The London Mathematical Society

UK Mathematical Sciences – Research and Teaching in Symbiosis

A position paper of the London Mathematical Society

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EXECUTIVE SUMMARY

1. The UK's mathematical sciences research base is strong. About 50% of research activity in the mathematical sciences was rated as internationally excellent or world-leading in Research Assessment Exercise (RAE) 2008.

2. The UK mathematical sciences research base has increased by about 18% in the past decade. 1632 (full time equivalent) mathematical sciences researchers were submitted to RAE2001 and 1924 to RAE2008. During the same period the numbers in some cognate subjects such as physics and chemistry remained constant or declined.

3. Numbers of UK mathematical science students have greatly increased in recent years. The total number of full time equivalent students (FTEs) on undergraduate (UG) and postgraduate (PG) courses in the mathematical sciences increased by over 13% between 2003 and 2008, when the total was 34,120. During this period the number of students in UK Higher Education (HE) increased by 4.8%. The number of UK UG students graduating in the mathematical sciences is about double that in either physics or chemistry.

4. Students with mathematical sciences degrees are in high demand in the UK economy. This is demonstrated, for example, by statistics on starting salaries for undergraduates.

5. Activity in UK mathematical science has become more concentrated geographically over the past decade. There were 112 submissions in the mathematical sciences to RAE2008, compared with 151 to RAE2001.

6. EPSRC funding for research in the mathematical sciences is dwarfed by support for cognate subjects such as physics and chemistry. Thus the mathematical sciences research fund allocation was £14.3M in 2009-10, the corresponding sum for the physical sciences was £81.6M. The difference is much too great to be explained by differences in equipment costs.

7. Despite the strong and crucial performance of UK mathematical sciences in both teaching and research, the trends outlined here are likely – if allowed to continue – to lead to the closure of a number of first-rate UK mathematical sciences departments. In a climate dominated by cuts and of grants awarded with full economic costing, university administrations find themselves under severe pressure to invest only in areas capable of generating high levels of research grant income, and withdraw support from other areas. Such decisions are blind to wider economic and social imperatives, and thus can have unintended consequences that are deeply damaging to the needs of the country.

This paper has been endorsed by the Institute of Mathematics and its Applications.

Introduction

The first finding of the EPSRC International Review of Mathematics 2004[1] stated:

> *The mathematical sciences are increasingly playing a more central role in the development of advanced modern societies because frequently advanced technology relies on sophisticated mathematical content. Mathematical research has to be considered from this angle, and therefore must be developed ambitiously. It must be properly connected to advanced training to meet the needs for properly trained personnel for the future, in industry and in academia.*

This report summarises the context in which research and training in the mathematical sciences is currently carried out in UK higher education. The focus here is on research and postgraduate training, but we emphasise from the outset that this cannot be separated from undergraduate education, both of those studying for degrees in mathematics and of those many students studying other disciplines for which the mathematical sciences are a crucial language and tool.

The past 20 years have seen a great flowering of mathematics world-wide: Wiles' proof in the 1990s of Fermat's Last Theorem in Number Theory has been followed by Perelman's 2003 proof of the 1904 Poincaré conjecture, one of the Clay Mathematics Institute's $1 million Millennium Problems. While wonderful feats of individual brilliance, both these achievements depended essentially on the efforts of many other mathematicians, working over many years. The full impact of these major breakthroughs on science, technology and on mathematics itself may not be realised for decades. A (very) long-term payback from fundamental research is often characteristic of the mathematical sciences: examples include results in number theory which led to the RSA algorithm which now underpins the security of much electronic communication and commerce, and results of Radon in 1917 and Fritz John in 1938 which provided the theoretical basis for two and three dimensional tomography respectively, over 40 years later[2].

