LONDON MATHEMATICAL SOCIETY
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NEWSLETTER

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ALGORITHMS FOR KIDNEY DONATION

THE EVOLUTION OF PDEs IN THE UK

THE WORK OF SIMON DONALDSON

EDITORIAL OFFICE

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COVER IMAGE

Detail from a painting of Donaldson sailing through mathematics, by Nathalie Wahl. See page 30.

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CONTENTS

Upcoming LMS Events

The following events will take place in the next two months:

Mary Cartwright Meeting. London, 2 March 2018; tinyurl.com/ycv3ssfq

Society Meeting in Honour of Maryam Mirzakhani (1977–2017). Warwick University, 22 March 2018; tinyurl.com/y8ge9pgf

A full list of upcoming LMS events can be found on page 54.

Abel Prize Committee Visits the UK

The Abel Prize Committee reception at the Norwegian Ambassador's Residence in Kensington

The Abel Prize committee meets three times a year. The first meeting, in the autumn, is held in Oslo. The second is a telephone call, and the third and final meeting (at which the prizewinner is chosen) takes place in the home country of one of the committee members. I was very happy to have been able to persuade the Norwegian Academy of Science and Letters that they should come to the UK this January, and the final meeting was held in De Morgan House by kind permission of the London Mathematical Society. The meeting lasted all weekend, and on the following Monday a symposium *Abel in Oxford* took place at the Mathematical Institute. This featured colloquium-style talks by Sir Andrew Wiles (Abel Prize winner in 2016), Irene Fonseca (member of the committee) and John Rognes, the outgoing chair of the committee. The logistical annoyance of moving the whole entourage to Oxford for a day was easily outweighed by the superior culinary options available in London over the weekend.

Naturally, with the winner of the prize kept secret until March, I am not at liberty to comment on much of what went on at the meeting. Suffice it to say that we examined a very large number of letters from respected mathematicians around the world. These make remarkably interesting reading, but of course confidentiality is taken extremely seriously, and nothing is done via email. With a consensus having been reached by the end of Saturday, the committee spent a good deal of Sunday writing the citation. This task has taken several hours in both of the years I have been on the committee. We also had the pleasure of a meeting with Alex Bellos, who will be writing an account of the winner's work for a general audience.

It is a pleasure to thank the LMS for allowing us to use De Morgan House, and LMS President Caroline Series for hosting a reception and dinner for the committee on Friday evening. Look out for the announcement of the winner at 11 am on March 20th!

Ben Green
University of Oxford

Abel reception: report from the LMS

The 2018 Abel Prize Committee reception was held at the Norwegian Ambassador's Residence in Kensington and hosted by Ragnhild Imerslund, Minister, Deputy Head of Mission and Head of the Political and Economic Section, in the absence of the Ambassador Monica Juul. The reception was an opportunity for members of the STEM community to meet with the Abel Prize Committee and the Norwegian Ambassador's staff.

Professor Marie-France Vignéras, Institut de Mathématiques de Jussieu, Paris and a member of the Abel Prize Committee also welcomed guests on behalf of the Committee and presented LMS President, Professor Caroline Series FRS, with a copy of a publication containing three Niels Henrik Abel manuscripts. This will be available for members to view in the De Morgan House library in the Members Room.

John Johnston
Society Communications Officer

New Books About Ada Lovelace

Ada Lovelace is famous for her prescient 1843 account of the unbuilt Analytical Engine, a mechanical computer designed by Charles Babbage, which would have had the same computational abilities as a modern general purpose computer. The paper contains an account of the design, and of how the machine might be programmed using punched cards. It is illustrated with a large table, often called 'the first computer programme', displaying the calculation of the Bernoulli numbers. Lovelace's far sighted remarks about whether the machine might think, or compose music, still resonate today.

Ada Lovelace: The Making of a Computer Scientist by Christopher Hollings, Ursula Martin and Adrian Rice will be published in April 2018 by the Bodleian Library, in partnership with the Clay Mathematics Institute. It presents the first accessible account of Lovelace's scientific and mathematical education, showing how she learned as well as what she learned. It is copiously illustrated with extracts from her remarkable mathematical correspondence course with Augustus De Morgan, and even a page of mathematical doodles which includes the Königsberg bridges problem, and a magic square (tinyurl.com/y7ezvl66).

Also published this year, Miranda Seymour's *In Byron's Wake* is a portrait of Ada Lovelace, and her mother, Annabella Milbanke, and shows how the spirit of Lovelace's absent father, Lord Byron, governed and haunted their lives. Seymour is a novelist and critic, and biographer of Ottoline Morrell, Robert Graves and Mary Shelley (tinyurl.com/y8mqnsyr).

Ursula Martin and Miranda Seymour will be talking about these complementary new books at 4 pm on 19 March 2018 at the Oxford Literary Festival: tinyurl.com/y97zgcsd

Mathematika: Klaus Friedrich Roth special issue

Alex Sobolev, Managing Editor, writes about the special issue of Mathematika (volume 63, part 3, December 2017: tinyurl.com/y92arxsr) dedicated to Klaus Friedrich Roth — an LMS member and 1983 recipient of the De Morgan Medal.

The editors of Mathematika are pleased to announce a special issue of the journal dedicated to Klaus Friedrich Roth, which is now available on Cambridge Core. Roth, who passed away on 10 November 2015 at age 90, published during his distinguished career five papers in *Mathematika*, including three of his most influential:

- The 1955 paper 'Rational approximations to algebraic numbers' was his legendary solution of the famous Siegel conjecture concerning approximation of algebraic numbers by rationals, for which he was awarded the Fields Medal in 1958.
- The 1965 paper 'On the large sieves of Linnik and Rényi' was a huge breakthrough and, together with the almost contemporaneous paper of Bombieri published also in Mathematika in the same year, continues to have a profound impact on the development of analytic number theory today.
- The 1954 paper 'On irregularities of distribution', although cited much less than the other two, was nevertheless considered by Roth to be his best work, and paved the way for what is now known as geometric discrepancy theory, a subject at the crossroads of harmonic analysis, combinatorics, approximation theory, probability theory and even group theory.

The editors of *Mathematika* resolved in early 2016 to dedicate an issue of the journal to celebrate the life and work of Roth. William Chen and Robert Vaughan act as special editors for this project and, thanks to their efforts, many excellent contributions were received from friends and colleagues of Roth, making this a very special issue.

It was a coincidence that the famous abc-conjecture was first formulated in an open problem session during the Symposium on Analytic Number Theory held at Imperial College in 1985 on the occasion of Roth's sixtieth birthday. As no formal proceedings were published and the very brief records were only available to the participants of the symposium, this has caused much inconvenience to the many authors who wish to cite it.

The special editors have invited David Masser to write some brief anecdotes on this occasion. His note 'Abcological Anecdotes' opens this issue, and will serve as a reference point for this historical moment in the history of our subject.

PEOPLE

New Year Honours for Mathematical Scientists

SIR BERNARD SILVERMAN, FRS
PROFESSOR CHRISTL DONNELLY, FRS
HOWARD GROVES

The London Mathematical Society would like extend its congratulations to Sir Bernard Silverman FRS, Professor Christl Donnelly CBE FRS and Howard Groves MBE for the honours they received in the 2018 New Year Honours list.

Sir Bernard, ex-Chief Scientific Adviser at the Home Office receives a Knighthood for Public Service and Services to Science.

Professor Donnelly, Professor of Statistical Epidemiology at Imperial College London, receives a CBE for services to Epidemiology and the Control of Infectious Diseases.

Mr Groves, Member of the Senior Mathematical Challenge Problems Group and Member of the UK Mathematics Trust Challenges Sub Trust, receives an MBE for services to Education.

Other notable honours include Professor Philip Nelson, Chief Executive Officer, Engineering and Physical Sciences Research Council and Chair, Research Councils UK, who receives a CBE for services to UK Engineering and Science. The full list of Honours is available at tinyurl.com/y7bfqplu.

2018 King Faisal International Prize

PROFESSOR JOHN BALL

Sir John Ball FRS (University of Oxford), past President (1996–1998) of the London Mathematical Society, has received the 2018 King Faisal International Prize (Science). The Society extends its congratulations to Sir John on receiving this award. The Prize is awarded by the King Faisal Foundation and the Science subcategories cover a range of disciplines, alternating between physics, chemistry, biology and mathematics, with the Prize for Science first awarded in 1984.

The Prize is awarded to individuals who have 'carried out and published original scientific research on the prize's topic, with major benefits to humanity, and meeting one or more of the prize's objectives as determined by the respective Selection Committee'.

Professor Caroline Series FRS, LMS President, said: "I am delighted by the news that Professor John Ball has been awarded this prestigious international prize. Besides his distinguished research achievements, John has worked tirelessly for the benefit of the mathematical community, not only as LMS President but also as the President of the International Mathematical Union 2003-2006."

More information about the prize is available at tinyurl.com/y9lo3bfk.

Breakthrough Prize in Mathematics

PROFESSOR CHRISTOPHER HACON
PROFESSOR JAMES MCKERNAN

The 2018 Breakthrough Prize in Mathematics has been awarded to Professor Christopher Hacon (University of Utah) and Professor James McKernan (University of California, San Diego). The Prize ceremony was held in Silicon Valley on 3 December 2017. Each Breakthrough Prize award is $3 million, the largest individual monetary prize in science.

Professors Hacon and McKernan receive their awards for 'transformational contributions to birational algebraic geometry, especially to the minimal model programme in all dimensions'. The LMS congratulates both on receiving the award. The Breakthrough Prize in Mathematics rewards significant discoveries across the many branches of the subject. The prize was founded by Mark Zuckerberg and Yuri Milner and was announced at the 2014 Breakthrough Prize ceremony.

The LMS is also delighted to announce that Professor James McKernan gave a presentation at the LMS Meeting at the Joint Mathematics Meetings in San Diego on Wednesday 10 January 2018. The report about the meeting is on page 16.

MATHEMATICS POLICY ROUNDUP

UK Research and Innovation

Sir John Kingman has been announced as Chair of UK Research and Innovation (UKRI). Sir John is currently interim Chair and will take up the permanent role in April this year. More information is available at tinyurl.com/y8x55u7g.

Industrial Strategy

This government white paper *Industrial Strategy: Building a Britain fit for the future*, published at the end of November 2017, sets out a long-term plan to 'boost the productivity and earning power of people throughout the UK'. The full strategy document is available at tinyurl.com/ycrts52w.

Research Excellence Framework

The UK higher education funding bodies have made decisions on the operation of the next Research Excellence Framework process. The conclusions, which follow a year of consultation, confirm the arrangements for returning research staff and outputs to the exercise. More information is available at tinyurl.com/yab7cd99.

Education Select Committee

The government has responded to the House of Commons Education Select Committee report *Exiting the EU: challenges and opportunities for higher education*. The report is available at tinyurl.com/y958aoon.

Graduate Employment Gap

The Chartered Institute of Personnel and Development (CIPD) has analysed the latest data from the Higher Education Statistics Agency (HESA) on graduate outcomes six months after graduation and reveals that almost half of new graduates are working in non-graduate jobs. The analysis also shines a light on graduate gender pay disparities, with female graduates earning less than their male counterparts even when subject choice and institution are factored in.

The report's *Executive Summary* also states that 'given the importance placed by the government on increasing the number of people with STEM skills, it appears that individuals with those skills are not doing particularly well as they account for all but one of the subject areas with the highest proportion of unemployed graduates six months after graduation. Previous research has suggested that many STEM graduates lack the experience and "soft" skills employers are looking for.

It seems that by international standards the UK has a healthy supply of STEM graduates relative to other countries; the UK's share of natural sciences, mathematics and statistics graduates is more than double that of the OECD average'.

The full report entitled *The graduate employment gap: expectations versus reality* is available at tinyurl.com/ya4ux46h.

John Johnston
Joint Promotion of Mathematics

EUROPEAN

Message from the EMS President

Another calendar page turns, and another year heads for the history books. We may have to wait a decade for the next prime numbered year, but many mathematical and mathematics-related events will occur much sooner. To begin with, the year we are entering has been declared The Year of Mathematical Biology, and I think this is a good omen. We are all now convinced that mathematics is omnipresent, but this truth has dawned in different ways in different fields.

It took over two centuries from its first successful marriage with physics, before mathematics started to be applied seriously to biological and social systems. The explosive growth, over the last six or seven decades, of our knowledge of biological mechanisms provides strong motivation to focus on questions which mathematicians and biologists could address together.

There will be many events over the year, such as the European Conference on Mathematical and

Theoretical Biology in Lisbon (the programme of which includes the annual lecture jointly organized by the European Mathematical Society (EMS) and the Bernoulli Society), the EMS Joint Mathematical Weekend in Joensuu, summer schools, and a great deal more.

The year 2018 will be also important in the life of our society. As happens every second year, the EMS Council (our highest authority) will meet to discuss both our achievements and our next goals. The June Council Meeting in Prague will need to elect the new EMS president, to make decisions about the society's budget, and to settle other important questions related to the EMS's mission and smooth running.

It is satisfying to note that the number of EMS members, both individual and corporate, continues to grow steadily on average. The conditions in which we live are not all the same, of course, and it is encouraging indeed when the national society of a country plagued by serious political and economic problems once again fulfils its membership duties. (They could serve as an example to other corporate members whose approach is more – shall we say – relaxed.) The EMS is not a rich society even compared to some of our members, to say nothing of our partners overseas, but we are doing well financially and are delighted to be able to support more summer schools, conferences, distinguished speakers, and other activities than ever before.

As usual, the turn of the year brings a renewal of our standing committees, the backbone of the society's work. In some, changes are minimal, in others substantial (in part due to the eight-year cap on committee service). This year, the Applied Mathematics and Ethics Committees will undergo the biggest changes, with at least a half of their membership renewed. Let me take this opportunity to thank all departing committee members for their hard work, and to wish all newcomers success and satisfaction in working towards common goals. Our gratitude is owed also to all those who work for the broader European mathematical community, in a wide variety of roles including for the European Digital Mathematics Library, on the boards of mathematical journals and research centres, prize committees, and in a multitude of other ways.

The coming year also heralds exciting mathematical events worldwide, principally the International Congress of Mathematicians at the beginning of August in Rio de Janeiro, to which we are looking forward. By that time, we will also know the location of the 2022 ICM. As representatives of all mathematicians on our continent, we express no preference on the competition between Paris and Saint Petersburg, but we have no doubt that either choice will lead to a wonderful meeting. We are glad that, either way, after sixteen years the congress will return to the continent of its birth. We may be European patriots, but at the same time we do not forget that there is a single world of mathematics, and we continue working to make connections all over the globe. The EMS has recently completed our list of cooperation agreements with major mathematical societies on other continents, by signing an agreement with the Chinese Mathematical Society. We hope it will lead to exciting joint ventures.

The Chinese note brings to mind yin and yang, and with cooperation naturally comes competition. To give one example, let me mention zbMATH. We are pleased that FIZ Karlsruhe, our partner in this enterprise, has had its financial support renewed, guaranteeing that the healthy competition with MathSciNet, beneficial for the whole mathematical community, will continue. We have just signed a new agreement fixing the EMS's involvement in the future development of zbMATH, and we are ready to work on further improvements of this great reference tool.

A year ago, I mentioned here the worrying state of the world, and I have to say that the situation has not improved – rather the opposite, with no need to list all the neuralgic points of the globe. We can do little to influence those political tectonic processes, but it is important to preserve and strengthen the *esprit de corps*, and to oppose the regular calls to ostracize some or other part of our community. A good example of such an attitude was the Second Caucasian Conference which convened (after a one-year delay) in the east of Turkey. I thank those who attended despite pressure they faced at home, and I hope this tradition will continue.

Having said that, I wish all of you good health and a lot of interesting mathematics in 2018.

Pavel Exner
EMS President

EMS Council

The EMS Council meets every second year. The next meeting will be held in Prague, 23-24 June 2018 at Balling Hall at the National Library of Technology. More information can be found from the Council webpage at tinyurl.com/yaap75sz.

EMS Meeting of Presidents

The traditional gathering of the national societies' presidents will convene this year on 14-15 April in Maynooth following a kind invitation from the Irish Mathematical Society. This gathering serves mainly as a forum where the EMS corporate members can share their experiences, plans, and worries, and discuss them with the EMS officers. The program conventionally opens with the presentation of the host society and information about EMS activities; usually also some societies prepare presentations with information they want to share. The main part of the meeting is a general discussion, this year probably on the relations between the EMS and the national societies. Reports on past meetings can be seen at tinyurl.com/ybapafyv.

EMS Simons for Africa

The African Continent is very diversified and the development of a career in mathematics faces different and sometimes difficult progression. The EMS Committee for Developing Countries, with the support of the Simons Foundation, has opened a program of research visits to foster research opportunities for young and established researchers, open to all areas of pure and applied mathematics and statistics and directed to fellows based in Africa. The 2018 deadlines are 15 May, 15 September and 15 November. More information can be found at tinyurl.com/ycefz6mw.

Institut Henri Poincaré

Sylvie Benzoni is the new Director of the Institut Henri Poincaré in Paris. Founded in 1928, the IHP is a highly prestigious centre of mathematics and an ERCOM Centre (European Research Centres in the Mathematical Sciences). Sylvie Benzoni is an internationally recognised expert in partial differential equations as they relate to fluid dynamics and phase transitions. She has been a professor at the Université Claude Bernard in Lyon since 2003, and director of the Institut Camille Jordan since 2016. She is also a serving member of the EMS Committee for Raising Public Awareness of Mathematics.

European Solidarity Committee

Solidarity travel grants are reserved for European based researchers, with special priority for countries where travel support can be more difficult to obtain. A total of 10 grants were awarded this year. There are two application periods with deadline 31 March and 30 September. More information can be found at tinyurl.com/y75o4kml.

Prize Miguel Catalán 2017

David Pérez García (Instituto de Ciencias Matemáticas, Universidad Complutense de Madrid, Spain) has been awarded the prize Miguel Catalán 2017, in the area of Science, in recognition for his contributions in the field of quantum technologies. More information can be found at tinyurl.com/yak6b85b.

David Chillingworth
LMS/EMS Correspondent

OPPORTUNITIES

ICMS Funding Opportunities

The International Centre for Mathematical Sciences in Edinburgh is funded by EPSRC to support research in all areas of the mathematical sciences and its connections to other disciplines and industry. The new ICMS grant starts in April 2018. Proposals are externally refereed and organisers are encouraged to contact ICMS to discuss their proposal before the final submission. Funding is available for:

Research Workshops: typically five days for 50-80 people to engage in a focussed workshop across all areas of the mathematical sciences and their applications, including cross-disciplinary research and engagement with problems from the public sector and industry. There are two calls each year.

Early Career Workshops: ICMS workshops with an organizing committee of ECRs, enabling ECRs to take leadership roles in their area. ICMS can provide mentorship from people with experience of running successful meetings at ICMS to advise the organising committee.

Strategic Workshops: typically up to five days for 20-50 people to help the mathematical sciences community to engage in cross disciplinary funding opportunities and policy driven initiatives where timing is important. These can also be used to provide a rapid response to create communities in emerging areas, particularly where this involves working across disciplines. There are four calls each year although where necessary ICMS will consider proposals out of cycle; a short preliminary outline is needed to show that the idea fits the strategic workshop category.

Research in Groups: typically 2-6 people for two to three weeks. RiGs brings together an international group to work on a particular problem leading to new research or grant proposals. There are four calls each year.

Research Partnerships with Industry: similar to RiGs but with an industrial partner, leading to new research, a proof of concept, or a report for the industrial partner. There are four calls each year.

Follow-on Grants: designed to enable groups of researchers to re-evaluate work arising from workshops held at ICMS or elsewhere.

ICMS also organises stand-alone Knowledge Transfer and Public Engagement activities, and hosts workshops with external funding. Details and application instructions can be found at icms.org.uk.

Undergraduate Essay Prize

The British Society for the History of Mathematics is pleased to invite submissions for its 2017-18 undergraduate essay prize. The essay may be on any topic within the history of mathematics and should be no more than 2,500 words in length (excluding references). The competition is open to any person who is enrolled as an undergraduate in a UK or Irish university during the academic year 2017-18. The value of the prize will be £100, plus free membership of the Society for three years.

Details about how to enter can be found on the Society website bshm.ac.uk/ug-essay. The deadline for receipt of submissions is 21 June 2018.

Oxford Mathematics: UK-Networking Scheme

The Mathematical Institute invites applications from UK-based mathematicians under its UK-networking scheme. Applications are invited from individuals who have no other support, for funds up to £500,

- to participate in events at the Oxford Mathematical Institute;
- to invite members (including visitors) of the Oxford Mathematical Institute to visit their own institutions.

