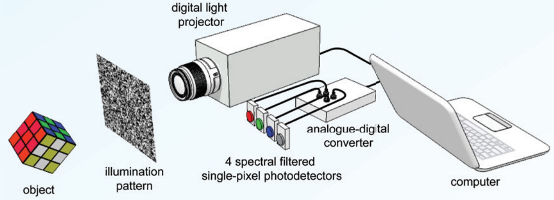

mathsinside

PIXELS

Producing images with the help of mathematics



These days most phones and tablets have cameras with resolutions measured in megapixels. This means the images they produce are made out of millions of separate squares that form the picture. A camera that took photos with a single pixel would condense the entire scene into a single square. How would a camera with a single pixel work? Surely all it can measure is how bright the whole scene is?

The single pixel camera works by projecting light in know patterns onto an object and recording the amount of light that is returned. The more light from the pattern that falls on the object the greater the amount that is reflected to the sensor. By shining a number of patterns and recording the level of returned light an image of the object can be reconstructed. This reconstruction uses a technique from mathematics known as inverse problem solving.

An example of an inverse problem that you're likely to be familiar with is the game Twenty Questions - a game in which you have to guess which person someone else is thinking of by asking twenty yes/no questions. If you know the solution — who the person is — then it's easy to answer yes or no to any questions asked about them. The tricky part of the inverse problem is that you know the answers to a few questions and must work out the solution.

Imagine you were playing twenty questions, but instead of guessing a person you had to guess a number between 1 and 1000. There are a couple of strategies that you could take: you could ask questions like, 'is it the number 240?',which is not a very good strategy; or you could ask questions like 'is it less than 500?'. In the first case if the answer was no you still wouldn't know very much, whereas in the second case it doesn't matter if the answer is yes or no: you would have ruled out half the numbers straight away.

When the single pixel camera shines a pattern of light on the object and measures the level of light reflected we've asked and answered a question, "how closely does this pattern resemble the object?". Using random patterns is like choosing the second strategy described above: instead of checking a single square of the picture and getting the precise answer for that square alone, we check a pattern of squares and get some, but not all, of the information about them. If we repeat this a numbers of times we can then infer what the object must look like in order to produce the measured reflections. This strategy produces an answer much more quickly, but less precisely (at lower resolution) than checking squares individually.

The picture is formed by layering the random patterns on top of each other using a weighted average. The **weighted average** is similar to the usual average but each value is first multiplied by a number special to that value, in this example the weight is the relative level of light reflected when the pattern was shone on the object.

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