In their conclusions to the 2004 International Review of UK Research in Mathematics, the Review Panel wrote:

> *Mathematical research is a people intensive activity with competition at an international scale. From that point of view the panel sees some dangers:*
>
> *• six years from entering university to getting a Ph.D. almost inevitably induces a narrow training; bringing the British system in line with the perspective offered by the Bologna agreement should be considered;*
>
> *• difficulties in recruitment and retention of qualified personnel have appeared and are likely to worsen;*
>
> *• the age distribution of some disciplines such as statistics is very unbalanced, and may heavily hamper future developments;*
>
> *• the UK cannot afford to concentrate its advanced training in mathematics, which has to be nurtured by the most up-to-date research, in a small number of highly competitive universities.*

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[1] http://www.cms.ac.uk/irm/irm.pdf
[2] Mathematics Today 42 pp. 132–134 and 168–169 (2006)

We hope that the information collected in this document will help in examining the extent to which these dangers have been overcome in the period since 2004, or whether some of them remain as serious threats to the future health of mathematical science in the United Kingdom.

2. UK Mathematical science - scale and distribution of research

(i) Research volume: The volume of research activity in the mathematical sciences in the UK is high, and showed significant increases over the early years of this century. First, as shown in Table 1, there was a significant increase in the number of academic staff submitted to the mathematical sciences sub-panels in the 2008 Research Assessment Exercise (RAE) over the number submitted in 2001. Second, Table 2 gives a comparison of research grant spend in 2001-02 and 2006-07 for grants from the UK research councils (RCUK) and other sources. It is not easy to make a direct comparison of research grant funding over this period, because new research council grants have been funded under a "full economic cost" (FEC) model since 2004. However, it appears that there was a significant increase in Research Council support for the mathematical sciences over this period.

Subject (UoA)	2000 FTE staff	2007 FTE staff	% increase
Pure	510	685	34%
Applied	735	850	16%
Statistics+OR	387	389	0.5%
Math. Science (= P+A+S)	1632	1924	18%

Table 1: FTE numbers

	Research spend in 2001-2 (£M)		Research spend in 2006-7 (£M)	
	From RCUK	**Non RCUK**	**From RCUK**	**Non RCUK**
Pure	2.7	1.7	6.3	2.1
Applied	10.1	6.8	16.9	6.4
Statistics+OR	2.2	3.7	5.8	3.8
Math. Science (= P+A+S)	14.9	12.3	29.1	12.3

Table 2: RCUK grant income

While the overall trend revealed in the above tables is one of a steady increase, there are two aspects where this is not the case:

- Submitted FTEs in statistics and OR remained essentially constant. It is possible, though unlikely, that these figures hide an increase in numbers of statistics researchers who were submitted to the RAE in subjects other than statistics. A more likely explanation is that these figures simply reflect the continuing difficulty in recruiting academics to statistics and OR posts in the UK.
- The figure for non-RCUK research spend in the mathematical sciences is constant across the period, at £12.3M.

(ii) **Concentration of research activity**: The expansions of the last decade in Research Council funded research activity and in numbers of researchers were accompanied by a parallel concentration of activity into a smaller number of larger departments. This is shown in Table 3:

Subject (UoA)	**Number of submissions in 2001**	**Number of submissions in 2008**	**% [#]of researchers in largest 4 submissions in 2001**	**% [#]of researchers in largest 4 submissions in 2008**	**% [#] of researchers in largest 8 submissions in 2001**	**% [#] of researchers in largest 8 submissions in 2008**
Pure	47	37	26% [130]	27% [186]	39% [200]	44% [302]
Applied	58	45	21% [157]	25% [210]	36% [263]	40% [338]
Statistics	46	30	17% [65]	27% [107]	31% [118]	47% [182]

Table 3: Concentration of research

Tables 1 and 3 demonstrate clearly that:

- while the number of active researchers has been increasing, research in the mathematical sciences in the UK is being concentrated in fewer places.

Powerful forces underlie this development:

- Research Council Grant structures favour large units[3];
- Research Council rules for Doctoral Training Centres favour large units.

A further reason is probably also important, but is more difficult to quantify: universities, under pressure to maximise research income, may focus limited funds in areas where external grant income is more plentiful, rather than basing investment decisions on educational or academic grounds, or having regard to the UK's strategic needs.

These pressures run counter to the high popularity (with both students and with employers) of undergraduate degrees in the mathematical sciences across the breadth of the UK[4]. The

[3] Note for example the introduction by EPSRC of Platform grants in the mathematical sciences for 2010-11 – see http://www.epsrc.ac.uk/plans/funding/Pages/maths.aspx

[4] See Section 4(i) below.

buoyant demand (from many employment sectors) for highly trained young mathematical scientists[5] makes it vital to maintain a wide geographic spread of top-quality mathematicians and statisticians across the UK's higher education institutions.