Applications outside the remit but in the spirit of the above are welcome. For more details see tinyurl.com/j62fbyy.

Mentorship Programme — ECMTB

The 11th European Conference on Mathematical and Theoretical Biology (ECMTB 2018) will be held in Lisbon, Portugal, from 23 to 27 July 2018. The ECMTB 2018 Organizing Committee is setting up a Mentorship Programme to facilitate research and career interactions between junior and senior scientists attending the meeting. Participants of ECMTB 2018 can sign up to be part of the mentorship programme, either as a mentee, a mentor, or both. If you wish to participate in the Mentorship Programme at ECMTB 2018, fill in the registration form no later than 10 May 2018. You can find more information about the ECMTB 2018 Mentorship Programme at tinyurl.com/yad323dr.

PROMYS Europe 2018

The Program in Mathematics for Young Scientists (PROMYS) was founded in Boston in 1989. PROMYS Europe, a challenging six-week residential summer programme at the University of Oxford, is seeking pre-university students from across Europe who show unusual readiness to think deeply about mathematics. PROMYS Europe is designed to encourage mathematically ambitious students who are at least 16 to explore the creative world of mathematics. Participants tackle fundamental mathematical questions within a richly stimulating and supportive community of fellow first-year students, returning students, undergraduate counsellors, research mentors, faculty, and visiting mathematicians.

First-year students focus primarily on a series of very challenging problem sets, daily lectures, and exploration projects in Number Theory. There will also be a programme of talks by guest mathematicians and the counsellors, on a wide range of mathematical subjects, as well as courses aimed primarily at students who are returning to PROMYS Europe for a second or third time.

PROMYS Europe is a partnership of Wadham College and the Mathematical Institute at the University of Oxford, the Clay Mathematics Institute, and PROMYS. The programme is dedicated to the principle that no one should be unable to attend for financial reasons. Most of the cost is covered by the partnership and by generous donations from supporters. In addition, full and partial needs-based financial aid is available, which can cover the fee and travel costs, in part or in full.

The application and application problem set are available on the PROMYS Europe website, www.promys-europe.org. The deadline for first-year student applications is 18 March. PROMYS Europe 2018 will run from 15 July to 25 August at the University of Oxford.

International Mathematics Competition for University Students

The 25th IMC, being held from 22-28 July 2018 in Blagoevgrad, Bulgaria, is organized by University College London and hosted by the American University in Bulgaria, Blagoevgrad. Universities are invited to participate and to send several students and one teacher. Individual students are welcome. The competition is planned for students completing their first, second, third or fourth year of university education and will consist of two sessions of five hours each. Problems will be from the fields of Algebra, Analysis (Real and Complex), Geometry and Combinatorics. The maximum age for participants is 23 years of age at the time of the IMC. The working language will be English. Over the past twenty four competitions there have been participants from over 200 universities from over 50 countries.

The IMC is a residential competition and all student participants are required to stay in the accommodation provided by the hosts. For further information visit the website at imc-math.org.uk. Register participation on-line at tinyurl.com/ybkvkp66 and arrival details online at tinyurl.com/ya3pzyge. Further details may be obtained from John Jayne (j.jayne@ucl.ac.uk).

LMS Durham Symposia 2019: Call for Proposals

The London Mathematical Society invites proposals for Durham Symposia in 2019, and intends to support two Symposia to take place in August 2019. The Symposia began in 1974, and have now become an established and recognised series of international research meetings. They provide an excellent opportunity to explore an area of research in depth, to learn of new developments, and to instigate links between different branches. The format is designed to allow substantial time for interaction and research. The meetings are by invitation only and held in August, lasting five days, with up to 50 participants, roughly half of whom will come from the UK. They are held at the University of Durham.

Prospective organisers should send a formal proposal to the Durham Representative, Dirk Schuetz (dirk.schuetz@durham.ac.uk) by 20 March 2018. Proposals should include:

- A full list of proposed participants, divided into specific categories (see the guidance on submission of proposals at lms.ac.uk/events/durham-symposia for more details). Proposers are encouraged to actively seek to include women speakers and speakers from ethnic minorities, as well as consult the 'LMS Advice on Diversity at Conferences and Seminars'.
- A detailed scientific case for the symposium, which shows the topic is active and gives reasons why UK mathematics would benefit from a symposium on the proposed dates.
- Details of additional support from other funding bodies.

The Durham Representative will provide an estimated cost for accommodation for the symposium and estimated travel costs for each participant.

Before submitting, organisers are welcome to discuss informally their ideas with the Durham Representative (dirk.schuetz@durham.ac.uk) and/or the Chair of the Research Grants Committee, Dr Francis Clarke (grants@lms.ac.uk). For further details about the Durham Symposia, visit the Society's website: lms.ac.uk/events/durham-symposia.

LMS Grant Schemes

The deadline for applications for the following grant schemes is 15 May 2018:

- Conferences (Scheme 1): grants of up to £7,000 available
- Visits to the UK (Scheme 2): grants of up to £1,500 available
- Research in Pairs (Scheme 4): grants of up to £1,200 available
- International Short Visits (Scheme 5): grants of up to £3,000 available

- Postgraduate Research Conferences (Scheme 8): grants of up to £4,000 available
- Celebrating New Appointment (Scheme 9): grants of up to £600 available

For details and application forms, see lms.ac.uk/grants/research-grants. Please address queries to Anthony Byrne, grants administrator: grants@lms.ac.uk.

Applications are also invited for Computer Science grants (Scheme 7; up to £500 available per grant). The deadline is 15 April 2018. Details at lms.ac.uk/grants/computer-science-small-grants-scheme-7. Address queries to Katherine Wright: lmscomputerscience@lms.ac.uk.

VISITS

Visit of Jaeyoung Byeon

Professor Jaeyoung Byeon will be visiting the University of Oxford from 22 April to 16 June 2018. He is a member of the Department of Mathematical Sciences at KAIST, South Korea. His research interests are PDE, calculus of variations and nonlinear analysis. During his visit he will give lectures at:

- ICMS, Edinburgh, 27 April (contact Beatrice Pelloni: b.pelloni@hw.ac.uk)
- University of Swansea, 10 May (contact Vitaly Moroz: v.moroz@swansea.ac.uk)
- University of Oxford, 11 June (contact Luc Nguyen: nguyenl@maths.ox.ac.uk)

For further details contact Jonathan Whyman (oxpde-manager@maths.ox.ac.uk).

The visit is supported by an LMS Scheme 2 grant.

Visit of Tony Samuel

Dr Tony Samuel (California Polytechnic State University, USA) will be visiting Dr Jamie Walton at Durham University from 21 March to 1 April 2018. Dr Samuel's current research focuses on applications of ergodic theory, noncommutative geometry and potential theory to fractals and aperiodic patterns (also known as quasicrystals).

For further information contact Dr Jamie Walton (james.j.walton@durham.ac.uk). The visit is supported by an LMS Scheme 4 Research in Pairs grant and the Department of Mathematical Sciences at Durham University.

Visit of Albert Shiryaev

Professor Albert Shiryaev is a world-leading Russian mathematician active in the fields of probability, stochastic analysis and financial mathematics. He is a member of the Russian Academy of Sciences and doctor honoris of Albrecht-Ludwig University of Freiburg, of the Amsterdam University, of the Anger University (France) etc. Professor Shiryaev will be visiting the UK between 28 April and 10 May 2018. Details of the talks during his visit are:

- London School of Economics, 1 May, *Notion of randomness, von Mises collective and Kolmogorov complexity* (contact Umut Çetin: u.cetin@lse.ac.uk)
- University of Liverpool, 3 May, *Optimal stopping procedures in financials models with disorder of trends* (contact Alexey Piunovskiy: piunov@liv.ac.uk)
- University of Liverpool, 4 May, *On the Kolmogorov forward and backward equations for general jump processes* (contact Alexey Piunovskiy: piunov@liv.ac.uk).
- University of Warwick, 7 May, *Around the Cameron-Martin/Girsanov theorem* (contact Vassili Kolokoltsov: v.kolokoltsov@warwick.ac.uk)
- Heriot-Watt University, 9 May, *Notion of randomness, von Mises collective and Kolmogorov complexity* (contact Anke Wiese: A.Wiese@hw.ac.uk)

For further details contact Alexey Piunovskiy (piunov@liv.ac.uk). The visit is supported by an LMS Scheme 2 grant.

Annual Elections to LMS Council

The LMS Nominating Committee is responsible for proposing slates of candidates for vacancies on Council and vacancies on its own membership. The Nominating Committee welcomes suggestions from the membership.

Anyone who wishes to suggest someone for a position as an Officer of the Society or as a Member-at-Large of Council (now or in the future) is invited to send their suggestions to Professor John Toland, the current Chair of Nominating Committee (nominations@lms.ac.uk). Please provide the name and institution (if applicable) of the suggested nominee, her/his mathematical specialism(s), and a brief statement to explain what s/he could bring to Council/Nominating Committee.

It is to the benefit of the Society that Council is balanced and represents the full breadth of the mathematics community; to this end, Nominating Committee aims for a balance in gender, subject area and geographical location in its list of prospective nominees.

Nominations should be received by 20 April 2018 in order to be considered by the Nominating Committee.

In addition to the above, members may make direct nominations for election to Council or Nominating Committee. Direct nominations must be sent to the Executive Secretary's office (nominations@lms.ac.uk) before noon on 1 September 2018. For details on making a direct nomination, see lms.ac.uk/about/council/lms-elections.

The slate as proposed by Nominating Committee, together with any direct nominations received up to that time, will be posted on the LMS website in early August.

LMS Society Meeting at the ICM 2018

Marta Sanz-Solé

The London Mathematical Society is pleased to announce that the LMS Lecturer during the International Congress of Mathematicians 2018 (ICM 2018) will be Professor Marta Sanz-Solé. Marta Sanz-Solé is Professor at Universitat de Barcelona and her research interests are in stochastic analysis, in particular stochastic differential and partial differential equations.

Professor Sanz-Solé was awarded with a Narcis Monturiol Medal of Scientific and Technological Excellence by the Generalitat of Catalonia in 1998. She was elected Fellow of the Institute of Mathematical Statistics and was President of the European Mathematical Society from 2011 to 2014.

The ICM 2018 will take place in Rio de Janeiro from 1 to 9 August 2018. Further details on how to register can be found at: icm2018.org

At each ICM, the LMS hosts a scientific meeting. This year, the meeting will be held on Tuesday 7 August from 6.00 to 7.00 pm and is open to all ICM participants.

There will also be an LMS Reception after the Society Meeting from 7.00 to 9.00 pm for LMS Members and guests. A ticket is required for the reception only and members can request a ticket by emailing Elizabeth Fisher (lmsmeetings@lms.ac.uk).

LMS Hardy Lecture Tour 2018

The Hardy Lectureship, founded in 1967 in memory of G.H. Hardy in recognition of outstanding contributions to mathematics and the Society, is a lecture tour of the UK by a mathematician with a high reputation in research.

The 2018 LMS Hardy Fellow is Professor Lauren Williams (UC Berkeley). Professor Williams' research is highly interdisciplinary and has had a broad impact on several areas of mathematics, ranging from combinatorics and algebra to probability and mathematical physics, with applications to particle processes and shallow water waves.

Professor Williams will visit the following locations:

- BMA House, Tavistock Square London, 29 June, 3.30 pm (organiser: LMS)
- Oxford, 2 July; 4.30 pm (organiser: Lionel Mason)
- Cambridge, 4 July (organiser: Mark Gross)
- Kent, 5 July (organisers: Clelia Pech, Stephane Launois)
- Edinburgh, 9 July (organiser: Milena Hering)
- Leeds, 10 July (organiser: Robert Marsh)
- Birminham, 11 July (organiser: Marta Mazzocco)

Contact the local organisers for further information on attending each lecture. For general enquiries about the Hardy Lectures, contact Elizabeth Fisher (lmsmeetings@lms.ac.uk).

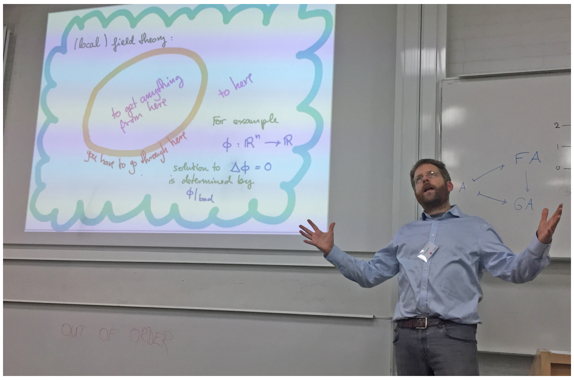
REPORTS OF THE LMS

Report: LMS South West and South Wales Regional Meeting

The 2017 South West and South Wales Regional Meeting took place at Cardiff University on 13 December, preceded by a Graduate Morning and followed by a workshop on Algebraic Structures and Quantum Physics (14-15 December).

The meeting was the first to be combined with a Graduate Morning where PhD students could present their research. Also, as part of the Graduate Meeting Professor Edwin Beggs (Swansea) and Dr Ulrich Pennig (Cardiff) gave excellent introductions to Hopf algebras and topological quantum field theory so as to prepare the students for the Regional Meeting talks. These talks were greatly appreciated by the students who attended.

The Regional Meeting began with the signing of the LMS Members' Book, including by one of the speakers, Professor Shan Majid, and two of the organisers, Gandalf Lechner and Simon Wood. After that the two speakers gave lectures on the connections between algebra and physics, the topic of the workshop.



Ingo Runkel

Professor Shahn Majid (Queen Mary University) began with an overview of the problems encountered when studying quantum gravity, explaining how quantisation requires one to replace classical symmetries groups by their deformations: quantum groups. This was followed by remarkable new results on the use of braid diagrams to construct new quantum groups and their implications for non-commutative geometry.

Professor Ingo Runkel (University of Hamburg) gave a beautiful introduction — sprinkled with wit and well-chosen examples ensuring accessibility to novices and experts alike — to the concept of categorification and its implications for physics, in particular topological quantum field theory. He then proceeded to invert this concept to show how de-categorification can be used to relate high dimensional topological quantum field theories to lower dimensional ones.

This set the tone for a highly stimulating workshop over the next two days, with talks delivered by experts from the UK and abroad, where it became apparent that the same algebraic language and tools are used in many different areas relating to applications in quantum field theory, lattice systems, quantum information theory, string theory, and computer science.

Mathew Pugh, Gandalf Lechner, Simon Wood
Cardiff University

Report: LMS Meeting at the Joint AMS Meeting

Vice-President John Greenlees and Publications Secretary John Hunton

An LMS meeting was held in the Marriott Marquis San Diego Marina Hotel on Wednesday 10 January 2018. The meeting was part of the program at the annual Joint Mathematics Meeting organized by the American Mathematical Society and the Mathematical Association of America. There were an estimated 6,400 mathematicians in attendance. Other organizations hosting affairs at the San Diego JMM include the Association for Symbolic Logic, Association for Women in Mathematics, Society for Industrial and Applied Mathematics, National Association for Mathematics, National Science Foundation, Rocky Mountain Consortium, and Pi Mu Epsilon, the national undergraduate mathematics honor society. There were a number of invited addresses, invited paper sessions, contributed paper sessions, poster sessions, panel discussions, minicourses, meeting of special interest groups, a short course, and an exhibit hall with over sixty vendors. The meeting also included an extensive social program.

The LMS meeting was chaired by the LMS Vice-President John Greenlees. International members of the Society were given an opportunity to sign the Members' Book. The induction ceremony was followed by a lecture given by Professor James McKernan (University of California San Diego). The speaker was a recipient of the Clay Research Award (2007), the AMS Frank Nelson Cole Prize (2009) and the Breakthrough Prize in Mathematics (2018).

Professor McKernan's talk was titled *How many equation does one need to define a variety?* In particular, his talk focused on algebraic varieties which were the zero locus of a collection of polynomials. The speaker began by giving several straightforward examples of these types of algebraic varieties. He noted that Hilbert had shown that any such variety is defined by finitely many polynomials. In fact, if the dimension is one less than the ambient space then one polynomial equation always suffices. He mentioned an intriguing conjecture of R.C. Hartshorne that states that if the dimension is two less than the dimension of the ambient space then two polynomials suffice, provided the number of variables is at least seven. The remainder of the talk focused on this fascinating conjecture.

The talk was followed by a well-attended wine reception in the Mission Hills Room of the Marriott Marquis Hotel, where we were joined by the LMS Vice-President, John Greenless, the LMS Publications Secretary, John Hunton, and LMS representatives Elizabeth Fisher and Suzanne Abbott.

J.J. Tattersall
Providence College

LETTERS TO THE EDITOR

The Moore–Penrose Inverse

The fascinating survey of the vast corpus of Penrose's work on theoretical physics and cosmology in the *Newsletter's* issue 473, prompts this short note on what has become known as the Moore-Penrose inverse. We therefore concentrate on two papers by Penrose published when he was a doctoral student (*Proc. Cam. Phil. Soc.*, 51 (1955) 406–413; 52 (1956) 17–19).

Penrose showed that given an arbitrary matrix A there exists a matrix, denoted A^+, uniquely determined by A and called the *Moore-Penrose inverse* of A, such that

(i) $AA^+A = A$, (ii) $A^+AA^+ = A^+$,
(iii) $(AA^+)^* = AA^+$, (iv) $(A^+A)^* = A^+A$.

(Of course if A is invertible then A^+ coincides with the ordinary inverse A^{-1}.) He further proved that for matrices conformable for multiplication

$$||AXB - C||_2 \geq ||AA^+CB^+B - C||_2, \qquad (1)$$

where $||\cdot||_2$ denotes the Euclidean norm on finite matrices.

Immediately after the appearance of Penrose's work, Rado pointed out (in *Proc. Cam. Phil. Soc.* 52 (1956) 600–601) that, unbeknown to Penrose, as early as 1920, E.H. Moore had formulated a generalized 'reciprocal' with properties equivalent to Penrose's (in *Bull. Amer. Math. Soc.*, 26 (1920) 394–395). Hence, the curious nomenclature 'Moore-Penrose'.

Unlike Moore's work, Penrose's had an immediate impact on many areas of enquiry: for instance, control theory, econometrics, game theory, linear programming. But this enormous range of application, specifically of finite-dimensional, matrix theory, delayed mathematical developments: certain results came later than they should have done.

If the underlying Hilbert Space is infinite-dimensional we proceed as follows: let A_0 be the restriction of A to the orthogonal complement $(\ker A)'$ so that A_0 maps $(\ker A)'$ bijectively onto $\operatorname{Ran} A$ and so has a bounded inverse A_0^{-1}; then provided $\operatorname{Ran} A$ is closed, let P be the projection onto $\operatorname{Ran} A$ and let $A^+ = A_0^{-1}P$. Then it is trivial to verify A^+ here, so defined, is the Moore-Penrose inverse of A.

Now we can extend (1) to infinite-dimensions. This inequality has been extended to the von Neumann-Schatten class with norm $||\cdot||_p$ provided $p \geq 2$. Corresponding results have appeared for unital C^*-algebras. (These results are due to the writer.) And most recently, (1) was extended to the supremum norm on $L(H)$, a result whose proof could have been done 60 years ago. Thus, the power of Roger Penrose's achievements can also be seen sharply in (retrospectively speaking) his early excursion into pure mathematics.

Philip Maher
philipmaher1@yahoo.com

Letters may be edited for style and space.

Records and Proceedings at LMS meetings
Ordinary Meeting, 10 January 2018

The meeting was held at the Marriott Marquis San Diego Hotel, San Diego, during the Joint Mathematics Meeting 2018. Around 40 members and guests were present for all or part of the meeting.

The meeting began at 6.00 pm with the Vice-President, Professor John Greenlees, in the Chair.

There were no members elected to Membership at this Society Meeting.

Four members signed the Members' Book and were admitted to the Society.

Professor Greenlees introduced the LMS Lecture given by Professor James McKernan FRS, University of California, San Diego, on *How many equations does one need to define a variety?*

Professor Greenlees expressed warm thanks to Professor McKernan and invited the audience to ask questions.

Afterwards, a wine reception was held in the Mission Hills Room at the Marriot Marquis San Diego Hotel and attended by 60 members and guests.

Records and Proceedings at LMS meetings
South West & South Wales Regional Meeting, 13 December 2017

The regional meeting was held at the University of Cardiff as part of a workshop on *Algebraic Structures and Quantum Physics* (13-15 December 2017). Over 35 members and visitors were present for all or part of the meeting.

The meeting began at 3.00 pm with the Vice-President, Professor John Greenlees, in the Chair.

No members were elected to membership.

Three members signed the book and were admitted to the Society.

