
mathsinside
www.mathspromotion.org.uk

SEEING THE DETAIL

Using mathematics to uncover the secrets of the universe

angular resolution | Rayleigh criterion | wavelength | radians | aperture | radiowaves | array

Top image: ALMA antennas under the Milky Way, ESO/José Francisco Salgado (josefrancisco.org). Bottom image: ALMA site, ALMA (ESO/NAOJ/NRAO)/L. Calçada (ESO).

In space what appears as one object can actually be two objects really close together. Take stars: seven out of 10 have a companion too close to be seen with just your eyes. To see the two stars you need a telescope with a high enough *angular resolution*.

To work out the angular resolution of a telescope, often denoted by the Greek letter θ (Theta), astronomers use a mathematical equation called the *Rayleigh criterion* which says that:

$$\theta = 1.22 \frac{\lambda}{D}$$

where λ is the *wavelength* of light the telescope sees and D is the diameter of the telescope's *aperture* – the maximum extent of the telescope that light falls on; the resolution in this equation is measured in *radians* (180 degrees is equal to /pi radians, so 1 radian is approximately 57 degrees). Changing the size of the aperture changes a telescope's angular resolution.

Astronomers look at the universe in many different types of light. The Atacama Large Millimetre *Array* (ALMA) detects *radiowaves* with a wavelength of about 1 mm. This is about a thousand times longer than the wavelength of visible light. Hence to achieve the same resolution as an optical telescope 8 m in diameter, the Rayleigh resolution equation tells us we need to build a telescope that is 8000 m in diameter.

Of course we can't build a single telescope 8 km in diameter. Instead, ALMA is made up of lots of radio antennas, which are combined together to simulate the effect of a single large telescope: the effective diameter D is set by the maximum distance between the separate antennas. By moving the antennas around, it is possible to vary the effective size D and hence the telescope's angular resolution.

In their closest configuration, with a maximum diameter of 150 m, ALMA's antennas have a relatively poor angular resolution but give astronomers a good overview of large objects, such as a pair of colliding galaxies; but in their largest configuration of 15 km, ALMA delivers ultra-high resolution images, 10 times better than the Hubble Space Telescope: this is ideal for imaging smaller objects like discs of gas and dust around newly formed stars. At this highest resolution, a tiny 0.000002 degrees, ALMA could resolve the main features on a human face 100 km away. Even at a midway separation of 6 km, the array could help spot Jupiter-like planets forming in the dusty discs around young stars.

UNIVERSITY OF CAMBRIDGE

LONDON MATHEMATICAL SOCIETY

Institute of mathematics & its applications

selected The Royal Society summer science exhibition 2012

mathsinside www.mathspromotion.org.uk

Design: www.blitzbranding.co.uk