3. UK mathematical science – research quality

UK research activity in the mathematical sciences was rated highly in RAE2008[6], with around 50% of research activity[7] in each of the three relevant subject areas being rated as internationally excellent (3*) or world-leading (4*). This is shown in Table 4, in which two further features are also worthy of note:

- Performance is very similar across all three disciplines.
- Excellence is widespread, not exclusively concentrated in a small number of departments.

It is worth recalling again here the scale of the activity: as table 3 records, there were 112 submissions to the three mathematical sciences Units of Assessment (UoAs) in RAE2008, and 21 universities made submissions to all three of the mathematical sciences UoAs.

	%4*	**%3***	**%2***	**%1***
Pure: all submissions	14	39	39	6
Pure: biggest 4 submissions	30	43	26	1
Pure: biggest 8 submissions	23	39	30	6
Applied: all submissions	12	37	38	13
Applied: biggest 4	26	45	29	0
Applied: biggest 8	24	43	31	2
Statistics: all submissions	14	40	37	9
Statistics: biggest 4	22	45	29	4
Statistics: biggest 8	22	44	30	3

Table 4: Average profiles in RAE2008

Citation indices provide a second indicator of the strength in depth of UK mathematical sciences in an international context at the start of the 21st century – the Thomson Reuters

[5] See Table 6, page 7.
[6] For a brief description of RAE2008 including an explanation of how profiles were created, see section 3.1 of the LMS Position Paper, "UK Funding mechanisms for mathematical sciences research"; or, for full details, http://www.rae.ac.uk/
[7] Averages in the table below are taken over submissions – that is, they are *not* weighted by the size of submissions.

Citations Top 20, for mathematical sciences 1998-2008, shows Scotland in second place, England in fourth[8].

4. Teaching in the mathematical sciences

(i) **All students**: The total number of students studying mathematical sciences at UK Higher Education institutions has shown a steady increase throughout the most recent period for which HESA data is available[9], as shown in Table 5:

	2003-4	**2004-5**	**2005-6**	**2006-7**	**2007-8**	**% increase in 5 years**
Math. Sciences students in UK HE	30105	30560	31570	32950	34120	13.3%

Table 5: Number of UK mathematical science students

The increase shown in Table 5 is not simply a consequence of the increase in total numbers of UK students over the period – there were 2.20M students in UK HE in 2003-4, and 2.31M in 2007-8, an increase of 4.8%.

Mathematical sciences students are highly valued by UK employers. The most recent data[10] on the mean salaries of first degree graduates entering their first jobs in 2008 is given in Table 6:

[8] See http://www.timeshighereducation.co.uk/story.asp?sectioncode=26&storycode=406463 Denmark and the USA are respectively first and third. Volume of work produced in Wales and N. Ireland was insufficient to allow inclusion. Ranking is based on citations per paper, for papers published between 1/1/98 and 31/12/08. While citation data certainly need to be treated with great caution, they may be of some value when used to make large-scale comparisons within a given field.

[9] See http://www.hesa.ac.uk/index.php/content/view/1541/161/ Details can be found there of how students are counted – not all counted here are doing UG or PG degrees in mathematical sciences alone.

[10] This is HESA data, published in http://www.thecompleteuniversityguide.co.uk/single.htm?ipg=6371 We give only the highest 19 graduate categories, and the mean.

Subject	Starting salary
Dentistry	£29,805
Medicine	£28,896
Chemical Engineering	£28,415
Economics	£25,726
General Engineering	£25,455
Veterinary Medicine	£25,206
E. and S. Asian Studies	£24,769
Building	£24,755
Civil Engineering	£24,473
Mechanical Engineering	£24,446
Social Work	£23,816
Mathematics	**£23,644**
Aero and Man. Eng.	£23,464
Physics & Astronomy	£23,275
Electronic Engineering	£23,123
Librarianship & Inf. Man.	£22,935
Russian	£22,568
Geology	£22,282
Computer Science	£22,278
Mean across sector	**£21,286**

Table 6: Starting salary after first degree

(ii) **Education of PhDs**: The number of PhDs being awarded annually in the mathematical sciences in the UK appears to have remained fairly constant over the last 15 years. This is shown by Table 7. The figures here are taken from the RAE data for 2001 and 2008, and hence are certainly somewhat lower than the correct numbers, since a number of students will have completed PhDs in mathematical science with supervisors who were not returned under mathematical sciences in the RAE.