Dr Mathew Pugh introduced the first lecture given by Professor Shahn Majid (QMUL) on *Braided algebra and dual bases on quantum groups*.

After tea, Dr Simon Wood introduced the second lecture by Professor Ingo Runkel (Hamburg) on *Categorification and field theory*.

The Vice-President thanked the speakers for their talks and he also thanked the speakers at the Graduate Student Meeting, which was held in the morning and which was the first ever Graduate Student Meeting associated with a Regional Meeting. The Vice-President expressed the thanks of the Society to the local organisers for putting on such an interesting meeting.

Afterwards, Dr Mathew Pugh invited participants to the Society Dinner, which was held at the Jury's Inn.

Algorithms for Kidney Donation

DAVID MANLOVE

Sometimes a patient requiring a kidney transplant has a willing donor who, owing to blood- and/or tissue-type incompatibilities, cannot donate. The introduction of the Human Tissue Act allowed people to 'swap' donors and for multiway kidney exchanges to take place. We describe the algorithm used by the NHS Blood and Transplant to find optimal sets of kidney exchanges, which has identified 760 actual kidney transplants to date.

Introduction

Kidney disease is a life-threatening illness that is prevalent in society: as of 31 December 2014, 58,968 adults in the UK were receiving treatment for kidney failure, with 27,804 of those on dialysis. Not only does dialysis present major lifestyle obstacles for a patient, long-term survival rates are much better when renal replacement therapy is in the form of transplantation rather than dialysis.

Needless to say, there is a shortage of donors. In the UK, as of 31 March 2017, there were 5,233 patients on the active transplant list (also known as the deceased donor waiting list or DDWL) waiting for a donor kidney. Between 1 April 2016 and 31 March 2017, 2,238 kidney transplants from deceased donors took place. The median waiting time to transplant is 864 days for an adult and 266 days for a child (based on patient registrations between 1 April 2010 and 31 March 2014). Kidneys used for transplantation can come from both deceased and living donors, with living donor transplantation generally having better long-term outcomes compared to deceased donor transplantation. In the UK, around 30% of all kidney transplants between 1 April 2016 and 31 March 2017 came from living donors.

It is often the case that a patient requiring a kidney transplant has a willing donor, but due to blood- and/or tissue-type incompatibilities, the transplant cannot take place. However, in the UK, the Human Tissue Act 2004 and the Human Tissue (Scotland) Act 2006 introduced, among other things, the legal framework required to allow the transplantation of organs between donors and recipients with no genetic or emotional connection. Essentially this meant that, provided that certain conditions are fulfilled (including the absence of any financial inducement), it is now possible for a patient to receive a kidney from someone they have never met, which was not permitted previously. This has given rise to new possibilities for living kidney donation. For example, with the introduction of the Human Tissue Act, a patient with an incompatible donor can now 'swap' their donor with that of another patient in a similar position, via a "kidney exchange" that involves two or more incompatible donor–recipient pairs, so that every recipient can obtain a compatible kidney. This is known as *paired kidney donation*.

A *pairwise (kidney) exchange* involves two incompatible donor–recipient pairs (d_1, r_1) and (d_2, r_2), where d_1 is compatible with r_2, and d_2 is compatible with r_1. In the exchange d_1 donates a kidney to r_2 in exchange for d_2 donating a kidney to r_1. *3-way exchanges* extend this concept to three pairs in a cyclic manner (see Figure 1). In view of the cyclic nature of these structures, we will sometimes loosely refer to pairwise and 3-way exchanges as *cycles*.

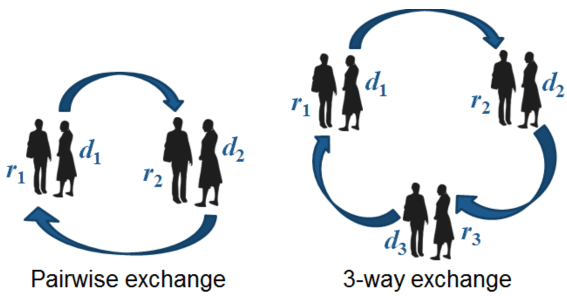


Figure 1: Paired donation: a pairwise exchange and a 3-way exchange

Another consequence of the Human Tissue Act is that an *altruistic donor* a_1 (also referred to as a *non-directed donor*), who wishes to donate a kidney, but does not have a particular identified recipient, can benefit either a single patient on the deceased donor waiting list or can trigger a *domino paired donation chain* (henceforth abbreviated to *chain*) involving one or more incompatible donor–recipient pairs: here a_1 donates to a recipient r_2 in exchange for r_2's donor donating to the recipient r_3 in the next pair in the chain, and so on, with the final donor donating to the deceased donor waiting list. A chain is *short* (resp. *long*) if it consists of one (resp. two) incompatible donor–recipient pairs). See Figure 2.

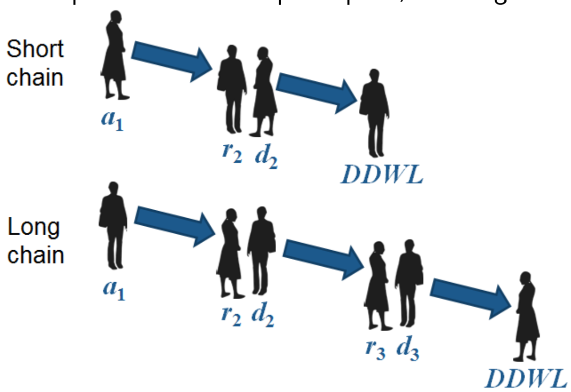


Figure 2: Altruistic donation: a short and long chains

In a number of countries, centralised programmes (also known as *kidney exchange matching schemes*) have been introduced to help optimise the search for *kidney exchanges*, which are sets of cycles and chains satisfying various criteria. These countries include the USA, the Netherlands, the UK, South Korea, Romania and many others around the world.

Following the introduction of the Human Tissue Act, in early 2007 the UK established what has now become the UK Living Kidney Sharing Scheme (UKLKSS), administered by the National Health Service Blood and Transplant (NHSBT). The purpose of the UKLKSS is to identify optimal sets of cycles and chains at regular intervals (currently every quarter) based on donors and recipients currently in the pool. There is a precise definition of "optimal" that is determined by NHSBT (and in particular its Kidney Advisory Group); we will elaborate on this below.

In general, it is seen as logistically challenging to carry out the transplants involved in a cycle when the number of pairs involved in a single such exchange is large. This is mainly because all operations have to be performed simultaneously due to the risk of a donor reneging on his or her commitment to donate a kidney after their loved one has received a kidney. But also, longer cycles / chains involve more participants, and therefore carry a higher risk that the whole cycle / chain will break down if one of the donors or recipients involved becomes ill. Mainly for these reasons, at the present time, optimal solutions constructed for the UKLKSS are restricted to contain pairwise and 3-way exchanges, and short and long chains. 3-way exchanges, for example, require substantial coordination, involving six simultaneous operating theatres and surgical teams (for three nephrectomies and three transplants). They attracted national media attention when they were first introduced in the UK (for example, see the BBC's coverage on tinyurl.com/y8y99pc7).

Algorithms for constructing optimal solutions for the quarterly matching runs of the UKLKSS have been developed by the author and his colleagues, and have been used by NHSBT since July 2008. The aim of this article is to describe in mathematical terms the problem to be solved, give an overview of the solution techniques employed by the UKLKSS algorithm, present a summary of results from recent matching runs and conclude with some avenues for future work.

Definition of the problem model

The problem of finding an optimal set of cycles and chains essentially corresponds to computing an optimal cycle packing in a weighted directed graph. Suppose we have some number of incompatible donor–recipient pairs (d_i, r_i) (i.e., d_i is the willing but incompatible donor for r_i), and some number of altruistic donors a_j.

We consider the case without altruistic donors first. For this form a directed graph whose vertices are the incompatible donor–recipient pairs $v_i = (d_i, r_i)$. Add a directed edge, or *arc*, from $v_i = (d_i, r_i)$ to $v_j = (d_j, r_j)$ whenever d_i is a compatible donor for r_j (with respect to blood- and tissue-type, and other medical factors such as age restrictions).

Now suppose there are altruistic donors. For each such donor a_j form a pair $v_j = (d_j, r_j)$ where d_j represents the altruistic donor and r_j is a dummy recipient who is compatible with each (non-altruistic) donor. (In practice the dummy recipient indicates a patient on the deceased donor waiting list.) Add the vertices (d_j, r_j) to the graph just constructed, and again add an arc from $v_j = (d_j, r_j)$ to $v_k = (d_k, r_k)$ whenever d_j is a compatible donor for r_k. Likewise add an arc from v_k to v_j whenever d_k is a non-altruistic donor. The resulting directed graph is called the *kidney exchange digraph*. The following proposition is then straightforward.

Proposition 1. *Let $D = (V, A)$ be a kidney exchange digraph. A 2-cycle (resp. 3-cycle) in D not involving an altruistic donor corresponds to a pairwise (resp. 3-way) exchange, whilst a 2-cycle (resp. 3-cycle) in D involving an altruistic donor corresponds to a short (resp. long) chain.*

Each arc from $v_i = (d_i, r_i)$ to $v_j = (d_j, r_j)$ (where d_j is not an altruistic donor) in the kidney exchange digraph is assigned a positive real-valued weight that arises from a scoring system employed by NHSBT to measure the potential benefit of a transplant from d_i to r_j. Factors involved in computing this weight include: waiting time (based on the number of previous matching runs the participant has been unsuccessfully involved in), sensitisation (based on calculated HLA antibody reaction frequency), HLA mismatch levels between a donor and recipient (which roughly speaking corresponds to levels of tissue-type incompatibility) and points relating to the difference in ages between d_i and d_j (see [4] for more details). If d_j is an altruistic donor then the weight of the arc (v_i, v_j) is 0. The weight of a cycle is the sum of the weights of the individual arcs in it. Higher weights are better than lower ones.

Define a *set of exchanges* $\mathcal{S}$ to be a vertex-disjoint collection of cycles in D, each of length at most 3. Proposition 1 then indicates how the cycles in $\mathcal{S}$ correspond to pairwise / 3-way exchanges and short / long chains. If $v_i \in V$ is incident to a cycle in $\mathcal{S}$ then v_i is said to be *matched*, otherwise v_i is *unmatched*. Suppose some $v_i = (d_i, r_i) \in V$ is unmatched. If d_i is not an altruistic donor then each of d_i and r_i is said to be *unmatched* and will not participate in a cycle or chain. On the other hand if d_i is an altruistic donor then d_i is also said to be *unmatched*, but in practice he/she will donate directly to the deceased donor waiting list. For this reason, we define the *size* of $\mathcal{S}$ (corresponding to the number of transplants yielded by this set of exchanges) to be the number of vertices matched by $\mathcal{S}$ plus the number of unmatched vertices corresponding to altruistic donors. The following theorem is then immediate.

Theorem 1. *Let $D = (V, A)$ be a kidney exchange digraph with weight function w. A maximum size (resp. weight) set of exchanges in D gives a set of kidney exchanges for the corresponding donors and recipients with the maximum number of transplants (resp. maximum weight).*

Imposing additional objectives

In practice there are additional objectives (beyond maximising size and weight) to be satisfied by an optimal set of exchanges constructed for the UKLKSS. These are mainly associated with mitigating the risk associated with long chains and 3-way exchanges, and we elaborate on these objectives now.

Given a 3-cycle C in D with arcs (v_i, v_j), (v_j, v_k),(v_k, v_i), we say that C contains a *back-arc* if without loss of generality $(v_j, v_i) \in A$. In such a case we say that C contains an *embedded 2-cycle* involving arcs $(v_i, v_j), (v_j, v_i)$. A 3-cycle with a back-arc and an embedded 2-cycle can be found in the digraph shown in Figure 3, comprising vertices (d_2, r_2), (d_5, r_5) and (d_4, r_4). An *effective 2-cycle* is either a 2-cycle or a 3-cycle with at least one back-arc.

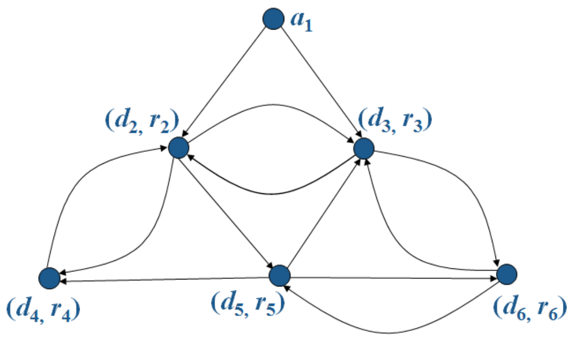


Figure 3: Example kidney exchange digraph

A back-arc can be seen as a form of fault-tolerance in a 3-cycle. To understand why, consider again the 3-cycle in the kidney exchange digraph shown in Figure 3 involving vertices (d_2, r_2), (d_5, r_5) and (d_4, r_4). If either d_5 or r_5 drops out (for example due to illness), then the pairwise exchange involving (d_2, r_2) and (d_4, r_4) might still be able to proceed. On the other hand, if either of the pairs (d_2, r_2) or (d_4, r_4) were to withdraw, then this pairwise exchange would have failed anyway. Thus the risk involved with a 3-way exchange, due to the greater likelihood (as compared to a pairwise exchange) of the cycle breaking down before transplants can occur, is mitigated with the inclusion of a back-arc.

We now present the definition of an *optimal* set of exchanges for the UKLKSS, as determined by the Kidney Advisory Group of NHSBT.

Definition 1. A set of exchanges $\mathcal{S}$ is *optimal* if:

(1) the number of effective 2-cycles in $\mathcal{S}$ is maximised;

(2) subject to (1), $\mathcal{S}$ has maximum size;

(3) subject to (1)-(2), the number of 3-cycles in $\mathcal{S}$ is minimised;

(4) subject to (1)-(3), the number of back-arcs in the 3-cycles in $\mathcal{S}$ is maximised;

(5) subject to (1)-(4), the overall weight of the cycles in $\mathcal{S}$ is maximised.

We give some intuition for Definition 1 as follows. The first priority is to ensure that there are at least as many 2-cycles, and 3-cycles with embedded 2-cycles, as there would be in an optimal solution containing only 2-cycles. This is to ensure that the introduction of 3-way exchanges is not detrimental to the maximum number of pairwise exchanges that could possibly take place. Subject to this we maximise the total number of transplants (this is the number of unmatched altruistic donors, plus twice the number of pairwise exchanges and short chains, plus 3 times the number of 3-way exchanges and long chains). Subject to this we minimise the number of 3-way exchanges. Despite Objective 1, this is still required (see [5] for an example to show why this might be the case). Next the number of back-arcs in 3-way exchanges is maximised (note that a 3-way exchange could contain more than one back-arc). Finally we maximise the sum of the cycle weights.

As an example, relative to Figure 3, an optimal set of exchanges comprises the short chain involving a_1 and (d_3, r_3), the 2-cycle involving (d_2, r_2) and (d_4, r_4), and the 2-cycle involving (d_5, r_5) and (d_6, r_6) (assuming that each arc has unit weight).

Algorithms for finding optimal solutions

When only short chains and pairwise exchanges are permitted, an optimal set of exchanges (with respect to size and/or weight) can be found in polynomial time using maximum weight matching in a general graph (see, e.g., [2] for more details). However when 3-way exchanges are allowed (and even in the absence of altruistic donors), the problem of finding a set of exchanges that maximises the number of transplants is NP-hard [1] and indeed APX-hard [2].

Our UKLKSS algorithm uses a sequence of Integer Programming (IP) formulations (each extending the so-called *cycle formulation* first proposed by Roth et al. [9]) to find an optimal set of kidney exchanges. After each run of the IP solver, we use the optimal value calculated at that iteration to enforce a constraint that must be satisfied in subsequent iterations. This ensures that once Objectives 1 to r in Definition 1, for some $1 \le r \le 4$, have been satisfied by an intermediate solution, they continue to hold when we additionally enforce Objective $r+1$.

At the outset, we deal with Objective 1 (i.e., maximise the number of effective 2-cycles). Construct an undirected graph $G = (V, E)$ corresponding to the kidney exchange digraph D, where the vertices in G and D are identical, and an edge in G corresponds to a 2-cycle in D (i.e., $\{v_i, v_j\} \in E$ if and only if $(v_i, v_j) \in A$ and $(v_j, v_i) \in A$). Compute N_2, the size of a maximum cardinality matching in G using the Micali-Vazirani implementation of Edmonds' algorithm [6].

Objective 2, maximising the number of transplants, is dealt with next. This is done using an IP model which extends the cycle formulation of Roth et al. [9] in order to enable unmatched altruistic donors to be quantified. See "The IP model for Objective 2".

The IP model for Objective 2

Let $\mathscr{C} = \{C_1, C_2, \ldots, C_{n_C}\}$ denote the set of all possible cycles of lengths 2 and 3 in the digraph D. Without loss of generality, suppose that the 2-cycles in $\mathscr{C}$ are $C_1, \ldots, C_{n_2}$, the 3-cycles in $\mathscr{C}$ are $C_{n_2+1}, \ldots, C_{n_2+n_3}$, and the 3-cycles with back-arcs in $\mathscr{C}$ are $C_{n_2+1}, \ldots, C_{n_2+n_b}$ (so $n_C = n_2 + n_3$). Let x be an $(n_C + n_A) \times 1$ vector of binary variables $x_1, x_2, \ldots, x_{n_C+n_A}$, where n_A is the number of altruistic donors, and for $1 \le j \le n_C$, $x_j = 1$ if and only if C_j belongs to an optimal solution, and for $1 \le j \le n_A$, $x_{n_C+j} = 1$ if and only if altruistic donor a_j is unmatched. Let n_P denote the number of incompatible donor–recipient pairs in D. The IP model is then

$$\max\left(\sum_{j=n_2+n_3+1}^{n_C+n_A} x_j + 2\sum_{j=1}^{n_2} x_j + 3\sum_{j=n_2+1}^{n_3} x_j\right) \tag{2}$$

subject to:

$$\sum_{C_j \in \mathscr{C}: v_i \in C_j} x_j \le 1 \quad (1 \le i \le n_P) \tag{3}$$

$$x_{n_C+i} + \sum_{C_j \in \mathscr{C}: v_{n_P+i} \in C_j} x_j = 1 \quad (1 \le i \le n_A) \tag{4}$$

$$\sum_{j=1}^{n_2+n_b} x_j \ge N_2 \tag{5}$$

We now provide some intuition for the IP model. Objective function (1) maximises the number of transplants (2 times the number of selected 2-cycles plus 3 times the number of selected 3-cycles plus the number of unmatched altruistic donors). Constraints

Matching run		2016				2017				Total
		Jan	Apr	Jul	Oct	Jan	Apr	Jul	Oct	to date
Properties of D	#vertices	255	271	284	273	271	269	269	279	
	#altruistic donors	11	5	7	10	12	11	11	13	
	#arcs	4304	3966	4804	3964	4292	4459	4621	5964	
	#2-cycles n_2	279	148	194	151	292	321	214	381	
	#3-cycles n_3	1946	782	1487	993	2538	2806	1573	3815	
Identified solution	#pairwise exchanges	3	2	4	2	4	5	2	10	136
	#3-way exchanges	8	5	7	8	7	12	12	10	201
	#short chains	2	0	0	4	1	1	3	2	92
	#long chains	8	4	7	5	9	9	7	8	73
	size	58	31	50	51	58	75	67	78	1278
	weight	5835	3086	5125	4154	6863	6913	5858	8138	
Actual transplants	#pairwise exchanges	4	2	4	4	4	3	2	?	104
	#3-way exchanges	4	2	3	5	3	5	3	?	96
	#short chains	7	2	1	5	3	6	3	?	84
	#long chains	1	2	5	3	2	2	5	?	32
	total	37	20	34	42	29	39	34	?	760

Table 1: Results arising from matching runs between January 2016 and October 2017, and from all runs.

(2) and (3) ensure that each donor and recipient is included in at most one selected cycle, whilst Constraint (3) also ensures that if no cycle containing a given altruistic donor is selected, then that donor is set to be unmatched. Finally Constraint (4) ensures that the number of effective pairwise exchanges is maintained as being optimal. For descriptions of the IP models for Objectives 3-5 in Definition 1, the interested reader is referred to [5].

Results obtained from UKLKSS matching runs

Prior to our involvement, NHSBT used an in-house algorithm that identified only pairwise exchanges. With the need to find both pairwise and 3-way exchanges, a new software application was developed based on the algorithm described above. At its heart the application uses the COIN-Cbc IP solver to solve each of the IP problems involved. COIN-Cbc was chosen due to its open licence agreement and the need to deploy the application commercially.

