LONDON MATHEMATICAL SOCIETY

Statement on the teaching and assessment of mathematics degrees

Following requests from many colleagues, the Education Committee of the London Mathematical Society (LMS) has produced this statement to support everyone teaching mathematics in Higher Education. It aims to explain to non-mathematical colleagues some of the distinctive features of the assessment and teaching of mathematics, both as a subject in its own right and in a service teaching context.

This statement highlights, amplifies and interprets material from the Quality Assurance Agency's (QAA) subject benchmark statement for undergraduate programmes in mathematics, statistics and operational research (2015), which can be found at http://www.qaa.ac.uk/en/Publications/Documents/SBS-Mathematics-15.pdf

We do not quote directly from the subject benchmark statement, but instead note the paragraphs in the statement most relevant to each of the points below. We encourage the reader to read this statement alongside the subject benchmark statement.

We bring out four main points below.

Point 1 A student who fails a small number of individual modules, but has an overall satisfactory average, should not be deemed to have failed a degree programme in mathematics.

This point arises from paragraphs 4.3, 4.15 and 4.16 of the mathematics subject benchmark statement.

Despite, in many cases, the best efforts of those designing and delivering their modules and programmes of study, some students will utterly fail to grasp a particular topic by the time they are assessed on it, and a few never will. It is quite possible, and not uncommon, for students to write many pages of what (to the inexpert eye) appears to be mathematics, all of it however devoid of significant meaning and value, and impossible to deem worthy of examination marks.

Of course, a good written examination will incorporate some of the easiest, most elementary material – and yet, in a raw assessment of the proportion of the material understood, especially immediately or soon after their first exposure to it, some students will achieve little.

Neither will any conversion from raw marks to a university scale, defined by qualitative descriptors, be able to turn such a performance into a passing one. We note here that, on such scales, many qualitative subjects have the property that the majority of marks often fall into a relatively narrow middle ranging from 50% to 75%. The situation in mathematics and other quantitative subjects is very different, as raw marks over 90% and under 20% are not at all uncommon.

Yet this poor performance in a small number of modules may be, for these students, an aberration. They may well achieve a passing mark overall in their degree programme. The problem is that the competence-based assessment of a fine, modular sub-division of the programme material, while appropriate for a vocational course, is not appropriate for a difficult academic discipline.

A mathematics student has to engage bravely with substantial, interconnected, daunting constructions. To an academic mathematician, it is the student's having done so as a whole during their programme, the *whole degree programme* learning outcome, which is important. If asked under what circumstances a student's failing one or a few modules should make him/her unworthy of a degree, they will typically reply – 'never'. The same is clearly not true of, say, a medical practitioner's ability to calculate drug doses correctly. Credit-framework models, however appropriate for some programmes, will nearly always be viewed as inappropriate for the assessment of mathematics.

We are deeply concerned about the tendency in some universities to impose on mathematics and other significantly quantitative programmes the requirement that students pass essentially all modules, regardless of their overall average score. This changes the nature of the study of mathematics in our country; it encourages the removal of all challenging material from modules, and encourages the students to 'play safe' in their choice of modules.

Point 2 On occasion, a specific module should be available to more than one year of a mathematics degree.

This point arises from paragraph 3.3 of the mathematics subject benchmark statement.

Because of the cumulative, sequential nature of mathematics, it can sometimes be difficult to assign a clear level – a natural year of study in the undergraduate programme, say – to a particular topic. In a sense a mathematical topic fits quite naturally into a modular system: it has its pre-requisites, and in turn other modules, perhaps many, may require it. But it may be inappropriate to fix its horizontal level in this web. A module may quite naturally be studied in different years for different students and on different programmes, and yet still make appropriate contributions to different students' differing sub-webs of connected modules.

Point 3 Integrated masters and stand alone masters programmes in mathematics should not necessarily be obliged to reach the frontiers of knowledge.

This point arises from paragraph 2.4 of the mathematics subject benchmark statement.

Given the sequential nature of mathematics and its development over a very long time span, it is unsurprising that in some areas of the subject the frontier could require five or more years of rigorous study to reach. There are no royal roads, no short-cuts: to study the higher levels students must have the experience and expertise that come with having re-constructed each lower level for themselves.

