created elsewhere in the UK.

[39] `www.epsrc.ac.uk/newsevents/news/2011/Pages/projectstudentships.aspx`.

Complaints were made about inadequate technology for remote transmission of TCC courses and about access difficulties for students from outside institutions, but the panel assumes that these problems can be fixed relatively easily as technology improves. However, the panel was unhappy to learn that, for some TCCs, policies about course assessment and PhD student attendance vary with participating departments, with the result that the quality of the courses is not reviewed and some students do not take advantage of the TCCs. A review[40] of the TCCs commissioned by EPSRC in the summer of 2010 had a favourable impression but concluded, as does the present panel, that it is too early to determine whether TCC-trained students are more competitive with respect to students trained outside the UK. The EPSRC review also noted with apparent disapproval the lack of a uniform policy about TCC course attendance and assessment.

A different approach to making UK PhDs more competitive is represented by Centres for Doctoral Training (CDTs), awarded to specific universities in scientific areas within EPSRC's remit (not just in the mathematical sciences). Fifty new CDTs were funded by EPSRC in 2009, each admitting about 10 students per year. The students admitted to a CDT have access to taught courses and also complete a PhD dissertation within the centre.

Several concerns about CDTs were expressed during site visits—that they lead to an undue concentration of funding in individual institutions, and that the options for PhD research topics might be overly influenced by funding availability, where the latter concern arises because a relatively large proportion of the available PhD assistantships in a subfield might be available from one institution.

Based on these concerns (which the panel sees as valid), the panel strongly recommends that consortia of universities be permitted to compete for Centres for Doctoral Training (or a similar programme with a different name); this is effectively impossible at present because students must be trained together in one place. If smaller universities without CDT funding cannot attract PhD students, the best faculty will either leave the smaller institutions or else be hampered in living up to their full research and educational potential, in either case damaging the institutional diversity valued by the panel (see Section 4 and Recommendation R-1).

In response to Question H-7, it is too early to tell whether the measures taken will develop an appropriate balance in UK PhDs between depth and breadth of knowledge in the mathematical sciences.

16.2.3 Funding of PhD students

A major change since the 2004 IRM has been that EPSRC now funds PhD students, on average, for 3.5 years rather than for only 3 years. This is unquestionably an improvement in the effectiveness of UK mechanisms for supporting talented individuals (Question H-3), and the panel heard essentially universal support for the extended time of PhD student funding.

As to whether the length of EPSRC funding should be increased, comments at the site visits, as well as private comments made to panel members, were mixed. Some people favoured moving to assured four-year funding, while others argued that it was better to support more students for 3.5 years, asserting that their universities and departments could almost always find funding for an additional half-year if needed.

16.2.4 A new kind of Master's degree

A system containing additional options for PhD student training is appealing to the panel, and in this spirit we recommend that UK universities consider establishment of a PhD programme that begins with a one-year *research Master's*, followed by three years of PhD education and training. The special 'Master's' would contain advanced taught courses, appropriately assessed, and a project with a significant research component, to be carried out at a university or in industry. For students in such a programme who decide not to pursue a PhD, an MSc degree could be awarded. Such a structure is, in effect, a compromise with the Bologna agreement,[41] usually described as $3 + 2 + 3$. Its purpose is not simply to introduce early-stage PhD students to taught courses, but also to provide a taste of research in either an academic or industrial setting to ensure that students are committed (and able) to conduct research.

[40] `www.epsrc.ac.uk/pubs/reports/Pages/maths.aspx`.
[41] `www.hefce.ac.uk/learning/inter/`.

16.3 PhD student concerns

Stiff competition on the international job market for UK PhDs in mathematical sciences is a fact of life today, and it is too early to tell whether the changes discussed in Section 16.2.2 will lead to improved prospects. In discussions with panel members at the site visits, many PhD students said that they are very worried about finding an academic job after completing their degree. The only exceptions were PhD students who had been supported by a CASE studentship (see Section 7), who felt that they were likely to find a rewarding job in industry.