The application can either be accessed programmatically through a web API or alternatively manually via a web interface (kidney.optimalmatching.com). The former version (along with several prototypes) has been used by NHSBT to find an optimal solution in each of the UKLKSS matching runs (occurring at roughly quarterly intervals), since July 2008. The kidney exchange digraph corresponding to the July 2015 matching run, together with an optimal solution, is shown in Figure 4.

Table 1 summarises the input to, and output from, each matching run between January 2016 and October 2017, and from all matching runs since the UKLKSS began. In each case an optimal solution was returned within 3 seconds (on a Linux Centos 5.5 machine with a Pentium 4 3GHz single core processor with 2Gb RAM).

The table shows key attributes, for each matching run, organised into three categories, concerning: (i) the kidney exchange digraph D, (ii) the optimal solution identified by the algorithm, and (iii) the actual transplants arising from this solution. In Category (i), we indicate the number of vertices and arcs in D, the number of altruistic donors (which contribute towards the vertex count), and the numbers of 2-cycles and 3-cycles in D. In Category (ii) we break the identified solution down into the numbers of pairwise and 3-way exchanges, and short and long chains, showing the overall size and weight. Finally in Category (iii), we show the numbers of pairwise and 3-way exchanges, and short and long chains, that actually occurred, together with the total number of actual transplants (the final figures from the October 2017 runs are unknown at the time of writing).

To date, in total 1278 potential transplants have been identified, resulting in 760 *actual* transplants taking place (752 since July 2008). In general not all the identified transplants proceed to surgery for a number of reasons, including last-minute tissue-type incompatibilities being discovered, and donors and recipients becoming too ill for transplant [4].

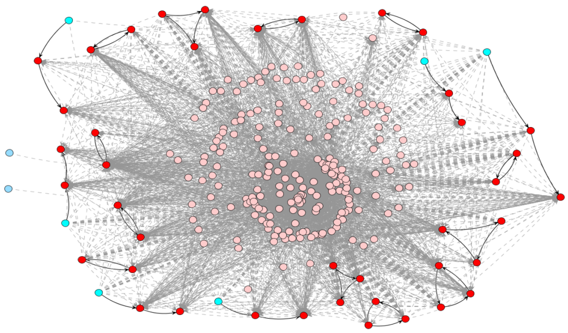


Figure 4: The kidney exchange digraph corresponding to the July 2015 matching run, together with an optimal solution. Turquoise and blue vertices are altruistic donors (the former trigger long chains whilst the latter are unmatched). Turquoise and red vertices belong to cycles and chains, whilst pink vertices represent unmatched donors and recipients. The software for visualising this digraph was written by Tommy Muggleton.

Going forward

Algorithms for paired and altruistic kidney donation will need to anticipate new challenges that may arise in the future. Firstly, longer chains may become feasible, given that the transplants do not have to be conducted on the same day in this case. Secondly, European collaboration as part of COST Action CA15210 (ENCKEP: European Network for Collaboration on Kidney Exchange Programmes), for which the author is Vice-Chair, will lay the foundations for future transnational collaboration, giving opportunities for countries to share their pools of donors and recipients so that optimal solutions can yield more transplants, and better quality matches. Longer chains and larger pools lead to more challenging computational problems, which kidney exchange algorithms must anticipate. Some preliminary work along these lines has already given encouraging results [3].

Acknowledgements

The research conducted was financially supported by the National Health Service Blood and Transplant (NHSBT) between 1 April 2010 – 30 June 2011, and between 1 January 2012 – 30 June 2013 (project 10-11-ODT), and also by the Engineering and Physical Sciences Research Council through grants EP/E011993/1, EP/K010042/1, EP/K503903/1 and EP/N508792/1. Collaboration with current and former University of Glasgow colleagues Péter Biró, Gregg O'Malley and James Trimble, and with NHSBT colleagues Rachel Johnson, Iain Harrison and Matthew Robb, is gratefully acknowledged. Any views expressed here are those of the author and not necessarily those of NHSBT.

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David Manlove

David Manlove is a Senior Lecturer in Computing Science at the University of Glasgow. His research interests lie in algorithms and complexity, and in particular he studies algorithmic aspects of matching problems with and without preferences. He and his son are season ticket holders for Hibernian Football Club. On 21 May 2016 they watched their team win the Scottish Cup for the first time in 114 years.

Initiatives for the Mathematical Sciences in Africa

PATRICK DOREY, STEPHEN HUGGETT, JOHN HUNTON, AND FRANK NEUMANN

We describe several current initiatives aimed at supporting mathematics in Africa, and draw attention to opportunities for UK mathematicians to become involved.

The African Institute for Mathematical Sciences

A class at AIMS South Africa in 2011

The African Institute for Mathematical Sciences (AIMS) is a network of centres for the mathematical sciences across Africa. The first AIMS centre was founded in 2003 by Neil Turok in Muizenberg, just outside Cape Town in South Africa; since then centres have been opened in Senegal, Ghana, Cameroon, Tanzania and, most recently, Rwanda. A principal focus of these centres is the teaching of a one-year Masters degree, entry to which is open, on a competitive basis, to graduating students from all over the continent.

Courses are delivered in three-week blocks by volunteer lecturers, and in the past many of these lecturers have come from mathematics departments in the United Kingdom. Lecturing a course at an AIMS centre is a very rewarding experience, giving an opportunity to meet students from a wide variety of backgrounds, all of whom are highly motivated to learn. The year is split into three phases: the skills courses on a set range of topics, followed by review courses on a wide variety of subjects which vary from year to year depending on the proposals received, and then an essay phase during which students work with an individual supervisor. Anyone interested in lecturing a course at an AIMS centre should visit the course proposals page at the website of AIMS-NEI, the umbrella organisation of the AIMS network, at www.nexteinstein.org/apply/teach,where they can also find more information about the whole AIMS programme. There are also opportunities for graduating Masters or (more usually) PhD students to spend a whole year at an AIMS centre, working as a tutor; more details of these are at www.nexteinstein.org/apply/teaching-assistants.

International Short Visits (LMS Scheme 5)

This grant scheme supports visits for collaborative research, either by the grant holder to a developing country,[1] or by a named mathematician from such a country to the grant holder.

The detailed criteria can be found on the LMS website at www.lms.ac.uk/grants/international-short-visits-scheme-5, but here is a summary.

For visits to the UK, the maximum award is £3,000 towards actual expenses for travel (both international and within the UK) and for accommodation and subsistence (up to a maximum of £2,000, with a limit of £100 per day). For visits from the UK, the maximum award is £2,000 towards actual expenses for travel, accommodation and subsistence.

Algebraic Geometry Summer School in Mombasa, 2013

The applicant should be a mathematician based in the UK, and non LMS members will need to ask an

[1]The LMS uses the definition in www.mathunion.org/cdc/grants/cdc-definition-for-developing-countries.

LMS member to countersign their application. Visits to the UK are expected to last at least 14 days. Incoming visits sometimes include attending a conference, but the justification for the grant cannot include this, the grant cannot be used to cover any expenses incurred during the period of the conference, and the visit should be for a period of 14 days in addition to any time spent at the conference. Here is a table of recent outgoing awards:

Year	Amount	Africa	Number	Africa
2012/13	£7,000	£5,700	4	3
2013/14	£13,330	£11,600	9	8
2014/15	£6,079	£6,079	4	4
2015/16	£7,103	£5,900	4	3
2016/17	£4,442	£4,442	3	3
Total	£37,954	£33,721	24	21

Here is a table of recent incoming awards:

Year	Amount	Africa	Number	Africa
2012/13	£5,500	£3,000	2	1
2013/14	£5,274	£5,274	2	2
2014/15	£7,150	£1,450	3	1
2015/16	£3,615	£0	2	0
2016/17	£4,210	£2,410	2	1
Total	£25,749	£12,134	11	5

Michael Wemyss: a visit from Uganda

Dr David Ssevviiri, Head of School at Makerere University in Uganda, visited the UK on a scheme 5 LMS grant for four weeks during May 2017. He spent the majority of his time visiting me in Glasgow, although he also spent significant time in Oxford, Bath, Edinburgh and Warwick. One publication has resulted from his visit, but perhaps the main outcome was all the new mathematicians he met, and the new mathematical connections made between David and the UK mathematical community.

Throughout Africa, enormous administrative loads do not allow much opportunity for research, or for career development. David wants to develop the capacity to graduate PhD students in Uganda, and he is very aware of all the steps needed to achieve this. We have one ongoing application, through the IMU, to jointly supervise PhD students, and pending further funding, we hope to organise a conference in Uganda in 2019.

Volunteer Lecturer Program

The goal of the Volunteer Lecturer Program (see www.mathunion.org/cdc/volunteer-lecturer-program) is to offer mathematics departments in universities in the developing world lecturers for intensive 3–4 week courses in mathematics at the advanced undergraduate or master's level. The volunteer lecturer is chosen by the host institution from an online database of mathematician volunteer lecturers maintained by the Commission for Developing Countries of the International Mathematical Union (IMU).

LMS Publications

Mathematics is a global activity. To be fully part of that activity a mathematician needs access to research articles, and researchers need the opportunity to publish in international journals to let the world know what they are doing. The *Transactions of the LMS* publishes research articles under the 'gold OA' model (author's institution or funder pays) and so its content is freely available to everyone; authors from developing countries would normally receive a waiver and would publish in this journal for free.

The gold OA model however represents only a tiny proportion of mathematical publishing, the vast majority being under the traditional subscription model — free for anyone to publish, but institutions pay for access for their mathematicians. This is the principal structure the *Bulletin*, *Journal* and *Proceedings of the LMS* operate under, and as such brings in the main part of the Society's annual income, allowing the existence of the Society's various grant schemes (including Scheme 5 discussed earlier in this article) and much of the other work of the LMS.

To mitigate the access problems for mathematicians in the developing world, LMS journals are available through a number of schemes and initiatives that provide free access to institutions from poorer regions. A good example is the ARDI initiative, part of the Research4Life partnership[2]. ARDI operates in conjunction with a number of major publishers, including John Wiley who on behalf of the LMS currently produce, distribute and market the *Bulletin*, *Journal*, *Proceedings* and *Transactions of the LMS*, as well as the LMS owned *Journal of Topology*. The LMS manage *Composito Mathematica* and *Mathematika* on behalf of the Compositio Foundation and UCL respectively; these are published by CUP and are also included in the ARDI umbrella, as is the Institute of Physics, with whom the LMS jointly publish *Nonlinearity* and three Russian translation journals (the English versions of *Sbornik*, *Izvestiya* and *Russian Mathematical Surveys*). Institutions in over 70 countries (including much of sub-Saharan Africa) can sign up for free access to all journals in these publishers' lists, and another 50 or so countries can gain access at very reduced rates.

The LMS journals are of course freely available to LMS members, whatever country they live in.

Niels Laustsen: a visit to South Africa

In January 2017 a scheme 5 grant from the LMS enabled me to spend three weeks visiting two South African colleagues, Professors Sonja Mouton and Heinrich Raubenheimer at the Universities of Stellenbosch and Johannesburg, respectively. The main aim of my visit was to initiate collaborative research, which has led to a joint paper with Mouton and her former research student Benjamin, while a paper with Raubenheimer is nearing completion.

During my visit, I was invited to give research seminars at the two host universities, as well as North-West University in Potchefstroom. My hosts also very kindly showed me some of the attractions of their beautiful country, including the vineyards of Western Cape, a rescue centre for cheetahs and the famous 'cradle of humankind', where some of the world's most important discoveries of hominid skeletons have been made.

Mentoring African Research in Mathematics

The Mentoring African Research in Mathematics (MARM) programme is an initiative sponsored by the LMS and the International Mathematical Union (IMU) in association with the African Mathematics Millennium Sciences Initiative (AMMSI). MARM links African academics with their UK and European counterparts via professional mentoring partnerships. In doing so, MARM provides the means and opportunities for African mathematicians to develop international working relationships while also improving the quality of academic provision within their home institution.

NIMS Ghana on KNUST campus in Kumasi, Ghana in 2014

The MARM programme has now been running since 2006 and was originally funded by the Nuffield Foundation and the Leverhulme Trust. So far 20 MARM partnerships have been set up with African institutions in Cameroon, Congo, Ethiopia, Ghana, Ivory Coast, Kenya, Malawi, Morocco, Nigeria, Rwanda, South Africa, Tanzania and Uganda[3].

There are many different challenges to overcome in each of the African countries and the individual Europe–Africa partnerships collaboratively seek to address the specific needs and priorities of the host institutions in Africa within their very diverse educational environments. Over the years, the MARM programme has helped mentoring and supporting African postgraduate students and early career academics in research and wider academic skills. This includes joint supervision of PhD students, support of international workshops and research schools, consultancy on developing postgraduate programmes and improvements in local library and IT resources. An important emphasis is also the support of female

[2]See www.wipo.int/ardi/en/about.htmland www.research4life.org.

[3]More information on the programme and details of the particular partnerships can be found on the MARM webpages of the LMS at www.lms.ac.uk/grants/mentoring-african-research-mathematics.

An example of MARM impact: NIMS in Kumasi, Ghana

The National Institute for Mathematical Sciences (NIMS) is a research and training centre based on the KNUST campus in Kumasi and supported by all major Ghanaian universities.

The first round MARM partnership between the mathematics departments of the University of Leicester and KNUST was instrumental to bring NIMS to Kumasi and to obtain substantial external funding to support a new postgraduate programme in mathematics and applications at NIMS. The programme with focus on scientific computing and industrial mathematical modelling is currently sponsored until March 2018 by the Norwegian Petroleum Geo-Services (PGS) company and administered through the Norwegian Academy of Science and Letters (DNVA).

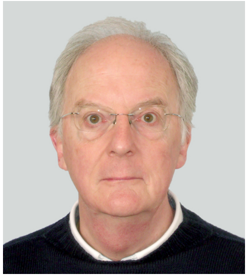
students and researchers at all stages in their careers as there is a large gender imbalance within the mathematical sciences in Africa.

Even a small MARM grant of £10,000 for each partnership for an initial period of two years can make a difference and helps to build new research links and networks so crucial for the development of mathematical sciences in Africa and to counter the brain drain of young gifted African researchers. The impact of the MARM programme for the mathematical sciences in Africa has been substantial. For example, it led directly to the leveraging of external funds into the National Institute of Mathematical Sciences (NIMS) at Kwame Nkrumah University of Science and Technology (KNUST) in Kumasi, Ghana[4]. It also gave substantial support to the Kenyatta University International Mathematics Conference series in Nairobi, Kenya encouraging networking and attracting high-level international speakers to the African continent. The initiative also resulted in several co-authored publications by African and European researchers as well as African PhD theses completed under joint supervision.

The MARM partnerships have proved to be fertile seeds for a growing network of academic initiatives and links in the mathematical sciences across the continent and have a lot of potential for substantially enhancing the existing research landscape in Africa.

Patrick Dorey

Patrick Dorey is a Professor of Mathematics at Durham University, specialising in mathematical physics. He has lectured at AIMS centres in South Africa, Ghana, Cameroon and Tanzania, and is currently Chair of the Academic Council of AIMS Ghana.

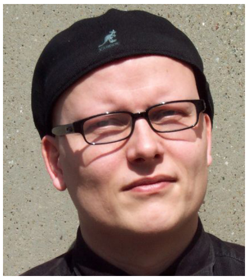


Stephen Huggett

Stephen Huggett is Professor of Pure Mathematics at Plymouth University. His research interests include twistor theory and graph theory. He is General Secretary of the LMS. He does not always look as vacant as in his passport photograph.

John Hunton

John Hunton is Professor of Pure Mathematics at Durham University. His research is in Algebraic Topology, particularly in the study of Aperiodic Patterns. He has been Publications Secretary of the LMS since 2013.



Frank Neumann

Frank Neumann is Associate Professor of Pure Mathematics at the University of Leicester. His research interests are in algebraic topology and algebraic geometry. He has been a MARM research mentor in Ghana and currently serves as a member for the MARM board of LMS, IMU and AMMSI.

[4]More details about NIMS and its programmes and activities can be found on the NIMS website web.nims.edu.gh.

The Work of Simon Donaldson

IVAN SMITH AND RICHARD P. THOMAS

Simon Donaldson is a giant of modern geometry and analysis who has had a huge impact on subjects ranging from topology and algebraic geometry to theoretical physics. We give a broad-brush description of a very small subset of some of his work.

1. Introduction

We have been set the daunting task of trying to explain Simon Donaldson's research to (say) an undergraduate in applied mathematics. The approach we have taken is to spend most of the time describing some of the prerequisites, such as de Rham cohomology and moduli spaces. This is both to make them accessible, but also to see them as toy models for some of his work.

2. Moduli spaces

A "*moduli space*" is a fancy name for a parameter space. It parameterises something — all geometric structures of a certain type, for instance, or all solutions of a certain equation. It is the set of solutions of some problem, but it is more than a set, it has an appropriate geometric structure on it, like a topology or an algebraic structure. So, for example,

$$\mathcal{M} \;=\; \big\{(m,c)\colon\, m,c\in\mathbb{R}\big\} \;\cong\; \mathbb{R}^2$$

is a moduli space for non-vertical straight lines $y = mx + c$ in the plane. It is also the moduli space of solutions of a differential equation

$$\mathcal{M} \;=\; \left\{y(x)\colon\, \frac{d^2y}{dx^2} = 0\right\}.$$

It has a natural topology on it because we can make sense of two lines being "close". From its second description, it also has a natural linear structure on it (solutions of linear differential equations can be added or multiplied by scalars). So in this example it is a vector space with the usual topology.

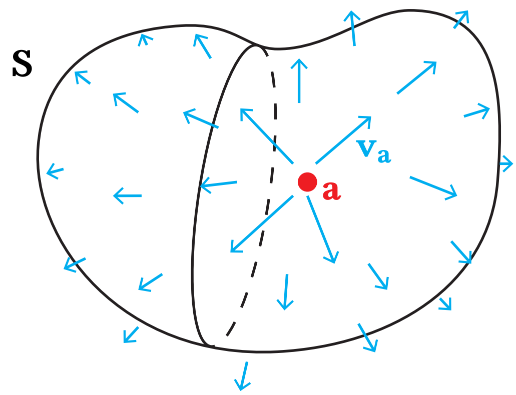
Mathematicians have studied moduli spaces extensively in different settings. In algebraic geometry there is a long history of starting with a space X (an "algebraic variety") and studying the geometry of moduli spaces $\mathcal{M}_X$ associated to it (the moduli space of all algebraic curves in X, or of all line bundles on X, or ...).

Donaldson turned this procedure on its head, and used the geometry of a moduli space $\mathcal{M}_X$ (of solutions of the "self-dual Yang-Mills equations" on X) to study the original space X. Before explaining this, we show how a linear version (replacing the nonlinear Yang-Mills equations by the linear Maxwell equations) is already familiar to first year undergraduates: studying the moduli space of solutions of standard vector calculus equations on a space X tells us about the topology of X.

de Rham theory

Consider a fluid (or electric or gravitational field), emitted from a source at $\mathbf{a} \in \mathbb{R}^3$, flowing with vector field

$$\mathbf{v_a}(\mathbf{x}) = \frac{1}{4\pi^2}\cdot\frac{\mathbf{x}-\mathbf{a}}{|\mathbf{x}-\mathbf{a}|^3} \quad \text{on } \mathbb{R}^3\backslash\{\mathbf{a}\}.$$



This is incompressible flow, so $\nabla.\mathbf{v_a}(\mathbf{x}) = 0$ away from $\mathbf{a}$ and the flow across any surface not enclosing $\mathbf{a}$ is zero by the divergence theorem:

$$\int_{\partial D} \mathbf{v_a}.\, d\mathbf{S} \;=\; \int_D \nabla.\mathbf{v_a}\, dV \;=\; 0.$$

Here $D \subset \mathbb{R}^3$ is an open set not containing $\mathbf{a}$ and ∂D is its boundary. Really

$$\nabla.\mathbf{v_a}(\mathbf{x}) \;=\; \delta_{\mathbf{a}}$$

is the Dirac delta source at $\mathbf{a}$, so that when a surface S encloses $\mathbf{a}$ we have

$$\int_S \mathbf{v_a}.\, d\mathbf{S} \;=\; 1,$$

i.e. the flow across S is the flow emitted by the source at $\mathbf{a}$ inside it. In this way $\mathbf{v_a}$ "detects" the "hole" in the space $\mathbb{R}^3\backslash\{\mathbf{a}\}$ it is defined on. Similarly

$$\mathbf{v} = \sum_{i=1}^{n} \alpha_i \mathbf{v}_{\mathbf{a}_i}$$

detects and distinguishes all of the holes in $X = \mathbb{R}^3\backslash\{\mathbf{a}_1, \ldots, \mathbf{a}_n\}$: letting S_i be a small surface enclosing only $\mathbf{a}_i$ we have

$$\int_{S_i} \mathbf{v}.\, d\mathbf{S} \;=\; \alpha_i.$$

By Stokes' theorem, adding the curl of another vector field to $\mathbf{v}$ doesn't change this integral, so elements of the vector space

$$\frac{\{\mathbf{v}\colon \nabla.\mathbf{v} = 0\}}{\{\mathbf{v} = \nabla \wedge \mathbf{w}\}} \qquad (1)$$

detect holes. In fact de Rham's theorem states that for any oriented manifold X the vector space (1) of divergence-free vector fields on X (modulo curls of vector fields) is the *dual* of the vector space

$$H_2(X, \mathbb{R})$$

of 2-dimensional "holes" on X. Formally $H_2(X, \mathbb{R})$ is made up of real linear combinations of surfaces S (without boundary) in X modulo those surfaces $S = \partial D$ which are the boundary of a 3-dimensional subspace D of X. Intimidatingly, it is called the 2-dimensional *homology* of X, while its dual (1) is called its 2-dimensional *cohomology* and is denoted $H^2(X, \mathbb{R})$.

