## LONDON <br> MATHEMATICAL SOCIETY <br> EST. 1865

NEWSLETTER
Issue: 474 - January 2018


TILTING THE
CLASSROOM

SIMULATING
GALAXY
FORMATION

MODELLING
OUR SENSE OF SMELL

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The Newsletter welcomes submissions of feature content, including mathematical articles, career related articles, and micro-theses from members and non-members. Submission guidelines and LaTeX templates can be found on newsletter.Ims.ac.uk/submissions.

Feature content should be submitted to the editor-in-chief at iain.moffatt@rhul.ac.uk.

News items should be sent to newsletter@lms.ac.uk

Notices of events should be prepared using the templates on newsletter.Ims.ac.uk/submissions and sent to calendar@lms.ac.uk.

For advertising rates and guidelines see newsletter.Ims.ac.uk/rate-card.

## NEWSLETTER WEBSITE

The Newsletter is freely available electronically through the LMS's website newsletter.Ims.ac.uk.

## COVER IMAGE

Figure 1 of J. Schaye, et. al., The EAGLE project: simulating the evolution and assembly of galaxies and their environments, Mon. Not. R. Astron. Soc. 446.

## MEMBERSHIP

Joining the LMS is a straightforward process. For details see Ims.ac.uk/membership.

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## LMS Council 2017-18

The results of the 2017 LMS Elections to Council and Nominating Committee were announced at the LMS Annual General Meeting on 10 November 2017. Council membership is as follows:

## PRESIDENT:

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Professor A. Dancer (University of Oxford);
*Professor D.E. Evans (University of Cardiff);
Dr A.D. Gardiner;
Professor B. Nucinkis (Royal Holloway);
Professor G. Stallard (Open University);
Dr A. Vdovina (University of Newcastle);
*Professor S. Zerbes (University College London).
*Members continuing the second year of their twoyear election in 2016.

## LMS NOMINATING COMMITTEE:

Also at the AGM, Professor H. Dugald Macpherson (University of Leeds) and Dr M. Mathieu (Queen's University Belfast) were elected to the Nominating Committee for three-year terms of office.

Continuing members of the Nominating Committee are: Professor J. Toland (Chair), Professor M. Mazzocco, Professor R. Heath-Brown, Professor S. Rees and Professor U. Tillmann.

## LMS Moves to Online Grants Applications

The Society is pleased to announce that as part of an exercise to facilitate the process of applying for LMS grants, applications for the first Scheme in the Society's portfolio of grants may be carried out electronically via the LMS website.

Applications for 'Visits to the UK' (Scheme 2) research grants should now be submitted online using a dedicated web-form. The new online form is accessible on the 'Visits to the UK' (Scheme 2) webpage (Ims.ac.uk/grants/visits-uk-scheme-2). The existing downloadable PDF grant application forms will still be available on the webpage for submissions to the upcoming January deadline if preferred, however, after this date all applications will have to be completed and submitted electronically via the web-form.

This change to the application process for Scheme 2 grants is part of a wider exercise by the Society to digitise the application processes for all LMS grants in an effort to make applying for Society funding more accessible for all UK-based mathematicians. Development is ongoing for the similar digitisation of Conference Grants (Scheme 1); Joint Research Groups (Scheme 3); Research in Pairs (Scheme 4); International Short Visits (Scheme 5); Postgraduate Research Conference Grants (Scheme 8) and Celebrating New Appointments (Scheme 9). Web-forms for these and other Society grants will be rolled out as development work completes in each area.

Any queries regarding the changes to research grant application forms should be sent to grants@lms.ac.uk.

## ICM 2018: LMS Travel Grants for Early Career Researchers

The London Mathematical Society has set aside funds to be used for making grants to support the attendance of UK-based Early Career Research mathematicians at the ICM 2018 in Rio de Janeiro from 1-9 August 2018 (icm2018.org/portal/en/).

The grants are intended to contribute to the costs of attending the ICM 2018, not to meet them entirely. The grants are not to support attendance at Satellite meetings. Applicants should be Early Career Researchers, defined as within five years of PhD completion (excluding career breaks), based at a UK institution. PhD students whose research would benefit from attending the meeting may also apply but their applications should be strongly supported with a clear mathematical case by their supervisor.

To apply, complete the application form (which can be downloaded from the Society's website: www.lms.ac.uk/grants/lms-travel-grants-icms-and-ecms) and return to Elizabeth Fisher by email:
ecr.grants@lms.ac.uk or by post: ICM 2018 Travel Grants for Early Career Researchers, London Mathematical Society, De Morgan House, 57-58 Russell Square, London WC1B 4HS. Deadline is 2 February 2018. Applicants will be informed of the outcome by mid-March. You do not need to be an LMS member to apply.

NB: The LMS has also set aside funds to be used for grants to support the attendance of other UKbased mathematicians at the ICM 2018 in Rio de Janeiro from 1-9 August 2018. Further details of this scheme are also available on the Society's website: Ims.ac.uk/grants/lms-travel-grants-icms-and-ecms.

## PEOPLE

CMS Board Meeting


Ruth Kaufman, Caroline Series, Ineke de Moortel

The President of the Operational Research Society (ORS), Ruth Kaufman, the President of the London Mathematical Society (LMS), Professor Caroline Series, FRS, and the President of the Edinburgh Mathematical Society (EdMS), Professor Ineke de Moortel, attended a Council for the Mathematical Sciences (CMS) Board Meeting at De Morgan House in October 2017. Ruth Kaufman finished her term as ORS President in December 2017 and Professor Caroline Series took over the office of LMS President at the LMS Annual General Meeting on 10 November 2017.

## Philip Leverhulme Prize Winners

The 2017 Prizes for Mathematics and Statistics have been awarded to Dr Anders Hansen, Dr Oscar Randal-

Williams and Dr Carola-Bibiane Schönlieb all of the University of Cambridge; Professor Dominic Vella (University of Oxford) and Dr Hendrik Weber (University of Warwick). The Society congratulates all the award winners, in particular LMS members Dr Oscar Randal-Williams, also a 2017 LMS Whitehead Prize winner and Dr Carola-Bibiane Schönlieb, the 2018 LMS Mary Cartwright Lecturer.

## Bertrand Russell Prize 2018



Christiane Rousseau (Université de Montréal) will receive the inaugural Bertrand Russell Prize of the American Mathematical Society (AMS) in recognition of her many contributions furthering human values and the common good through mathematics.

Throughout her career, Professor Rousseau has inspired people of all ages and diverse backgrounds through her lectures, publications, and a wide range of activities reaching out to the general public. In particular, through her visionary leadership of the thematic year Mathematics of Planet Earth 2013, Professor Rousseau has mobilized mathematicians to take on world challenges, advancing the discipline and making her a most appropriate recipient of the first Bertrand Russell Prize of the AMS.

## Blaise Pascal Medal

Professor Felix Otto of the Max Planck Institute, Germany has been awarded the Blaise Pascal Medal in Mathematics of the European Academy of Sciences for seminal contributions on stochastic homogenization, calculus of variations, functional analysis and applications to thin-film micro magnetism. For details see eurasc.org/.

## 2018 Steele Prize

The 2018 Steele Prize for Mathematical Exposition will be awarded to Martin Aigner and Günter M. Ziegler of the Freie Universität Berlin, for Proofs from THE BOOK. For more details and as well as reflections by the recipients about THE BOOK see tinyurl.com/y7ma29qr. The prize will be awarded in January 2018 at the AMS Meeting in San Diego.

## MATHEMATICS POLICY ROUNDUP

## Autumn Budget 2017

The Budget was announced on 22 November 2017 with areas relevant to STEM R\&D and mathematics teaching. The following is taken from government documents.

Supporting the government's ambition of increasing $R \& D$ investment in the economy to $2.4 \%$ of GDP by 2027, the Budget confirms that the $£ 4.7$ billion National Productivity Investment Fund (NPIF) investment in science and innovation announced at Autumn Statement 2016 will grow by a further $£ 2.3$ billion of additional spending in 2021-22, taking total direct R\&D spending to $£ 12.5$ billion per annum by 2021-22. The Industrial Strategy White Paper will provide further detail on what this funding will support, including:

- support for our creative and digital industries by developing pioneering immersive technology for creative content, and launching a new Al and machine-learning programme targeted at the services sector;
- new support to grow the next generation of research talent and ensure that the UK is able to attract and retain the best academic leaders globally.

Given the crucial role of mathematics in preparing the next generation for jobs in the new economy, the government will:

- give more children the opportunity to be taught using world-leading techniques by providing $£ 27$ million to expand the successful Teaching for Mastery maths programme into a further 3,000 schools;
- reward schools and colleges who support their students to study maths by giving them $£ 600$ for every extra pupil who decides to take Maths or Further Maths A levels or Core Maths with over $£ 80$ million available initially, and no cap on numbers;
- nurture top mathematical talent by delivering its commitment to open maths schools across the country. The Budget commits $£ 18$ million to fund an annual $£ 350,000$ for every maths school under the specialist maths school model, which includes outreach work;
- test innovative approaches to improve GCSE Maths resit outcomes by launching a $£ 8.5$ million pilot, alongside $£ 40$ million to establish Further Education Centres of Excellence across the country to train maths teachers and spread best practice.

The full Budget document is available at tinyurl.com/ybl4pl8d.

## House of Commons Select Committee Chair writes to Chancellor

The Chair of the House of Commons Science and Technology Committee, Norman Lamb MP wrote to the Chancellor of the Exchequer, following the Committee's evidence session on 'The Science Budget and Industrial Strategy'. The Chair raised a number of issues.

- Commit now to UK's full participation in the 'Horizon-2020' research programme throughout the relevant research projects and throughout the Brexit implementation period, as well as EU's successor 'Framework Programme 9'
or offer an alternative vision for future close collaboration.
- Acknowledge that a further science uplift will be needed within the next 10 years of at least a further $£ 2.4 \mathrm{bn}$ a year to deliver government's $2.4 \%$ of GDP target for overall R\&D expenditure - and signal such an increase to be made within the current Spending Review period.
- Clarify the rationale for the selection of the 'challenges' on which the Industrial Strategy Challenge Fund is based and how these will evolve.
- Consider how the R\&D Tax Credit system might be better targeted (for example, to spread science and innovation to the regions).
- Extend place-based research and innovation interventions, including locating future new research institutes outside the 'golden triangle'.
- Publish the Connell review of the 'Small Business Research Initiative' without delay, and the government response, and establish a central fund and management for the Initiative and encourage all government departments to deploy it.

More information is available at tinyurl.com/ya4xjs28.

## Review of Knowledge Exchange in the Mathematical Sciences

A review of knowledge exchange in the Mathematical Sciences, chaired by Prime Minister's Council for Science and Technology member, Professor Philip Bond was set up with support from EPSRC and Innovate UK's Knowledge Transfer Network (KTN). The outcomes from the Review will be launched in the House of Lords in spring 2018. More information is available at tinyurl.com/kfyc2yc.

## STEM Strategy for Scotland

The Scottish government has published its STEM Education and Training Strategy for Scotland. 'The Strategy aims for everyone to be encouraged and supported to develop their STEM skills throughout their lives, enabling them to be inquiring, productive and innovative. This is in order both to grow

STEM literacy in society and drive inclusive economic growth in Scotland.' The Strategy is available at tinyurl.com/ybrrh5gx.

## New Education and Skills Measures

Education Secretary Justine Greening announced a series of measures to provide 'opportunities for all and ensure that the government is building the skills needed to secure the nation's prosperity'. This includes piloting new style bursaries in mathematics, with upfront payments of $£ 20,000$ and early retention payments of $£ 5,000$ in the third and fifth year of a teacher’s career. Increased amounts of $£ 7,500$ will also be available to encourage the best mathematics teachers to teach in more challenging schools.

It was also announced that there would be $£ 6$ million further investment to expand Maths Hubs to more challenging areas, to 'help spread excellence in mathematics teaching'. More information is available at tinyurl.com/y7cvb89y.

## 2018 will be the Year of Engineering

The government has announced that it will work with industry partners to make 2018 the Year of Engineering - and will work with them to offer a 'million direct and inspiring experiences of engineering to young people throughout the year'. More information is available at tinyurl.com/ya3p8xru.

## EPSRC Chief Executive to Serve as Executive Chair

Professor Philip Nelson will serve as the Executive Chair of the Engineering and Physical Sciences Research Council (EPSRC) when it becomes part of UK Research and Innovation (UKRI). Professor Nelson is currently Chief Executive of EPSRC and has agreed to serve an additional six months beyond his existing term when UKRI comes into existence on 1 April 2018.

## New Head of Mathematics Theme at EPSRC

The outgoing Head of Mathematics Theme at EPSRC, Philippa Hemmings, will hand over to Katy Blaney in January 2018.

John Johnston
Joint Promotion of Mathematics

## 6th Heidelberg Laureate Forum

The 6th Heidelberg Laureate Forum (HLF) will take place in Heidelberg, Germany during 23-28 September 2018. At HLF all winners of the Fields Medal, the Abel Prize, the ACM A.M. Turing Award, the Nevanlinna Prize, and the ACM Prize in Computing are invited to attend. In addition, young and talented computer scientists and mathematicians are invited to apply for participation. Applications are open in three categories: Undergraduates, PhD Candidates, and PostDocs. Online application and further information is available at tinyurl.com/y7obwzxb.

Editor's note: see the HLF report on page 17.

## Fellowships for Women in Science

The L'Oréal and UNESCO founded For Women in Science programme recognises the achievements of exceptional female scientists and awards them with fellowships to help further their research. In 2018, five awards of $£ 15,000$ each will be offered to outstanding female post-doctoral researchers in the UK \& Ireland. The sponsors are keen to encourage applications from mathematicians and Professor Gwyneth Stallard (former chair of the LMS Women in Mathematics committee) is on the judging panel. Past winners, including mathematicians, can be seen at tinyurl.com/ycrkawk6. Apply by 16 February 2018 at womeninscience.co.uk.

## CIMPA Programmes

CIMPA is the Centre Internationale de Mathématiques Pures et Appliquées. Its mission is the training of mathematicians mainly from developing countries by means of study visits during the university academic year and summer schools. The seat of CIMPA is at Nice, its host university being the University of Nice Sophia Antipolis. There are currently two open calls for CIMPA programmes:

Support for Training in Research: This programme consists of the organisation of series of researchlevel courses in mathematics within the geographic areas of activities of CIMPA (Africa, Central and South America, Asia). Proposals for the period March 2018 August 2018 must be submitted by 3 January 2018, and those for the period September 2018 - February 2019 by 3 July 2018.

CIMPA-CARMIN: Applications for CIMPA-CARMIN funding are now made on the CIMPA website. These projects consist of trimester programmes to be held at the Institut Henri Poincaré (Paris). For further details of these and other CIMPA activities see www.cimpa.info.

## Fields Institute Director

The Fields Institute for Research in Mathematical Sciences (Toronto) invites applications or nominations for the position of Director for a three- to five-year term (renewable once) beginning 1 July 2018 or as soon as convenient afterwards. Applications or nominations will be considered until the position is filled, but the Search Committee plans to examine dossiers starting 15 January 2018. See fields.utoronto.ca/ for more information.


## The LMS Website's News Section

The Council of the LMS is looking for two editors to help develop and manage the news, events and features appearing on the front page of the Society's website https://www.Ims.ac.uk. These roles are editorial rather than technical. The main responsibility will be in keeping the news items that appear on the the front page of the LMS's website up-to-date, relevant and interesting to its members.

Expressions of interests or informal enquiries can be made to the Newsletter's editor-in-chief via iain.moffatt@rhul.ac.uk.

## Call for Nominations: Ramanujan Prize 2018

The Ramanujan Prize for young mathematicians from developing countries has been awarded annually since 2005. The Prize is now funded by the Department of Science and Technology of the Government of India (DST), and will be administered jointly by the Abdus Salam International Centre for Theoretical Physics (ICTP), the International Mathematical Union (IMU) and the DST.

The prize winner must be less than 45 years of age on 31 December of the year of the award, and have conducted outstanding research in a developing country. Researchers working in any branch of the mathematical sciences are eligible. The Prize carries a $\$ 15,000$ cash award. The winner will be invited to the ICTP to receive the Prize and deliver a lecture. The Selection Committee will take into account not only the scientific quality of the research, but also the background of the candidate and the environment in which the work was carried out. The deadline for receipt of nominations for the 2018 Prize is 1 February 2018. Nominations should be made through the online system: e-ramanujan.ictp.it/nominator.

## LMS Grant Schemes

## Schemes 1-5 (Research Grants Committee)

The following grant schemes are offered by the LMS Research Grants Committee. The deadline for grant applications under Schemes 1-5 is 22 January 2018 (for events and visits intended to be held June September 2018).

## Scheme 1: Conference Grants

'Conference Grant' awards are made to the organisers of conferences to be held in the UK. Priority is given to the support of meetings where an LMS grant can be expected to make a significant contribution to the viability and success of the meeting. Support of larger meetings of high quality is not ruled out, but for such meetings an LMS grant will normally cover only a modest part of the total cost.

## Scheme 2: Visitors to the UK

'Visitors to the UK' awards aim to provide funding to UK-based mathematicians to partially support visitors to the UK; the visitors are expected to give lectures in at least three separate institutions.

## Scheme 4: Research in Pairs

'Research in Pairs' awards aim to provide partial support to UK-based mathematicians to help support visits for collaborative research with mathematicians from within the UK or abroad.

## Scheme 5: International Short Visits

'International Short Visit' awards are intended to provide funding to UK-based mathematicians to support visits for collaborative research, either to or from a country in which mathematics is considered to be in a disadvantaged position. Applicants unsure if the proposed country is eligible under a Scheme 5 grant is welcome to contact the Grants Team for further advice.

## Scheme 7 (Computer Science Committee)

Scheme 7 aims to provide support for visits to undertake collaborative research at the interface of mathematics and computer science. The deadline for applications in the next round of Scheme 7 grants is 15 April 2018.

## Schemes 8 \& 9 (Early Career Research Committee)

The following schemes are offered by the LMS Early Career Research Committee; the deadline for submission of applications is 22 February 2018 (for events intended to be held June - September 2018).

## Scheme 8: Postgraduate Research Conferences

'Postgraduate Research Conference’ awards are made to provide support to postgraduate research conferences - organised by and for postgraduate research students - to be held in the UK.

## Scheme 9: Celebrating New Appointments

'Celebrating New Appointments’ awards are made to provide partial support for meetings held in the UK to celebrate the appointment of a new lecturer in mathematics at a UK institution. The aim of the grant award is to embed the new lecturer in their home institution and the local mathematical community.

For full details of the grant schemes offered by the Society, and for information on how to make an application, please visit Ims.ac.uk/grants or contact the Grants Team — grants@lms.ac.uk.

## Reminders

## LMS Prizes: call for nominations

Details at tinyurl.com/lmsprizes. The deadline for nominations is 26 January 2018.

## Christopher Zeeman Medal 2018: call for nominations

Details at tinyurl.com/zeemanmedal. The deadline for nominations is 28 February 2018.

Louis Bachelier Prize 2018: call for nominations Details at tinyurl.com/bachelier. The deadline for nominations is 31 January 2018.

## Cecil King Travel Scholarship: call for applications

Details at tinyurl.com/cecil2018. The deadline for applications is 31 March 2018.

## LMS URBs 2018: call for applications

See tinyurl.com/undergradbursaries, search "LMS URB" or contact urb@lms.ac.uk for details. The deadline is 16 February 2018.

## LMS Invited Lectures Series 2019: call for proposals

The deadline for proposals is 2 February 2018. For details, visit tinyurl.com/invited2018.

LMS Research Schools 2019: call for proposals Information about the submission of proposals can be found at tinyurl.com/RS2019 along with a list of previously supported Research Schools. Proposals should be submitted to Elizabeth Fisher (research.schools@lms.ac.uk) by 31 January 2018.