Several PhD students who plan to finish within the next year asserted that there are not enough suitable postdoctoral positions in the UK, and that available positions are limited in number and location. The limited number of postdoctoral positions has, not surprisingly, a disproportionately negative effect on two-career couples.

16.4 Postdocs and junior faculty

Strikingly few of the postdocs and junior faculty whom the panel met had received their PhDs in the UK. (In a few cases, not a single postdoc or junior faculty member at the sub-panel lunch meeting had received his/her PhD in the UK.) Assuming that the early-career researchers whom the panel met are typical, the panel's response to Question H-6 is that the UK's process for preparing and supporting early-career researchers is not producing PhDs who come out on top when competing with PhDs trained elsewhere. Based on comments during the site visits from departments and researchers who had recently hired postdocs or junior faculty, the panel attributes this problem almost entirely to the more mathematically rich preparation of PhD students from other countries.

When asked why they had come to the UK from elsewhere, several postdocs stated vigorously that there are many more opportunities in the UK than in their home countries. This provides some (limited) evidence in response to Question H-5 that UK academic positions remain attractive in an international context.

The proportion of junior faculty from outside the UK, which seemed to have increased since the 2004 IRM, is a worrying trend because it implies a dwindling supply of top-quality home-grown talent (Question A-5). If other countries begin making a significant effort to attract leading mathematical sciences researchers at all levels, the UK will no longer be able to count on its ability to hire the best researchers from abroad. The panel regards this fragility as a serious potential risk to the UK's future international standing.

The panel observes with concern the apparent inconsistency of training PhDs under a system that, in part by design, does not prepare them well for postdoctoral or academic positions in the UK, which go instead to PhDs educated in other countries. A (presumed) goal of building a healthy pipeline is not, based on the panel's observations, satisfied by the present system. The panel encourages strong efforts to ensure that PhD training meets the highest international standards; it is in this spirit that the panel recommends the 'new' Master's described in Section 16.2.4.

16.5 Participation of women

Question H-9 in the Evidence Framework involves diversity of the UK mathematical sciences research community in terms of gender.

The panel can state that, compared to other countries, the proportion of women is strikingly small. The gender data presented by EPSRC (in Sections 6.5.11 and 7.6 in Part I of *Information for the Panel*; see Section 1.5.1) indicate that the proportion of women principal and co-principal investigators is improving slightly (in younger researchers), but the overall numbers are not encouraging.

One-fourth of the present panel members are women, but during our site visits, no more than 10% (sometimes much less than 10%) of those present were women, and very few women made presentations to the panel. The only contexts in which there were more women were the lunch meetings with early-career researchers; this is consistent with the trends in the data provided by EPSRC.

Possibly more worrying than the numbers was that, with a few notable exceptions, the people with whom we spoke did not seem to be particularly concerned about this issue. A typical attitude appeared to be, approximately, 'It's unfortunate, but there simply are very few women in the mathematical sciences'.

The panel was informed at some site visits about positive steps involving the London Mathematical Society—in particular, its Women in Mathematics Committee and a March 2008 statement by the LMS Council.[42] In 2009, the LMS and the Committee of Heads of Departments on Mathematical Sciences established a Good Practice Award[43] to help advance women's careers in university mathematical sciences departments.

Nonetheless, the overall impression from data and the panel's experiences during the review week is that action about gender diversity is not a sufficiently high priority in the UK. Panel members believe that this lack of attention will be damaging to the future research excellence of UK mathematical sciences research, and our experience suggests that significant changes will happen only when the issue is taken seriously.

In the US, seven NSF-funded mathematical sciences institutes[44] encourage outstanding women researchers (including early-career women) in various ways. In the panel's view, a similar strategy should be followed in the UK via activities of the institutes and learned societies (Sections 8 and 9), which could ensure that excellent-quality women at all levels attend (and, if appropriate, speak) at programmes held at UK institutes or organised by learned societies. (See Recommendation R-11, Section 2.)

Concern about participation of UK women in science is not limited to the mathematical sciences. The 2008 International Review of Materials panel stated:

> The Panel strongly believes . . . that there is an urgent need to improve diversity among groups designed as faculty leadership. [12, page 37]

In the same spirit, the present panel agrees, with special reference to the 'Developing Leaders' strategy in EPSRC's delivery plan [6]; see Recommendation R-11.

