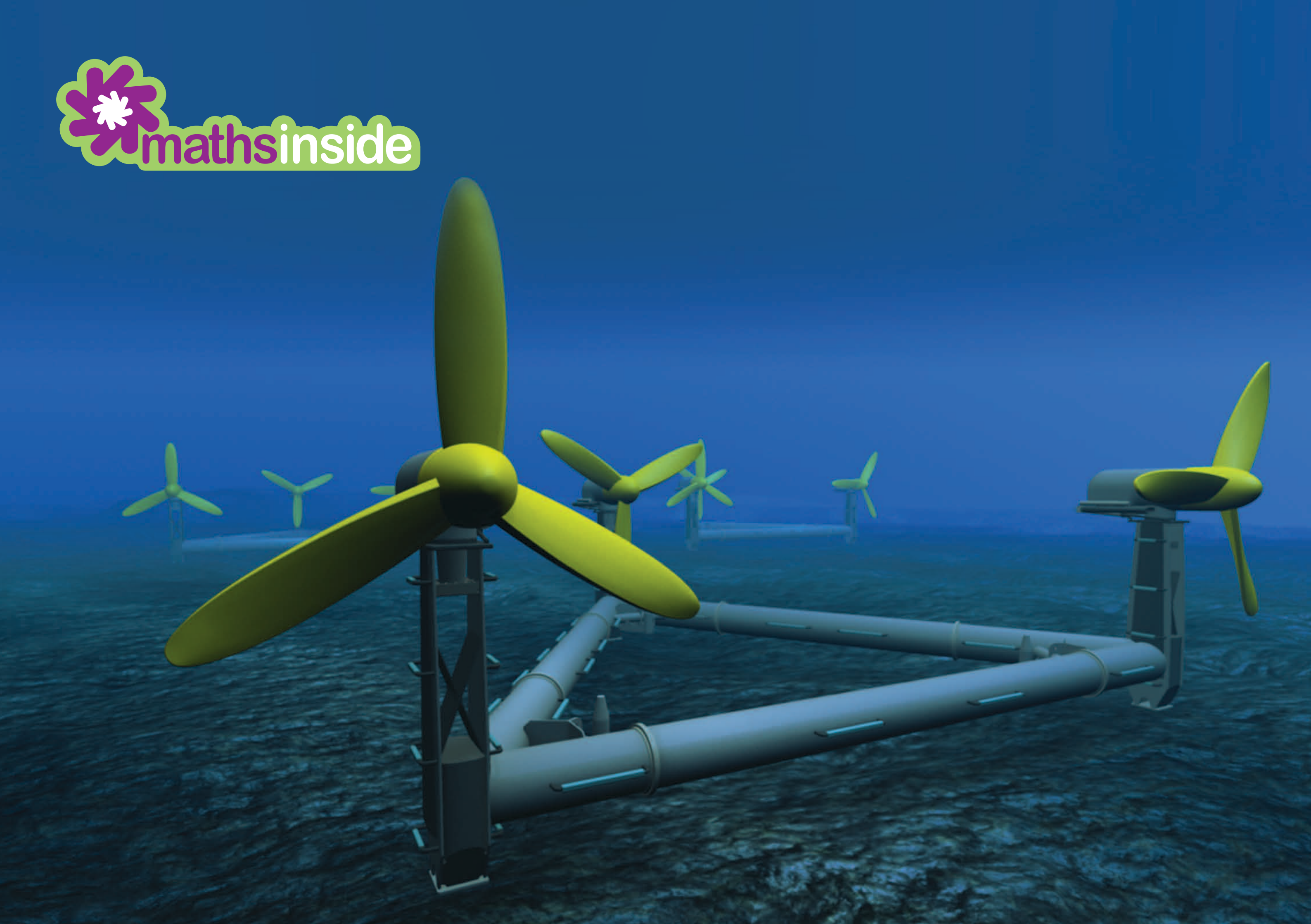
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# TIDAL TURBINES

## Using mathematics to develop new ways of producing energy

turbines | power | density | area | speed | coefficient | Betz's Limit

**The global need for energy is rapidly increasing but at the same time the fossil fuels that have been traditionally used to power the planet are running out.** Add in the fact that burning fossil fuels releases climate-altering carbon dioxide into the atmosphere and it is easy to see why the race is on to develop alternative ways to produce energy.

Mathematicians, physicists and engineers at Swansea University are working on marine **turbines** which operate in a similar way to wind turbines but are placed underwater. Here the blades are turned by the sea as the tide comes in and out each day. The tides are caused by the gravitational pull of the Moon and the Sun and so are very regular, making tidal power more predictable than solar or wind power.

Placing the turbines underwater is no easy task and so it is important that they generate as much power as possible once they are there. Mathematics plays a crucial part in doing this because the engineers use the following equation:

$$P = \frac{1}{2} \rho A v^3 c_p$$

where $P$ is the **power** generated, $\rho$ is the **density** of the water, $A$ is the **area** of the circle swept out by the turbine's blades, $v$ is the **speed** that the water moves through the blades and $c_p$ is a **coefficient** between 0 and 1 which is a measure of the amount of power in the moving water that is captured by the rotor as it turns.

So to get as much power out as possible, engineers want to design a turbine that makes the coefficient $c_p$ the highest it can be. This means optimising things like the shape and angle of the blades and how far apart they are spaced. There is, however, a limit to how high the engineers can get this number – known as the **Betz's Limit**, it is about 0.59. Current designs have been able to reach about 0.5.

The limit arises because the turbine cannot extract all available power from the water, otherwise the speed of the water exiting the turbine would drop to zero and the flow of the water behind it would be blocked. Using the fact that mass is conserved and that the work done on the turbines is equal to the change in kinetic energy of the fluid, Albert Betz was able to show that a turbine will extract the maximum amount of power when it slows the water down by a factor of three. This occurs when $c_p$ has a value of 0.59.

As the design of marine turbines improves it is hoped that in future the waters around the UK will hide the equivalent of underwater wind farms, exploiting the ebb and flow of the sea and generating energy from the tides. One day these turning blades could provide five per cent of the UK's electricity, offering another alternative to fossil fuels.

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