## The QJMAM Fund for Applied Mathematics

The Trustees of the Quarterly Journal of Mechanics and Applied Mathematics (QJMAM) are pleased to announce the establishment of a major new fund for the support of UK Applied Mathematics. The journal, created in 1948, is owned by the QJMAM Trust and published by Oxford University Press (OUP). An agreement between the Trustees and OUP means that we anticipate creating a fund that will distribute some tens of thousands of pounds per year. The fund will be administered by the Institute of Mathematics and its Applications, but decisions on the award of grants will be made by a panel appointed by the Trustees. It is expected that there will be three closing dates for applications each year, and it is hoped that the first will be in early 2018.
Applications will be invited under a number of headings, expected to include: Conference and workshop organisation, conference travel, collaborative research visits and academic-industrial collaborations. Priority will be given to applications that clearly enhance the fields of mechanics and applied mathematics and award recipients will be encouraged to report their research findings in QJMAM.
The Trustees also intend to award an annual prize (the QJMAM Prize) for the best paper in QJMAM in the previous calendar year; they will make the award on the basis of recommendations by the Executive Editors of the journal.

The Trustees (contact details below) welcome other suggestions for how the Fund may be used. It is planned to set up an online application procedure shortly. Details related to the Fund and applications to the Fund will be available and updated at https://ima.org.uk/support/grants/qjmam-fund/
The Trustees: John King (john.king@nottingham.ac.uk), Chris Linton (C.M.Linton@lboro.ac.uk), Andrew Norris (norris@rutgers.edu) and Tim Pedley (T.J.Pedley@damtp.cam.ac.uk).


## Professorship in M athematical Physics

 at E cole polytechnique fédérale de L ausanne (EPFL)The Institute of Mathematics of the School of Basic Sciences at the EPFL invites applications for open rank position in M athematical Physics.
W e are especially interested in mathematical areas related to string theory, including (but not restricted to) representation theory or algebraic geometry. This is an open rank search, and appointment is possible either at the tenured level (Full Professor or Associate Professor) or the tenure track level (A ssistant Professor), depending on the successful candidate's track record.

We seek candidates with an outstanding research record and the capacity to direct high quality research. We also expect a strong commitment to excellence in teaching at all levels.

Substantial start-up resources and research infrastructure will be made available. EPFL offers highly competitive salaries at an international level.

A pplications including a cover letter, a curriculum vitae, a list of publications, a concise statement of research and teaching interests, as well as the names and addresses (including email)
of at least 3 references for junior position and 5 for senior position. A pplication files should be submitted in pdf format via the website

## https://facultyrecruiting.epfl.ch/position/7962626

The evaluation process will begin immediately. A pplications submitted prior to February 1st, 2018 will be guaranteed consideration.

Enquiries may be addressed to:

## Prof. Assyr Abdulle

Chairman of the Search Committee
E-mail: math.hiring@epfl.ch
For additional information, please consult www.epfl.ch, sb.epfl.ch

EPFL is an equal opportunity employer and a family friendly university.

## VISITS

## Visit of Tiago Pereira

Dr Pereira will be visiting the Department of Mathematics, Imperial College London from 22 January to 12 February 2017. His research concerns dynamical systems with a focus on transitions to collective dynamics in complex networks. During his visit Dr Pereira will give lectures at:

- Imperial College London, 1 \& 2 February (contact Jeroen Lamb: jsw.lamb@imperial.ac.uk)
- University of Exeter, 5 February (contact Peter Ashwin: P.Ashwin@ex.ac.uk)
- University of Warwick, 7 February (contact Robert McKay: r.s.mackay@warwick.ac.uk)


## Visit of Lassina Dembele

Dr Lassina Dembele will be visiting the University of Sheffield from 1 to 28 of February 2018. Dr Dembele's research focuses on problems around the Langlands programme with a keen interest in computational approaches. During his visit Dr Dembele will deliver lectures at:

- University of Sheffield, 14 February
- University of Bristol, 21 February
- University of Nottingham, 28 February

For further details contact Jeroen Lamb (jsw.lamb@imperial.ac.uk). The visit is supported by an LMS Scheme 2 grant.

For further details contact Haluk Sengun (m.sengun@sheffield.ac.uk). The visit is supported by an LMS Scheme 2 grant.

## Visit of Martin Buhmann

Professor Martin Buhmann ScD will be visiting the UK between 2 and 15 March 2018. His research includes multivariate approximation theory, especially employing radial basis functions. During his visit Professor Buhmann will give the following lectures:

- University of Cambridge, 2 March (contact Carola-Bibiane Schönlieb: cbs31@cam.ac.uk)
- University of Bath, 7 March (public lecture) and 9 March 2018 (research lecture) (contact Chris Budd: C.J.Budd@bath.ac.uk)
- University of Leicester, 14 March (contact Jeremy Levesley: jl@leicester.ac.uk)
For further details contact Carola-Bibiane Schönlieb (cbs31@cam.ac.uk). The visit is supported by an LMS Scheme 2 grant.


## Visit of Dr Nicholas Touikan

Dr Nicholas Touikan (Stevens Institute of Technology, New Jersey, USA) will be visiting the UK between 9 and 18 March 2018. His field of specialty is Geometric Group Theory, with a particular interest in algorithmic problems and equations in discrete groups that have large scale non-positive curvature. During his visit Dr Touikan will give lectures at:

- University of Glasgow, 12 March (contact Alan Logan: Alan.Logan@glasgow.ac.uk)
- Heriot Watt University, 14 March (contact Laura Ciobanu: I.ciobanu@hw.ac.uk)
- University of Bristol, 16 March (contact Mark Hagen: mh17540@bristol.ac.uk)

For further details contact Alan Logan (Alan.Logan@glasgow.ac.uk). The visit is supported by an LMS Scheme 2 grant.

## Visit of Daniil Proskurin

Dr Daniil Proskurin (Taras Shevchenko National University of Kiev, Ukraine) will be visiting the UK between 13 and 26 May 2018. His main research interests are in operator algebras, in particular, $\mathrm{C}^{*}$ algebras and their representations. Details of Dr Proskurin's talks during his visit are:

- Swansea University, Thursday 17 May (contact Eugene Lytvynov: e.lytvynov@swansea.ac.uk)
- University of Sheffield, Wednesday 23 May (contact Vladimir Bavula: v.bavula@sheffield.ac.uk)
- University of York, Thursday 24 May (contact Alexei Daletskii: alex.daletskii@york.ac.uk)

For further details contact Eugene Lytvynov (e.lytvynov@swansea.ac.uk).

The visit is supported by an LMS Scheme 2 grant.

## Membership of the London Mathematical Society

The standing and usefulness of the Society depends upon the support of a strong membership, to provide the resources, expertise and participation in the running of the Society to support its many activities in publishming, grant-giving, conferences, public policy, influencing government, and mathematics education in schools. The Society's Council therefore hopes that all mathematicians on the staff of UK universities and other similar institutions will support mathematical research by joining the Society. It also very much encourages applications from mathematicians of comparable standing who are working or have worked in other occupations.
Benefits of LMS membership include access to the Verblunsky Members' Room, free online subscription to the Society's three main journals and complimentary use of the Society's Library at UCL, amongst other LMS member benefits (Ims.ac.uk/membership/member-benefits).
If current members know of friends or colleagues who would like to join the Society, please do encourage them to complete the online application form (Ims.ac.uk/membership/online-application). We do ask for current LMS members to act as proposers for applications for Ordinary membership and Associate Membership (open to undergraduates, postgraduates and those within three years of completing their PhD). LMS Representatives at UK mathematics departments can also act as proposers and the current list can be found at Ims.ac.uk/membership/lms-representatives.
Contact membership@Ims.ac.uk for advice on becoming an LMS member.

# LMS Council Diary: a Personal View 

At its meeting on 13 October 2017, Council heard updates from several Committees on activity in the months since Council's previous meeting at the end of June. The Education Secretary informed Council that the newly revamped Advisory Committee on Mathematics Education (ACME) was requesting that the Society and the Institute for Mathematics and its Applications jointly fund two of its 'contact groups' to cover the GCSE and A-level education stages, and Council agreed to continue its annual contribution to support the activity of ACME until 2020. Our Librarian June Barrow-Green presented a proposal regarding a potential project to digitise The Educational Times and Mathematical Questions with their Solutions from the Educational Times, and Council will consider this further. The Publications Secretary presented a new statement of the highlevel aims of the Society's publications for Council's consideration, and also informed Council that the Publications Committee was exploring possibilities for including computational input in the Society's main journals.

Vice-President Brown then provided an update on the progress of the data collection exercise to undertake a census of all UK postdoctoral fellowships and research assistantships. Data available from websites had now been gathered, and this data would be sent to Heads of Departments for checking. VicePresident Greenlees presented a proposal from the Women in Mathematics Committee for the Society to undertake an updated Benchmarking Survey, further to the survey undertaken in 2012. The purpose of the proposed survey is to benchmark progress, to gather in examples of effective good practice, and to provide some third party analysis of the Athena SWAN scheme. He noted the importance of the Society's leadership and influence in the area of women in mathematics, largely due to activities such as this. Council agreed to proceed with this project. Council's meeting on 10 November 2017 was the final one of the year, and tends to be shorter than usual as it is followed by the Society's Annual General Meeting, reception, and dinner. At the start of the meeting, the President gave an update on his various activities, including ongoing discussions with the Royal Society about possible uses of Chicheley Hall as a national centre for science which would include mathematics. Council also accepted a Statement on Principles of

Diversity at Conferences from the Women in Mathematics Committee that aims to provide practical suggestions for achieving diversity in mathematical activities.

Vice-President Brown gave an update from the Research Policy Committee, noting that VicePresident Greenlees would henceforth be taking up this role. Among other items, he noted that Professor Graham Niblo would be serving as the new chair of the Engineering and Physical Sciences Research Council's Mathematical Sciences Strategic Advisory Team. Vice-President Brown also pointed out that the Council for Mathematical Sciences would be nominating Subject Panel Members for the next Research Excellence Framework, and Heads of Department would be asked for suggestions.

When the Council meeting adjourned, members then proceeded to the British Medical Association for the Society's Annual General Meeting. Members heard presentations by Vice-President Brown describing the Society's ongoing activities in support of mathematics, and by Treasurer Rob Curtis highlighting aspects of the Society's annual budget report. The agenda also included the awarding of this year's Society prizes and Simon Tavaré's Presidential Address.

This year's AGM marked the end of several Council and Committee members' service to the Society, and all were thanked for their service. In particular, Simon Tavaré handed the Presidency to Caroline Series, while outgoing Education Secretary Alice Rogers stepped down after five years in that role, having on a previous occasion served two years as VicePresident, and Ken Brown had completed an eightyear tenure as Vice-President. The Society wishes them well, and your Diarist would like to extend her particular thanks to all for the great pleasure and privilege of working with them. At the same time, on behalf of all Council members, your Diarist would like to extend congratulations to newly-elected officers: President Series, Education Secretary Kevin Houston, and Vice-President Cathy Hobbs, who has just completed five years of service on Council as a Member-At-Large. We look forward to working with them and with all new Council and Committee members, and to the upcoming new year full of activity.

Tara Brendle

## LMS Longstanding Members

The following is a list of members who have completed 50 years or more of membership of the London Mathematical Society.

75 years in 2018:
Freeman J. Dyson.
71-74 years in 2018:
Eric L. Huppert, Walter K. Hayman.

70 years in 2018:
Godfrey L. Isaacs, Bernard Fishel.

## 66-69 years in 2018:

David Borwein, H. Peter F. Swinnerton-Dyer.

61-65 years in 2018:
John C. Amson, J. Vernon Armitage, Michael Atiyah, John F. Bowers, Aldric L. Brown, Ronald Brown, Daniel E. Cohen, David E. Edmunds, David A. Edwards, Hanafi K. Farahat, Ronald Harrop, loan M. James, Lionel W. Longdon, John M. Marstrand, Ismail J. Mohamed, David Monk, Brian H. Murdoch, Michael F. Newman, Roger Penrose, Roy L. Perry, Francis Rayner, Margaret E. Rayner, John R. Ringrose, Dennis C. Russell, Paul A. Samet, S. James Taylor, G.E. Wall, John E. Wallington.

60 years in 2018:
Bryan Birch, Gearoid De Barra, Dorothy M.E. Foster, Donald Keedwell, I.G. Macdonald.

## 56-59 years in 2018:

Patrick D. Barry, Benjamin Baumslag, Norman Blackburn, David A. Burgess, Lilian G. Button, Roger W. Carter, John H.E. Cohn, Hallard T. Croft, Roy O. Davies, lan M.S. Dey, Vlastimil Dlab, Alan J. Douglas, James O.C. Ezeilo, Matthew P. Gaffney, Richard K. Guy, Desmond J. Harris, Howard M. Hoare, Roland F. Hoskins, Glenys Ingram, John F.C. Kingman, Joseph F. Manogue, Alun O. Morris, Albert A Mullin, Alan R. Pears, John E. Peters, Frank Rhodes, Joseph B. Roberts, Stewart A. Robertson, John W. Rutter, Arthur
D. Sands, Eira J. Scourfield, Abe Sklar, Dona Strauss, Anthony C. Thompson, Ronald F. Turner-Smith, Terence C. Wall, Eric W. Wallace, Alan West, Sheila O. Williams.

## 51-55 Years in 2018:

J. Clifford Ault, Alan Baker, John C.R. Batty, Alan F Beardon, Homer Bechtell, Simon J. Bernau, Thomas S. Blyth, M.C. Bramwell, William Brown, Roger M. Bryant, Allan G.R. Calder, Munibur R. Chowdhury, R.F. Churchhoue, Michael J. Collins, Bruce D. Craven, Charles W. Curtis, P. Laurie Davies, M.A.H. Dempster, M.M. Dodson, Patrick Dolan, J. Keith Dugdale, Martin J. Dunwoody, Peter L. Duren, Roger H. Dye, L.C. Eggan, Barry G. Eke, K. David Elworthy, David Epstein, John Erdos, Edward A. Evans, W. Desmond Evans, Roger A. Fenn, James W.M. Ford, Cyril F. Gardiner, David J.H. Garling, Peter Giblin, Robin E. Harte, William J. Harvey, Philip Heywood, Keith E. Hirst, Terence H. Jackson, Otto H. Kegel, J. David Knowles, E. Christopher Lance, David G. Larman, Ronald Ledgard, W.B. Raymond Lickorish, Peter G. Lowe, Malcolm T. McGregor, John McKay, Ian M. Michael, Hugh Morton, Robert Moss, Roy Nelson, Peter M. Neumann, Frederick C. Piper, Oliver Pretzel, John S. Pym, John B. Reade, George A. Reid, John F. Rennison, Derek J.S. Robinson, W. John Robinson, H. Peter Rogosinski, James Edward Roseblade, Colin P. Rourke, Keith Rowlands, Stephan M. Rudolfer, Rodney Y. Sharp, Bruce L.R. Shawyer, Ernst H. Sondheimer, Ivar Stakgold, Brian F. Steer, Nelson M. Stephens, Bill Stephenson, W. Brian Stewart, Anthony E. Stratton, Wilson A. Sutherland, David Tall, Graham F. Vincent-Smith, Rabe R. von Randow, Grant Walker, Martin Antony Walke, John F. Watters, Bertram A.F. Wehrfritz, Alfred Weinmann, David J. White, Thomas A. Whitelaw, Joyce E. Whittington, Christopher M. Williams, Geoffrey V. Wood.

## 50 years in 2018:

Irene A. Ault, Anthony D. Barnard, Sheila Carter, Donald J. Collins, Colin R. Fletcher, Charles Goldie, Wilfrid A. Hodges, Graham J.O. Jameson, Michael E. Keating, Thomas J. Laffey, Earl E. Lazerson, David W. Lewis, Bernard L. Luffman, Bob Margolis, Peter McMullen, William Moran, Kung-Fu Ng, David R. Page, Philip Samuels, David Singerman, Brian Thorpe, David Tipple, Douglas R. Woodall.

## Retiring Members of Council

## SIMON TAVARÉ

President 2015-2017
After serving as LMS President for two years, Professor Simon Tavaré, FRS, FMedSci handed over the badge of office at the AGM on 10 November 2017.

Professor Tavaré took over the office of LMS President towards the end of the Society's 150th Anniversary celebrations, which showcased the importance of mathematics and mathematicians to society and the economy. Professor Tavare saw this as a vital opportunity for the LMS to continue its efforts in building a sustainable future for mathematics in the UK. Professor Tavaré oversaw a series of significant developments, both in the Society's governance structure and in its wider activities.

During his tenure the Society established two new committees: the Early Career Research Committee and the Society Lectures and Meetings Committee. The Society also restructured its Publications area to develop a robust and forward looking strategy, with its main partners John Wiley \& Sons and Cambridge University Press. It also developed the Directory of Mathematical Scientists in the UK, which it hopes will provide an invaluable resource for Mathematical Scientists within academia and industry, and across all the Mathematical Sciences, providing a network of contacts to encourage collaboration and enhance connectivity. Another major project in 2016-17 was the redesign of the LMS Newsletter. The first issue was published in September 2017, with a more modern design and exciting new mathematical features, articles and content.

Professor Tavaré has been an effective ambassador for the Society at a range of meetings in the UK and overseas, including Society Meetings and Joint Meetings with the Royal Statistical Society, the Institute of Mathematics and its Applications and the ICMS. He also led the delegation invited to the 7ECM in Berlin to hold talks with the German Mathematical Society.

Professor Tavaré chaired the continued LMS Spring Reception series, generously hosted by Dr Richard Golding, at which Sir John Kingman, Chair of newly formed UK Research and Innovation (UKRI) and son of the Society's 65th LMS President, gave an update on the development of UKRI, and Professor Philip

Bond, a member of the Prime Minister's Council for Science and Technology, outlined plans for the review of knowledge exchange in the mathematical sciences, emphasising the many ways the mathematical sciences contribute to the nation. The reception is a crucial networking event for those in the Mathematical Sciences to meet with policy makers and those from industry.

Professor Tavaré is recognised as an excellent communicator and he has attended meetings to help promote the Mathematical Sciences, including mathscon, as part of a panel including journalist Alex Bellos, to discuss How to make more people love mathematics.

He has continued the momentum built throughout 2015 and has presided over significant changes that will enhance the Society's standing in the mathematical sciences and wider STEM communities. The Society would like to thank Professor Tavaré for his dedicated service and wish him well for the future. At the AGM, Professor Tavaré handed over the badge of Presidential Office to Professor Caroline Series, FRS.

## KEN BROWN

Vice-President 2009-2017
After eight years Professor Ken Brown, Professor of Mathematics, University of Glasgow has retired as LMS Vice-President.

Professor Brown has been an influential part of the Society's activities for a number of years, with his experience and invaluable counsel on many issues across research policy. Professor Brown has been instrumental in providing input to consultations and his work for the LMS and the Council for the Mathematical Sciences (CMS), particularly with EPSRC, has helped highlight the important issues that affect the Mathematical Sciences people pipeline, primarily with funding for PhD students, Doctoral Training Programmes and Balancing Capability. In particular he ensured that evidence was gathered to support any case put forward - e.g., Centres for Doctoral Training. He also strongly supported the development of the Mathematical Sciences Directory and was instrumental in bringing the project to fruition.

Professor Brown has given many years of service to the LMS dating back to 1992. He was a member of LMS Council from 1992-2001 and Vice-President from 1997 to 1999 and from 2009 to 2017; Chair of Research Policy Committee 2010-2017; Chair of Personnel Committee 1999-2001 and 2009-2017; Editorial Adviser 2002-06.

He has also served on a wide range of other Committees and groups as a Member of the RAE Pure Maths Subpanel 1996 and 2001, Vice-Chair 2001, Chair of Pure Maths Subpanel 2008; Member REF Expert Advisory Group 2008-09; Member EPSRC College since 1995; Member National Advisory Board of the Isaac Newton Institute (INI) 1998-2002; Member Scientific Committee of the International Centre for Mathematical Sciences (ICMS) (Edinburgh) 2006-2015; Member EPSRC Math Sciences Strategic Advisory Team (SAT) 2013-2015 and Chair 2015-2017.

Professor Brown's wide experience has benefited the Society and the mathematics community for many years and the LMS is extremely grateful to him for the support he has given to the Society and the wider Mathematical Sciences community.