17 International Connections

The multipart Question C in the Evidence Framework asks for an assessment of collaborations between UK mathematical sciences researchers and world-leading researchers in other countries. Happily, this question is straightforward to answer, based on the panel's own knowledge and the many consultation responses provided by EPSRC in *Evidence for the Panel* (Section 1.5.1). Research in the mathematical sciences lends itself more readily than perhaps any other area to international collaboration because so much of its content represents general concepts (in contrast to requiring, for example, a personal visit to an experimental facility). Without doubt, a significant element in the collaborations of UK mathematical scientists with leaders from other countries is participation of the latter in programmes at the institutes and at conferences and workshops sponsored by the learned societies; see Sections 8 and 9.

Question C-2 focuses on collaborations with specified parts of the world. There is no question about the very high level of collaboration with the US and Europe, and, as documented by multiple consultation responses, collaborations with China, India, and Japan are increasing. A difficulty with collaborations in countries (including the latter three) that are far removed geographically is that face-to-face meetings and visits are needed to strengthen collaborations, but the associated expenses are high. In addition, as mentioned in Section 1.4, concerns were expressed to the panel about funding for visits as well as recent restrictions on visas.

[42] `ftp.lms.ac.uk/activities/women_maths_com/index.html`.
[43] `www.lms.ac.uk/content/good-practice-award`.
[44] `mathinstitutes.org`.

18 Annotated Evidence Framework

This section contains the Evidence Framework annotated with the section numbers in which each question is discussed. For some questions, the panel had no data or information on which to base an answer.

A. What is the standing on a global scale of the UK Mathematical Sciences research community both in terms of research quality and the profile of researchers?

   All the subsections of Section 5.

   A-1. Is the UK internationally leading in Mathematical Sciences research? In which areas? What contributes to the UK strength and what are the recommendations for continued strength?
   All the subsections of Section 5.

   A-2. What are the opportunities/threats for the future?
   All the subsections of Section 5.

   A-3. Where are the gaps in the UK research base?
   All the subsections of Section 5.

   A-4. In which areas is the UK weak and what are the recommendations for improvement?
   All the subsections of Section 5.

   A-5. What are the trends in terms of the standing of UK research and the profile of UK researchers?
   All the subsections of Section 5.

B. What evidence is there to indicate the existence of creativity and adventure in UK Mathematical Sciences research?

   All the subsections in Section 5, and Section 13.

   B-1. What is the current volume of high-risk, high-impact research and is this appropriate?
   Section 13.

   B-2 What are the barriers to more adventurous research and how can they be overcome?
   Section 13.

   B-3. To what extent do the Research Councils' funding policies support/enable adventurous research?
   Section 13.

C. To what extent are the best UK-based researchers in the Mathematical Sciences engaged in collaborations with world-leading researchers based in other countries?

   Section 17.

   C-1. Does international collaboration give rise to particular difficulties in the Mathematical Sciences research area? What could be done to improve international interactions?
   Section 17.

   C-2. What is the nature and extent of engagement between the UK and Europe, USA, China, India, and Japan, and how effective is this engagement?
   Section 17.

   C-3. How does this compare with the engagement between the UK and the rest of the world?
   Section 17.

D. Is the UK Mathematical Sciences community actively engaging in new research opportunities to address key technological/societal challenges?

   Sections 3.2, 6 and 10..

- D-1. What are the key technological/societal challenges on which Mathematical Sciences research has a bearing? To what extent is the UK Mathematical Sciences research community contributing to these? Are there fields where UK research activity does not match the potential significance of the area? Are there areas where the UK has particular strengths?

  Sections 6, 6.2, 6.4 and 10.

- D-2. Are there any areas which are under-supported in relation to the situation overseas? If so, what are the reasons underlying this situation and how can it be remedied?

  Not answered.

- D-3. Does the structure of the UK's mathematical science research community hamper its ability to address current and emerging technological/societal challenges? If so, what improvements could be implemented?