By the same reasoning we similarly find that curl free vector fields (modulo "conservative" vector fields — those which are the gradient of a function) detect 1-dimensional "holes" in any X:

$$\frac{\{\mathbf{v}\colon \nabla \wedge \mathbf{v} = 0\}}{\{\mathbf{v} = \nabla f\}} \;=:\; H^1(X, \mathbb{R}) \;\cong\; H_1(X, \mathbb{R})^*. \qquad (2)$$

Again the pairing is by integration, this time along (real linear combinations of) loops γ modulo those which are the boundary $\partial\Delta$ of a surface $\Delta \subset X$.

Donaldson sailing through mathematics, by Nathalie Wahl

Even more simply, the space of constant functions

$$\{f : X \to \mathbb{R} : \nabla f = 0\} \;=\; H^0(X, \mathbb{R}) \;\cong\; H_0(X, \mathbb{R})^*$$

is dual to the ($\mathbb{R}$-vector space on the set of) path components of X. The pairing is by evaluating a function at a point of the path component, i.e. integrating it on 0-dimensional "holes".

There is also an extension to all dimensions. Finally, instead of taking the quotient by a subspace in (1, 2), we can equivalently take their orthogonals (using the L^2 inner product, and ignoring the infinite dimensional technical issues in this survey). Since the orthogonal to $\operatorname{im} d$ is $\ker d^*$ (for any linear operator d with adjoint d^*) we can rewrite these groups as

$$\{\sigma\colon d\sigma \;=\; 0 \;=\; d^*\sigma\} \;\cong\; H^i(X, \mathbb{R})^* \qquad (3)$$

with d, d^* linear differential operators.

Thus we see topological invariants of X from differential equations; taking the dimension of the moduli space (3) of solutions of Maxwell's equations $d\sigma = 0 = d^*\sigma$ in a vacuum counts the holes in X.

Donaldson theory

Donaldson worked with a *nonlinear* generalisation of the Maxwell equations, called the self-dual $SU(2)$ Yang-Mills equations. They are defined on a compact oriented 4-dimensional manifold X (a topological space which looks locally like $\mathbb{R}^4$, but globally is curved and twisted).

The solution space is no longer a vector space, but the nonlinearities of the equations feed back in a positive way to stop solutions becoming too "big". (This is related to finiteness of energy in physics.) Building on work of Taubes and Uhlenbeck, this allowed Donaldson to compactify the space of solutions in a natural way, yielding a compact moduli space $\mathcal{M}_X$.

He then takes simple invariants of the resulting space $\mathcal{M}_X$ such as the *linear* invariants of the last section; i.e. its cohomology:

$$X \rightsquigarrow \mathcal{M}_X \rightsquigarrow H^*(\mathcal{M}_X).$$

Although the second transform here is linear or "simple", the first is highly complicated and nonlinear. Thus the end result is deep new invariants of the 4-dimensional manifold X containing much more information than linear invariants such as $H^*(X)$.

To obtain numbers one can integrate the resulting cohomology classes in $H^*(\mathcal{M}_X)$ against natural classes restricted from the space of all gauge connections. This amounts to cutting down the space of gauge fields to those satisfying some natural conditions, and then the Donaldson invariants simply *count how many satisfy the Yang-Mills equations*.

These invariants revolutionised 4-dimensional topology, distinguishing many manifolds thought to be the same, finding new phenomena which occur in no other dimension, and proving structural results. For instance he showed that the 4-manifolds arising from algebraic geometry cannot be decomposed as nontrivial unions ("connect sums") of other manifolds: the former have nonzero (in fact strictly positive) Donaldson invariants, whereas the invariants of connect sums all vanish.

Physics

By applying techniques from physics to pure mathematical problems, Donaldson's work reversed the usual interaction between the two subjects. Atiyah and Witten soon interpreted his work in terms of quantum field theory. This, and the development of string theory, has led to a celebrated and vigorous interaction between the two subjects over the last 30 years that shows no sign of slowing down. It is now standard for theoretical physicists to make predictions about pure mathematics that astonish us and are later proved to be true. Indeed whole areas of geometry are now devoted to computing "Donaldson-type invariants" derived from geometric moduli spaces, and understanding physics' predictions about them.

3. Donaldson's theorem

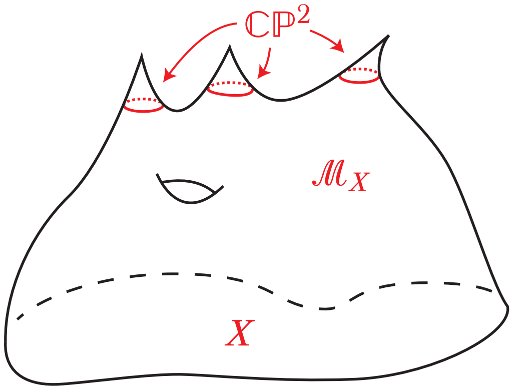
Donaldson's first big result — proved around 1982 during his DPhil with Atiyah and Hitchin in Oxford — is the one that still bears the name "Donaldson's theorem". It was his first use of the Yang-Mills moduli space, slightly different from that of Section 2.

He discovered that on compact orientable 4-dimensional manifolds X satisfying a simple topological restriction (X should be "simply connected and have self-dual intersection form") the moduli space $\mathcal{M}_X$ of Yang-Mills fields of charge one is a 5-dimensional manifold with two natural "ends" or boundaries that can be determined.

Firstly there are solutions of the Yang-Mills equations where the charge concentrates in a smaller and smaller patch about a point $x \in X$; in the limit it has Dirac delta field strength at x and is zero elsewhere. So it is natural to compactify $\mathcal{M}_X$ by adding in these singular Dirac-delta solutions, giving the 5-dimensional $\mathcal{M}_X$ a 4-dimensional boundary X.

At the other extreme, there are "reducible" solutions to the equations made from solutions of the linear Maxwell equations. These are easily understood using the de Rham theory of (3); there is one for every 2-dimensional subspace (or element of $H_2(X)$, in the language of Section 2) A of X whose self-intersection is +1 — that is, a perturbation of A intersects A transversely in a single point (or a finite number of points which add up to one, when counted with the sign of the intersection determined by the orientation).

Near such a solution $\mathcal{M}_X$ is singular, locally looking like a cone (whose vertex parameterises the reducible solution). Chopping off the vertex of the cone leaves a boundary which Donaldson showed is a standard manifold called $\mathbb{CP}^2$ (the complex projective plane described in Section 4). The upshot is that $\mathcal{M}_X$ looks schematically as follows.



Despite not knowing what $\mathcal{M}_X$ looks like in the middle, just knowing it is a smooth connected manifold means that $\mathcal{M}_X$ relates its incoming boundary X to its outgoing boundary given by a union of $\mathbb{CP}^2$s. (The technical term is that $\mathcal{M}_X$ is a *cobordism* from X to the disjoint unions of $\mathbb{CP}^2$s.) In fact for our self-dual X it means that the free abelian group $H_2(X)$ of 2-dimensional "subspaces" (cycles) has an *integral basis* of cycles whose self intersection is +1. This means the unimodular lattice $H_2(X)$ with its intersection pairing is the simplest possible, and not something exotic like the E_8 lattice.

In fact, a year before Michael Freedman had constructed compact topological 4-manifolds with *any* intersection form (like E_8). Donaldson's work uses differential equations, and so a *smooth* structure on the 4-manifold. So Freedman's 4-manifolds with exotic intersection forms could *not* be given a smooth structure — they can be made by patching together open sets in $\mathbb{R}^4$ by continuous maps, but not by differentiable maps. This phenomenon cannot occur in dimensions ≤ 3, and it had been assumed that eventually the same would be proved in 4-dimensions too. Moreover, Donaldson's work shows that $\mathbb{R}^4$ has a countably infinite number of different smooth structures (in all other dimensions $\mathbb{R}^n$ has only its standard smooth structure). For this work Freedman and Donaldson both received Fields medals in 1986.

4. Moment maps and infinite dimensional symplectic reduction

Consider the following set-up. It sounds boring at first, but bear with us! Let $\mathbb{C}^\times = \mathbb{C}\backslash\{0\}$ be the nonzero complex numbers, made into a group by multiplication. The unit complex numbers form the compact circle subgroup

$$S^1 \subset \mathbb{C}^\times.$$

These groups act on $\mathbb{C}^n$ by scaling: $\lambda \in \mathbb{C}^\times$ takes $\mathbf{z} \in \mathbb{C}^n$ to $\lambda\mathbf{z}$.

Consider the quotient space of orbits of the action. Since the origin is fixed by the $\mathbb{C}^\times$ action it is a"bad" orbit and the quotient cannot be Hausdorff there. Removing it we can form a "nice" quotient

$$(\mathbb{C}^n\backslash\{0\})\big/\mathbb{C}^\times. \tag{4}$$

It parameterises complex lines through the origin in $\mathbb{C}^n$ (i.e. 1-dimensional complex subspaces of the complex vector space $\mathbb{C}^n$ — the closures of the $\mathbb{C}^\times$-orbits) and is called complex projective space $\mathbb{CP}^{n-1}$. In the language of Section 2 it is the moduli space of lines through the origin. In fact it is a manifold — near any such line we can choose a complementary affine codimension-1 subspace which passes through one nonzero point of the line. Then intersection with the subspace gives a 1-1 correspondence between nearby lines and points of the subspace. Thus $\mathbb{CP}^{n-1}$ looks locally like a copy of $\mathbb{C}^{n-1}$.

Now we note a few features about this apparently mundane situation. Firstly, think of $\mathbb{C}^n$ as $\mathbb{R}^{2n}$ with a linear operator I which is multiplication by the imaginary number i. So $I\mathbf{v}$ is the rotation of $\mathbf{v} \in \mathbb{R}^{2n}$ through 90° inside the real plane given by the 1-dimensional complex subspace that $\mathbf{v}$ spans. Let $\mathbf{X}$ denote the vector field on $\mathbb{R}^{2n}$ that generates the circle action (i.e. it is the derivative of the circle action — the tangent vector to its orbits). Then

(1) The action of $(0, \infty) \subset \mathbb{C}^\times$ is generated by the vector field $I\,\mathbf{X}$. (This is equivalent to the $\mathbb{C}^\times$ action being holomorphic on $\mathbb{C}^n$.)

(2) This vector field $I\mathbf{X}$ is minus the gradient of an S^1-invariant function (or "hamiltonian") $h\colon \mathbb{R}^{2n} \to \mathbb{R}$,

$$I\,\mathbf{X} = -\nabla h.$$

For instance $h(\mathbf{v}) = \frac{1}{2}(|\mathbf{v}|^2 - r^2)$ will do, for any $r > 0$. Equivalently, $\mathbf{X}$ is the "symplectic gradient" of h,

$$\mathbf{X} = I\,\nabla h. \tag{5}$$

(3) Inside each "good" $\mathbb{C}^\times$ orbit (i.e. complex line in $\mathbb{C}^n$, minus the origin) the zeros of h form a

single orbit of the compact group S^1. This is the circle of vectors $\mathbf{v}$ in the associated complex line with $|\mathbf{v}| = r$. Conversely the "bad" orbit $0 \in \mathbb{R}^{2n}$ misses the zeros of h.

(4) On any fixed $\mathbb{C}^\times$ orbit, the zeros of h are the critical points of the S^1-invariant function $M(\mathbf{v}) := \frac{1}{4}|\mathbf{v}|^2 - \frac{r^2}{2}\log|\mathbf{v}|$, by the elementary calculation

$$D_{\mathbf{X}}\big(M(\mathbf{v})\big) = 0, \qquad D_{I\,\mathbf{X}}\big(M(\mathbf{v})\big) = -h.$$

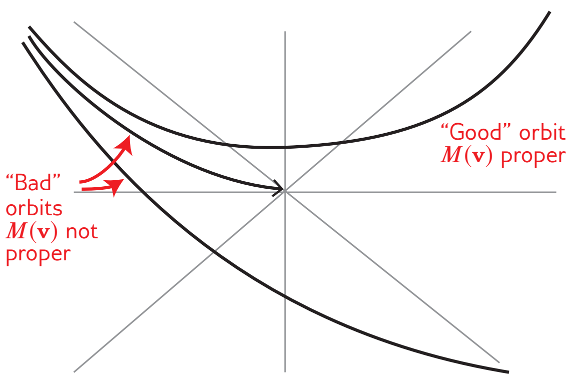
Furthermore this function $M(\mathbf{v})$ is *convex* on any $\mathbb{C}^\times$ orbit:

$$\begin{aligned} D_{I\,\mathbf{X}}D_{I\,\mathbf{X}}\big(M(\mathbf{v})\big) &= -D_{I\,\mathbf{X}}\,h \\ &= -I\,\mathbf{X}\cdot\nabla h \;=\; |\mathbf{X}|^2 \;\geq\; 0. \end{aligned}$$

The convexity in (4) proves the uniqueness of the S^1 orbit with $h = 0$ in (3). Thus (3) shows the quotient (4) of $\mathbb{C}^n\backslash\{0\}$ by the noncompact group $\mathbb{C}^\times$ is also the quotient of the compact sphere $S_r^{2n-1} := \{\mathbf{v}\colon |\mathbf{v}|^2 = r^2\} \subset \mathbb{R}^{2n}$ by the compact group $S^1 \subset \mathbb{C}^\times$:

$$\frac{\mathbb{C}^n\backslash\{0\}}{\mathbb{C}^\times} = \frac{h^{-1}(0)}{S^1}.$$

This is a prototype of the *Kempf-Ness theorem*. Given a space X (really a "Kähler manifold"; these recur in Section 5 acted on by a compact group K (our S^1), then analogues of (1)–(4) above hold under mild conditions. X inherits an action of the complexified group G (our $\mathbb{C}^\times$) and a K-equivariant vector-valued function or "moment map" m (our h). The key point is that an analogue of (4) holds: a (K-invariant) function M is *convex* on G-orbits and its critical values correspond to zeros of m. Therefore finding solutions of $m = 0$ (and proving their uniqueness, up to the action of the symmetry group K) reduces to the much easier problem of determining whether or not a G-orbit is closed, as the following picture "shows".



Standard examples of Kempf-Ness in action give

- A matrix is diagonalisable if and only it commutes with its adjoint. This comes from the action of $U(n)$ on matrices by conjugation, with moment map $m(A) = [A, A^*]$.
- A configuration of n points on the unit sphere $S^2 \subset \mathbb{R}^3$ can be moved by a Möbius map so that their centre of mass is at $0 \in \mathbb{R}^3$ so long as no $\lfloor \frac{n}{2} \rfloor$ of them are coincident. Here m is the ($\mathbb{R}^3$-valued) centre of mass of the points.

PDEs as moment maps

Even if a PDE can be seen as a moment map $m = 0$, we cannot immediately apply the finite dimensional Kempf-Ness theorem. But the same formal structures (1)-(4) mean we can flow down I times the vector field induced by m to try to minimise $|m|^2$. In an appropriate function space we can ensure there is a weak solution if the orbit is closed, or the convex ("Mabuchi") function M of (4) is proper. That convexity ensures uniqueness of the solution as before. So the formalism reduces solving the PDE to 2 steps.

- Describe the "good" orbits (those which are closed in an appropriate sense, or on which the function (4) is proper). This usually turns out to be an "algebraic" problem in some sense; much simpler than solving a PDE.

- Prove regularity of weak solutions. This is the harder, analytic problem which Donaldson has solved in various situations using deep analysis helped by control arising from the geometry of the problem in hand.

Infinite dimensions

A great theme in Donaldson's work is applying the Kempf-Ness theorem in infinite dimensional situations. Here solving $m = 0$ amounts to solving a nonlinear PDE. Examples include

- The (hermitian) Yang-Mills equations on Kähler manifolds. Here the moment map is a component of curvature (or field strength). See "PDEs as moment maps".

- The Einstein equations on Kähler manifolds. Here the moment map is the scalar curvature of the metric.

So for instance he (and Uhlenbeck and Yau) famously solved the Hermitian-Yang-Mills equations, and — most recently — with Chen and Sun he solved the Kähler-Einstein equations on manifolds which are "K-stable" (extending Yau's famous solution in the Ricci flat case). For this and other work, he received a $3 million Breakthrough prize in 2015.

5. Peaked sections

Basic complex analysis concerns holomorphic functions. Smooth complex algebraic varieties, being cut out by polynomial equations in $\mathbb{C}^n$ or $\mathbb{P}^n$, have lots of local holomorphic functions s, and so also lots of complex hypersurfaces (subvarieties one dimension smaller) $s^{-1}(0)$. These are key to making arguments inductive in dimension, as in Zariski's work on surfaces and Deligne's proof of the Weil conjectures via "Lefschetz pencils". A key point is that the zero-set of s is smooth provided the function vanishes transversely — where $s(x) = 0$, we want the Jacobian matrix $Ds_x \neq 0$.

A Kähler manifold has both a metric, which infinitesimally measures length, and a complex structure I (which rotates infinitesimal vectors by $90°$ as in Section 4). Together they define a *symplectic* structure, allowing us to measure infinitesimal areas spanned by two tangent vectors (apply I to one and take the inner product with other). These structures satisfy a compatibility or *integrability* condition — in particular there are lots of holomorphic functions satisfying the Cauchy-Riemann equations (6), despite this being an overdetermined PDE in higher dimensions.

Symplectic manifolds have the same data without the integrability condition. They describe the phase spaces of mechanical systems, amongst other applications. There are generally few (or no) holomorphic functions, so no local complex co-ordinates — the infinitesimal action of I needn't agree with multiplication by i in local balls $\mathbb{C}^n \subset X$. In particular, it was an important open problem whether these X should have any symplectic hypersurfaces. For example, it was wide open whether a symplectic 4-dimensional manifold contains any closed symplectic surface at all! Donaldson proved the existence of hypersurfaces in all dimensions by an ingenious patching construction.

There are no holomorphic functions on a ball which decay at infinity, but there are "twisted functions" (sections of line bundles) which decay: the prototype is $e^{-k|z|^2}$, which decays faster as k grows. These give "peaked" sections near any point of a symplectic or complex manifold, like holomorphic Dirac deltas, which can be patched together to give "approximately holomorphic sections". Donaldson showed these were good enough to build symplectic hypersurfaces in general.

Approximately holomorphic sections

A function $s : X \to \mathbb{C}$ is holomorphic if its derivative is $\mathbb{C}$-linear, so $Ds \cdot I = i \cdot Ds$. Using $i^2 = -1 = I^2$ this is the same as

$$Ds + i \cdot Ds \cdot I = 0. \qquad (6)$$

Approximately holomorphic means

$$|Ds + i \cdot Ds \cdot I| \ll 1 \qquad (7)$$

but $s^{-1}(0)$ can still be nasty. For $s^{-1}(0)$ to be smooth and symplectic we need

$$|(Ds - i \cdot Ds \cdot I)_x| > |(Ds + i \cdot Ds \cdot I)_x| \quad (8)$$

which makes Ds_x non-zero *by a definite amount*. For sums of local sections $e^{-k|z|^2}$, making k big makes (7) easy but keeping k small (to minimise errors from the patching procedure) makes (8) easier. Donaldson proved a local-to-global principle using ideas from real algebraic geometry to show how to play these conditions off against each other.

Similar ideas give *families* of approximately holomorphic functions, and so — by taking their zero loci — families of symplectic surfaces. For instance if we have two functions s_0 and s_∞ then we get a one-parameter family of functions $s_t := s_0 + ts_\infty$ by varying t. This is called a *Lefschetz pencil*, and allows one to employ in symplectic geometry the same dimension reduction arguments — deducing results about X from results about the hypersurfaces $s_t^{-1}(0)$ — that have been so successful in algebraic geometry.