## F. ALICE ROGERS <br> Education Secretary 2012-2017

After five years as Education Secretary Professor Alice Rogers has retired as LMS Education Secretary.

Professor Rogers is particularly well respected across the mathematics community for her education expertise and has been instrumental in shaping the national mathematics education landscape over a number of years.

Professor Rogers' involvement with the Society has spanned the past 15 years where she has provided support across a range of committee activities. This includes as a member of LMS Council from 2002 to 2009 and then from 2012 to 2017, being VicePresident from 2005 to 2009 and Education Secretary from 2012 to 2017; member of Personnel Commit-
tee 2007-2017 and Chair from 2002-2009; member of Education Committee 2005-2011 and Chair from 2012 to 2017; Mathematics Promotion Unit Steering Group Chair 2006-2009, member of the Women in Mathematics Committee 2000-2002 and Chair from 2002-2005.

Her expertise has been evident across a wide range of activities, in particular in education where she has guided the Society as Education Secretary with input into a vast array of consultations, including the national Curriculum, A-level and GCSE reform and representing the Society on external committees including as a Member of the Advisory Committee on Mathematics Education (ACME) from 2007-2011 (Deputy Chair from 2009) and interaction with the government Department for Education. She has been instrumental in tackling the issues facing women in mathematics, both as a member and as Chair of the Women in Mathematics Committee.

Professor Rogers' input to activities and events has helped to shape the Society's future direction and its influence in the wider mathematical community and the LMS thanks her for her many years of support.

## SAM HOWISON and DIANE MACLAGAN Members-at-Large

Professor Sam Howison and Dr Diane Maclagan stepped down as Members-at-Large of Council at the 2017 AGM. Both were elected to Council in 2015 and have made noticeable contributions to the business of Council and to the wider activities of the LMS. Professor Howison will remain an active member of the LMS Research Policy Committee. Dr Maclagan was a member of the LMS Programme Committee and continues to contribute to the Society's work as a member of the LMS Women in Mathematics Committee, and the newly formed Early Career Researcher Committee. The LMS thanks them for the broad and varied support that they have given to the Society in achieving its charitable aims and supporting the wider mathematical community.

## CONFERENCE FACILITIES <br> De Morgan House offers a $40 \%$ discount on room hire to all mathematical charities and $20 \%$ to all not-for-profit organisations. Call 02079270800 or email roombookings@ demorganhouse.co.uk to check availability, receive a quote or

 arrange a visit to our venue.

## REPORTS OF THE LMS

## Report: LMS-IMA Joint Meeting on Symmetry and Computation

This pioneering joint LMS-IMA event was introduced by Elizabeth Mansfield, Vice-President of the IMA, followed by the LMS President Simon Tavaré welcoming all to De Morgan House and hoping that this may be the first of many such meetings. The interests of the audience and of the speakers were wide-ranging, there being no indication of who were members of either society (or both, or none) and there was an air of anticipation to see what the day would bring.

The first speaker Gloria Marí Beffa (U WisconsinMadison) introduced her talk on Discrete geometry of polygons and soliton equations with discussion of vortex filament flow illustrated by captivating video footage of dolphins creating and playing with a very stable vortex ring plus a lab movie of a head-on collision of two vortex rings. The next talk by Kurusch Ebrahimi-Fard (Trondheim) dealt with algebraic structures on rooted trees, his title A fresh look at the Magnus expansion coming from the technique developed by Wilhelm Magnus in 1954 for expanding the solution $Y(t)$ to the linear differential equation $Y^{\prime}(t)=$ $A(t) Y(t)$ in terms involving integrals and Lie brackets of increasing complexity. After lunch Evelyne Hubert (INRIA Méditerranée) talked on Invariants of ternary forms under the orthogonal group and described a motivation arising from the study of diffusion along connecting fibres in the brain. Darryl Holm (Imperial College) next talked about Stochastic transport in fluid dynamics, beginning (as he advises students always to do) with motivation - in this case weather prediction. Peter Neumann (Oxford) rounded off the day with An introduction to computational group theory outlining what computing had been able to do, was doing, and would be likely to do for group theory.

This and other talks were given with a nice degree of gentle humour, and for your correspondents at least the day felt very well spent. Professor Caroline Series, incoming LMS President, thanked the meeting organisers and reiterated the hope that there would be future joint meetings of the IMA and the LMS.

Colin Campbell and David Chillingworth

## Report: Heidelberg Laureate Forum 2017



John Hopcroft talking about deep learning
The Heidelberg Laureate Forum (HLF) brings together laureates of the main awards in mathematics and computer science (the Abel Prize, the Fields Medal, the Turing Award and the Nevanlinna Prize) and selected young researchers from all over the world. The selection process aims at attracting the best young researchers in their fields, with a strong commitment towards gender, ethnic and cultural diversity.

The scientific programme of the HLF includes plenary talks by the laureates, a panel discussion (the topic of this year was quantum computing), student poster presentations and mini-workshops. The HLF week is structured in such a way to foster interaction between participants, and was carefully planned to make sure that young researchers were always in contact with laureates. During the social events, one could listen to Stephen Smale's thoughts on the time gap between the establishment of Poincaré's conjecture in dimensions greater than 5, 4 and 3; or share Martin Hellman's worries about an eminent nuclear war; or ask Alexei Effros why Paris looks like Paris according to machine learning; or help Whitfield 'Whit' Diffie conduct experiments on how many cents are necessary to equilibrate a helium balloon (the answer is different in euros, pounds and dollars, and pouring water is a more promising approach).

Some clear hot topics popped out of the talks. Quantum computing was the "official" hot topic. Jay Gambetta (IBM) presented a python package with which
one can really operate a 16-qubit quantum computer! In his talk, Seth Lloyd (MIT) asked what quantum computing can do to data science and machine learning. Despite all exciting recent advances, there are still great challenges for quantum technologies. The more crucial one is arguably quantum error-correction, or the ability to cope with "noise" in quantum systems. This is an inherently interdisciplinary research topic picturing connections to various branches of mathematics, from topology to functional analysis. Another frequently discussed topic was deep learning (well represented by John Hopcroft, Jeff Dean and Alexei Effros), which has unleashed a plethora of recent applications with disruptive impact in society. Yet, in the words of John Hopcroft, "although many people are successful in applications, very few understand what is going on". A complete understanding of the mathematical machinery behind deep learning is
still a genuine research direction of great theoretical interest.

The HLF provided the perfect environment to mingle with supernaturally smart researchers, to establish partners for future collaborations and to make friends who are genuinely interested in tackling the next big research challenges.

I would like to end this note by thanking the London Mathematical Society for awarding me a travel grant that enabled my attendance in the forum, and encourage all young researchers (postdocs and PhD students) in mathematics and computer science to apply for the next editions of the HLF. (Editor's note: see the HLF call on page 8.)

Antonio Campello Imperial College London

## Records of Proceedings at LMS meetings <br> Ordinary Meeting, 12 October 2017

The meeting was held at De Morgan House in London as a joint meeting with the Institute of Mathematics and its Applications (IMA). Over 45 members and visitors were present for all or part of the meeting.
The meeting began at 11.00 am with the President, Professor Simon Tavaré FRS, FMedSci, in the Chair.
No members were elected to Membership.
One member signed the book and was admitted to the Society.
Professor Tavaré handed over to the President of the IMA, Professor Chris Linton, for the welcome from the IMA.

Professor Linton then handed over to Professor Elizabeth Mansfield who introduced the first lecture given by Professor Gloria Marí Beffa (U. Wisconsin-Madison) on Discrete Geometry of Polygons and Soliton Equations.

Professor Mansfield introduced the second lecture given by Professor Kurusch Ebrahimi-Fard (Trondheim) on A Fresh Look at the Magnus Expansion. After lunch, Professor Arieh Iserles introduced the third lecture by Professor Evelyne Hubert (INRIA Méditerranée) on Invariants of Ternary Forms Under the Orthogonal Group.

Professor Iserles then introduced the fourth lecture given by Professor Darryl D. Holm (Imperial College, London) on Stochastic Transport in Fluid Dynamics. After tea, Professor Peter Clarkson introduced the final lecture, which was given by Dr Peter M. Neumann, OBE (Oxford) on An Introduction to Computational Group Theory.

At the end of the meeting, Professor Mansfield thanked the speakers on behalf of the IMA. Professor Mansfield then handed over to the LMS President Designate, Professor Caroline Series, FRS, who thanked the speakers on behalf of the LMS and closed the meeting.

A reception was held at De Morgan House. A Joint Society Dinner was hosted by the IMA and the LMS at Antalya Restaurant.

## Records of Proceedings at LMS meetings Annual General Meeting and Society Meeting, Friday 10 November 2017

The meeting was held at the British Medical Association House, London. About 110 members and visitors were present for all or part of the meeting. The meeting began at $3: 00 \mathrm{pm}$, with the President, Professor Simon Tavaré, FRS FMedSci, in the Chair. Members who had not yet voted were invited to hand their ballot papers to the Scrutineers, Professors Chris Lance and Rodney Sharp.

The Vice-President, Professor Ken Brown, presented a report on the Society's activities and the President invited questions.
The Treasurer, Professor Rob Curtis, presented his report on the Society's finances during the 2016-17 financial year and the President invited questions. Copies of the Trustees Report for 2016-17 were made available and the President invited members to adopt the Trustees Report for 2016-17 by a show of hands. The Trustees Report for 2016-17 was adopted.
The President proposed Messrs Kingston Smith be re-appointed as auditors for 2017-18 and invited members to approve the re-appointment by a show of hands. Messrs Kingston Smith were re-appointed as auditors for 2017-18.

Forty-three people were elected to Ordinary Membership: Dr Chinnaraji Annamalai, Dr Irina Biktasheva, Dr Mark Bloomfield, Professor Ghassan Chammas, Professor Alessio Corti, Professor Mihalis Dafermos, Dr Christopher Daw, Mr Matthew Dieruf, Dr Sander Dommers, Dr Isobel Falconer, Professor Alistair Fitt, Dr David Fletcher, Dr lan Flood, Dr Christopher Frei, Mr Lawrence Habahbeh, Mr David Hallakoun, Professor Deirdre Hollingsworth, Dr Thomas Hudson, Mr Gautam Kakar, Dr Derek Kitson, Dr Deepak Laxmi Narsimha, Dr Omar Leon Sanchez, Dr Marina Logares, Mr Fouad Mardini, Mr Michael Murray, Dr Nicola Pagani, Dr Florian Pausinger, Mr Junyang Peng, Dr Karl-Mikael Perfekt, Professor Malcolm Perry, Mr Thomas Roper, Mr Charles Sagayaraj A C, Mrs Tatyana Shipulina, Dr Steven Sivek, Dr Greg Stevenson, Dr Hala Taha, Mr Abdou Ben Ali Tcheikh Said, Dr Vijayantee Teeluck, Dr Alan Thompson, Mr Nuhu Tsaku, Dr Angharad Ugonna, Professor Prudence Wong, Dr Argyrios Zolotas. Sixty-four people were elected to Associate Membership: Mr James Aaronson, Mrs Jehan Al-Ameri, Mrs Stamatina Alexandropoulou, Mrs Peter Banks, Dr Stuart Barton, Miss Luciana Basualdo Bonatto, Mr Diego Berdeja Suarez, Mr Simon Bergant, Mr Isaudin Bin Ismail, Miss Candida Bowtell, Mr Lucas Branco, Mr Asad Chaudhary, Mr Sebastjan Cizel, Miss Rhianwen Davies, Dr Taysir Emhemed Dyhoum, Mr Allen Fang, Mr Cameron Foreman, Mr Guy Fowler, Dr Stefan Frei, Mr Alexander Gietelink Oldenziel, Ms Carla Groenland, Mr Jacob Gross, Mr Andre Guerra, Ms Johan Henriksson, Dr Alexandra Hogan, Mr Patrick Hough, Mr Yucong Huang, Mr Marc Isern, Mr Aashraya Jha, Mr M Syafiq Johar, Mr Tom Johnston, Mr Carlisle King, Mr Lukas Koch, Mr James Kohout, Mr Rohon Kundu, Mr Seungjai Lee, Mr Todd Liebenschutz-Jones, Mr Victor Lisinski, Mx Barbara Mahler, Dr Celine Maistret, Mr Matei Mandache, Mr David Marchant, Dr Elena Marensi, Ms Vlad Margarint, Mr Michael McAuley, Miss Arzoo Mustafi, Mr Omefe Omavuezi, Mr Adilet Otemissov, Mr Andrea Petracci, Mr Nils Rochowicz, Ms Emily Roff, Mr Matthew Schrecker, Ms Alice Schwarze, Mr Sam Shepherd, Mr Andreas Sojmark, Mr Jan Steinebrunner, Miss Neslihan Suzen, Dr Roberto Svaldi, Mr Matija Tapuskovic, Mr Michael Taylor, Mr Christopher Turner, Mr Graham Van Goffrier, Mr Andrei Velicu, Mr Oliver Vipond, Mr Oliver Whitehead, Dr Billy Woods, Mr Ka Man Yim, Mr Thomas Zielinski.

Three people were elected to Reciprocity Membership: Mr Mamadou Alouma Diallo, Mr Viacheslav Ivanov, Mr Eric Linn.

One hundred and eighty-three people were elected to Associate Membership for Teacher Training Scholars: Miss Kerry Ackerman, Mr Christian Agbodza, Mr Syed Akhtar, Miss Breerah Alam, Miss Elena Antoniou, Miss Jordanne Armstrong, Mr James Ashmead, Miss Kirsty Atkinson, Mr Matthew Atkinson, Miss Della Avery, Miss Murryum Azeem, Mr Henry Baggs, Miss Charlotte Barker, Miss Emily Barker, Miss Amy Bennett, Mr Joe Berwick, Mr Matthew Brackstone, Miss Chloe Elizabeth Broad, Miss Georgina Brown, Dr David Brown, Mr Christian Brown, Miss Katarina Buntic, Miss Freya Bushnell, Mr Andrew Bussell,

Miss Jessica Cawdron, Miss Eva Cheng, Ms Joy Moi Yan Cheung, Mr Mufeed Choudhury, Miss Sophie Churchard, Miss Holly Clark, Mr Joe Clarke, Miss Lucy Cooke, Miss Helen Coombes, Mr Brandon Cooper, Mrs Joanne Cooper, Miss Alannah Cowley, Miss Caitlain Cox, Mr Elliot Crouch, Miss Amy Dai, Mr James Davies, Mr Rhys Davies, Miss Bruna de Almeida Araujo, Mr Andrew Dickson, Miss Nicole du Preez, Mr Francis Edwards, Miss Ebony Edwards, Mr Daniel Eggleton, Dr Claire Ellison, Mr Armin Farangi, Mrs Lauren Finch, Mr Joshua Forster, Mrs Maria Foster, Mr Owen Garrity, Mr Tom Gatens, Mr Dave Gee, Miss Holly Gibbons, Mrs Fiona Glavin, Mr Matthew Gooch, Mr Amar Gorania, Mr Scott Gregory, Miss Laura Groves, Mr Botond Hajdara, Mr Liam Hallam, Miss Demi Hatahet, Miss Victoria Hawksworth, Miss Alice Hebditch, Mrs Kim Helme, Miss Bethany Henderson, Miss Laura Hendley, Ms Lauren Hennessy, Mr John Hewetson, Miss Chloe Hill, Miss Gemma Hill, Miss Emma Hird, Miss Emily Hodgson, Miss Katherine Howells, Mr Dean Hubbard, Mrs Tania Hudson (nee Fitzgerald), Mr Kashaf Hussain, Ms Maria Monica Hy, Miss Juvayriyah Ikram, Miss Kimberley Irving, Ms Wahida Jabarzai, Miss Haneen Jaidy, Miss Pinar Jandauria, Mrs Zoe Jayhanie, Mr Tony Johnson, Mr William Johnson-Vaughan, Mr Evan Jones, Miss Danielle Kay, Dr Camilla Kerr, Mr Zain Kiani, Miss Olivia King, Miss Zara Knappy, Mr Martin Knight, Mr Scott Knowles, Mrs Adi Kremnizer, Mrs Sally Kurpierz, Mr Younous Laaouini, Ms Clare Lake, Mr William Lamb, Miss Katy Langley, Mr Zbynek Loebl, Mr Hamada Mahdi, Miss Nichola Makepeace, Miss Ines Makonga, Miss Elizabeth Marsden, Mrs Chinyere Mbanefo, Ms Kirsten McGarrie, Miss Francesca Meakin, Mr Ben Mercer, Mr Alexander Merrills, Mrs Aveline Joan Meyn, Miss Martha Minall, Mr Mohammed Imran Mir, Miss Samantha Mortimer, Miss Emma Moulton, Dr Azadeh Neman, Miss Kate O'Donnell, Mr Tiernach O'Reilly, Ms Alexandra Paivana, Mrs Victoria Pang, Miss Hye Yun Park, Miss Cordellia Parker, Mrs Nita Patne, Miss Emma Patterson, Ms Sarah Pearce, Mr Malcolm Pearce, Mr Thomas Percy, Miss Isabelle Perrin, Mr Benjamin Pethybridge, Miss Lydia Philpott, Mr Aidan Pittman, Mrs Emma Playfair, Miss Megan Plowman, Mr Frederick Priestley, Mr Muhammad Aminur Rahman, Ms Anitha Rajkumar, Mr Mohammed Rashid, Miss Amelia Ratsma, Mrs Claire Redmond, Miss Paige Retalic, Mr Joseph Ridge, Mr Richard Robbins, Mr Ben Robbins, Mr Maxwell Robertson, Miss Jade Sadler, Miss Christina Sanderson, Miss Mariam Sattar, Mr Luke Savin, Miss Rebecca Schorah, Mr Stephen Shackleton, Mr Kirtan Shah, Mr Moshin Raza Shah, Mrs Akhi Sikder, Mr Mitchell Silverthorne, Mr Sean Sims, Mrs Suja Sivadass, Mrs Aniko Antonietta Somi, Mr Graeme Strang, Mr Daniel Summers, Miss Hayley Swinyard, Miss Shona Tate, Mr Lee Thompson, Mr Matthew Timpson, Dr Matthew Toogood, Miss Claire Tranter, Mr Alister Trendell, Dr Georgios Tzovlas, Mrs Richa Vaid, Mrs Barbara Vassalluzzo, Miss Rebecca Walker, Mrs Tatiana Wanietikina, Miss Sameera Warsame, Mrs Anna Welfield, Mrs Gara Whittaker, Miss Jessica Wilcox, Mr Stephen Williams, Miss Gabrielle Williams, Miss Erica Williams, Miss Candi Sze Ching Wong, Miss Carmen Wood, Mr Tawer Zadok.

Eight members signed the book and were admitted to the Society.
The President announced that the next meeting of the Society would be at Cardiff on 13 December as part of the South West \& South Wales Regional Meeting on Algebraic Structures and Quantum Physics. The following meeting would be at the Joint Mathematics Meeting in San Diego on 10 January 2018.

The President, on Council's behalf, presented certificates to the 2017 Society Prize-winners:
Pólya Prize: Professor Alex Wilkie, FRS (University of Oxford)
Senior Whitehead Prize: Professor Peter Cameron (University of St. Andrews)
Senior Anne Bennett Prize: Professor Alison Etheridge, FRS (University of Oxford)
Naylor Prize \& Lectureship in Applied Mathematics: Professor John Robert King (University of Nottingham) Whitehead Prizes: Professor Julia Gog (University of Cambridge), Dr András Máthé (University of Warwick), Dr Ashley Montanaro (University of Bristol), Dr Oscar Randal-Williams (University of Cambridge), Dr Jack Thorne (University of Cambridge), Professor Michael Wemyss (University of Glasgow)
The winner of the Berwick Prize, Dr Kevin Costello (Perimeter Institute, Canada), was unable to attend to collect his prize.

The President also announced that the Society and the Institute of Mathematics and its Applications (IMA) had jointly awarded the David Crighton Medal to Professor I. David Abrahams. The David Crighton Medal would be presented to David Abrahams at a joint ceremony on 15 March 2018 at The Royal Society.