  Nothing in the structure of the research community hampers its ability to address current and emerging technological/societal challenges.

  Sections 14.1 and 14.2.

- D-4. Are there a sufficient number of research leaders of international stature in the Mathematical Sciences in the UK? If not, which areas are currently deficient?

  All the subsections of Section 5.

E. Is the Mathematical Sciences research base interacting with other disciplines and participating in multidisciplinary research?

All the subsections of Section 6 and of Section 10.

- E-1. Is there sufficient research connecting mathematical scientists with investigators from a broad range of disciplines including life sciences, materials, the physical sciences, finance, and engineering? What is the evidence?

  All the subsections of Section 6 and of Section 10.

- E-2. Where does the leadership of multidisciplinary research involving mathematical sciences originate? In which other disciplines are the mathematical sciences contributing to major advances?

  Sections 6, 10 and 14.

- E-3. Are there appropriate levels of knowledge exchange between the Mathematical Sciences community and other disciplines? What are the main barriers to effective knowledge and information flow, and how can they be overcome?

  Section 14.

- E-4. Have funding programmes been effective in encouraging multidisciplinary research? What is the evidence?

  Section 14.

F. What is the level of interaction between the research base and industry?

Sections 7 and 11.

- F-1. What is the flow of trained people between industry and the research base and vice versa? Is this sufficient and how does it compare with international norms?

  Sections 6.4, 11 and 11.3.

- F-2. How robust are the relationships between UK academia and industry both nationally and internationally, and how can these be improved?

  Section 11.3.

- F-3. To what extent does the mathematical sciences community take advantage of opportunities, including research council schemes, to foster and support this knowledge exchange? Is there more that could be done to encourage knowledge transfer?

  Section 11.3.

F-4. Nationally and internationally, what is the scale of mathematical sciences research and development undertaken directly by users? What are the trends? Are there implications for the UK mathematical sciences research community, and how well-positioned is it to respond? Is there any way that its position could be improved?
Section 11.3.

G. How is the UK mathematical sciences research activity benefiting the UK economy and global competitiveness?

Section 11.

G-1. What are the current and emerging major advances in the mathematical sciences area which are benefiting the UK? Which of these include a significant contribution from UK research?
See the list of topics at the beginning of Section 11.

G-2. How successful has the UK mathematical sciences community (academic and user-based) been at wealth creation (e.g. spin-out companies, licenses, etc.)? Does the community make the most of opportunities for new commercial activity? What are the barriers to successful innovation based on advances in the mathematical sciences in the UK, and how can these be overcome?
Sections 10 and 11.3.

H. How successful is the UK in attracting and developing talented Mathematical Sciences researchers? How well are they nurtured and supported at each stage of their career?

Section 16.

H-1. Are the numbers of graduates (at first and higher degree level) sufficient to maintain the UK Mathematical Sciences research base? Is there sufficient demand from undergraduates to become engaged in Mathematical Sciences research? How does this compare to experience in other countries?
Section 16.1.

H-2. Is the UK producing a steady stream of researchers in the required areas or are there areas of weakness in which the number of researchers should be actively managed to reflect the research climate? What adjustments should be made?
Section 16.4.

H-3. How effective are UK funding mechanisms at providing resources to support the development and retention of talented individuals in the mathematical sciences?
Sections 16.2 and 16.2.3.

H-4. How does the career structure for researchers in the Mathematical Sciences in the UK compare internationally?
Not answered.

H-5. Is the UK able to attract international researchers to work in the UK? Is there evidence of ongoing engagement either through retention within the UK research community or through international linkages?
Section 16.4.

H-6. Are early career researchers suitably prepared and supported to embark on research careers?
Sections 16.2, 16.3 and 16.4.

H-7. Is the balance between deep subject knowledge and ability to work at subject interfaces and boundaries appropriate?
Sections 11.3 and 16.2.

H-8. How is the UK community responding to the changing trends in the UK employment market?
Not answered.

H-9. How diverse is the UK mathematical sciences research community in terms of gender and ethnicity and how does this compare with other countries?
Section 16.5.

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19 Annexe A: Steering Committee

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