We recall the classical situation of 2 dimensions. Here the quotient s_0/s_∞ of two holomorphic functions on a genus g surface Σ_g maps it to the Riemann sphere $S^2 = \mathbb{C} \cup \{\infty\}$. (Really s_0, s_∞ are sections of line bundles, or meromorphic functions with poles.) This defines a branched cover $p : \Sigma_g \to S^2$. Over an open set U about the generic point of S^2 this exhibits Σ_g as d disjoint copies of U (d is called the *degree* of the map p). But over a finite number of *branch points* $p_i \in S^2$ two of these sheets can come together. The local model is $\mathbb{C}$ double covering $\mathbb{C}$ by the map $z \mapsto z^2 =: w$; here two sheets coalesce over the origin $0 \in \mathbb{C}$. As we wind once around the origin (by say $w = \exp(i\theta)$, $\theta \in [0, 2\pi]$) the preimages $\pm\exp(i\theta/2)$ in the cover start off (at $\theta = 0$) at ± 1 and come back (at $\theta = 2\pi$) to ∓ 1.

This phenomenon is called *monodromy* — as we wind once around a branch point, the two sheets are exchanged. Globally this monodromy is described in purely topological terms by a homomorphism from the fundamental group $\pi_1(S^2 \backslash \{p_1, \ldots, p_n\})$ of the complement of the branch points p_i — i.e. loops in $S^2 \backslash \{p_1, \ldots, p_n\}$ up to deformations — to the symmetric group Σ_d of permutations of the d sheets. This homomorphism has the property that the generators (anticlockwise loops around the p_i) map to transpositions. Conversely, such data determines a covering p by using it to glue together the sheets.

Donaldson's work means we can do something similar for symplectic 4-manifolds. A pair of approximately holomorphic functions s_0, s_∞ define a map $s_0/s_\infty : X \dashrightarrow S^2$, away from the finite set B where $f = 0 = g$. The fibres (preimages of points) are symplectic 2-manifolds $\Sigma_g \backslash B$, and X is determined by how these surfaces are twisted by monodromy as we loop around the critical points of p in S^2 (the points over which the fibre becomes singular). That is, the map is determined by the monodromy homomorphism from $\pi_1(S^2 \backslash \{p_1, \ldots, p_n\})$ to the *mapping class group* of automorphisms of $\Sigma_g \backslash B$. There is an analogue of the simple branching condition above (that at the critical points of s_0/s_∞ it looks like a nondegenerate quadratic form) for which we can determine the local form of the monodromy (the generating loops should map to "positive Dehn twists"). Conversely, given such data, we can reconstruct X and the map s_0/s_∞.

Thus we get an entirely algebro-topological characterisation of symplectic 4-manifolds, reducing them to a problem of group theory. Furthermore, one can use also these symplectic pencils to prove Donaldson's four-manifold invariants of Section 2 are always non-zero on symplectic four-manifolds.

Quantisation

Donaldson used related techniques to produce a striking numerical scheme for approximating geometric PDEs like the Kähler–Einstein equation. The method can be described as a form of (geometric) quantisation, replacing the points of a space X by wave functions peaked about that point. Donaldson uses the above approximately holomorphic peaked sections as his wave functions; finite dimensional spaces of these give rise to natural "algebraic" metrics on the space X. These approximate any given metric on X in the limit where the dimension goes to infinity (which corresponds to tending the peaked wave functions to Dirac deltas), just as smooth functions can be approximated by polynomials.

Donaldson then finds a quantised version of the Kähler–Einstein equation on these finite dimensional spaces which he approximates the original PDE in the infinite dimensional limit. Using the finite dimensional quantised equation amounts to a new, more efficient, form of numerical analysis adapted to this problem. It has been coded up to give numerical solutions which are used in physics.

FURTHER READING

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The Evolution of PDEs in the UK

PETER J. BUSHELL AND DAVID E. EDMUNDS

In the last fifty years there has been a transformation in the way UK mathematicians view partial differential equations. This article discusses these changes and the part played by Sussex in bringing them about.

At the beginning of the 20th century mathematical analysis in Britain was in a poor state. Ignorance of striking European developments in the subject had led Hardy, in his elementary book *Pure Mathematics*, first published in 1908, to write "like a missionary talking to cannibals", as Littlewood put it. The position was rectified by Hardy and Littlewood themselves, together with their students, and by 1940 the overall standing of British analysis was appropriately high. However, in the UK the study of differential equations, as a branch of analysis, was still regarded with suspicion. Outside the excellent work of Titchmarsh and his group, too much of the British work resembled a chaotic assembly of seemingly unrelated facts. Broad lines of development were hard to discern; lack of knowledge of important advances made in other countries underlined the insular approach to the subject that Hardy had earlier attacked in a more general context. Those intrepid enough to give seminars involving PDEs at British universities even as late as 1966 faced considerable difficulties: the pure mathematicians in the audience were prone to view the subject as a branch of engineering with little respectability and less interest, while the applied mathematicians felt that such questions as existence, uniqueness and regularity of solutions of differential equations were trivialities, comparable to the study of angels dancing on pins. Fortunately, help was at hand from the Science Research Council (SRC), a predecessor of the EPSRC.

Aware of the anomalous position of differential equations in UK pure mathematics, the SRC formed a Differential Equations Panel in the 1960s to advise what should best be done. One of its initial recommendations was that a conference on differential equations be held in Edinburgh with leaders in the field giving plenary courses. Immediately before coming to Sussex, David Edmunds had spent the academic year 1965–66 at the University of Minnesota in Minneapolis at the invitation of Jim Serrin. Once in Sussex he became aware of the SRC initiative and persuaded Serrin to come to Sussex for a year if supporting funds were available. The SRC seized this opportunity, with the outcome that when the Edinburgh meeting took place in September 1967, Serrin delivered one of the main series of lectures, with background material provided by Edmunds and Edward Fraenkel. The meeting was a considerable success, and after it Serrin spent the rest of the academic year at Sussex, where he wrote one of his most celebrated papers, on the Dirichlet problem for quasilinear elliptic operators. All this had a very positive effect: the SRC concluded that it was well worthwhile to allocate a considerable amount of money to support conferences and research appointments in the area. Even better, from our point of view, they regarded Sussex as a place in which such topics might blossom. An important feature in the formation of this attitude was that Serrin's good opinion of Sussex had got around and influenced leaders in the field. Soon a substantial SRC grant was awarded to support a 'PDE year' at Sussex, involving extended visits by prominent people and culminating with a Symposium in 1970 attended by many of the world leaders in the subject.

Such developments naturally had internal repercussions at Sussex in the shape of allocation of posts. The existing set of analysts (Peter Bushell, Robin Dyer and David Edmunds) was strengthened by the appointments to Lectureships of Bert Peletier, Charles Stuart and, a little later, Bruno Moscatelli. Companion to this was the SRC Research Fellowship awarded to Mark Thompson. Moreover, the greater visibility of the subject led to an increase in the number of gifted research students eager to work in the area. Prominent among these were John Ball, John Toland and Jeff Webb. Some came through Sussex, others from a variety of universities in the UK, Europe or even Japan. All benefited from the lectures given in the MSc course in Analysis and Differential Equations that was run with SRC support. The range of topics covered in it was ambitious and unique at that time in the UK: it included toplogical degree theory, monotone operators and their generalisations, variational inequalities, linear elliptic and parabolic equations, the theory of Sobolev spaces and spectral theory. This quite sophisticated instruction had a trickle-down effect with the consequence that the level of undergraduate Analysis courses at Sussex was comparable to that in the best UK Universities.

The Conference photograph from the Symposium on Partial Differential Equations, 1976

During the PDE year 1969–70 visits to Sussex of between 1 and 3 months were made by F. V. Atkinson, F. E. Browder, G. Fichera, R. Finn and G. Stampacchia. All gave short courses on their recent work. The high spot of the year was the Symposium held in June 1970, supported by the SRC and sponsored by the LMS. Lectures were given by those just mentioned and by T. Kato and J.-L. Lions, both of whom spent a week at Sussex. The impact on British mathematics was substantial: on the pure side, it could no longer be doubted that the study of partial differential equations as a branch of analysis was of central importance, attracting the attention of absolutely first-rate mathematicians. Moreover, it led to a change in the way applied mathematics is done in the UK: hostility to the use of theorems and matters of exact proof in applied problems slowly began to soften as the advantages of such procedures became clearer; and now applied analysis is flourishing in the UK. This transformation of the British scene naturally had local consequences, the most significant of which was the decision of Edward Fraenkel, then at Cambridge, to spend lengthy periods in the early 1970s at Sussex as visiting Professorial Research Fellow, supported by the SRC. In 1975 this visiting role became more permanent with his appointment first as an SRC Professorial Research Fellow from 1975 to 1978, after which he was appointed as Professor of Mathematics at Sussex.

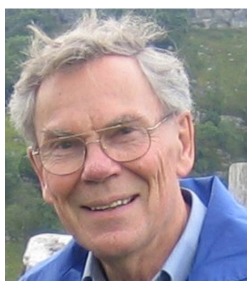
So that the efforts of these years would not be dissipated, a Symposium on Partial Differential Equations was held at the University of Durham from 12 to 23 July, 1976. Organised by Edmunds and Fraenkel and supported by the SRC and the LMS, this was almost certainly the most significant PDE meeting held in the UK up to that time. Not only did it attract a very large number of world leaders in the field, but even more remarkably many of the younger people present went on to achieve real distinction themselves. To justify this statement it is merely necessary to list the participants (other than those then at Sussex): H. Amann, C. J. Amick, M. F. Atiyah, J. F. G. Auchmuty, J. M. Ball, T. B. Benjamin, H. Brezis, F. E. Browder, K. Brown, J. Carr, C. M. Dafermos, J. J. Duistermaat, N. du Plessis, M. S. P. Eastham, J. Eells, W. D. Evans, R. Finn, W. Forster, F. G. Friedlander, L. Gårding, E. Giusti, F. Goldman, V. W. Guillemin, J. Hale, P. Hess, L. Hörmander, R. J. Knops, J. Leray, A. Lichnewsky, R. H. Martin, J. B. McLeod, R. Melrose, M. Miranda, V. J. Mustonen, L. Nirenberg, J. Norbury, J. Ockendon, L. A. Peletier, A. Plant, A. J. B. Potter, P. H. Rabinowitz, J. C. Saut, J. B. Serrin, I. Singer, R. A. Smith, G. Stampacchia, R. F. Streater, C. A. Stuart, L. Tartar, M. Thompson, J. F. Toland, N. S. Trudinger, J. R. L. Webb, H. Weinberger and T. J. Willmore. The extraordinary nature of this meeting is further underlined by the fact that those attending include (as of 2017) two Fields Medallists, five Members of the US National Academy of Sciences, nine Fellows of the Royal Society, three Abel Prize winners, one Crafoord Prize winner and two Wolf Prize winners.

All this activity kept the Sussex Mathematics Department in the news as the only one in the UK in which the study of PDEs was a major research topic. As might be expected, the growing reputation of members of the group led to departures for promotion elsewhere: Bert Peletier and Charles Stuart were among those who were elected to chairs elsewhere, in Leiden and Lausanne respectively. In ordinary, stable circumstances the strength of the group would have ensured that high-class replacements were provided, but by the late 1970s the heady optimism of universities in the 1960s had been replaced by growing financial constraints. Together with competing claims from other sections of the Department, this prevented effective and speedy action being taken to combat the difficulty of attracting and retaining high-quality faculty in such a competitive field. Despite this setback Sussex continued to lead the way in PDE activity: an LMS-supported 2-day meeting was held at Sussex in May 1983, the principal speakers including L. Hörmander, P.-L. Lions and L. Nirenberg. Some relief was provided by the appointment of the harmonic analyst A. Carbery in 1985, but as the 1980s wore on the Sussex Mathematics Department suffered from a decrease in student numbers and the growing practice of other Science departments to do their own mathematics teaching. In 1988, as part of a strategic review, the University considered the funding of the Mathematics Department. The practical outcome was that pressure was put on various faculty members to take early retirement; and some yielded to this. Moreover, Edward Fraenkel decided in 1988 to leave so as to take a Chair in Bath. However, despite all this turmoil the flood of first-rate visitors to the Analysis group continued, establishing strong contacts with the groups at Jena, Prague, Tbilisi and Wroclaw; the frequent lengthy visits of Hans Triebel were of particular significance.

A new and exciting phase began in 1991 with the appointment of Dima Vassiliev as lecturer in Mathematics: credit goes to Jacqueline Fleckinger (Toulouse) for having suggested to Dima that he should apply for a post at Sussex. Dima was rapidly promoted, first to a Readership and then to a Chair. His presence proved to be a magnet for other high-quality Russian analysts, as a result of which the department became in the 1990s once again a major centre of PDE activity in the UK: those appointed to permanent posts were L. Parnovski, E. Shargorodsky and A. V. Sobolev, while M. Levitin, A. Pushnitsky and Yu. Netrusov occupied research positions. In their wake came very distinguished Russian visitors, notably Mikhail Birman, Michael Solomyak and Mark Vishik. Again the impact was not only on the pure side, for several of the new analysts had a deep understanding of applied topics. The departure of Vassiliev in 1999 to a Chair in Bath, coupled with the attraction of the capital city for the others, led to an exodus of devastating significance, and before long Chairs in London were occupied by Parnovski and Sobolev at UCL, and Pushnitsky and Shargorodsky at King's; Levitin went to Cardiff and is now Professor of Mathematics at Reading; Netrusov has a Readership at Bristol. Of course a good test of the quality of a department is provided by the proportion of faculty that leave for senior positions elsewhere, but these changes, coupled with natural retirements and the departure of Carbery in 1995 to a Chair at Edinburgh, were too large and happened too quickly for an adequate reaction by Sussex to be possible. The subsequent decline of the department was almost inevitable, and in the most spectacular way the department lost its position as one of the strongest in the country in analysis. Faculty numbers dwindled and the very existence of the department seemed in question by 2010. Happily, the corner was turned and now, with student numbers buoyant, the immediate future seems more secure. The interests of the department as a whole have shifted to applications, reflecting a tendency in the country and perhaps in the world, but the analysis and PDE flag is currently kept flying by a lively group.

What should be made of the last 50 years of Differential Equations at Sussex? One measure of its standing is the impressive list of those analysts accepting honorary doctorates from Sussex: John Ball, Jim Serrin, John Toland and Hans Triebel. The Sussex Analysis group played a significant role in bringing about the recognition in Britain that the study of PDEs was of central importance, offering challenges to the very best mathematicians. Although the current national policy of concentration of British research support in large universities means that the PDE torch is now carried mainly in such institutions, it was the Sussex group that lit the flame, took the vital first few steps and continues to be involved.

We are indebted to John Ball and John Toland for very helpful comments, and to Simon Eveson for his aid.



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David Edmunds is an analyst with interests in functional analysis and elliptic equations. He is now an Emeritus Professor at the University of Sussex.

Reciprocal Societies: The Indian Mathematical Society

The Indian Mathematical Society (IMS) was founded on 4 April 1907, by late Shri V. Ramaswami Aiyar, a Deputy Collector in the services of the then Madras Province, with twenty founding members. The Headquarters of the Society is at Pune (India). The Indian Mathematical Society (IMS) is the oldest scientific society in India devoted to the promotion of study and research in mathematics. The society has around 3,000 life members.

One of the main activities of the society is organization of IMS Annual Conferences. During every Annual Conference, the following five Memorial Award Lectures are arranged as a part of the Academic Programme: P.L. Bhatnagar Memorial Award Lecture (instituted in 1987), Srinivasa Ramanujan Memorial Award Lecture (instituted in 1990), V. Ramaswamy Aiyer Memorial Award Lecture (instituted in 1990), Hansraj Gupta Memorial Award Lecture (instituted in 1990) and Ganesh Prasad Memorial Award Lecture (instituted in 1993 and delivered every alternate year). A Special Session of Paper Presentation Competition is organized in the conference and Six IMS Prizes, AMU prize and V.M. Shah Prize are awarded to the best research papers presented in various areas.

The IMS has instituted five prizes for best research papers in order to encourage researchers in India. The prizes include viz. A.K. Agarwal Award (given for best paper published in the areas of Number Theory, Combinatorics, Discrete Mathematics, Analysis and Algebra), A.M. Mathai Award (for best publication in Applicable Mathematics), Satish Bhatnagar Award (for best paper in the area of History of Mathematics), A. Narasinga Rao Memorial Prize (awarded to the author of the best research paper published in the *Journal of the Indian Mathematical Society* or *The Mathematics Student*) and P.L. Bhatnagar Memorial Prize (awarded to the top scorer of the Indian team at the International Mathematical Olympiad). Each prize consists of a certificate and a cash amount. These prizes are presented during the Inaugural Session of the Annual Conference of the Indian Mathematical Society.

Postage stamp issued by the Government of India in celebration of the Centenary of the IMS

The Society has a reciprocity agreement with eighty mathematical societies in the world (including LMS) which allows exchange of the journals. The Society publishes three periodicals viz. *The Journal of the Indian Mathematical Society*, *The Mathematics Student* and the *IMS Newsletter*.

Srinivasa Ramanujan (Published with permission from the Master and Fellows of Trinity College Cambridge)

The Indian Mathematical Society (IMS) and the London Mathematical Society (LMS) have close association through Srinivasa Ramanujan and G.H. Hardy. Ramanujan's earliest contributions were published in the *Journal of the Indian Mathematical Society* (1911-1917) and *Proceedings of the London Mathematical Society* (1915). Ramanujan and Hardy's papers appeared in the *Proceedings of the London Mathematical Society* (1917-1919). More information about the IMS can be found on its website: indianmathsociety.org.in.

N. K. Thakare (General Secretary)
M.M. Shikare (Administrative Secretary)
Indian Mathematical Society

Microtheses and Nanotheses provide space in the Newsletter for current and recent research students to communicate their research findings with the community. We welcome submissions for this section from current and recent research students. See newsletter.lms.ac.uk for preparation and submission guidance.

Microthesis: Eco-Evolutionary Food Web Modelling

GAVIN M. ABERNETHY

We extend a model of the evolutionary development of predator-prey relationships and food webs, with explicit spatial structure. The networks generated by this model are used to re-examine previous results in food web theory including the link-species relationship and the stability of the evolved networks.

Eco-evolutionary Models

Predicting the effects of challenges such as climate change on the earth's ecosystems requires models that can simulate the ecological and evolutionary responses to perturbations of complex networks. Simulated networks should possess realistic structures that not only could exist, but could have been reached by evolution from a first species. Eco-evolutionary models accomplish this by using stochastic elements and mutation to assemble the network, adding species which are then subject to predator-prey dynamics. The researcher thus specifies the evolutionary and ecological rules of the system, and allows it to select the number, type and relationships of the surviving species as emergent properties.

Spatial Webworld Model

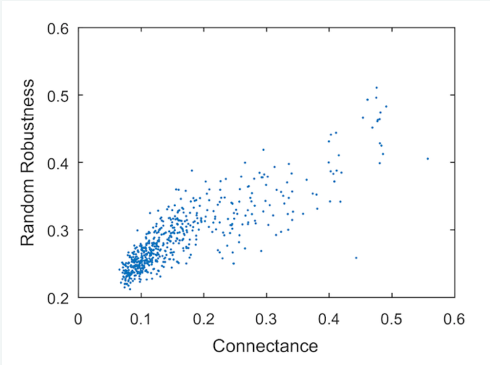
The Webworld eco-evolutionary model [1] was chosen as the starting point for this research. In this model, species are assigned binary traits which have random scores against each other, and the combined trait scores for a pair of species determine the strength and direction of feeding relationships and competition. This informs the ecological loop, which determines population densities, N_i, and is iterated using the balance equation shown using a ratio-dependent functional response $g_{i,j}$. The parameter λ controls ecological efficiency.

$$\frac{\mathrm{d}N_i}{\mathrm{d}t} = -N_i + \lambda \sum_{j=0}^{n} N_i g_{i,j}(t) - \sum_{k=1}^{n} N_k g_{k,i}(t)$$

In our spatial version of this model, each iteration of feeding and reproduction is followed by migration between connected habitats.

Stability in population models

The stability of population dynamics in response to perturbation has been a contentious topic in ecological modelling, complicated by multiple definitions of perturbation and stability. For the networks constructed by the Webworld model we studied "robustness": the fraction of the species which must be manually deleted (in random order) followed by population dynamics in order to result in the extinction of at least 50% of species.