Subject (UoA)	PhDs awarded per year (average), 1996-2000	PhDs awarded per year (average), 2001-2007
Pure	104	102
Applied	186	162
Statistics+OR	94	73
Math. Science (= P+A+S)	384	337

Table 7: PhD numbers

The number of PhDs per year increased fairly steadily through the period of RAE2008 – for example 261 were awarded in 2001, and 376 in 2006. The RAE returns record a total of 2191 PhDs in the mathematical sciences in the period of RAE2008.[11]

5. Comparison with sister disciplines

The growth in the number of UK mathematical sciences researchers over the last decade has already been noted above. During the same period, activity in some cognate subjects has remained approximately constant or even declined. This is reflected both in the numbers of academics in each discipline, and in the numbers of undergraduates produced, as Table 8 shows:

	FTE staff 2001	FTE staff 2007	Graduating students (07-08)
Chemistry	1300	1151	2860
Mathematical Sciences	1632	1924	5300
Physics	1668	1686	2515

Table 8: Changes in scale of discipline

There is a stark contrast between the levels of activity shown above and the amounts of Research Council funding. Thus, for example[12],

> *in 2008-9 EPSRC's total spend on research grants in the Mathematics Programme (which also covered Public Engagement) was £15.7M, whereas the corresponding figure for Physical Sciences was £97.8M, and that for Information Technology was £84.1M.*

Comparative allocations for 2009-10 show a similar pattern; here, for example, are the EPSRC programme allocations for mathematical sciences and for physical sciences:

[11] HESA figures for the number of PhDs in the mathematical sciences are somewhat higher – presumably because some students are doing mathematical science PhDs with supervisors not submitted to a mathematical science UoA in the RAE. This is one illustration among many of the difficulties besetting UK HE statistics.
[12] See http://www.epsrc.ac.uk/about/facts/Pages/budget.aspx

EPSRC Programme 2009-10	DTA[13] allocation	CDT[14] allocation	Research Funding
Mathematical Sciences	£11.1M	3 CDTs, cost £10.8M	£14.3M
Physical Sciences	£32M	5 CDTs	£81.6M

Table 9: EPSRC programme allocations, 2009-10

A part of the mismatch shown in the above figures between Research Council support for mathematics and that for cognate subjects is a consequence of the fact that research in (for example) the physical sciences is more expensive than research in the mathematical sciences. However, even after the costs of equipment are removed, the imbalances are likely still to be striking.

Other G8 countries do *not* accord the mathematical sciences the same Cinderella status as the UK in regard to the share of research support. Thus, for example[15], the US National Science Foundation's 2010 budget for the mathematical sciences is $246.4M, while for physics and chemistry combined it is $535.1M. In Germany, the Deutsche Forschungsgemeinschaft[16] (DFG) awards substantially more each year than the EPSRC to mathematical sciences (€70.6M in 2009 having risen year on year from €36.4M in 2006).

6. Conclusions

There is a vibrant mathematical science community in the UK, which

- produces a large annual cohort of well-trained graduating students in the mathematical sciences;
- helps to give the necessary mathematical training to those graduating in other disciplines, thereby crucially supporting the development of future scientists, engineers, financial and IT specialists;
- carries out a large volume of research of high international quality;
- produces around 400 PhDs per year in the mathematical sciences.

This activity is widely spread geographically; this indeed is a crucial feature, since the current numbers of undergraduate mathematicians, the support teaching for other subjects, and the variety of mathematical research will all be impossible to maintain if UK mathematical science is yet further concentrated into fewer centres.

[13] DTA – Doctoral Training Account.
[14] CDT – Centre for Doctoral Training.
[15] AAAS REPORT XXXIV RESEARCH AND DEVELOPMENT FY 2010.
[15] Deutsche Forschungsgemeinschaft, Jahresbericht 2009, Aufgaben und Ergebnisse.