Professor Zoubin Ghahramani, Professor of Information Engineering at the University of Cambridge and Chief Scientist at Uber, gave a lecture on Bayesian Statistics, Non-Parametrics, Neural Networks, and Artificial Intelligence.

After tea, Professor Sharp announced the results of the ballot. The following Officers and Members of the Council were elected.

President: Caroline Series; Vice-Presidents: Cathy Hobbs, John Greenlees; Treasurer: Robert Curtis; General Secretary: Stephen Huggett; Publications Secretary: John Hunton; Programme Secretary: lain A. Stewart; Education Secretary: Kevin Houston; Members-at-Large of Council (for 2 year terms): Mark A.J. Chaplain, Andrew Dancer, Tony Gardiner, Brita Nucinkis, Gwyneth Stallard and Alina Vdovina; Member-at-Large (Librarian): June Barrow-Green.

Five Members-at-Large who were elected for two years in 2016 have a year left to serve: Alexandre Borovik, Tara Brendle, Francis Clarke, David E. Evans and Sarah Zerbes.

The following were elected to the Nominating Committee: H. Dugald Macpherson and Martin Mathieu. The continuing members of the Nominating Committee are: John Toland (Chair), Marta Mazzocco, Roger Heath-Brown, Sarah Rees and Ulrike Tillmann.

Professor Simon Tavaré handed over the Presidential badge of office to Professor Caroline Series, FRS. The new President thanked members for the honour and privilege of being elected as President and promised to fulfill the Charter, Statutes and By-laws of the Society.

The newly-elected President, Professor Caroline Series, took the Chair.
Professor Simon Tavaré, FRS, FMedSci gave the Presidential Address on The Magical Ewens Sampling Formula.

After the meeting, a reception was held at De Morgan House, followed by the Annual Dinner, which was held at the Montague Hotel and attended by 100 people.

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# Tilting the Classroom 

LARA ALCOCK


#### Abstract

This article describes and illustrates 12 simple ways to make large mathematics lectures more engaging. These include a variety of short-and-snappy activities, framed by organisational practices that support concentration and maintain a positive atmosphere. These practices can be implemented individually or in combination, with no need for a wholesale classroom restructure.


## Introduction

I borrowed this article's title from Calvin Smith, who told me that his classroom, while not flipped, is tilted. This perfectly captures my own approach to lecturing. My lectures are in a sense traditional: students sit in rows, listen to me, and take notes. But they also engage with a variety of conceptual reasoning tasks. I do not claim that this approach is perfect, and I do not intend to be prescriptive - I have opinions, based on research in undergraduate mathematics education, but I think that good teaching is partly about authenticity and there is no single way to do it right.

What I do think important is that lecturers are free to try out new ideas on a small scale and without pressure for radical innovation. Radical innovation is currently fashionable: teaching development schemes often require it, and lecturers are encouraged to flip their classrooms, experiment with new technologies, and so on. But I find this troubling. I am all for trying new things, but innovative teaching is timeconsuming and can easily fail. Radical changes are risky by nature, and traditional teaching can excel.

With that in mind, this article describes 12 practices that I use in lectures, each of which requires minimal effort to implement. I have applied these practices most recently in a real analysis course for 200 firstand second-year students. Like any such course, this is difficult. Its fundamental definitions are logically complex - no-one deals with triply quantified statements in everyday life or in earlier mathematics and it is completely different from procedure-based learning. I can't work miracles, and I do not know how to make it intelligible to every student. But I can help many to engage with the complex ideas and to recognise their own development. In this article, I frame the twelve practices with three background principles and some thoughts about influencing students toward effective study habits.

## Principles

The first principle is that there is no point in the lecturer covering the material if the students don't. That is simplistic, of course: I teach to the curriculum, and I only partially control what is learned - students need to work after class on the more difficult ideas. But I also offer numerous opportunities to engage and re-engage during lectures.

The second principle is that students are not inherently lazy or bad people. This can be hard to remember - I certainly have moments at which it is not uppermost in my thoughts. But my experience is that the vast majority of students, the vast majority of the time, have good intentions. They also have moments of weakness, and they respond poorly to sensations of failure. But that is not because they are students, it is because they are people.

The third principle is that learning results from student activity more than from lecturer activity. As I gain experience, I think less about what I will say, and more about what students will do both in lectures and in independent study.

## Practices: Organisation

I want all of the students' intellectual energy available for mathematics. And I want all of their emotional energy available for maintaining resilience in the face of struggle. So the first four practices are about setting up the environment so that everyone feels secure and can invest their energies wisely.

## 1. Announcements

In the changeover before each lecture, I put handwritten announcements on the visualiser. These announcements say boring things like this.

Good morning.
Please pick up a set of notes.
Turn to page 54. What is your answer to this morning's question?

This helps students to feel confident about practicalities, which is particularly important for first years. It helps them to help one another - the half who read the announcements can answer questions when the other half ask. Consequently, it dramatically cuts the amount of time I spend repeating myself.

## 2. Break

Around middle of each lecture, I use a natural break in the content to give a two-minute breather. The time is a bit different each day because I don't want anyone clock-watching. And I don't care what students do in the break. Some use it to review what we have just covered, others get out their phones. I think the only question to consider about breaks is: which graph of attention against time do you want?


## 3. Notes

I use gappy notes (or skeleton or partially populated notes), distributed weekly. Students have copies, and I have one that I use at the visualiser. I cover about four pages per lecture, and the amount of pre-printed material varies considerably. Each week's notes have a problem sheet attached to the back, so I don't have to distribute these separately. And page numbers for the whole course are contiguous, so anyone who mixes up their paperwork can reorder it easily.

## 4. Routine

My lectures are currently on Monday (1lam and 5pm) and Wednesday (1lam). On Monday morning, students pick up notes on the way in. Between Monday
and Wednesday they are expected to read a few preprinted pages, where I make clear that this should take less than an hour and that I advise setting a regular reading time. Wednesday's lecture starts with ten true/false questions. After that lecture, completed notes go on the virtual learning environment (ours is based on moodle), followed on Friday by problem solutions. The actual routine doesn't matter, of course, it just matters that there is one. Like all of these organisational practices, this helps students to know where everything is and what they are supposed to be doing, so that they can focus their energies on learning.

## Practices: Study guidance

There is substantial evidence that students - and people in general - hold erroneous and unhelpful beliefs about learning [3]. First years, certainly, know little about what is expected in undergraduate study. Some have been micromanaged by earlier teachers, and have not developed good planning skills or self-discipline. Some have found earlier mathematics fairly easy, and do not know how to handle themselves in the face of a challenge. The next three practices offer practical advice and encouragement.

## 5. Clarifying expectations

The week 2 reading begins with information on what real analysis is like. Here is a short sample.

Here is what happens when I teach Analysis. In week 1 , everyone is in a good mood because they're starting something new. In weeks 2 and 3, there is a buildup of increasingly challenging material. In week 4 , the mood in the lecture theatre is dreadful. The whole class has realized that this is difficult stuff and that it isn't going to get any easier. Everyone hates Analysis and, by extension, quite a few people hate me. I am not fazed by this, though, because I have taught Analysis about twenty times now and I know what will happen next... (from [1]).

Someone needs to say this, because new students who experience difficulty will believe that they are failing, and some will respond with avoidance rather
than redoubled effort. Reading tasks are good for such content because labouring it in lectures takes time and can seem patronising. This reading goes on to discuss strategies for keeping up, how much time I expect students to spend studying notes and trying problems, and what to try and where and how to seek help when stuck.

## 6. Self-explanation training

The week 3 reading is a research-based booklet providing self-explanation training adapted for mathematics students (see setmath.lboro.ac.uk). Selfexplanation training teaches students to read effectively, and has been used across a range of academic subjects and mathematical levels [6]. This self-explanation training states that when reading mathematics, students should explicitly relate each line to earlier material and to their existing knowledge, questioning their own understanding. It teaches them to differentiate self-explanation from monitoring ('Yeah, yeah, I get that') and from paraphrasing. Experimental and eye-movement studies have established that it leads to better proof comprehension and more expert-like reading behaviour $[2,5]$.

## 7. Early feedback opportunity

After nearly 20 years of lecturing, I finally do what training courses say that you should: at the end of the first main topic, I give out big sticky notes and ask students to write down something they like about the course, and something that they don't like or are concerned about or didn't understand. The positive responses are straightforward and predictable. The negative things are more varied - everyone is unhappy in their own way - and include things like:

- Analysis is difficult.
- Pace is too fast.
- Worried about constructing proofs.
- A few don't like interactive discussions.
- Would like lecture capture used.
- Would like more worked examples.

Each year I put a full list on the visualiser and take ten minutes to discuss it. How many elements of the course do you think I say I will change? That's right: none. I know a lot more than undergraduates
do about teaching and learning mathematics. But the value of such feedback is not in finding things to change, it's in arranging an opportunity to explain why things are as they are. It helps students to see that not everyone wants the same things, and that some requests are mutually exclusive - you can't have both more examples and slower delivery. And I stress that the concerns are reasonable, which helps the students to feel understood.

## Practices: Activities

In-lecture mathematical activities can provide students with opportunities to be wrong, opportunities to be right, and opportunities to feel unsure. I believe that all of these are important for engagement and a sense of progress. And, handled well, a large class is ideal for generating emotional investment. Instead of an unmemorable 'Yeah okay, yeah okay', I want students to experience a more memorable 'Oh I know that... Oh no wait, maybe I don't. . . Gosh that is harder than it looks... Oh I get it now!'.

Gappy notes are great for this. They allow me to pre-print information that I want to record but not write. They facilitate variety and short-and-snappy tasks, which is important because momentum is easily lost. I don't have students do routine calculations; these take too long, and if there is one thing that new undergraduates can do on their own, it's routine calculations. I use lecture time instead to develop conceptual understanding by having students articulate their thinking to one another. Here are some things that I ask them to do and discuss.

## 8. Filling things in

If something can be filled in by students without my assistance, I think it should be. This works for routine extensions, applications to examples, and conceptual thought about mathematical claims. For instance, students can complete this theorem.

$$
\lim _{n \rightarrow \infty} x^{n}= \begin{cases}\infty & \text { if } \\ 1 & \text { if } \\ 0 & \text { if }\end{cases}
$$

Everyone can get this right, and it requires thought about the roles of $x$ and $n$, which are less likely to seem important if I print or write the full theorem.

Similarly, if provided with definitions of bounded above, upper bound, and supremum, students can complete definitions of bounded below, lower bound, and infimum. And, of course, they can fill in tables that provide examples related to these concepts.

My favourite filling-in task is about the axioms for the real numbers. I didn't want to write these out that list is long. But I knew that printing them would not prompt much thought. I toyed with the idea of printing the axioms and writing in the names ('commutativity of addition'). Then I had a brainwave: I now print the list of axioms and the list of names, and have students match them up. This is a few weeks into the term so, after some initial hesitation, most people can get them all right. More importantly, they have to think about the meanings of commutativity, distributivity, and so on. And that's key for all of these activities. They are quick and doable, but they require thought about meaning.

## 9. Deciding

Another type of activity is deciding. My lectures often include several decision tasks, which start with 30 seconds or a minute or three minutes for thinking and discussion. I then ask for a vote, using the oldschool technology of raised hands. The hand-raising works because I raise my own hand, right up in the air, for both answers ('Votes for true [raise hand. . . lower hand]... Votes for false [raise hand]'). Before every vote, I say I I don't care who is right or wrong, I just care that you're thinking and that you're willing to change your mind if someone gives you good reason to'. If there are not enough votes, I say 'That is not enough votes', and give the students a minute to think some more and vote again.

My favourite decision questions are those that I know will split the class 50:50. I draw attention to these by asking everyone to vote again and look around the room. Then I say 'Whatever you think, half of the class disagrees. Do you want to change your mind?'. The room then comes alive: everyone knows that their peers are not stupid, yet apparently half of them are wrong. This dramatically increases everyone's motivation to work out whether they might have overlooked a crucial idea.

A useful type of decision question is: What symbol goes in the gap in this theorem? $\Rightarrow, \Leftarrow$, or $\Leftrightarrow$ ? Here are some theorems for which that works.

$$
\text { - }\left(a_{n}\right) \rightarrow a \quad\left(\left|a_{n}\right|\right) \rightarrow|a|
$$

$$
\begin{aligned}
& \text { - }\left(a_{n}\right) \text { is convergent } \quad\left(a_{n}\right) \text { is bounded. } \\
& \text { - } \sum_{n=1}^{\infty} a_{n} \text { is convergent } \quad\left(a_{n}\right) \rightarrow 0
\end{aligned}
$$

The last takes two or three rounds of voting because, even when we have just studied the series $\sum 1 / n$, about $75 \%$ answer incorrectly - the intuition that a series converges if its terms tend to zero is tough to dislodge. But that's the point. My drawing attention to a counterintuitive result is not enough. Being wrong a couple of times is more memorable.

Another useful question type involves a true/false decision, which can set up what is coming next. Here are some of those, with the set-up that $A \subseteq \mathbb{R}$ has a supremum $\sup A$.

- $\sup A \in A$.
- If we define $-A=\{-a \mid a \in A\}$, then $\sup (-A)=-\sup A$.
- $\forall \varepsilon>0, \exists a \in A$ such that $\sup A-\varepsilon<a \leq \sup A$.

After considering these, students are more ready to hear my comments. And this is true whether or not they have made much progress. Those who struggle to interpret a quantified statement learn as much about interpretation as they do about the result.

## 10. Reading and explaining

A third type of activity is reading something and explaining it to your neighbour. This, in my view, is worth doing: independent reading is an important skill, and if something is important then it merits lecture time. And explanation tasks can be short. I often ask students to read a definition, theorem or proof and to use gestures, diagrams or examples to explain what it means (and why it is true or valid). This, again, requires thought about meaning. Of course, some definitions, theorems and proofs are difficult, so I adjust for this. For the definition of sequence convergence, for instance, I first give students enough time to try to understand it and realise that they don't. I then offer an extended explanation, building up a diagram and an informal verbal expression. I then ask them to explain to one another what I just said. When they realise that they can't quite do that either, I say that l'll run through it once more and give them another go. Attention, by that point, is high.

## 11. True/false questions

Wednesday's lecture starts with ten true/false questions, printed on one side of paper with space for each response. Here are a few examples.

- The number $\sqrt{47}$ is irrational.
- The number $47 / 225$ has a non-repeating decimal expansion.
- The set of even numbers is countably infinite.
- For all $x \in \mathbb{N}, 4\left|x^{3} \Rightarrow 4\right| x$.
- If $x \in \mathbb{Q}$ and $y \notin \mathbb{Q}$ then $x+y \notin \mathbb{Q}$.

The instruction is to state whether each statement is true or false and, if it is false, to give a counterexample or a brief reason. I give about seven minutes for individual, silent attempts, about three for students to discuss their answers, and about two for them to consult their notes. Then I run through the answers.

I originally intended these questions to encourage students to do the reading - those who haven't done it spend a few minutes feeling uncomfortable. But their real value is in providing retrieval practice, which is important because repeated retrieval is known to strengthen memory [3]. And, ironically, they provide individualised feedback - many students comment that the true/false questions highlight what they need to review.

## 12. Tests

Three times during the term, the true/false quiz is replaced by a 20 -minute for-credit test. This contains ten true/false questions with the usual instructions, and two or three more challenging questions. The challenging questions are published a week in advance so that students can prepare. They can work together and look up whatever they want, but they are not allowed to ask tutors or staff in our mathematics learning support centres. This allows me to ask questions that go beyond what has been covered in lectures, while holding everyone accountable for producing their own answers; those who want to cheat have to remember what their clever friend said, not just copy it out.

## Influencing students

My overall aim is to be a positive influence on student behaviour, and in this l've been guided by the book Influencer [4]. Its authors argue that there are six sources of influence, sorted into a three-by-two grid. The columns are motivation (do I want to do it?) and ability (can I do it?), and the rows capture individual, social and structural influences.

Individual motivation sounds straightforward. My students, after all, have chosen to study mathematics. But every lecturer knows that desire to obtain a degree is not directly linked to desire to engage with difficult ideas in the day's sixth lecture. Fortunately, I think there are two sources of individual motivation, one of which is often overlooked. Some students are interested in real analysis. Some are not. But everyone is interested in their own intellectual development. Everyone likes to be right, and most are pretty happy to be wrong and then right, having gained an insight. Activities can engage that.

Individual ability, counterintuitively, is easier to manipulate. Some abilities can be improved: study planning and mathematical reading can be addressed directly [1, 2, 5]. And perhaps more important is perception of ability. Students at this level often can fill in definitions, explain theorems and proofs, and get most of our true/false questions right. That provides a sense of progress and developing capability, which makes the difficult things more palatable.

Social motivation is a strange one. Many undergraduates tell one another that they don't need to study if the first year doesn't count for credit, and I can't generate a comprehensive culture shift. But I can create an environment in which it is clear that the vast majority are, in fact, keen to do well, and willing to work hard and support one another.

Social influences on ability can be direct: my students are encouraged to help one another, both to understand the mathematics and to keep going when it gets tough. Or they can be indirect: students who struggle in isolation can have skewed ideas about what it means to do well, whereas students who see regular evidence that no-one else knows all the answers either tend to have a better calibrated sense of their own performance, and to suffer less worry.

Structural influences on motivation are tricky. The Influencer authors stress that carrots and sticks are not effective replacements for individual and social motivation. In academia, for instance, tests can make
people study, but I do not believe that they make them want to study. So I am leery of set-ups involving frequent assessed work: I want students to develop deep understanding of a body of mathematics, not to chase after bits of credit. Because of this, I am content that my true/false questions are formative only, and that tests form a minor part of my strategy.

Structural supports for ability are easier. They often involve simply removing rocks from the path, and my organisational practices are designed for this. Gappy notes enable students to keep up while still thinking. A consistent routine minimises time-wasting confusion about what is happening when. And tired, tense and irritable students find it difficult to concentrate; a friendly atmosphere of mutual support can loosen the tension and help them engage.

## That all sounds nice, but...

The material in this article raises consistent concerns among mathematicians, so I conclude by addressing some common questions ${ }^{1}$.

Some concerns are about time. Many mathematicians can't imagine having time to include activities in lectures. But time problems are caused by writing out longhand everything that you want to say. Of course, writing is valuable for students, for practice and because it is hard to pay attention when there is nothing physical to do. But I am selective about what merits this treatment. Do I want students to spend two minutes copying a definition? Maybe, if I want to draw attention to an aspect of its formulation. But maybe l'd rather they spend two minutes explaining it to their neighbour.

Other concerns are about participation. Some worry that not all students will engage with in-lecture tasks. That is certainly true: some will talk about something else or check social media feeds. But it is worrying only if you think that they won't do that otherwise. Some worry that students will provide one another with incorrect mathematical explanations. That, also, is true, but less so than you might think. And again it is worrying only if they would fully understand a lecturer - talking is an imperfect way to learn, but so is listening. Some worry about regaining attention after a task. But if you can do that at the beginning of a lecture, you can do it again later. Finally, some
mathematicians ask why I don't use clickers to gather responses - these, after all, permit full anonymity. The answer used to be laziness: the set-up takes some work and I never got around to it. But then I realised that one crucial thing for undergraduates to learn is that mathematics requires persistence, and that struggle is normal. Students in my class often get things wrong or have to admit that they don't know. Familiarity and ease with that is exactly what I want to encourage.

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# Uncertainty Analysis for Heavy Simulations of Galaxy Formation 

IAN VERNON


#### Abstract

Extremely complex simulations of the universe are now being performed in order to study galaxy formation. The responsible use of such simulations presents a huge challenge as it requires a comprehensive uncertainty analysis: a seemingly impossible task. We present a framework to address this challenge, based on state-of-the-art Bayesian methodology.