Robustness

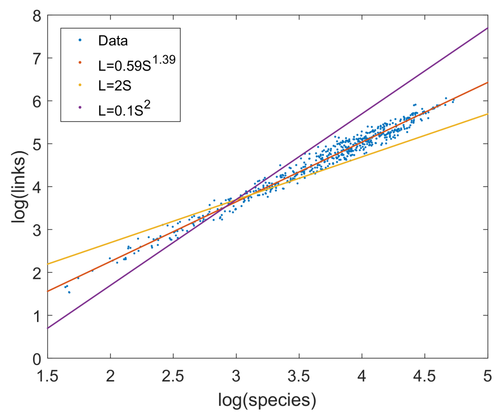
We find that robustness positively correlates with connectance (Coefficient 0.855), coherent with findings from empirical data, and the static Cascade, Generalised Cascade, Niche and Nested-Hierarchy Models [2].

When an equilibrium is reached, or enough time has elapsed, an invasion event occurs: a species is selected and its "child" species is introduced with small population and sharing all but one of the parent's traits. The population dynamics then decide whether it joins the network. Beginning with a resource and a single species in each of several discrete habitats, this process grows a food web metacommunity (a spatial network of species networks).

While the lifespan of existing species appears to grow over time, enabling migration between previously disconnected communities makes it more difficult for mutant species to invade or for established species to be displaced. It also tends to raise average trophic levels in each habitat. Long simulations of 2×10^7 evolutionary timesteps demonstrate that, contrary to previous reports for this model, the network does not necessarily tend towards a permanent general structure, as it can collapse down to a single species even after millions of evolutionary events.

Link-Species Relationship

Consider a relationship of the form $L = aS^b$, where S is the number of species in a network, L the number of predation links, and a, b are constants. Ecologists have sought a general law for this relationship, with some proposing exponents of $b = 1$ (constant L/S) or $b = 2$ (where connectance, defined as the fraction of realised feeding links, is constant).



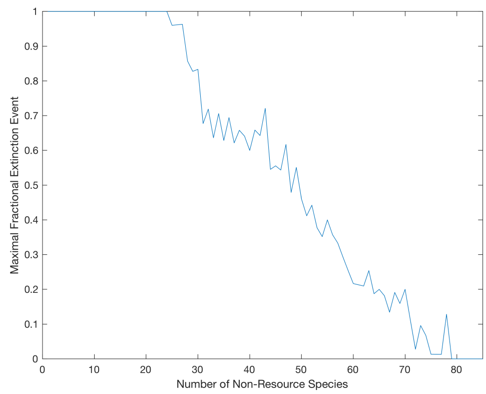
Link-Species Relationship

Data from 600 simulations supports neither the constant connectance hypothesis, nor the link-species scaling law, but rather a best fit of $L = 0.59S^{1.39}$. This is coherent with some empirical data that supports an exponent of $b = 1.5$ [3].

Diversity and Extinction Events

The stability-diversity relationship of food webs has been of interest since May's result [4] that, using linear stability analysis, larger networks are less stable. See "Stability in population models" for another measure of stability.

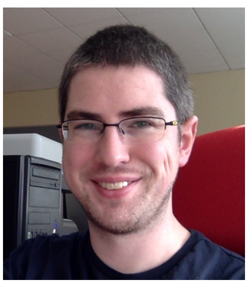
We test the number of extinctions that result from the perturbations caused by invasion events. From almost 4×10^8 total invasion events, the maximal fractional extinctions declines as a function of prior species diversity, so in this sense larger networks have greater stability.



Maximal fraction of food web extinction

FURTHER READING

[1] B. Drossel, P. G. Higgs, A. J. McKane, The influence of predator–prey population dynamics on the long-term evolution of food web structure, J. Theor. Biol. 208 (2001) 91–107.
[2] J. Dunne, R. J. Williams, Cascading extinctions and community collapse in model food webs, Philo. Trans. Royal Soc. London B: Biological Sciences 364 (2009) 1711–1723.
[3] J. Dunne, The network structure of food webs, in: Ecological networks: linking structure to dynamics in food webs (2006) 27–86.
[4] R. M. May, Will a large complex system be stable?, Nature 238 (1972) 413–414.



Gavin Abernethy

Gavin Abernethy is a PhD student at Ulster University. He obtained his undergraduate degree in Mathematics at the University of St Andrews. He is currently researching evolutionary food web models and playing Overwatch.

This section is for Early Career Researchers. Please send suggestions for questions or topics you would like to see covered to newsletter@lms.ac.uk.

Excelling at Interview

"Dear X, I am a PhD student/postdoc. I'm applying for a permanent academic job or a fellowship. Can you suggest ways I can do well at interview? What sort of questions should I expect?" — We invite perspectives from academic mathematicians with experience as interviewer and as interviewee.

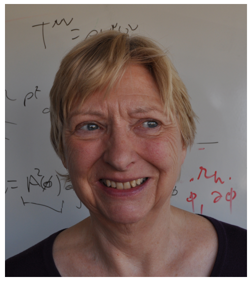
Graham Niblo is Professor of Mathematics at the University of Southampton. He works in geometric group theory and non-commutative geometry.

A lot of what the panel want to know about you is already on paper — your publications list, your references and your history have already qualified you for the interview, but the panel will want to explore any possible weaknesses and, for a first permanent academic job, how you are likely to transition from a life of independence and research to wider engagement with the Faculty.

Panellists are likely to have a range of backgrounds, not all in your area, so if you are asked to explain your research there will be more interest in the context than in technical detail. That will give you an opportunity both to explain to the non-experts why your research is interesting, and to demonstrate that you can communicate and interact with a range of colleagues. Make sure you know what other research is done in the department so you can discuss possible collaborations outside your immediate comfort zone.

You will probably be asked about your teaching experience, and it is good to lead with things you are enthusiastic about. Students find enthusiastic teaching more engaging and in the world of league tables engagement is key.

A good panel will try to give you the chance to demonstrate what you have to offer. They won't be trying to trip you up, so don't be defensive. Assume that they are genuinely interested in you (which they are or you wouldn't be there).



Anne Davis is Chair of Mathematical Physics 1967 at the University of Cambridge. She has been Chair of the Faculty Board of Mathematics, Head of High Energy Physics and University Gender Equality Champion in STEMM.

You will probably be asked to give a talk on your research. The panel will try to understand your research, even if it's not in their area. You can put in technical details, but start with an introduction explaining the context, what you are doing and why. If you are asked a technical question on your research by the expert, answer the question technically. The expert can explain to the rest of the panel if necessary.

You will probably be asked which courses you could teach. You should give the panel a list of courses, from all years of the course and not just the specialist courses. If you are asked if you could teach X do not say 'no', instead say 'I will need to spend some time preparing it, but yes'. Do not talk about teaching the brightest students only.

You may well be asked about PhD and MSc projects. Be prepared to give some examples.

You might be asked for what you see yourself doing in the next 5–10 years. You may also be asked where you see yourself in 5 years' time. Be ambitious here. You will apply for grants from research councils and other sources to build your research group with PhD students and postdocs.

The most important thing is to prepare yourself for the interview at that particular university and not some arbitrary place, so look at their website, their personnel and their undergraduate courses.

Anitha Thillaisundaram is a Lecturer in Algebra at the University of Lincoln. She obtained her PhD in Group Theory at the University of Cambridge in 2011.

For most permanent academic jobs, teaching is just as important as research, perhaps even more important, depending on the university. You should demonstrate a willingness to teach courses outside your research area and to cater for weaker students. Student satisfaction is also a priority nowadays. Additionally most UK universities require academics to take on administrative roles, so you should be prepared to be asked about that, as well as about applying for grants.

It is good if your research overlaps or fits in nicely with the existing group, but this depends on the university's long-term goals: whether they want to strengthen their existing research group or branch out in a different direction.

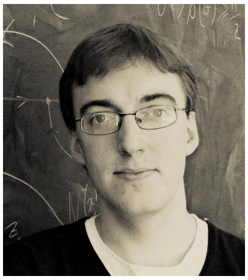
I was asked in my interview where I see myself in five years. So your future goals could be something to think about. Also most interviews end with the interviewers asking if you have any questions – it would be good to have one or two! Lastly, remember that the university is looking to hire someone for a permanent position, so it would be good to show enthusiasm or a liking for the place, town or city, to give further evidence that you're keen to take up the position and move there in the long term.

Charles Walkden is Reader in Pure Mathematics and Director of Teaching & Learning in the School of Mathematics, University of Manchester. He received his PhD in Ergodic Theory at the University of Warwick.

We look for academics who are outstanding teachers as well as outstanding researchers and you need to be able to demonstrate this. You could be asked 'Briefly describe your existing teaching experience'. Don't just list courses or classes you've given (the panel will have already read this on your CV); instead, use this as an opportunity to describe your approach to teaching, what you've done that worked, what you've done that didn't work and what you would change in the future.

Many maths departments teach mathematics to students on other degree programmes. Expect questions like 'Suppose you were teaching maths to engineering students, what do you think the main challenges would be?' or 'How would you keep a class of 100 materials science students interested in maths?'

A great (and hard!) interview question I heard is 'Suppose you met my aunt, who knows no mathematics. How would you explain in three sentences what your research is about?' Don't try to explain your most impressive theorem! Instead I'd be looking to see if you can connect your research (even if it's in a highly abstract area of pure mathematics!) to something tangible that a layperson could appreciate.



Andrew Treglown is a Senior Birmingham Fellow at the University of Birmingham. He got his PhD from the same institution in 2011.

For a permanent position, the interview panel will be looking at what new expertise and skills you can bring to the department. Perhaps you can foster new collaborations with academics at the university, or maybe there is a module you could design in a subject that is currently underrepresented in the teaching. Invest some time before the interview to develop a picture of the research and teaching within the department.

You should convey precisely why your research is interesting (remember, often there are non-mathematicians on interview panels!). What is the most important research idea you have had? For permanent positions you should have a clear research 'vision'. What problems will you be working on in the coming years?

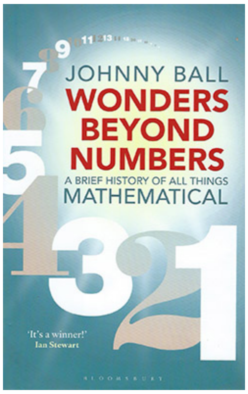
British universities also take seriously metrics such as the National Student Survey (NSS), Teaching Excellence Framework (TEF) and Research Excellence Framework (REF). I've had an interview panel ask me what a 4* REF paper looks like and ask me to self-evaluate my own papers!

Enthusiasm also goes a long way. Make it clear you are passionate about research and teaching and excited about joining the university!

Wonders Beyond Numbers

by Johnny Ball, Bloomsbury, 2017, hardback, 480 pages, £16.99,
ISBN 978-1-4729-3999-9.

Review by Norman Biggs



Johnny Ball is famous in the UK for his TV programmes on mathematics. He is renowned for his blend of knowledge and enthusiasm, and his ability to explain mathematical ideas in simple (but not simplistic) terms. The last of these talents is rare, and not always appreciated by the makers of TV programmes.

This book contains several recollections of Johnny's career in television, the most recent being a rather depressing illustration of the current state of the art (page 402). In 2016 a feature of the famous Chelsea Flower Show was a 'mathematical garden', and Johnny was invited to present a piece on it. The mathematical garden attracted a great deal of interest from visitors, and the BBC devoted several hours of prime-time coverage to the Flower Show. Nevertheless, the time allocated to Johnny's piece was — three minutes.

It is against this background that the book under review should be assessed. The intended readership is not that of the *Newsletter*. It is the general public, who deserve to be better informed about truly important ideas, how they emerged, how they evolved, and how they will affect our future.

The book has many excellent features. I particularly liked the chapters (7,8,9) on the worldwide origins of mathematics, the medieval scholars, and the exploration of the world — despite the omission of Gerbert of Aurillac and the mysterious appearance of Dick Whittington. Readers of the *Newsletter*, mathematicians and historians, may well ask for a little more information about the book from a professional standpoint, even though it is not intended for us. The history of mathematics (in particular) has traditionally been an easy target for 'fake news'. I therefore decided to compare Ball's book with the famous *Men of Mathematics* by E.T. Bell, a book much read by aspiring mathematicians of my own generation. The books of Ball and Bell are separated by about 80 years, and their authors come from very different backgrounds. Bell gave us insights into the lives and works of people who had contributed to the kind of mathematics that was fashionable in the 1930s. His account was therefore unbalanced, mathematically, historically, and geographically. Ball has the benefit of the work of modern scholars on the development of mathematics, as opposed to the lives of mathematicians, and so we do learn from him about the Chinese, Indian, and Islamic contributions. But, like Bell, he is still rather too willing to accept the pretty stories of the Ancient Greek world that have emerged from millennia of classical scholarship. For example, in both books we are warned that Pythagoras was, to some extent, a mythical character, but we are then informed that he was personally responsible for several groundbreaking advances in mathematics. Among other things, Bell tells us he discovered the irrationality of $\sqrt{2}$, and Ball that he discovered the regular icosahedron and dodecahedron. Of course, the total absence of evidence does not necessarily imply that he did not do these things, but we should be careful. Nevertheless, Johnny Ball's book is full of delightful insights into how mathematics was really done. A good read!

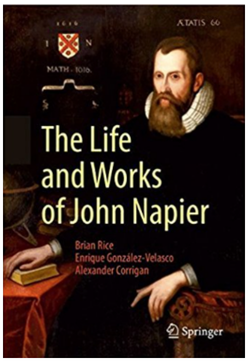
Norman Biggs

Norman Biggs is Emeritus Professor of Mathematics at the London School of Economics. He has written many books and papers on mathematics, and has also written on its history and its links with metrology and numismatics. He has contributed to the work of the LMS in many ways, for example as its General Secretary from 2002 to 2006. His book *Quite Right: The Story of Mathematics, Measurement and Money* was published by Oxford University Press in 2016.

The Life and Works of John Napier

by Brian Rice, Enrique González-Velasco & Alexander Corrigan, Springer, hardback, pp994, £123, ISBN 978-3-319-53281-3.

Review by Edmund F. Robertson



John Napier (1550-1617) is best known today as the inventor of logarithms, a device which had a huge impact on science for several centuries. He himself would almost certainly have said that his most important work was *A plaine discovery of the whole revelation of Saint John* (1594) which is a no-holds-barred attack on the Roman Catholic Church. In fact he was widely acclaimed in his own lifetime for this work, written in English but soon translated into Dutch, French and German.

The book under review publishes for the first time all five of Napier's published works in one volume. The *Plaine Discovery* is given in Napier's original English. Napier's *Mirifici logarithmorum canonis descriptio* (1614), in which he gives a description of the use and applications of logarithms, is given in the English translation made in Napier's lifetime. The other three works are: the *Rabdologiae seu numerationis per vigulas libri duo* (1617), which describes three mechanical calculation devices invented by Napier, given in the 1990 English translation; the *Mirifici logarithmorum canonis constructio* (1619), which describes Napier's method for constructing logarithm tables, given in an 1889 English translation; and *De arte logistica Joannis Naperi* (1839), on arithmetic, geometry and algebra, given in a slightly modified version of an English translation of 1982. There are three introductory chapters written specially for this book, a 60-page biography of Napier, a 36-page introduction to the *Plaine Discovery*, and an 84-page introduction to Napier's four mathematical works.

The *Plaine Discovery* is in two parts. The first contains 36 propositions set out in a mathematical style. These aim to set up a dating system for the world which, Napier 'proves' in Proposition 14, will end between 1688 and 1700. His other aim is to prove that the Pope is the Antichrist, which he 'proves' in Proposition 26. From the style of the arguments, it looks as though Napier might be trying to use his mathematical skills to blind his readers. Certainly he changes dates at times to make 'facts' fit his chronology. The second part consists of Napier's version of the Book of Revelation, together with his paraphrase of it and his interpretation as predicting history. The whole is an extreme attack on the Church of Rome. At times Napier appears careful and calculating, at others he appears carried away by his own passion. Andrew Corrigan, in his chapter *Revealing the Plaine Discovery*, writes, "Napier's life, personality, worldview and mathematical achievements cannot be understood fully unless one understands something of this …" The reviewer agrees with this comment and Corrigan's chapter plus Brian Rice's biographical chapter add greatly to understanding Napier's character and the historical context.

The first of Napier's two works on logarithms is the *Descriptio*, in two parts, in the first of which he gives the necessary definitions and examples of their use, and in the second part, applications to trigonometry. Much of this work consists of 90 pages of his logarithm tables which must have involved an enormous amount of work. The *Descriptio* contains no indication of how Napier computed his logarithm tables but this is contained in the *Constructio*, written by Napier but only published two years after his death. The *Mathematical Introduction* by Enrique González-Velasco assists greatly in showing Napier's thinking and the stages he went through in devising a practical method of calculating his logarithm tables. González-Velasco shows convincingly that Napier had the basic idea of logarithms around 20 years before the work was published and also that he has clear priority over Jost Bürgi.

The *Rabdologiae* describes three mechanical devices invented by Napier to assist calculating. These are the rod, strips in a box, and multiplication on a chess board abacus. Napier favoured the second of these, which he actually discovered after the other two. Some claim that this is worthy of being called "the first calculating machine". These clever machines never became popular for calculation, mainly because logarithms became the universally used calculating tool.

In many ways *De arte logistica* shows Napier's clear mathematical thinking best of all. It covers arithmetic, geometry and algebra although the geometry part is either largely lost or was never written. The ease with which he works with negative numbers is impressive. Although aware of imaginary numbers, he does not calculate with them. In dealing with equations, he takes all the terms to one side and equates them to zero. He was ahead of his time in this aspect and one can only wish that he had continued to work on these areas of mathematics for surely he would have made significant advances.

This is an excellent book and bringing all Napier's works together with three excellent commentaries has been an immense, but very worthwhile, undertaking. In the *Plaine Discovery* and the *Descriptio*, the long s has been kept as has the interchanging of u and v, together with archaic spelling. Although probably the correct decision, it does make reading these chapters difficult for anyone not experienced in reading texts of that era. A disappointment was the lack of a commentary on *De arte logistica*. González-Velasco writes, "...the contents of this work does not fall within the limits of this Introduction's title," a strange comment in a chapter entitled *Mathematical Introduction*. However, this is a small criticism and the authors are to be congratulated on producing an important work.

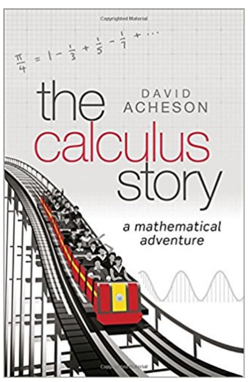
Edmund Robertson

Edmund F. Robertson is Emeritus Professor of Mathematics at the University of St Andrews. In addition to research in computational group theory, he is one of the two developers of the MacTutor History of Mathematics web archive. Since retiring in 2008, he has made the MacTutor Archive a full-time hobby.

The Calculus Story: A Mathematical Adventure

by David Acheson, Oxford University Press, 2017, hardback, pp208. £11.99, ISBN 978-0-19-880454-3.

Review by Mark McCartney



David Acheson's previous book aimed at a wide audience, *1089 and All That* (OUP, 2002), rightly got rave reviews for its combination of clear explanation, broad sweep, nice historical context and quirky 1950s illustrations. It was a wonderful book. However, the problem with authors writing wonderful books is that readers' expectations are high when it comes to the next book they publish.

Thus, when my copy of *The Calculus Story* arrived in the post I had already set the bar high, and was almost a little nervous that it wouldn't live up to my enjoyment of *1089*. I was reminded of the story told of an irate reader at a literary festival confronting the author Joseph Heller with the words 'Mr Heller, since *Catch-22* you haven't written anything else as good!' Apparently Heller replied 'No, you're right....but then, neither has anyone else.'

I'll return to the question of whether David Acheson has found himself in a similar 'Catch-22' situation at the end of this review.

As its title suggests, *The Calculus Story*, gives readers a gentle introduction to calculus, and while this is its clear focus, Acheson also covers topics such as planetary motion, infinite series and complex numbers. The book sets calculus is its historical context with a deceptive lightness of touch. Newton, Leibniz, Euler and others all make appearances, along with nicely chosen illustrations from sources, such as the page in John Wallis's *De Sectionibus Conicis* where the symbol for infinity first appears (p.42), or a section of a Newton manuscript where he 'discovers' Taylor's theorem (p.147).

As Acheson states in the introduction (p.5), 'this little book is more ambitious than it looks'. After introducing the reader to the idea of how to evaluate the gradient of a tangent to the curve $y = x^2$, he moves on to the general rule for differentiating x^n; the fundamental theorem of calculus; the derivation of the product and quotient rules; a derivation of Taylor's theorem, and even a nod towards the use of integration by substitution. Further, the book does not limit itself to introductory single variable calculus, and there is very brief coverage of partial derivatives, differential equations, the calculus of variations and chaos.