Thus, while both integrated masters and stand alone masters programmes will have developed the students' mathematical maturity, and given them some sense of the nature of the research frontier, they will certainly not have reached it along a broad front. Some may, in a few modules, in specific examples in a lecture, or in a final-year project, have reached it in isolated thrusts, but it is unrealistic to expect more than this.

Point 4 Despite the agreed importance of modern, computer-based teaching and learning, lectures delivered using clearly visible boards should continue to play an important role in the teaching of mathematics.

This point arises from paragraph 4.10 of the mathematics subject benchmark statement.

We make no specific claim that chalk boards are preferable to white boards. We also acknowledge that there are other methods of presentation, such as visualizers or data projectors for displaying prepared presentations. This allows the lecturer use the board to illustrate and expand on material being presented via the display equipment, to the benefit of the students.

However, for those rooms equipped with both boards and display equipment, it remains important that the boards and screens do not overlap, so that the lecturer can use the board at the same time as the screen without overlap.

Computer packages, such as Matlab, Mathematica, Maple and R, allow students to solve complicated problems, gain insight and explore proofs and conjectures, and their use should be encouraged.

Nevertheless, it is crucial to recognise that the teaching of the underlying mathematics requires a substantially different and active approach, which must be emphasized continually throughout the students' programme of study. It is not sufficient to leave it to chance by requiring that theorems are studied passively by working through printed notes or a textbook, which may be altogether too hard a task.

Rather one needs to see someone else, the lecturer, working through and *creating* the results. During all years of study the good lecturer will develop theorems by constructing the argument for the students, incrementally and in real time, and will describe at various levels what is being achieved. Like a good general, they will explain not only the details but also the tactics and the overarching strategy, and thus will help the students to gain a fuller understanding.

The lecturer must be able to create, to write out, during the lecture itself, a large body of argument, and many lecturers prefer to move around the theatre, sharing the students' view of the material. Ideally, much of what is written during a fifty-minute lecture will still be visible at the end: the lecturer will often be referring back to earlier material, not only to the detail but also to the thrust and tenor of it.

Such a board-based teaching style naturally results in an appropriate pace, suitable for the students to take their own notes and aiding engagement with the material. To deliver this style of lecturing the lecturer will require several boards, with a large total area, on which writing is clearly visible to all of the students.

Universities are increasingly putting in place the technology to record lectures. We acknowledge the value for students of being able to replay a lecture, particularly allowing them to review those parts of a lecture that they initially didn't understand. We take no position against the recording of lectures per se. However, we do feel that the lecturer needs to retain the freedom to move during the lecture, making good use of the board and the material written on the board.

Effective lecturing of mathematics requires having large areas on which to write, and large areas within which to move. Therefore, we cannot support the mandatory recording of lectures until such time as the technology of lecture recording allows for lectures to be recorded as the lecturer feels they need to be delivered. The desire to record lectures should not dictate the way in which lectures are given, and should not lead to compromises in the quality of the lecture experience of the students who attend the lecture.

Nevertheless it is important that students also have easy access to summaries of lecture notes or appropriate text books, where the results of the lecture are clearly and correctly stated (noting that students often make mistakes when copying from the presentation)

We do not encourage a general policy of provision to the students of full and detailed printed versions of the lectures in advance of the lecture (for example, by making them available on the internet or a virtual learning environment): this can give some students the impression that a quick glance at these notes is a satisfactory alternative to attending the lecture and engaging with the process of creating the material.

However, we are sensitive to the legislative changes that have occurred since 2010, particularly with regard the requirement that universities make reasonable adjustments for students with disabilities. Some universities may require the provision of printed lecture notes in advance of lectures, and in such cases, we can only ask that lecturers of mathematics follow their university requirements and adapt their teaching as appropriate. For instance, lecturers may make skeletal notes available to students before lectures, to be annotated by students during the lectures.

We learn mathematics by doing mathematics, and the lecture material will therefore be accompanied by problems and examples which tease out the context of the argument - why it works, where it does not, its generalizations and specializations. In the end the lecturer will have created a set of course materials which embody a perfect argument, and the students will have seen and learned about the construction of that argument, and its uses and implications.

Our conclusion (for the reasons outlined above) is that, when used in conjunction with the technological and other developments associated with learning mathematics, lecture boards remain an important technology for teaching mathematics in an exciting and interactive way, leading to a good understanding of the subject.

Education Committee
London Mathematical Society
Revised May 2017