## Simulating galaxies and universes

"We have some good news", my collaborator announced as I wandered into his office one morning in early 2017. "We've been granted 60 million CPU hours to run possibly the largest hydrodynamic simulation of the Universe ever!". "Well, I guess that is good news" I said, uncertainly, "What's the bad news?". "Er, I didn't mention there was any bad,... well, OK, we kind of want you to choose the location in parameter space to run the model at", he said. This wasn't wholly unexpected. "How long in real time will it take to run 60 million CPU hours on the given facility?" I enquired, curiously. "Real time? Oh, about one and a half years...". Several unprintable expletives then followed.

The model in question is the EAGLE simulation, which is indeed one of the most complex models of galaxy formation ever run. My collaborator is Prof Richard Bower, a member of the Institute of Computational Cosmology here at Durham University, and one of the core members of the EAGLE group and of the VIRGO consortium (www.virgo.dur.ac.uk) that created and ran EAGLE. The facility in question is run by PRACE, the Partnership for Advanced Computing in Europe (www.prace-ri.eu). I am a Bayesian statistician, with a background in theoretical physics, who specialises in the uncertainty analysis of computer models of complex physical systems - an area that overlaps with, and (some more contentious than myself would say) has a far wider and deeper scope than the recently fashionable area commonly termed "Uncertainty Quantification".

## The EAGLE simulation

EAGLE stands for the Evolution and Assembly of GaLaxies and their Environments which, aside from implying that someone really wanted an acronym that spelt EAGLE, means that its purpose is to understand
how large numbers of galaxies form, collide and evolve. The simulation models a cosmological volume of ( 100 Megaparsecs) ${ }^{3}$, which is about ( 326 million light years $)^{3}$, a volume large enough to contain approximately 10,000 galaxies the size of the Milky Way or larger. The simulation starts prior to the formation of any stars or galaxies, when the Universe was still very uniform, and uses nearly 7 billion particles in combination with the well-known fundamental physical laws of gravity and hydrodynamics. It models the effects of dark matter, allowing large galaxy-sized structures to grow; baryonic matter, forming stars; and that of the cosmological constant, causing that causes cosmic acceleration. The results of the simulation can then be compared to various detailed but complex observed data sets that measure a variety of galaxy features: common ones include the stellar mass function (the distribution of galaxies relative to their stellar mass), and the overall distribution of galaxy sizes. Examples of the output from EAGLE can be seen in figure 1. See icc.dur.ac.uk/Eagle for more details, including some rather beautiful movies.

Some example scientific questions that EAGLE seeks to answer are:

- How do galaxies stop growing? Is it because of the activity of the central black hole? Is it because they collide and merge? Is it because they are in a crowded environment?
- How typical is our own Milky Way? Are we in a normal galaxy in a normal part of the Universe or is there something special about where we live?
- How do the different gas flows affect the formation of galaxies?
- How does the presence of gas affect the observations of halo masses, lensing or dark matter?


Figure 1. Left panel: a slice through the EAGLE simulation volume, showing the intergalactic gas colour-coded from blue to red with increasing temperature. The inset zooms into a galaxy similar to the Milky Way, showing first its gas and then its stellar disc, which looks remarkably similar to observed spiral galaxies. Right panel: another slice through the EAGLE simulation showing the hot gas content (top), the dark matter density (bottom right) and what the simulation would look like in the visible spectrum (bottom left). (Image courtesy of the VIRGO consortium.)

A single run of the 100 Megaparsec (Mpc) model was performed in 2015 , which took 1.5 months using 4064 processors (a substantial proportion of the VIRGO consortium's computational resources at the time). This showed that EAGLE is of sufficient accuracy to attempt to answer many of the above questions, and led to a large number of publications, the first of which [1] has obtained over 580 citations, and was one of the most cited papers on astro-ph that year. Now the plan is to run even larger volumes: perhaps up to 15 times larger, as described in the slightly melodramatic opening paragraph above.

## A major challenge

In a word, the problem with running such a simulation is uncertainty. Now that the huge amount of work developing and efficiently coding up the current EAGLE version has been completed, we can perform a single model evaluation, using admittedly substantial computational resources and a lot of patience. This would be sufficient, were there only one possible way to run EAGLE. However, EAGLE features several uncertainties, many in the form of parameters related to hard to model 'sub-grid' processes. In short, galaxy formation critically depends on processes spanning wildly different scales: for example black holes at the centre of galaxies draw in gas on scales of 0.01 parsecs, but the energy produced by this process affects the whole galaxy and possibly its host halo up to a scale of 1 Megaparsec, effectively spanning 8 orders of magnitude in spatial resolution.

EAGLE itself, commendably one should say, spans over 5 orders of magnitude in resolution. To give some feel for this scale (although such comparisons should be treated with extreme caution, as there are many complexities here), if one managed a similar level of spatial resolution attempting to model the Earth's atmosphere, for use for example in a climate model, each cubic grid cell would be less than 26 metres across. However, EAGLE's impressive resolution is still nowhere near high enough to accurately represent either the effect of central black holes nor various other important small scale phenomena that affect galaxy formation, such as the impact of supernovas (massive stars that explode and drive gas out of the galaxy). Hence these processes have to be modelled via sub-grid scale models, that are parameterised using a modest number of physical input parameters, representing uncertain aspects of the processes in question. EAGLE also possesses additional cosmological parameters, but these are a little more understood and usually set to the values as measured to reasonably high accuracy by the Planck satellite. Seven sub-grid parameters of interest have been identified as strongly influential and hence form the core of the current study. The remaining parameters are thought to be somewhat sub-dominant, but their effects will be taken into account, in a less detailed form, in our analysis below.

To really understand the scientific ramifications of EAGLE, one inevitably has to explore its uncertain behaviour over this 7-dimensional parameter space. As each step in this parameter space takes 1.5 months
to complete, using 4064 processors, one can see the problem: standard search techniques are utterly impractical. To reiterate this point: a 7-dimensional hypercube has 128 corners, so visiting these alone would take the current version of EAGLE over 64000 years. Inevitably, more detail would be required in practice, so a 7 -dimensional grid with 10 points in each direction, 10 million in total, might be sufficient: this would take over 5 billion years to evaluate. Somewhat ironically, this is well over a third of the current age of the universe.

Critically, we must go even further: as EAGLE produces many different outputs that can be compared with a range of observed data sets, our real goal is to identify all the choices of the input parameters that will lead to acceptable matches between the model output and observed data (or to find that no such choices exist), hence requiring a detailed parameter search. Note that only finding a single acceptable match may be scientifically highly misleading. This is sometimes referred to as an inverse problem, a Bayesian calibration problem, or a history matching problem (we prefer the latter, for various somewhat subtle reasons: for details see [2]). Finally, we may want to use our understanding of the input parameter space to choose the input parameters for a single future, even larger, EAGLE run, or perhaps to design a limited set of slightly smaller runs chosen to be at highly informative locations across the parameter space. To address the above problems, we really require the use of Bayesian statistics.

## Bayesian analysis of computer models of complex physical systems

The reason the above general problem structure, as faced by the EAGLE collaboration, is of interest to Bayesian statisticians is not just because of the fascinating scientific questions EAGLE hopes to answer, but because it has many of the attributes of a type of problem that is currently occurring in a wide variety of scientific disciplines. Due to the increase in mathematical modelling and corresponding computing power, many scientific areas are developing ever more complex, high-dimensional and computationally expensive models of physical systems. Helpfully, an area of (Bayesian) statistics has developed over the last 25 years, designed specifically to combat the challenges posed by this kind of problem, the general form of which we now describe.

A model is created for a particular real world system of interest, that describes how a vector of various system properties $x$ affects a vector of system behaviour, given by the model as $f(x)$. So for example, for all of EAGLE's complexity, it is just a function $f(x)$ that maps a 7 -dimensional $x$ to a high-dimensional vector of galaxy property outputs $f$. The model or simulator is, however, imperfect, and the real system properties (suitably defined, an interesting question all by itself) are given by the vector $y$. We may wish to explicitly model the gap between reality $y$ and the model $f(x)$ evaluated at its best input $x^{*}$ for example via $y=f\left(x^{*}\right)+\epsilon$, where $\epsilon$ is now a random vector, with a possibly complex joint structure, representing the unknown structural deficiencies of the model. We can of course measure a subset of the system properties, but with error: these measurements are given by a vector of data $z_{p}$, and correspond to past system properties $y_{p}$ with $y$ partitioned as $y=\left(y_{p}, y_{f}\right)$, where $y_{f}$ represents possible future properties of interest, that we may want to predict. Again, we may make the gap between measurements and real system explicit for example via say $z=y+e$, where $e$ is a random vector representing measurement error.

We wish to answer various scientific questions, while accounting for all the uncertainties that exist in the above setup. For example we may wish to:

- Explore the model's behaviour $f(x)$ over a defined input space $x \in X$.
- Learn about acceptable values of $x$ (or perform full Bayesian inference on $x$ ) by comparing the model $f(x)$ to observed data $z$.
- Explore the accuracy of the model for reproducing various outputs, and hence assess its adequacy for the task at hand.
- Use the model combined with past observations $z_{p}$ to make predictions of future outputs $y_{f}$.
- Use the model along with the assessed uncertainties in some decision theory calculation, for example, to help aid policy makers.

However, the model or simulator $f(x)$ is usually extremely computationally expensive to evaluate, relative to the dimension of $x$, preventing the evaluation of any of the above calculations. Hence we have some major problems which can be grouped roughly as follows:

- The speed problem: the model is far too slow to be used to explore its input parameter space in naive ways. For example, we cannot plug it into standard optimisers or more sophisticated algorithms that usually require vast numbers of model evaluations.
- The general uncertainty problem: the answers to whatever scientific questions we wish to pose will critically depend upon the assessment of all the various uncertainties in the problem. In particular the multivariate nature of the structural discrepancy $\epsilon$, the observational errors $e$, and input parameter uncertainty $x$ may have a major impact.


## Solving the speed problem: Bayesian Gaussian process emulation

Firstly, we must acknowledge the underlying problem: except at a small number of input locations where we actually decide to run the model, we will always be uncertain as to the true value of the EAGLE function $f(x)$. In the Bayesian setting, we can incorporate this uncertainty naturally, by simply treating $f(x)$ at unevaluated $x$ as another random vector. Secondly, we then ask what do we know about this uncertain function $f(x)$ ? For example, many physical functions are in some sense smooth, in that although small changes to the input parameters could in principle greatly affect the outputs, this may (in the domain expert's view) be deemed unlikely based on consideration of the fundamental equations, and the physical nature of the system under investigation. Such considerations facilitate the construction of Bayesian emulators, which are specifically employed to combat the speed problem. A Bayesian emulator is a fast statistical function built to mimic the behaviour of the EAGLE function $f(x)$ over the input space $X$. The emulator provides both an expectation as to the value of $f(x)$ at an as yet unevaluated $x$, and critically an $x$-dependent uncertainty statement as to the emulator's accuracy at this point, which can be naturally incorporated into a Bayesian analysis. Most importantly, the emulators are very fast to evaluate and are usually multiple orders of magnitude faster that the model itself. In this application, they are between $10^{9}-10^{12}$ times faster than EAGLE (depending on which version of EAGLE we compare to), the kind of speed increase that tends to turn heads in most scientific communities.

A popular statistical model for the Bayesian emulator for $f(x)$, which has individual outputs $f_{i}(x)$, $i=1 \ldots q$, is structured as follows (see for example [2] for details):

$$
\begin{equation*}
f_{i}(x)=\sum_{j} \beta_{i j} g_{i j}\left(x_{A_{i}}\right)+u_{i}\left(x_{A_{i}}\right)+w_{i}(x) \tag{1}
\end{equation*}
$$

where the active variables $x_{A_{i}}$ are a subset of the inputs $x$ that are most influential for output $f_{i}(x)$. The first term on the right hand side of the emulator equation (1) is a regression term, where $g_{i j}$ are known deterministic functions of $x_{A_{i}}$, a common choice being low order polynomials, and $\beta_{i j}$ are unknown scalar regression coefficients. The second term, $u_{i}\left(x_{A_{i}}\right)$ is a Gaussian process ${ }^{2}$ over $x_{A_{i}}$, which means that if we choose a finite set of inputs $\left\{x_{A_{i}}^{(1)}, \ldots, x_{A_{i}}^{(s)}\right\}$, the uncertain outputs $u_{i}\left(x_{A_{i}}^{(1)}\right), \ldots, u_{i}\left(x_{A_{i}}^{(s)}\right)$ will have a multivariate normal distribution with a covariance matrix constructed from an appropriately chosen covariance function, a popular form being:

$$
\begin{equation*}
\operatorname{Cov}\left(u_{i}\left(x_{A_{i}}\right), u_{i}\left(x_{A_{i}}^{\prime}\right)\right)=\sigma_{u_{i}}^{2} \exp \left\{-\left\|x_{A_{i}}-x_{A_{i}}^{\prime}\right\|^{2} / \theta_{i}^{2}\right\} \tag{2}
\end{equation*}
$$

where $\sigma_{u_{i}}^{2}$ and $\theta_{i}$ are the variance and correlation length of $u_{i}\left(x_{A_{i}}\right)$ which must be specified a priori. The third term $w_{i}(x)$ is a nugget, a white noise process with variance $\sigma_{v_{i}}^{2}$, uncorrelated with $\beta_{i j}, u_{i}\left(x_{A_{i}}\right)$ and itself, that represents the effects of the remaining inactive input variables.

Given a set of $n$ carefully chosen runs $D_{i}=$ $\left(f_{i}\left(x^{(1)}\right), f_{i}\left(x^{(2)}\right), \ldots, f_{i}\left(x^{(n)}\right)\right.$ ), we can update our prior beliefs about $f(x)$ at unevaluated $x$ by $D_{i}$ using either Bayes' theorem (which requires full probability distributions) or the computationally efficient Bayes linear update (which only requires expectations and variances). The latter provides the adjusted expectation and variance of $f(x)$, denoted $\mathrm{E}_{D_{i}}\left(f_{i}(x)\right)$ and $\operatorname{Var}_{D_{i}}\left(f_{i}(x)\right)$. The following images show an example of a 1d emulator of a deterministic toy model (a simple sine function).

The speed of the emulators allows us to comprehensively explore the input parameter space $X$ and identify regions of $X$ that may lead to acceptable matches to the observed data $z$. We do this by using implausibility measures, the simplest form of which is, for output $i$

$$
\begin{equation*}
I_{i}^{2}(x)=\frac{\left(\mathrm{E}_{D_{i}}\left(f_{i}(x)\right)-z_{i}\right)^{2}}{\operatorname{Var}_{D_{i}}\left(f_{i}(x)\right)+\operatorname{Var}\left(\epsilon_{i}\right)+\operatorname{Var}\left(e_{i}\right)} \tag{3}
\end{equation*}
$$

[^1]

Figure 2. An example emulator for a 1-dimensional toy model where $f(x)=\sin (2 \pi(x-0.1) / 0.4)$, for the 1st wave, using just 6 runs (left panel), and for the 2 nd wave, using 2 additional runs (right panel). The emulator's expectation $\mathrm{E}_{D}[f(x)]$ and credible intervals $\mathrm{E}_{D}[f(x)] \pm 3 \sqrt{\operatorname{Var}_{D_{i}}\left(f_{i}(x)\right)}$ are given by the blue and red lines respectively, with the observed data $z$ that we wish to match to as the black horizontal line (with errors). The implausibility $I(x)$ is represented by the coloured bar along the $x$-axis, with dark blue implying $I(x)>3$, light blue $2.5<I(x)<3$ and yellow $(I(x)<1)$. These emulators are for deterministic models, but stochastic equivalents of course also exist.

Usually we perform the exploration in iterations or 'waves', using the emulators and implausibility measures to rule out parts of the current space $X_{k}$ that are obviously poor (which have high $I_{i}(x)$ for a subset of the outputs), before performing further runs of the model in the not-yet-ruled-out region $X_{k+1}$ say, and reconstructing new, more accurate emulators defined only over $X_{k+1}$. This divide and conquer approach is very powerful. The $x$-axis of figure 2 is coloured by implausibility, showing the obviously bad parts of the input space with high $I(x)>3$ in dark blue, that correctly suggest $f(x)$ will be far away from the data $z$, given with error as the horizontal black lines.

Addressing the full general uncertainty problem is of course context dependent. However, we have successfully applied this style of Bayesian emulation uncertainty analysis across multiple scientific disciplines, and have developed methodology for assessing the uncertainties of $\epsilon$ and $e$ and for combining them with emulators that solve the speed problem. See e.g. [2], [3] and references therein, the latter of which was awarded the Mitchel Prize by JASA/ISBA for the best applied Bayesian article worldwide.

## Taming exceedingly slow simulators: multilevel emulation

Even given the above emulation technology, EAGLE at its current size of 100 Mpc is still too slow to perform enough runs to construct even a moderately accurate emulator over 7-dimensional space. Things
seem a little hopeless until we ask if there are faster, approximate versions of EAGLE available, that we can use for a process known as multilevel emulation. Helpfully there are, EAGLE can indeed be run over smaller volumes of the Universe, and has been set up to run on cubes of size $12.5 \mathrm{Mpc}, 25 \mathrm{Mpc}, 50$ Mpc and the full 100 Mpc , which we will refer to as levels 1 to 4 respectively. Each level is thought to be approximately 8 times faster than the next, although levels 1 and 2 gain additional speed as they don't have to simulate very large galaxies.

There are, however, two important differences between the levels: a) levels 1 and 2 only model relatively small numbers of galaxies, and so we encounter noise in many of the outputs due to finite galaxy counts, b) the lower levels are physically different from the level 4 simulation, in that due to periodic boundary conditions the largest galaxies simply cannot form inside a 12.5 Mpc or even a 25 Mpc box, leading to possibly substantial systematic differences between runs at different levels for the same input $x$. Multilevel emulation can usually handle such issues. All we need is for the lower levels to be informative for the higher levels (so biases, systematic differences, etc. are fine).

We begin by building an emulator $f^{(1)}(x)$ for level 1 , summarised by the uncertain quantities $\left\{\beta_{i j}^{(1)}, u_{i}^{(1)}\left(x_{A_{i}}\right), w_{i}^{(1)}(x)\right\}$ from equation (1), based on a carefully chosen set of 60 runs. We then construct a prior emulator for level 2 by specifying a representation for $\left\{\beta_{i j}^{(2)}, u_{i}^{(2)}\left(x_{A_{i}}\right), w_{i}^{(2)}(x)\right\}$ based on their level

1 counterparts, say by modestly inflating the level 1 uncertainties and by including any additional physical structure or suspected systematic differences we are aware of. We now require far fewer level 2 runs (here we used 20) to update this relatively well informed prior level 2 emulator. We will then repeat the process for levels 3 and 4, but now focus on the parts of $X$ that may yield good matches to observed data (so that we do not squander runs in uninteresting parts of the parameter space). We are currently in the process of designing the set of level 3 runs.


Figure 3. The implausibility of the 7-dimensional input space of the EAGLE simulation, shown as all possible two-dimensional projections (the 7 input parameters are named down the main diagonal: the first 5 describe supernova, and the last two central black holes). The colour scheme is consistent with figure 2 so that dark blue shows regions we would discard, light blue gives borderline regions $(2.5<I(x)<3)$ that we would wish to explore further in the next wave, while the green/yellow regions suggest that the emulators currently think that good matches between the 25 Mpc level 2 version and the stellar mass function data could be found (but this may change with more runs). Note that the low implausibility points are plotted last, allow one effectively to see through the less interesting parts of the space. The pink dot is the location of the previous 100Mpc EAGLE run.

Figure 3 shows the results of the level 2 emulator and corresponding (maximised) implausibility measure based on the important stellar mass function outputs, over the full 7 -dimensional space (shown as all possible 2-dimensional projections). This used 400000 emulator evaluations, completing in minutes.

The dark blue areas will be ruled out as implausible. The light blue/green/red areas will need a second
wave of runs to investigate further, but look likely to produce moderate to good fits to the observed data set. The pink dot is the location of the single 100 Mpc run performed in 2015. It can be seen that it is in a good part of the input space as judged from several 2-dimensional projections, however its location could be improved.

This project is ongoing, but once we have performed a small number of level 3 and 4 runs, we will be in a position to answer the original question and propose a suitable parameter location for a single massive 'level 5' run, or to suggest a set of locations for slightly smaller runs, designed to resolve some of the key scientific questions outlined above. Then we will just have to wait.

## Acknowledgements

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## FURTHER READING

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# Modelling Our Sense of Smell 

THIBAULT BOURGERON, CARLOS CONCA, AND RODRIGO LECAROS

A first step in our sensing of smell is the conversion of chemical odorants into electrical signals. This happens when odorants stimulate ion channels along cilia, which are long thin cylindrical structures in our olfactory system. Determining how the the ion channels are distributed along the length of a cilium is beyond current experimental methods. Here we describe how this can be approached as a mathematical inverse problem.

## The olfactory system

The first step in sensing smell is the transduction (or conversion) of chemical information into an electrical signal that goes to the brain. Pheromones and odorants, which are small molecules with the chemical characteristics of an odour, are found all throughout our environment. The olfactory system (part of the sensory system we use to smell) performs the task of receiving these odorant molecules in the nasal mucosa, and triggering the physical-chemical processes that generate the electric current that travels to the brain. See Figure 1 and "Transduction of olfactory signals".

What happens next is a mystery. Intuition tells us that the electrical wave generated gives rise to an emotion in the brain, which in turn affects our behaviour. Of course, the workings of our other four senses is similarly a mystery. And so, we quickly come to perhaps one of the most fundamental questions in neurosciences for the future: how does our consciousness process external stimuli once reduced to electro-chemical waves and, over time, how does this mechanism lead us to become who we are?

How can we approach this problem with mathematics? Biology is synonymous with "function", so the study of biological systems should start by understanding the corresponding underlying physiology. Consequently, to obtain a proper mathematical representation of the transduction of an odour into an electrical signal we must first detect which atomic populations are involved in the process and identify their respective functions.

## Kleene's experimental procedure

The molecular machinery that carries out this work is in the olfactory cilia (see "Transduction of olfac-
tory signals"). Experimental techniques for isolating a single cilium (from a grass frog) were developed by biochemist and neuroscientist Steven J. Kleene and his research team at the University of Cincinnati in the early 1990s [3, 4]. One olfactory cilium of a receptor neuron is detached at its base and stretched tight into a recording pipette. The cilium is immersed in a bath of a chemical known as cAMP (by its chemical initials). This substance diffuses through the interior of the cilium, opening the so-called GNC channels as it advances (Figure 2), and generating a transmembrane electrical current. The intensity of the total current is recorded.

Although the properties of a single channel have been described successfully using these experimental procedures, the distribution of these channels along the cilia still remains unknown. lonic channels, in particular CNG channels are called "micro domains" in biochemistry, because of their practically imperceptible size. This makes their experimental description using the current technology very difficult.

## An integral equation model

Given the experimental difficulties, there is a clear opportunity for mathematics to inform biology. Determining ion channels distribution along the length of a cilium using measurements from experimental data on transmembrane current is usually categorized in physics and mathematics as an inverse problem. Around 2006, a multidisciplinary team (which brought together mathematicians with biochemists and neuroscientists, as well as a chemical engineer) developed and published a first mathematical model [2] to simulate Kleene's experiments. The distribution of CNG channels along the cilium appears in it as the main unknown of a nonlinear integral equation model.

## Transduction of olfactory signals



Figure 1. Odorants reaching the nasal mucus (left) \& Structure of an olfactory receptor neuron (right)
Cilia are long, thin cylindrical structures that extend from an olfactory receptor neuron into the nasal mucus (Figure 1). The sensing of an odour begins with pheromones or odorants binding to specific receptors on the external membrane of cilia, initiating a signalling cascade. These type of receptors are known as $G$-protein coupled receptors and pheromones are the first messenger in this signalling process. First messengers cannot physically cross the cellular membrane in order to initiate changes within the cell, and therefore require a signal transduction mechanism to propagate the signal intracellularly.

When an odorant molecule binds to an olfactory receptor on a cilium membrane, a conformational change occurs in the receptor, which activates a G-protein in the intracellular side of the membrane (see Figure 2). The active G-protein in turn activates an enzyme that synthesizes the second messenger of the signalling cascade, in this case a neurotransmitter called cyclic adenosine monophosphate (cAMP) that is able to amplify
the original signal and trigger specific physiological changes. cAMP can diffuse through the cell's cytoplasm and activate cyclic nucleotide-gated (CNG) ion channels allowing the flow of extracellular inorganic ions, mainly $\mathrm{Ca}^{2+}$ and $\mathrm{Na}^{+}$as illustrated in Figure 2, depolarizing the olfactory cell. This depolarization is characterized by a voltage difference between the intra and extracellular sides of the membrane. The increase in intracellular $\mathrm{Ca}^{2+}$ in turn activates a chloride flux that helps amplify this depolarization, generating an electric current that is conducted from the cilia to the axon of the olfactory neuron and transmitted to the olfactory bulb depicted in Figure 1. The G-protein deactivates, cAMP concentration diminishes and the system can return to its resting state. This is the overall process that human beings share with all mammals and reptiles to smell and differentiate odours.


Figure 2. Signal transduction mechanism for the olfactory system. A: In the absence of stimulus channels are closed, system is at resting state. B: Binding of odorants triggers cAMP synthesis and opening of CNG channels.

The essential nonlinearity in the previous model arises from the binding of the channel activating ligand (a ligand is a chemical messenger, in this case a cAMP molecule) to the CNG ion channels as the ligand diffuses along the cilium. In 2007, mathematicians D. A. French and C. W. Groetsch introduced a simplified model, in which the binding mechanism is neglected, leading to a linear Fredholm integral equation of the first kind with a diffusive kernel. The inverse mathematical problem consists of determining a density function, say $\rho=\rho(x) \geq 0$ (representing the distribution of CNG channels), from measurements
in time of the transmembrane electrical current, denoted $\mathrm{I}_{0}[\rho]$. This mathematical equation for $\rho$ is the following integral equation: for all $t \geq 0$,

$$
\begin{equation*}
\mathrm{I}_{0}[\rho](t)=\int_{0}^{L} \rho(x) \mathbb{P}(c(t, x)) \mathrm{d} x \tag{4}
\end{equation*}
$$

where $\mathbb{P}$ is known as the Hill function of exponent $n>0$ (see Figure 3). It is defined by:

$$
\forall w \geq 0, \quad \mathbb{P}(w)=\frac{w^{n}}{w^{n}+K_{1 / 2}^{n}}
$$

In this definition, the exponent $n$ is an experimentally determined parameter and $K_{1 / 2}>0$ is a constant which represents the half-bulk (i.e., the ligand concentration for which half the binding sites are occupied); typical values for $n$ in humans are $n \simeq 2$.


Figure 3. The Hill function $\mathbb{P}$
Besides, in the linear integral equation above, $c(t, x)$ denotes the concentration of cAMP that diffuses along the cilium with a diffusivity constant that we denote as $D ; L$ denotes the length of the cilium, which for simplicity is assumed to be one-dimensional. Here, by concentration we mean the molar concentration, i.e., the amount of solute in the solvent in a unit volume; it is a nonnegative real number.

Hill-type functions are extensively used in biochemistry to model the fraction of ligand (the chemical messenger) bound to a macromolecule as a function of the ligand concentration and, hence, the quantity $\mathbb{P}(c(t, x))$ models the probability of the opening of a CNG channel as a function of the cAMP concentration. The diffusion equation for the concentration of cAMP can be explicitly solved if the length of the cilium $L$ is supposed to be infinite. It is given by:

$$
c(t, x)=c_{0} \operatorname{erfc}\left(\frac{x}{2 \sqrt{D t}}\right)
$$

where $c_{0}>0$ is the maintained concentration of cAMP with which the pipette comes into contact at the open end $(x=0)$ of the cilium (while $x=L$ is the closed end). Here, erfc is the standard complementary Gauss error function,

$$
\operatorname{erfc}(x):=1-\frac{2}{\sqrt{\pi}} \int_{0}^{x} e^{-\tau^{2}} \mathrm{~d} \tau
$$

Accordingly, it is straightforward to check that $c$ is decreasing in both its variables and that it remains bounded: for all $(t, x), 0<c(t, x) \leq c_{0}$.

Despite its elegance, thanks to the simplicity of its formulation, this new model does not overcome the difficulties found in its non-linear version. In fact the mathematical inverse problem associated to model (4) can be shown to be ill-posed.

## Non-diffusive kernels

Moreover, it can be shown that any model based on a first-order integral equation with a diffusive smooth kernel necessarily leads the problem of recovering the density from measurements of the electrical current to be ill-posed.

One way to overcome the ill-posedness of the inverse problem in (4) consists of replacing the kernel of the integral equation with a non-smooth variant of the Hill function. (See [1] for another approach.)

Specifically, let $a \in\left(0, c_{0}\right)$ be a given real parameter. A discontinuous version of $\mathbb{P}$ is obtained by keeping the original Hill function $\mathbb{P}$ in the interval $[0, a]$, and by forcing a saturation state for higher concentrations. By doing so, one is led to introduce the following disruptive variant of $\mathbb{P}$ (shown in Figure 4):

$$
\mathbb{H}(c)=\mathbb{P}(c) \mathbb{1}_{c \leq a}+\mathbb{1}_{a<c \leq c_{0}}
$$

where $\mathbb{1}_{J}$ denotes the characteristic function of the interval $J$. The mathematical problem that recovers $\rho$ from the electrical current data is therefore modelled using the following integral equation:

$$
\begin{equation*}
\mathrm{I}_{1}[\rho](t)=\int_{0}^{L} \rho(x) \mathbb{H}(c(t, x)) \mathrm{d} x \tag{5}
\end{equation*}
$$

where $c(t, x)$ is still defined as before. The introduction of this disruptive Hill function can be understood mathematically as follows: as $t \rightarrow \infty$, the factor $x / \sqrt{D t}$ in the complementary error function defining the concentration tends to 0 , and consequently $c(t, x)$ tends pointwise to $c_{0}$.


Figure 4. A disruptive variant of $\mathbb{P}(a=0.157)$

An inverse mathematical problem and a direct problem are associated with both models (4) and (5). In the first, the electric current is measured and the unknown is the density $\rho$ of ion channels, while in the direct problem the opposite is true. Since these are Fredholm equations of the first type, it is natural to tackle them using convolution. Once the variable $\rho$ has been extended to $[0, \infty)$ by zero, the Mellin transform is revealed as being the most appropriate tool for carrying out this task (see "Mellin transform").

## A general convolution equation

The Mellin transform is the appropriate tool to study model (5). It allows reduction in a convolution equation of the Mellin type (see "Mellin convolution"). To do so, the key observation is the fact that $\mathbb{H}(c(t, x))$ can be written in terms of $\frac{\sqrt{t}}{x}$. Indeed, defining $G$ as

$$
G(z)=\mathbb{H}\left(c_{0} \operatorname{erfc}\left(\frac{1}{2 \sqrt{D} z}\right)\right)
$$

we have $\mathrm{I}_{1}[\rho](t)=\int_{0}^{L} \rho(x) G\left(\frac{\sqrt{t}}{x}\right) \mathrm{d} x$. Thus, by extending $\rho$ by zero to $[0, \infty)$, and rescaling time $t$ in $t^{2}$, we obtain

$$
\mathrm{I}_{1}[\rho]\left(t^{2}\right)=\int_{0}^{\infty} x \rho(x) G\left(\frac{t}{x}\right) \frac{\mathrm{d} x}{x}=(x \rho(x)) * G
$$

which is a convolution equation in $x \rho(x)$.
Taking Mellin transform on both sides and using its operational properties,

$$
\begin{equation*}
M \rho(s+1)=\frac{1}{2} \frac{M \mathrm{I}_{1}[\rho](s / 2)}{\mathcal{M} G(s)} \tag{6}
\end{equation*}
$$

## Mellin transform

Austrian mathematician Robert Hjalmar Mellin (1854-1933) gave his name to the so-called Mellin transform, whose definition and properties are recalled below. The interested reader is referred to E. Lindelö [5] for a summary of his work, and proof of the main results around this transform.

For $q \in \mathbb{R}, q+i \mathbb{R}$ will denote the vertical line $\{q+i t, t \in \mathbb{R}\}$ of the complex plane having abscissa $q$, and for $p \in \mathbb{R}(p \geq 1)$, $\mathrm{L}^{p}\left([0, \infty), x^{q}\right)$, or simply $\mathrm{L}_{q}^{p}$, will stand for the Lebesgue space with the weight $x^{q}$, i.e.,

$$
\mathrm{L}_{q}^{p}=\left\{f:[0, \infty) \rightarrow \mathbb{R} \mid\|f\|_{\mathrm{L}_{q}^{p}}<+\infty\right\}
$$

where $\|f\|_{\mathrm{L}_{q}^{p}}=\left(\int_{0}^{\infty}|f(x)|^{p} x^{q} \mathrm{~d} x\right)^{1 / p} . \mathrm{L}_{q}^{p}$, endowed with this norm, is a Banach space. Let $f$ be in $\mathrm{L}^{1}\left([0, \infty), x^{q}\right)$. The Mellin transform of $f$ is a complex-valued function defined on the vertical line $q+1+i \mathbb{R}$ by

$$
M f(s)=\int_{0}^{\infty} x^{s} f(x) \frac{\mathrm{d} x}{x}
$$

From its very definition, it is observed that the Mellin transform maps functions defined on $[0, \infty)$ into functions defined on $q+1+i \mathbb{R}$. Like in the Fourier transform, $M f$ is continuous whenever $f$ is in $\mathrm{L}^{1}\left([0, \infty), x^{q}\right)$. Specifically, we have

Theorem (Riemann-Lebesgue). The Mellin transform is a linear continuous map from $\mathrm{L}^{1}\left([0, \infty), x^{q}\right)$ into $\mathscr{C}^{0}(q+1+i \mathbb{R} ; \mathbb{C}) \hookrightarrow$ $\mathrm{L}^{\infty}(q+1+i \mathbb{R} ; \mathbb{C})$; its operator norm is 1 .

The following table summarizes the main operational properties of the Mellin transform:

| function | Mellin transform |
| :---: | :---: |
| $f(a t), a>0$ | $a^{-s} M f(s)$ |
| $f\left(t^{a}\right), a \neq 0$ | $\|a\|^{-1} M f\left(a^{-1} s\right)$ |
| $f^{(k)}(t)$ | $(-1)^{k}(s-k)_{k} M f(s-k)$ |

where, $\forall x \in \mathbb{R}$ and $\forall k \geq 1,(x)_{k}$ stands for the so-called Pochhammer symbol, which is defined by
$(x)_{k}=x \cdots(x-k+1)=\prod_{j=0}^{k-1}(x-j)$ if $k \geq 1$, and $(x)_{0}=1$, where $x$ is in $\mathbb{R}$.

## A priori estimates

Seeking continuity and observability inequalities for model (5) is then reduced to find lower and upper bounds for $\mathcal{M} G(\cdot)$ in suitable weighted $\mathrm{L}_{q}^{p}$ spaces. Doing so, one obtains the following.

Theorem (A priori estimates). Let $k=0$ or 1 and $r \in \mathbb{R}$ be arbitrary. Assume that the Mellin transforms of $\rho$ and $I_{1}[\rho]$ satisfy (6), then

$$
C_{\ell}^{k}\|\rho\|_{\mathrm{L}_{r}^{2}} \leq\left\|\left(\mathrm{I}_{1}[\rho]\right)^{(k)}\right\|_{\mathrm{L}_{2 k+\frac{r-3}{2}}^{2}} \leq C_{u}^{k}\|\rho\|_{\mathrm{L}_{r}^{2}}
$$

where

$$
\begin{aligned}
C_{\ell}^{k} & =\sqrt{2} \inf _{s \in \frac{r-1}{2}+i \mathbb{R}}\left|\left(\frac{s}{2}\right)_{k} M G(s)\right|>0 \\
C_{u}^{k} & =\sqrt{2} \sup _{s \in \frac{r-1}{2}+i \mathbb{R}}\left|\left(\frac{s}{2}\right)_{k} M G(s)\right|<+\infty
\end{aligned}
$$

## Observability of CNG channels

The a priori estimates of theorem above also allow us to determine a unique distribution of ion channels along the length of a cilium from measurements in time of the transmembrane electric current.

Theorem (Existence and uniqueness of $\rho$ ). Let $a>0$ and $r<1$ be given. If $\mathrm{I}_{1} \in \mathrm{~L}^{2}\left([0, \infty), t^{\frac{r-3}{2}}\right)$, $\mathrm{I}_{1}^{\prime} \in \mathrm{L}^{2}\left([0, \infty), t^{2+\frac{r-3}{2}}\right)$ and $a$ is small enough, then there exists a unique $\rho \in \mathrm{L}^{2}\left([0, \infty), x^{r}\right)$ which satisfies the following stability condition:

$$
\left\|\mathrm{I}_{1}\right\|_{\mathrm{L}^{2}\left([0, \infty), t^{\frac{r-3}{2}}\right)}+\left\|\mathrm{I}_{1}^{\prime}\right\|_{\mathrm{L}^{2}\left([0, \infty), t^{2+\frac{r-3}{2}}\right.} \geq C\|\rho\|_{L_{r}^{2}}
$$

where $C>0$ depends only on $a$ and $r$.

## Unstable identifiability

Since the French-Groetsch model is also a Fredholm integral equation of the first kind, it is natural to apply a Mellin transform here too. This leads to interesting results: neither an observability inequality nor a proper numerical algorithm for recovering $\rho$ can be established. However, an identifiability result holds whenever the current is measured over an open time interval (see the Identifiability Theorem below).

## Mellin convolution

For two given functions $f, g$, the multiplicative convolution $f * g$ is defined as follows

$$
(f * g)(x)=\int_{0}^{\infty} f(y) g\left(\frac{x}{y}\right) \frac{\mathrm{d} y}{y}
$$

Theorem (Mellin transform of a convolution)
Whenever this expression is well defined, we have

$$
M(f * g)(s)=M f(s) M g(s)
$$

Finally, the classical $L^{2}$-isometry has its Mellin counterpart.

Theorem (Parseval-Plancherel) The Mellin transform can be extended in a unique manner to a linear isometry (up to the constant $(2 \pi)^{-1 / 2}$ ) from $\mathrm{L}_{2 q-1}^{2}$ onto the classical Lebesgue space $\mathrm{L}^{2}(q+i \mathbb{R})$ :

$$
M \in \mathscr{L}\left(\mathrm{~L}_{2 q-1}^{2} ; \mathrm{L}^{2}(q+i \mathbb{R}, \mathrm{~d} x)\right)
$$

Defining $\tilde{G}$ as

$$
\tilde{G}(z)=\mathbb{P}\left(c_{0} \operatorname{erfc}\left(\frac{1}{2 \sqrt{D} z}\right)\right)
$$

and rescaling time $t$ in $t^{2}$, we obtain a convolution equation very similar to (6):

$$
\begin{equation*}
M \rho(s+1)=\frac{1}{2} \frac{M \mathrm{I}_{0}[\rho](s / 2)}{M \tilde{G}(s)} \tag{7}
\end{equation*}
$$

A close study of the transform of $\tilde{G}(s)$ allows us to establish the following two theorems, which provide information about the behaviour of the inverse problem associated with model (4).

Theorem (Non observability). Let $r<1$ be fixed. For every non-negative integer $k$ there exists no constant $C_{k}>0$ such that the observability inequality:

$$
\left\|\left(I_{0}[\rho]\right)^{(k)}\right\|_{L^{2}\left([0, \infty), t^{\left.2 k+\frac{r-3}{2}\right)}\right.} \geq C_{k}\|\rho\|_{L_{r}^{2}}
$$

holds for every function $\rho \in L^{2}\left([0, \infty), x^{r}\right)$.
Note that this result shows that $\mathrm{I}_{0} \in \mathscr{L}\left(L_{r}^{2} ; L_{\frac{r-3}{2}}^{2}\right)$, and that if the inverse problem were identifiable (i.e, $\mathrm{I}_{0}$ were injective), then $\mathrm{I}_{0}^{-1}$ could not be continuous.

Theorem (Identifiability). Let $r<0$ and $\rho \in$ $L^{1}\left([0, \infty), x^{r}\right)$ be arbitrary. If there exists a nonempty open subset $U$ of $(0, \infty)$ such that for all $t \in$ $U, I_{0}[\rho](t)=0$, then $\rho=0$ almost everywhere on $(0, \infty)$.

## A path forward

The Mellin transform has been successful in mathematically analyzing models (4) and (5), allowing us to answer questions of existence (observability), uniqueness and identifiability of the distribution of ion channels along a cilium, as well as stability issues associated with both direct and inverse problems in these models. However, the big question does not seem to be exactly this. Rather, it is about whether, by using and studying these models, Mathematics truly helps to improve our understanding of the olfactory system and, in general terms, the Real World. In this sense, Kleene's experiments have been a great contribution, albeit insufficient. Much stronger validation of the models is required, which can only be achieved by forming multidisciplinary teams and designing ad-hoc experiments.

## Acknowledgements

The authors are very grateful to Dr. Ziomara Gerdtzen, a biochemist at the University of Chile, for her valuable contributions to the section on transduction of olfactory signals.

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## Thibault Bourgeron

Thibault is a postdoctoral research fellow at The National Center for Scientific Research (CNRS), housed at Ecole Normale Supérieure de Lyon. His research interests lie in mathematical and numerical analysis of partial differential equations. Outside of the office he likes to travel and hike.


## Carlos Conca

Carlos is a professor of Mathematics at the University of Chile. His research interests lie in between mathematical physics, mechanics, biomathematics and partial differential equations. When he is not teaching math, he is outdoors working in the garden or taking care of his various pets, which include three turtles and five colorful Chinese koi fish.


## Rodrigo Lecaros

Rodrigo is a profesor of Mathematics at the Universidad Técnica Federico Santa María of Chile. His research interests lie in control theory, inverse problems and numerical analysis of partial differential equations. In his free time he likes to swim and watch science fiction movies.

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## New in MATHEMATICS from Oxford



# Reciprocal Societies: The Swedish Mathematical Society 



The Swedish Mathematical Society (SMS) was founded in 1950 and has a bit over 500 members. For a long time Swedish mathematics was concentrated at the universities in Lund and Uppsala, but by the end of the 19th century Stockholm, and later Gothenburg, were growing into large departments as well. The SMS helped bring together members from these, and later many more, universities, as well as a number of mathematics teachers from schools around the country. The first President of the society was Arne Beurling, who served from 1950 to 1952. The tradition has become to elect a new President every two years and always pick the new president from a different department than the current one.

The SMS has two member meetings per year, the larger annual meeting in late May or early June, and an autumn meeting in November. Each meeting has two parts, first a scientific one with talks on different mathematical topics, and after that a business

## Bulletinen

 meeting where the members vote on various issues. The annual meeting hosts the award ceremony for the Wallenberg Prize, awarded for research by a promising younger mathematician. The 2017 Prize was awarded to Maurice Duits, based at KTH in Stockholm, for his work in the theory of random matrices. The prize winner is also the main speaker of that year's autumn meeting, which is held the day before the finals of the Mathematical Competition for Schools. This is an annual mathematics competition for school children, grades 10 to 12, which the SMS has arranged since 1961.

Apart from these regular meetings the SMS also arranges conferences and other scientific meetings
together with other mathematical societies. The largest of these is the Nordic Congress of Mathematicians, which is arranged together with our sibling societies from the other Nordic countries. The different countries take turns in hosting the four day congress. The most recent congress was held from 16-20 March 2016 in Stockholm.


Participants at the joint meeting with the Catalan and Spanish mathematical societies

This meeting had approximately 550 participants and was part of the centenary celebration for Institut Mittag-Leffler, the mathematical research institute situated just outside Stockholm. As an example apart from the recurring congresses, from 12-15 June 2017 the SMS arranged a joint meeting with the Catalan and Spanish mathematical societies. This meeting was arranged at Umeå University and had approximately 180 participants, with equal participation from the three societies. Three times per year the society publishes a member bulletin, primarily in Swedish and now in electronic format. The Society also awards a number of yearly conference travels grants for PhD students. The SMS website is www.swe-math-soc.se.

Klas Markström
President of the Swedish Mathematical Society

Editor's note: the LMS and the SMS have a reciprocity agreement meaning members of either society may benefit from discounted membership of the other.

Microtheses and Nanotheses provide space in the Newsletter for current and recent research students to communicate their research findings with the community. We welcome submissions for this section from current and recent research students. See newsletter.lms.ac.uk for preparation and submission guidance.

## Microthesis: Homology of Coxeter and Artin groups

RACHAEL BOYD

Symmetric and braid groups are fundamental objects in mathematics and physics. Their generalisations in the form of Coxeter and Artin groups also have many applications in these areas. In my PhD, I have undertaken two projects on the homology (an important group-theoretical invariant) of Coxeter and Artin groups.

## Coxeter groups

A Coxeter group is a group generated by reflections, and 'braid type' relations (generalisations of the relations in the braid group). A Coxeter diagram encodes these group relations. It has one vertex for every generator and the edges describe the relations:


The braiding relations, encoded in a Coxeter diagram

Alongside the relations shown in the diagram we require all generators $s$ to satisfy $s^{2}=e$, that is the generators are 'reflections'. The symmetric groups are the simplest examples of Coxeter groups. These are generated by transpositions which have order 2. Here are two examples of Coxeter groups, including the diagram, the group relations and some geometric intuition on how these groups can be imagined:


[^2]

This infinite Coxeter group tiles the plane into triangles
There is a classification of finite Coxeter groups, due to Coxeter himself in 1935. This was integral to my work and is described below.


Every finite Coxeter group is represented either by one of these diagrams, or a product of two or more.

## Low dimensional homology of Coxeter groups

The homology of a group is a well studied invariant with important links to other areas of mathematics. In the first project of my PhD, [1], I calculated
the $2 n d$ and $3 r d$ homology groups of an arbitrary Coxeter group. My formulas have as input the Coxeter diagram, which they cut up and change into many smaller diagrams, loosely corresponding to subgroups of the Coxeter group. The output is then a sum of homologies of these small diagrams, which by some machinery (a spectral sequence) gives the homology of the original group.

## Artin groups and monoids

For every Coxeter group there is a related Artin group. In these the generators are no longer reflections but have infinite order, and the same braiding relations hold. Therefore the same diagrams represent the Artin groups. For the symmetric group, its corresponding Artin group is the braid group:


Diagram for the symmetric group (Coxeter case) and the braid group (Artin case)

The corresponding Artin monoid is the monoid with the same presentation as the Artin group, in particular its group completion is the Artin group. I am interested in sequences of Artin groups and inclusions, corresponding to the diagrams below, which naturally generalise some of the most studied 'finite type' sequences of Artin groups.


The sequence starts with any Artin group, and adds the generator and relations of a braid group with increasing number of strands

## Homological stability

Homological stability is a well-studied phenomenon which holds for many families of groups. A family of groups or monoids

$$
G_{1} \hookrightarrow G_{2} \hookrightarrow \cdots \hookrightarrow G_{n} \hookrightarrow \cdots
$$

is said to satisfy homological stability if the induced maps on homology $H_{i}\left(B G_{n}\right) \rightarrow H_{i}\left(B G_{n+1}\right)$ are isomorphisms for $n$ sufficiently large compared to $i$.

Homological stability holds for the symmetric groups and braid groups:


Sequences of symmetric and braid groups, and inclusions satisfying homological stability

More generally it holds for sequences of Coxeter groups with the diagrams shown previously [3].

## Homological stability for Artin monoids

In the second project of my PhD, [2], I proved that homological stability holds for sequences of Artin monoids whose diagrams have the form shown previously. There is an important conjecture in the theory of Artin groups called the $K(\pi, 1)$ conjecture, and where this conjecture holds my result proves homological stability for the corresponding sequences of Artin groups. The key step in the proof of this theorem is to show that a certain family of semisimplicial spaces on which the monoids act is highly connected.

## FURTHER READING

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## Rachael Boyd

Rachael Boyd is a PhD student at the University of Aberdeen, under the supervision of Richard Hepworth. Her main research interests are in algebraic topology and geometric group theory. Rachael is Scottish, sings for a rock band Mammuten and writes the blog Picture this maths, together with Anna Seigal who is a PhD student at University of California, Berkeley. You can find it at picturethismaths.wordpress.com.

This section is for Early Career Researchers. Please send suggestions for questions or topics you would like to see covered to newsletter@lms.ac.uk.

# Excelling at interview 

"Dear X, I am a PhD student/postdoc. I'm applying for jobs outside academia. Can you suggest ways I can do well at interview? What sort of questions should I expect?" - We invite perspectives from professionals with experience as interviewer and as interviewee.


Anna Railton is a Consulting Mathematician at the Smith Institute for Industrial Mathematics and System Engineering. She has a PhD in astrophysical fluid dynamics from Cambridge.

You should definitely expect to be asked to explain your PhD/postdoc research at some point. You will need to be able to give a concise and clear explanation of it to someone who is, almost certainly, not in your field and is also unlikely to be a mathematician. Practise both a short ( 1 minute) and longer version of your elevator pitch to friends and family and ask them for feedback. Being able to communicate your work clearly to a potentially non-technical audience is essential in industry and this is the perfect opportunity to prove you have these skills.

Employers will also want to know how you can apply your problem solving skills to their real world problems. So it is essential to research the sector and the companies where you are applying for jobs. Think about the sort of problems they may have and how you could solve them.

I have personally been caught out in interviews by forgetting some basics from first year undergrad. Avoid this frustration by brushing up on foundational topics you may not have given any thought to for a number of years. For example, can you still solve a differential equation, explain how you fit a curve to data, or solve simple probability/combinatorial problems? A small amount of research into the sector you are applying to can give some indication of what you might be asked about.


Tim Smith is a Fellow of the Institute and Faculty of Actuaries and has worked across pensions and life insurance. He has a BSc in Physics from Warwick, and an MSc in Actuarial Science from Imperial College.

There are three things that I would be particularly interested in exploring with you. The first is what your research could contribute to the role that you have applied for. This could be because it is directly relevant to the job, but more likely it is something tangential. The statistical techniques employed in PhDs from many fields are often a lot more advanced than those I see in the office for example, and it is worth thinking before the interview of areas where you think you could add value quickly.

Secondly, I would be interested in your motivation for changing direction in your career. After making a significant commitment to academia, a decision to move into the very corporate world of financial services is a big one. The mention of money as a motivating factor is often avoided, but I find it refreshingly honest if people acknowledge this. Go into the interview armed with a clear narrative of what has driven you to apply for the role and why you think you will enjoy it.

Finally, it is important that you understand the study requirements for the role you have applied for. If you are applying to become an actuary, for example, then you can expect another three to five years of study, a significant proportion of which will be in your own time. Do your research into everything the role entails, and be prepared to demonstrate this knowledge.

## Success Stories in Mathematics

What does it mean to be a successful mathematician? What is involved in a successful mathematical career? The LMS Success Stories project aims to celebrate the diversity of successful careers and mathematicians. We are always interested in new profiles! If you have an idea, or would like to submit your own profile, please email Success.Stories@lms.ac.uk.

## Name: Helen Webster

Job: Senior scientist in atmospheric dispersion \& air quality, Met Office


I loved mathematics from an early age, relishing the academic challenge and being absorbed by its beauty and logical structure. Choosing to pursue a Mathematics degree at Oxford University was, therefore, an easy decision. After my degree, I completed a PGCE in Secondary Mathematics before studying for a PhD in Applied Mathematics at the University of Kent in Canterbury.

At the Met Office, I am able to conduct scientific research and to see the practical benefits to society. As a research scientist in atmospheric dispersion, I seek to improve our ability to give good advice and predictions of the atmospheric transport of poten-
tially hazardous substances in the atmosphere. My work is interdisciplinary and involves applying my mathematical knowledge alongside physics, meteorology, environmental science and computing, to name but a few. I love the variety in the work and the fact that I am always learning new things. I also have the opportunity to present my work at scientific conferences and to publish in refereed journals. Recently, I spent four months on a secondment based at the US Geological Survey, collaborating with volcanologists to improve our modelling of volcanic ash clouds.

Aside from my day job, I am also a STEM ambassador which enables me to inspire young people and to promote STEM subjects by sharing my enthusiasm for my job and for mathematics. Outside of mathematics, I am a keen water skier and am actively involved with my local church.

## Name: Allison Henrich <br> Job: Associate Professor of Mathematics, Seattle University



I feel like I am successful because l've had amazing support and guidance from colleagues, collaborators, mentors, and professional development organisations. Without fail, my colleagues have advocated for me, to help me earn promotion and tenure, to help me win a national teaching award (the MAA's Alder Award), and to help me feel like I'm supported in general on a day to day basis. My collaborators across the world have been a constant source of inspiration for new research ideas. They've kept me excited about making time for my research, despite
the demands that teaching, service, and administration place on my time. My mentors have also been an invaluable resource. They have taught me how to win grants (like an REU grant from the National Science Foundation), how to mentor undergraduate researchers, and how to become more connected in the math community. I have been inspired to become a better teacher, mentor, and colleague through my involvement with the Mathematical Association of America, Project NexT, the Council on Undergraduate Research, and the Academy of Inquiry-Based Learning. This is all to say that nobody can become successful alone. I am constantly in awe of my good fortune, as I have been able to surround myself with people who support the work I do and help me push my own limits of success.

# The Real and the Complex: A History of Analysis in the 19th Century 

by Jeremy Gray, Springer, 2015, pp368. £24.99, ISBN 978-3-31-923714-5

Review by David Singerman



This book is based on a course on the history of mathematical analysis given at the University of Warwick. When I taught history of maths it was regarded a soft option; a third year pure course without a lot of highly technical mathematics. A student going to Jeremy Gray's course hoping for an easy ride would have been in for a shock! There is a lot of 19th century maths here which is quite difficult to follow for a modern reader and on top of that some deep historical content.

When a student begins studying analysis they usually get a shock when first presented with the definition of a limit - "For all $\varepsilon>0$ there exists $\delta>0$ such that...". On reading this book we realize how mathematicians grappling with the limit concept took a very long time to fully understand what they were doing. d 'Alembert (1754) wrote "One magnitude is said to be the limit of another magnitude when the second may approach the first within any given magnitude, however small, though the first may never exceed the magnitude it approaches so that the difference of such a quantity from its limit is absolutely unassignable." (Try teaching this to the first year!) This was rejected by Lagrange as being too vaguely geometric and geometry like motion was "foreign" to the very spirit of analysis. Lagrange then tried a more algebraic approach. It had to wait until Cauchy's 1821 Cours d'Anayse before something like our modern epsilon-delta definition began to emerge. Of course, mathematicians had been grappling with these ideas since Newton and Leibniz had introduced calculus in the seventeenth century. So perhaps students should be told that when they are grappling with the limit concept they are not the first who found the whole idea very difficult.

We have to wait until page 59 before complex variables appear. In 1825 Cauchy produced the integral and residue theorems but did not appreciate the depth of his discovery perhaps because he was not thinking geometrically about complex variables. In this history it is claimed that complex analysis really developed in a deep way when geometric ideas were introduced. This was mainly through the efforts of Riemann. In fact, Riemann plays a major role in his book. Chapter 15 is just called "Riemann", in which Riemann's paper on trigonometric series is discussed, including the Riemann integral. Chapter 16 is called "Riemann and complex function theory" which discusses his doctoral dissertation of 1851, where it is written that in it he gave a "complete, clear introduction of complex function theory as an autonomous domain in mathematics." There is a discussion of the Riemann mapping theorem and a description of Riemann surfaces. Chapter 17 is called "Riemann's later complex function theory" where he discusses Riemann's 1857 paper on Abelian functions (described as undoubtably one of the most important papers on mathematics published on the 19th century). Chapter 18 is called "Responses to Riemann's work". Chapter 19 and 20 are devoted to Weierstrass. Chapter 19 starts "A powerful algebraic alternative to Riemann's geometric complex function theory was developed by Weierstrass." Later in the book there are chapters on the construction of the real numbers due to Dedekind, Lebesgue's theory of integration and then Cantor's set theory and foundations with a description of the continuum hypothesis. These are only some of the mathematical ideas discussed.

As this book comes from a course on the history of mathematics, there are sections called "Revision and assessment". These often contain significant truths when approaching history. One I particularly like is "Just as we should not treat a famous mathematician from the past as a genius incapable of error, we should not treat them as failures for not seeing what later mathematicians saw, or what we value today.

They did not say everything and not everything they said was right: that is how research is done". Another useful piece of advice is "a good way to think historically is to try and imagine what things looked like in the past without the benefit of modern knowledge".

Students are encouraged to look at sources and for this reason there are valuable translations of sections of important papers. These are (1), Fourier's work on Fourier series (1822) An analytical Theory of Heat.
(2) Dirichlet's work on Fourier series On the convergence of trigonometric series... of 1829, (3) Riemann's Göttingen 1851 dissertation Foundations for a general theory of a variable complex quantity, (4) Riemann on the definition of an integral in his 1854 paper on the representability of a function by a trigonometric series, (5) Schwartz (1869) Über einige Abbildungsaufgaben where he found formulas for mapping triangles (say) onto the unit circle, thus illustrating the Riemann mapping theorem.

This is a deep book on the history of mathematics. There are some books on this topic which are easy reads. This is not one of them. Each mathematician discussed will have only a very short biography written about them. The mathematics is the important thing, not the personalities.

Summing up: an outstanding book which will profitably be read by anyone having to teach real or complex analysis.


## David Singerman

David Singerman is an emeritus professor at the University of Southhampton. His main interests have been on Fuchian groups and Riemann surfaces. He has recently retired as reviews editor for the Newsletter, but still serves on its editorial board.

# Leonhard Euler: Mathematical Genius in the Enlightenment 

by Ronald S. Calinger, Princeton University Press, 2016, pp696, £45.95, ISBN 978-0-69-111927-4

Review by Ciarán Mac an Bhaird



Ronald Calinger has written an imposing text on Leonard Euler (1707-83), one of the true giants of pure and applied mathematics. This book is very ambitious as it attempts to give a comprehensive overview of Euler's life and work, and the political, social and cultural influences on an institutional, national and international stage. The book's fifteen chapters follow a chronological order, for the most part, covering Euler's early life in Switzerland (chapter 1), his two periods in St. Petersburg (chapters 2-5, 13-15) and his 25 years in Berlin (chapters 6-12).

Throughout the book, in addition to Euler's extensive mathematical research, Calinger gives us a very real sense of Euler's extraordinary work ethic and his diverse set of skills. Euler seems to have been an excellent and willing administrator in the academies. His contributions ranged from ordering paper and ink, and selecting and ordering trees for academy avenues, to suggesting and pursuing new and suitable academy staff. He wrote important textbooks on school-level mathematics, appears to have been a dedicated teacher, and was assigned extra duties such as trying to fix fountains and reviewing bridge plan feasibility. In addition, we also get some insight into Euler's family life, his happy marriage to Katharina, the heart break of the death of so many children, his illnesses and how he managed with sight loss, which he apparently described as one less distraction.

We also get detailed descriptions, placed in context, of Euler's extensive correspondences. We read of his very close relationship with the Bernoulli family of mathematicians, particularly Johann I and Daniel. From a mathematical and scientific point of view, Euler's correspondences with Voltaire, D'Alembert, Clairaut, Lambert, Lagrange and Goldbach are fascinating. Significant sections of the book also provide exhaustive details on Euler's involvement in some of the major scientific and philosophical controversies of the time, for example debates on the validity of Newton's second law, and a long running disagreement over Wolffian philosophy and beliefs. Euler spent a considerable amount of time and effort on these issues, which was not always appreciated by others, and he experienced further professional frustrations with the Russian Orthodox Church during his first period in St. Petersburg and with Frederick II of Prussia.

The main focus of the text is on Euler's incredible research output. In mathematics, we get a taste of his significant contributions to analysis, calculus (including the calculus of variations) and differential equations, as well as his momentous work on infinite series. Furthermore, there are frequent discussions on Euler's interest and work on number theory, especially his interest in Fermat's conjectures and primes, which few at the time considered to be serious mathematics. Euler's work in these areas and in many others laid the basis for their treatment as modern mathematical topics. There is so much more that is touched on in this text, for example Euler's work on topology and his inspired treatment of complex numbers. There is also his definition, use and manipulation of functions which became central to much of his work, his role in using notation (for example, using $e$, promoting the use of $\Sigma$ and $\pi$, etc.) and his ability to write mathematics in a more modern and accessible manner.

Furthermore, we see how Euler applied his mathematical methods and insight to other areas of research. We have his extensive work on applied mathematics such as mechanics, optics and astronomy, and further work on cartography, shipbuilding and navigation, telescopes and ballistics, and his contributions to music theory and accounting practices. What is remarkable about Euler's research is not just the depth of his contributions, or the wide range of topics, but
also the extensive number of publications, over 850 (in addition to his correspondences). The significant delays in getting completed texts published in the 18th century are also very well highlighted by the author, though, in places, there are descriptions of Euler's works which are repeated unnecessarily.

Reading this book, you will be left in awe of Euler, and wonder where he found the time to do so much in his life. The author deserves praise for tackling such a serious project with huge amounts of detailed and complex material. One challenge for such a project is that it is almost impossible to deal with every specific topic in depth. The author, having decided to put details in chronological order, clearly had no choice but to focus on some areas more than others. As a result, some readers may not find the detailed analyses they might expect on certain mathematical topics, but that detail is available elsewhere in works that focus on specific aspects of Euler's life and work.

Despite these issues, the decision to write the book in chronological order still seems sensible, certainly it is difficult to determine a better approach for a book of this ambition. Once readers understand that the book provides an overview of Euler's life and work placed in historical context, I am sure that they will enjoy reading it as much as I did. When my students are studying Euler's mathematics, while they may be directed to other texts for detailed analyses of his mathematical methods, I will certainly recommend this book to them as an excellent reference resource on all matters related to Euler's life and work.


## Ciarán Mac an Bhaird

Ciarán Mac an Bhaird is a lecturer in mathematics at Maynooth University and Director of the Mathematics Support Centre. His current research interests are mostly in mathematics education, but he also conducts research in algebraic number theory and in the history of mathematics. Ciarán is from Co. Monaghan, plays sport on a regular basis and works on the family farm.

## Obituaries

William Preston Eames: 1929-2017



William Preston Eames, who was elected a member of the London Mathematical Society on 17 December 1959, died on 7 March 2017, aged 87.

The first in his family to go to university and funding himself completely on scholarships, Bill went to Brandon College for chemistry and mathematics. Here he made lifelong friendships, was the class valedictorian and was recognized with many gold medals. He found he loved the simplicity and creativity of mathematics and went on to University of Manitoba and subsequently to Queens, where he completed his PhD in analysis at the age of 24 . As part of the first group of international NSERC scholars, Bill moved to London to complete his postdoctoral work at King's College, University of London. Bill loved the city, the theatre, his Austin Healey, and his pottery teacher, Jane Coles. They married and lived in Blackheath, having two children, Gillian and Madeleine. In 1966 they moved back to Canada settling in Thunder Bay (Port Arthur), Ontario. Bill was Chair of the Mathematics Department at Lakehead University for many years where he had good friends and colleagues. He inspired many young people with his love of mathematics and creative ways of teaching.
[Extract from the Thunder Bay Chronicle Journal.]

$$
\text { T. Noel Murphy: } 1934 \text { - } 2017
$$



Noel Murphy, who was elected a member of the London Mathematical Society on 21 November 1968, died on 1 June 2017, aged 82.

Rosa Garrido writes: Born in Dublin, Noel left his home and his family at the age of 15 to pursue his studies in Liverpool. He went on to graduate from Liverpool University with a first-class honours BSc in 1961. After periods at the University of Liverpool
and Memorial University in St John's, Newfoundland, Noel was professor of Mathematics at Trent University, Ontario from 1966 to 1999. For many years, he collaborated with the analysis group at the Université Pierre-et-Marie Curie. While at Trent University, he served as chair of the Department of Mathematics and as President of the Trent University Faculty Association. Upon his retirement, he was made Professor Emeritus by the Trent University Senate.

## Christopher T.H. Baker: 1939-2017



Christopher Thomas Hale Baker, who was elected a member of the LMS on 18 January 1980, died on 20 August 2017, aged 78.

Nick Higham (University of Manchester) and Neville Ford (University of Chester) write: Christopher was born on the Isle of Thanet, Kent, and was educated at Colchester Royal Grammar School and Jesus College Oxford, where he held an Edwin Jones Scholarship and a State Scholarship.
He obtained his BA in 1961 and his MA and DPhil, in 1964, from the University of Oxford. Between 1964 and 1966 he held a Fulbright Award and was Instructor and PG Research Mathematician at UC Berkeley. From 1966 Christopher was lecturer, senior lecturer and then reader at the University of Manchester, becoming professor in 1989. He had periods of leave at the University of Toronto (in 1972 and 1976) and Oxford University.

Christopher served as head of the numerical analysis group for around ten years and served as Head of Department for three years from September 1995. Following his retirement in 2004, Christopher joined the University of Chester as a part-time member of the department, retiring from that role in 2016. At the time of his death he held the title of Emeritus Professor at both the University of Manchester and the University of Chester. Christopher was founding Director of the Manchester Centre for Computational Mathematics (MCCM), formed in 1992 by the numerical analysis groups at the University of Manchester and UMIST to build on existing collaborations. In his ten years as Director, the centre grew substantially in activity, as seen particularly in the Numerical Analysis Report series, and the MSc in Numerical Analysis and Computing. Christopher was instrumental in involving
external researchers in MCCM, notably the Chester numerical analysts.

His research interests included numerical solution of integral equations and functional differential equations (integro-differential and delay-differential equations), and parameter estimation in models. He is perhaps best-known for his monumental 1,034 -page monograph Numerical Treatment of Integral Equations (Clarendon Press, Oxford, 1977). He was able to expand some of the tools and techniques developed for integral equations into newly emerging fields of numerical dynamics and numerical methods for stochastic differential equations.

Christopher was a member of the 1992 Mathematics Assessment Panel in the UFC Research Assessment Exercise and of the Applied Mathematics panel in the 1996 Research Assessment Exercise. He chaired the Applied Mathematics panel in the 2001 Research As-
 sessment Exercise. Serving on three successive panels was a major service to the mathematics community. Some idea of this is given by Christopher's comment in the 2002 MCCM annual report, "During most of 2001, every flat surface at home and in my office was covered with RAE paperwork".

He was a Fellow of the Institute of Mathematics and its Applications and served as editor of the IMA Journal of Numerical Analysis from its foundation in 1981 to 1996. He was a dedicated editor, also giving long service to other journals including Journal of Computational and Applied Mathematics, Journal of Integral Equations and Applications and Advances in Computational Mathematics.

He had 15 PhD students (including the second author), from all around the world, and he continued collaborating with many of them. His careful supervision encouraged students to play to their strengths and to answer research questions which other people would find to be interesting. The second author remembers being challenged repeatedly by his question 'what do you mean by ...' (stability, for example) reflecting his determination to understand the underlying mathematics before venturing an opinion on a numerical scheme.

Christopher had heart bypass surgery in 1988 and the surgeon told him "We know these vein grafts last for 12 years". Thankfully, that was a severe underestimate, and Christopher maintained all his usual activities right until the end.

Christopher will be remembered as a kind, generous, and sociable colleague as well as for his leadership in applied mathematics and numerical analysis in Manchester, Chester, across the UK, and beyond. He is survived by his wife Helen, his children Deborah and Mark, and four grandchildren.

## Young Theorists' Forum

Location: Durham University
Date: $\quad 10-12$ January 2018
Website: tinyurl.com/y78da2la
The purpose of YTF is to bring together postgraduate students working in theoretical physics, providing them the opportunity to present their work to a friendly audience. For more information, visit the website or email durhamytf@gmail.com.

## Extreme Value Theory: Recent Challenges and Spatial Applications

Location: Cardiff University
Date: 6 February 2018
Website: tinyurl.com/y82r463l
The meeting will be a half-day workshop that takes a closer look at extreme observations in spatial data and different approaches to model their dependencies. The meeting is supported by an LMS Celebrating New Appointments Scheme 1 grant.

## Mathscon

Location: Imperial College London
Date: $\quad 10$ February 2018
Website: mathscon.com
Mathscon is a conference celebrating the beauty of mathematics and its varied applications. The day will comprise of interactive panel discussions and workshops on a range of topics. Tickets will be released soon; follow facebook.com/themathscon/ to stay updated.

## Mary Cartwright Lecture

Location: LMS, De Morgan House
Date: 2 March 2018
Website: tinyurl.com/cartwright18
The 2018 Mary Cartwright Lecture will be given by Carola-Bibiane Schönlieb (University of Cambridge) on Model-based learning in imaging. Attendance is free; to register, contact John Johnston (womeninmaths@lms.ac.uk). The meeting will be followed by a reception and dinner at $£ 35$ per head; see the website for details.

## Recent Trends in PDE

Location: King's College London<br>Date: 8-11 January 2018<br>Website: tinyurl.com/LondonPDE2018

This conference aims to bring together specialists and promising young mathematicians in the field of partial differential equations, with an emphasis on problems motivated by mathematical physics. Registration is free and some funding is available to junior participants.

## London Stringology Days and London Algorithmic Workshop (LSD \& LAW)

Location: Strand Campus, King's College London
Date: 8-9 February 2018
Website: tinyurl.com/yc6y6q3h
King's College London will be holding the 26th annual LSD \& LAW meeting, supported by an LMS Conference grant. There will be three invited talks. Abstract submission and registration deadlines are on 8 January.

## Mean-field Games, Energy and Environment <br> Location: Alan Turing Institute, London <br> Date: $\quad 12$ - 14 February 2018 <br> Website: tinyurl.com/y86j2zuk

This workshop aims to bring together leading experts in the field of mean-field games (MFG) and their applications. This event is supported by an LMS Conference grant, the Alan Turing Institute and King's College London. Registration deadline: 31 January 2018.

## David Crighton Ceremony and <br> Lecture

$\begin{array}{ll}\text { Location: } & \text { Royal Society, London } \\ \text { Date: } & \text { 6:30pm, } 15 \text { March } 2018 \\ \text { Website: } & \text { tinyurl.com/crighton17 }\end{array}$
The 2017 David Crighton Medal will be presented to Professor I. David Abrahams on 15 March 2018. A talk by Prof. Abrahams will then be followed by a reception. Admission is by ticket only; email Katherine Wright at prizes@lms.ac.uk by 1 March 2018.

## K60: Groups and Cohomology

Location: University of Southampton
Date: 19-21 March 2018
Website: tinyurl.com/yad7oql2
K60: Groups and Cohomology is a meeting to review progress in the areas of mathematics where Peter Kropholler has contributed, and to introduce a new generation to the techniques used and the problems that are still outstanding. The meeting is supported by an LMS Conference grant.

## Near-critical Stochastic Systems

Location: Royal Holloway, University of London
Date: 26-28 March 2018
Website: tinyurl.com/menshikov70
The focus of this meeting will be on presenting recent results and discussing prospective research directions in the study of non-homogeneous random walks and similar near-critical stochastic systems. Partly supported by an LMS Conference grant.

## Analysis Aspects of Dynamics

Location: Imperial College London
Date: April 2018
Website: tinyurl.com/analysisdynamics2018
The conference aims to bring together experts working on analysis and dynamics, and enhance the interaction between these fields. The meeting is partly funded by an LMS Conference grant. Limited funding is available for PhD students.

## Models in Population Dynamics, Ecology, and Evolution Location

Location: University of Leicester
Date: $\quad 9-13$ April 2018
Website: tinyurl.com/y8f9658k
This meeting, supported by an LMS Conference grant, will consider applications of mathematical modelling to explore processes and mechanisms in biological systems. Enquirues to S. Petrovskii: sp237@le.ac.uk.

## Young Functional Analysts' Workshop

Location: Newcastle University
Date: 21 - 23 March 2018
Website: tinyurl.com/yce6j3gy
This annual conference is for PhD students and other early-stage researchers in Functional Analysis and Applications. Registration opens on 20 January 2018; register your interest by emailing yfaw2018@gmail.com. Supported by an LMS Scheme 8 grant.

## BAMC 2018 <br> Location: University of St Andrews <br> Date: 26 - 29 March 2018 <br> Website: bamc.org.uk

See the website for full details of the plenary speakers for BAMC 2018. The organizers are grateful to the supporters of the meeting: LMS Conference grant, Edinburgh Mathematical Society, Royal Astronomical Society, STFC and IMA.

## Easter Probability

| Location: | University of Sheffield |
| :--- | :--- |
| Date: | $9-13$ April 2018 |
| Website: | tinyurl.com/yc2ygwcs |

This meeting will contain three mini-courses. Support is available for ten UK-based research students. The meeting is supported by an LMS Conference grant, the Applied Probability Trust and the Heilbronn Institute for Mathematical Research.

## Multiscale Biology

Location: University of Nottingham
Date: $\quad 16-18$ April 2018
Website: tinyurl.com/y95e9qsc
This conference will bring together scientists addressing multiscale phenomena across a range of biological systems. Direct enquiries to MSB2018@nottingham.ac.uk or reuben.odea@nottingham.ac.uk.

## LMS Meeting

## Northern Regional Meeting

25 May 2018, 2.00 pm, University of Northumbria, Newcastle

Website: tinyurl.com/ya27pwj8
Speakers:
T. Grava (Bristol \& SISSA)
N. Smyth (Edinburgh)
S. Lombardo (Loughborough)

These lectures are aimed at a general mathematical audience. All interested, whether LMS members or not, are most welcome to attend this event.

The meeting forms part of a workshop on Advances in the Theory of Nonlinear Waves on 23-25 May 2018.

The meeting will be followed by a reception and the LMS meeting dinner.

There are funds available to contribute in part to the expenses of members of the London Mathematical Society or research students registered at UK universities to attend the meeting and workshop. Requests for support, including an estimate of expenses, as well as all queries about the two events may be addressed to the organisers: Dr Benoit Huard (benoit.huard@ northumbria.ac.uk) and Dr Matteo Sommacal (matteo.sommacal@northumbria.ac.uk).

## LMS Meeting

Midlands Regional Meeting
4 June 2018, 1.30 pm, University of Leicester

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Website: tinyurl.com/y7xxdnp3
Email: Imsmrm2018@le.ac.uk
Speakers:
Fabrizio Catanese (University of Bayreuth)
Minhyong Kim (University of Oxford)
Leila Schneps (Institut de Mathématiques de Jussieu
Paris)
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These lectures are aimed at a general mathematical audience. All interested, whether LMS members or not, are most welcome to attend this event.

The meeting forms part of a workshop on Galois Covers, Grothendieck-Teichmüller Theory and Dessins D'enfants on 5-7 June 2018.

There are funds available to contribute in part to the expenses of members of the London Mathematical Society or research students registered at UK universities to attend the meeting and workshop. Requests for support, including an estimate of expenses, as well as all queries about the two events may be addressed to the organisers: Frank Neumann and Sibylle Schroll (Imsmrm2018@le.ac.uk).

## Society Meetings and Events

## January 2018

## August 2018

> 10 Society Meeting at the Joint AMS Meeting, San Diego, USA

## March 2018

2 Mary Cartwright Meeting, London
May 2018
25 Northern Regional Meeting, University of Northumbria

June 2018
4 Midlands Regional Meeting, Leicester
13 Society Meeting at the BMC, St. Andrews
29 General Society Meeting \& Hardy Lecture, London

7 LMS Meeting at the ICM, Rio de Janeiro

## October 2018

9 Joint Society Meeting with the Fisher Trust, Galton Institute, Genetics Society and RSS; Royal College of Surgeons, Edinburgh

## November 2018

9 Society and Annual General Meeting, London

## December 2018

17 South West \& South Wales Regional Meeting, Exeter

## Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website (www.Ims.ac.uk/content/calendar). Please send updates and corrections to calendar@lms.ac.uk.

## January 2018

4-6 British Postgraduate Model Theory Conference, Oxford
8-11 Recent Trends in PDE, King's College London (474)
10 Society Meeting at the Joint AMS Meeting, San Diego, USA
10-12 Young Functional Analysts' Workshop , Newcastle (474)
15-19 Theoretical and Algorithmic Underpinnings of Big Data INI Workshop, Cambridge (472)
24 Mathematics in Materials Science, Sussex
25 Scalable Statistical Inference Day, Sussex
26 Day on Markov Chains, Sussex

February 2018
6 Extreme Value Theory: Recent Challenges and Spatial Applications, Cardiff (474)

7 Indra's Pearls: A Mathematical Adventure, Lincoln (473)
8-9 London Stringology Days and London Algorithmic Workshop, King's College London (474)
10 Mathscon, Imperial College London (474)
12-14 Mean-field Games, Energy and Environment, Alan Turing Institute, London (474)

## March 2018

2 Mary Cartwright Lecture, London (474)
15 David Crighton Award and Lecture, David Abrahams, Royal Society London (474)
19-21 K60: Groups and Cohomology,
Southampton (474)
19-23 Statistics of Geometric Features and New Data Types, INI Workshop, Cambridge (473)

21-23 Young Functional Analysts' Workshop, Newcastle (474)
26-28 Near-critical Stochastic Systems: Workshop in Celebration of M. Menshikov's 70th Birthday, Royal Holloway University of London (474)
26-29 British Applied Mathematics Colloquium 2018, St Andrews (474)

## April 2018

$\begin{array}{ll}\text { 3-6 } & \text { British Congress of Mathematics } \\ & \text { Education, Warwick (471) }\end{array}$
4-6 Probability, Analysis and Dynamics '18, Bristol (473)
9-13 Easter Probability, Sheffield (474)
9-13 Models in Population, Dynamics, Ecology, and Evolution Location, Leicester (474)
12-14 NBFAS, Edinburgh (473)
16-18 Multiscale Biology Nottingham (474)
23-24 Stochastic Simulation, Uncertainty Quantification and Computational Imaging, ICMS Edinburgh (474)

## May 2018

21-22 Nonlinear Analysis and the Physical and
Biological Sciences, Edinburgh (473)
25 LMS Northern Regional Meeting, University of Northumbria (474)
28-29 NBFAS, Newcastle (473)

## June 2018

4 LMS Midlands Regional Meeting, Leicester (474)

4-7 Perspectives on the Riemann Hypothesis, Bristol (473)
13 Society Meeting at the BMC, St Andrews
13-15 Modelling in Industrial Maintenance and Reliability, Manchester Conference Centre (473)
29 General Society Meeting \& Hardy Lecture, London

## August 2018

1-9 ICM, Rio de Janeiro (473)
7 LMS Meeting at the ICM, Rio de Janeiro

## September 2018

2-4 Modern Mathematical Methods in Science and Technology, Kalamata, Greece
3-7 Dynamics Days Europe, Loughborough (473)

## October 2018

9 Joint Society Meeting with the Fisher Trust, Galton Institute, Genetics Society and RSS; Royal College of Surgeons, Edinburgh

## November 2018

9 Society and Annual General Meeting, London

## December 2018

17 LMS South West \& South Wales Regional Meeting, Exeter
11-14 Spain-Brazil Joint Meeting, Spain

## August 2019

4-9 Theory and Practice: an Interface or a Great Divide? Maynooth University

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[^0]:    ${ }^{1}$ This article is based on a talk given at Manchester, Reading, Leeds, the sigma conference, and the HEA/HoDoMs New Lecturers' Course. My thanks to those audiences for useful feedback.

[^1]:    ${ }^{2}$ It is worth noting that Bayesian-style Gaussian processes are now heavily used in the machine learning community, giving weight to the amusing, but perhaps unfair, quip that "machine learning is just doing Bayesian statistics on a Mac".

[^2]:    Dihedral groups are finite Coxeter groups