The book steps the reader through a few simple practical applications (for example the classic optimisation problem of maximising the area enclosed by a rectangular fence is described), but there are other slightly more advanced results which are stated without proof. Thus, for example the optimum distance from which to view Nelson on his column is stated, but no derivation given.

Despite covering such a wide terrain, this is not in any sense a textbook, but neither is it a book of 'popular mathematics'. After all, on nearly every page opened, the reader sees an equation. So who then is the likely audience? I think perhaps it is those who have already learnt a little differentiation and integration at school, but want to find out more about their full potential. Such a reader will have their confidence boosted by meeting some ideas they already know, discover interesting historical background, and be given a taster for the enormous power and application of calculus in mathematics and science.

So finally, is David Acheson likely to find himself in his own Joseph Heller moment? No. I doubt that he is. This is another wonderful book. *The Calculus Story*, though it does have some overlap in content with *1089 and All That*, is, if you like, a natural follow-on book for the reader who wants to learn more about mathematics. It is indeed an ambitious little book, but it delivers on those ambitions. It manages to give a broad survey of the power of calculus without becoming too detailed, all written in a style that carries the reader along and gently invites the curious to further study. If I were to have one complaint, it is that there are no quirky 1950s illustrations, which is a pity. But a chap can't have everything.

Mark McCartney

Mark McCartney lectures in mathematics at Ulster University. He has scholarly interests in nonlinear systems and the history of mathematics. He is of the view that most situations can be improved by reading a book and drinking tea.

Obituary

Shristi Dhar Chatterji: 1935–2017

Shristi Dhar Chatterji, who was elected a member of the London Mathematical Society on 11 March 1977, died on 28 September 2017, aged 82.

J.D.M. Wright writes: Professor Shristi Dhar Chatterji died unexpectedly, after a short illness, in Lausanne, Switzerland. He was born in Lucknow, India, on 29 July 1935. He attended the Indian Statistical Institute in Calcutta (ISI). Then he travelled widely in the USA and Europe before becoming a permanent professor at the Ecole Polytechnique Fédérale de Lausanne (EPFL). He was President of the Swiss Mathematical Society from 1986-87.

He was expert in probability theory and functional analysis and deeply knowledgeable over a wide range of mathematics. Among his major achievements was his work on martingales with values in a Banach space X. He showed that martingale convergence theorems work precisely when X has the Radon-Nikodym property. His extensive research publications in classical and functional analysis show his powerful originality combined with profound and subtle insights.

His epic three volume *Cours d'analyse* arose from his lecture courses at the EPFL. He also founded, and for many years edited, the journal *Expositiones Mathematicae*. His interest in the history of mathematics led him to translate Euler's *Letters to a German princess* from German into French. With H. Wefelscheid he edited *G.C. Young and W.H. Young: Selected papers*. Among his other historical writings he gives a masterly account of the life and work of Norbert Wiener.

Chat, as he was known to all his friends, was phenomenally gifted at languages. It was not just that he knew many languages — he had mastered them. His mother tongue was Bengali but his spoken English had no trace of a foreign accent and he spoke Italian like an Italian. Probably similar things could be said about his French, German, Danish and Hindi. At home he spoke Italian. This was natural since his wife, Carla, came from the Italian part of Switzerland. They married in 1967 and, just a few weeks before his death, celebrated their Golden Wedding Anniversary with a trip to Italy. He was delighted that their daughter, Indira, became a professional mathematician.

For many years Chat would come to London in November for the LMS AGM and Annual Dinner, and would make a side trip to Oxford. David Edwards wrote: "I always enjoyed his visits here enormously. He was not only an extremely distinguished mathematician but also wonderful company".

Chat was a connoisseur; of food, of wine, of literature, and of mathematics. As well as his creativity in producing new mathematics he had a passion for finding elegant, insightful proofs of important known results. He once said "The discoverer of a theorem hits it with any tools that come to hand in order to get a proof. Frequently the 'right' argument is only found later." This approach made him a superb teacher and expositor.

Chatterji was charismatic with prodigious intellectual gifts.

I wish to thank Professor Varadarajan and Professor Indira Chatterji for their help; in particular for sending me a copy of the memorial article which is soon to appear in *Expositiones Mathematicae*.

History of Computing Beyond the Computer

Location: Oxford University
Date: 21–22 March 2018
Website: tinyurl.com/y75nxgjy

This meeting will focus on the history underpinning modern programming, from Charles Babbage's hardware design language to the systematic exclusion of women. It is colocated with the Fourth Symposium on the History and Philosophy of Programming.

Frobenius Structures and Relations

Location: University of Glasgow
Date: 22–23 March 2018
Website: tinyurl.com/y9njetnz

A workshop on *Frobenius Structures and Relations* is part of a collaborative workshop series on Classical and Quantum Integrability, supported by an LMS Scheme 3 grant, involving Edinburgh, Glasgow, Heriot-Watt, Leeds, Loughborough and Northumbria Universities. Funds to support research student attendance and opportunities for contributed talks are available; email enquiries to Misha.Feigin@glasgow.ac.uk.

Operator and Spectral Theory

Location: University of Reading
Date: 4 April 2018
Website: tinyurl.com/ycz5evnx

This meeting aims to highlight the interplay between operator and spectral theory and illustrate modern problems relevant to both areas. Speakers: Ole Fredrik Brevig, Jonathan Partington, Alexander Pushnitski, Lucia Scardia and Karl-Mikael Perfekt. The meeting is supported by an LMS Celebrating New Appointments Scheme 9 grant.

Functor Categories for Groups

Location: University of Lincoln
Date: 13 April 2018
Website: tinyurl.com/y7wcpmn6

The category of totally disconnected locally compact groups is the theme for the 4th meeting of the research group 'Functor Categories for Groups'. FCG Research Group is supported by an LMS Joint Research Groups in the UK Scheme 3 grant. Limited funding is available for PhD students. To register, email athillaisundaram@lincoln.ac.uk.

Society Meeting in Honour of Maryam Mirzakhani

Location: Warwick University
Date: 22 March 2018
Website: tinyurl.com/y8ge9pgf

This meeting, commemorating the work of Fields Medallist and LMS Honorary Member Professor Maryam Mirzakhani (1977–2017), is part of the 2017–2018 Warwick EPSRC Symposium workshop on *Teichmüller Dynamics* (19–23 March).

Geometric and Topological Methods in Liquid Crystals

Location: De Morgan House, London
Date: 3 April 2018
Website: tinyurl.com/y7w83agb

This meeting will explore the interactions between geometry, topology and the physics of liquid crystals and related materials. Participants may display a poster and/or give a five-minute presentation. A £20 registration fee (waived for speakers) will cover facilities, coffees, teas and buffet lunch. See website for further details. Register at go.soton.ac.uk/9ax.

Distinguished Lecture Series 2018

Location: University of Bristol
Date: 11–13 April 2018
Website: tinyurl.com/y8p5wbtp

The 2018 Distinguished Lecture Series will be given by Ingrid Daubechies (Duke University). Register for the colloquium online by 11 April. Travel support for UK-based PhD students may be available, and applications for funding to support care costs will be considered: email heilbronn-coordinator@bristol.ac.uk by Monday 12 March 2018 for further information.

Young Mathematicians Colloquium 2018

Location: University of Birmingham
Date: 18 April 2018
Website: tinyurl.com/y7a4u8sw

This colloquium is aimed at pure and applied postgraduate mathematicians. The purpose is to provide an opportunity to give a talk and present their own research. We invite all students in the UK and abroad to submit an abstract: deadline 31 March 2018. Speakers: Kenneth Falconer, Valentina Grazian, Annika Heckel, Daniel Jones, Fabial Spill, James Sprittles.

Stochastic Simulation, Uncertainty Quantification and Computational Imaging

Location: ICMS, Edinburgh, UK
Date: 23–24 April 2018
Website: icms.org.uk/

This workshop will focus on sampling and simulation challenges arising in the field of Uncertainty Quantification, with applications in Computational Imaging. There will be short introductory courses in both uncertainty quantification and computational imaging.

UK-Japan Workshop on Analysis of Nonlinear Partial Differential Equations

Location: Swansea University
Date: 16–18 May 2018
Website: tinyurl.com/UKJapanPDE-2018

The aim of the workshop is to bring together a group of leading Japanese and UK nonlinear PDE experts for an exchange of ideas, with the aim of building world-leading research collaborations. It is intended as a springboard to launch new and lasting UK-Japan links. Supported by an LMS Conference grant.

Gregynog Welsh Maths Colloquium 2018

Location: Gregynog Hall, Newtown
Date: 21–23 May 2018
Website: tinyurl.com/yafq7skt

This long-standing annual event brings together mathematicians at Welsh HEIs, while also welcoming mathematicians from other parts of the UK. The three plenary speakers are Ruth King (Edinburgh), Jane White (Bath) and Ian Short (Open University). The Colloquium is supported by an LMS Conference grant.

Northern Regional Meeting

Location: University of Northumbria
Date: 25 May 2018
Website: tinyurl.com/ya27pwj8

This meeting forms part of a workshop on *Advances in the Theory of Nonlinear Waves* (23–25 May 2018). Requests for support, including an estimate of expenses, as well as all queries about the two events may be addressed to the organisers: Dr Benoit Huard (benoit.huard@ northumbria.ac.uk) and Dr Matteo Sommacal (matteo.sommacal@northumbria.ac.uk).

Collabor8.2

Location: Lancaster University
Date: 14–15 May 2018
Website: tinyurl.com/y926uqff

This meeting is designed to promote collaboration among early career researchers in the UK across the disciplinary divide of mathematics and physics. The focus is broad, encompassing theoretical problems from astronomy, cosmology, particle theory, string theory, condensed matter and quantum foundations. See the website for more details and to register.

Mathematics and Science: In Honour of Sir John Ball

Location: Oxford University
Date: 17–19 May 2018
Website: tinyurl.com/y9sxphm2

This will be a unique opportunity for researchers and current PhD students to learn about recent exciting developments in areas most associated with John Ball's name and influenced by his work. Limited funding is available for UK-based PhD students. Partly funded by an LMS Conference grant.

Lancaster Probability Days 2018

Location: Lancaster University
Date: 22–24 May 2018
Website: tinyurl.com/ycv2cjvs

This series of three collaborative one-day workshops, each on a different probabilistic/statistical theme, will feature plenary talks as well as short talks by junior researchers. Participants interested in contributing a poster or a talk should email probability@lancaster.ac.uk by 15 April 2018. Partly supported by an LMS Conference grant.

Recent Advances in Nonlinear Analysis

Location: Levico Terme, Italy
Date: 28–31 May 2018
Website: tinyurl.com/y8vt2la2

The conference focuses on the development of modern methods in Nonlinear Analysis and their connection with the broad area of Partial Differential Equations (PDEs). It marks the 60th anniversary of Professor Vicentiu Radulescu, an internationally recognised mathematician with major contributions in the field of Nonlinear Analysis and PDEs.

Midlands Regional Meeting

Location: University of Leicester
Date: 4 June 2018
Website: tinyurl.com/y7xxdnp3

The meeting forms part of a workshop on *Galois Covers, Grothendieck-Teichmüller Theory and Dessins D'enfants* on 5-7 June 2018. Requests for support and queries about the two events may be addressed to the organisers: Frank Neumann and Sibylle Schroll (lmsmrm2018@le.ac.uk). See the website for details of speakers.

Manifolds, Groups and Homotopy

Location: Sabhal Mòr Ostaig, Isle of Skye
Date: 18–22 June 2018
Website: tinyurl.com/ybwbdmfv

This international conference will be held on the scenic Isle of Skye, with the aim of stimulating activity in, and enhancing interactions between, the areas of manifold theory, group theory and homotopy theory. See the website for details of plenary speakers. Supported by an LMS Conference grant, Glasgow Mathematical Journal Trust and the EMS.

Future Challenges in Statistical Scalability

Location: Isaac Newton Institute, Cambridge
Date: 25–29 June 2018
Website: tinyurl.com/ybus7mbq

The aim is to end the meeting with a vision of the future. It will summarise how recent advances in mathematical sciences can be exploited by practitioners suffering from a data deluge, and present the outstanding challenges facing the discipline in coming years. Application closing date: 26 March.

General Meeting and Hardy Lecture

Location: BMA House, London
Date: 29 June 2018
Website: tinyurl.com/ycwz86yq

See the website for further details about the General Meeting and to register. The speakers will be Konstanze Rietsch (King's College, London); Hardy Lecture: Lauren Williams (UC Berkeley). A reception and Society dinner (cost: £40.00, including drinks) will be held after the meeting. Email lmsmeetings@lms.ac.uk to reserve a place at the dinner.

CHAOS 2018

Location: Rome, Italy
Date: 5–8 June 2018
Website: tinyurl.com/d2h38jp

The study of nonlinear and dynamical systems has emerged as a major area of interdisciplinary research. The principal aim of this conference is to expand the development of the theories of the applied nonlinear field, the methods, empirical data and computer techniques as well as the best theoretical achievements of chaotic theory.

An Analyst, a Geometer and a Probabilist Walk Into a Bar

Location: Cardiff University
Date: 25–29 June 2018
Website: tinyurl.com/ybpg5tkt

This workshop aims to bring together mathematicians working at the crossroads of analysis, geometry and probability. Allowing plenty of time for discussions, as well as funding for students and early career researchers, it is hoped that this workshop can inspire new projects and collaborations.

Numerical Linear Algebra and Optimization

Location: University of Birmingham
Date: 27–29 June 2018
Website: tinyurl.com/yb95bm67

The purpose of this 6th IMA conference on Numerical Linear Algebra and Optimization is to bring together researchers from both communities and to find and communicate points and topics of common interest. See the website for further details.

LMS Popular Lectures

Location: London and Birmingham
Date: 4 July and 19 September 2018
Website: lms.ac.uk/events/popular-lectures

The LMS Popular Lectures are free annual events which aim to present exciting topics in mathematics and its applications to a wide audience. All with an interest in mathematics may attend. The speakers for the 2018 Popular Lectures are Katie Steckles (*Maths's Greatest Unsolved Puzzles*) and Jennifer Rogers (University of Oxford; *Risky Business*).

European Conference on Mathematical and Theoretical Biology

Location: University of Lisbon
Date: 23–27 July 2018
Website: tinyurl.com/y7585tls

The 11th European Conference on Mathematical and Theoretical Biology (ECMTB 2018) will be a main event of the Year of Mathematical Biology. A Mentorship Programme has been set up to facilitate research and career interactions between junior and senior scientists attending the meeting.

Modern Mathematical Methods in Science and Technology 2018

Location: Elite Resort Hotel, Kalamata, Greece
Date: 2–4 September 2018
Website: tinyurl.com/y89rx26j

This three-day international conference is organised by the Department of Mathematics of the National and Kapodistrian University of Athens and aims to bring together researchers from a wide spectrum of applied mathematics. Conference themes include Differential Equations and Mathematical Models and Numerical Analysis; see the website for details. Abstract online submission is open until 30 April 2018.

Equivariant and Motivic Homotopy Theory

Location: Isaac Newton Institute, Cambridge
Date: 13–17 August 2018
Website: tinyurl.com/yc37rrrx

There are significant and subtle interactions between equivariant and motivic homotopy, with each providing intuition and strategies of proof for the other. This workshop will bring together researchers interested in the two areas. The closing date for applications is 13 May 2018.

LMS-IMA Joint Meeting: Noether Celebration

Location: De Morgan House, London
Date: 11 September 2018
Website: tinyurl.com/y9u82osz

See the website for further details and to register for a place. Speakers are Katherine Brading, Elizabeth Mansfield, Cheryl Praeger, Norbert Schappacherand and Reinhard Siegmund-Schultze. The LMS and IMA will host a Joint Society Dinner at a nearby venue. The cost of the dinner, including drinks, is £30.00 per person. To reserve a place at the dinner email Elizabeth Fisher (lmsmeetings@lms.ac.uk).

British Mathematical Colloquium 2018

11–14 June 2017, St Andrews University

Website: tinyurl.com/ybuv2we7

Note that this is a different time of year from usual for the BMC. Plenary speakers are Laura DeMarco (Northwestern University), Irit Dinur (Weizmann Institute), Martin Hairer (Imperial College London), Nalini Joshi (Sydney), Paul Seymour (Princeton) and Marcelo Viana (IMPA, Brazil). Julia Wolf (Bristol) will give a public lecture.

There will be 12 invited Morning Speakers, and five workshops: on Algebra, Analysis & Probability, Dynamics, Combinatorics, and History of Mathematics — details are on the website.

Three LMS Scheme 3 Networks will have meetings on the Thursday afternoon: North British Geometric Group Theory Seminar, North British Semigroups and Applications Network, and One Day Ergodic Theory Meetings. Details will be available on their websites.

Registration is now open on the website. The registration fee, which includes tea/coffee and lunches, is £50 (until 10 April) and £80 (from 11 April). £30 and £50 respectively for PhD students, who can also apply for travel support.

BMC 2018 is supported by the LMS, the Edinburgh Mathematical Society, the Glasgow Mathematical Journal Learning and Research Support Fund, and the Heilbronn Institute and is organised in partnership with the Clay Institute.

LMS Research School: *Mathematics of Multiscale Biology*

Exeter University; 2–6 July 2018

Biological systems often involve inextricably linked spatial and temporal scales. This multi-scale nature has entailed various mathematical developments, which constitute the theme of this Research School. Both theoretical and computational aspects will be covered in lectures and practical sessions on computers.

The three main lecture course topics are: *Mathematics of microscopic biological systems* (Professor Nick Monk, Sheffield); *Mathematics of macroscopic (super-cellular) biological systems*; and *Mathematics across biological scales*. The last two topics will be covered by Dr Bindi Brook, Dr Etienne Farcot, Professor John King, Dr Reuben O'Dea and Professor Markus Owen (Nottingham).

See tinyurl.com/multiscale for more information. Apply online at tinyurl.com/RS37app by **9 April 2018**.

LMS–CMI Research Schools

Homotopy Theory and Arithmetic Geometry: Motivic and Diophantine Aspects

Imperial College, London; 9–13 July 2018

The focus of this research school is on three major advances that have emerged lately in the interface of homotopy theory and arithmetic: cohomological methods in intersection theory, with emphasis on motivic sheaves; homotopical obstruction theory for rational points and zero cycles; and arithmetic curve counts using motivic homotopy theory.

The three main lecture course topics are: *Cohomological methods in intersection theory* (lecturer: Denis-Charles Cisinski, University of Regensburg; assistant: Christopher Lazda, University of Amsterdam); *Homotopical manifestations of rational points and algebraic cycles* (lecturer: Tomer Schlank, Hebrew University of Jerusalem; assistant: Ambrus Pal, Imperial College London); and Arithmetic enrichments of curve counts (lecturer: Kirsten Wickelgren, Georgia Tech; assistant: Frank Neumann, University of Leicester). The lecture courses will be supplemented by tutorial sessions and a guest lecture by Paul Arne Østvær (University of Oslo), Jon Pridham (University of Edinburgh) and Vesna Stojanovska (University of Illinois at Urbana-Champaign).

See tinyurl.com/homotopy for further information. Apply online at tinyurl.com/RS39app by **9 April 2018.**

New Trends in Analytic Number Theory

Exeter University; 13–17 August 2018

The three main lecture course topics are: Topics in Classical Analytic Number Theory (lecturers: Andrew Granville, UCL; Steve Gonek, University of Rochester, USA); Hardy and Littlewood Circle Method and Diophantine Geometry (lecturers: Tim Browning, University of Bristol; Trevor Wooley, University of Bristol); and Analytic Number Theory in Function Fields (lecturers: Zeev Rudnick, Tel-Aviv University; Jon Keating, University of Bristol).

Distinguished speakers: Chantal David (elliptic curves, Concordia University, Montreal); Ben Green (additive combinatorics, University of Oxford); Roger Heath-Brown (prime number theory, University of Oxford); Nina Snaith (random matrix theory, University of Bristol); Kannan Soundararajan (arithmetic functions, Stanford University, California).

See tinyurl.com/newtrends18 for further information. Apply online at tinyurl.com/RS36app by **14 May 2018.**

The LMS–CMI Research Schools are run in partnership with the London Mathematical Society and the Clay Mathematics Institute, and supported by the Heilbronn Institute of Mathematics Research.

Society Meetings and Events

March 2018

2 Mary Cartwright Meeting, London
22 Society Meeting in Honour of Maryam Mirzakhani, Warwick University

May 2018

25 Northern Regional Meeting, Northumbria

June 2018

4 Midlands Regional Meeting, Leicester
13 Society Meeting at the BMC, St Andrews
29 General Meeting and Hardy Lecture

July 2018

4 LMS Popular Lectures, London

August 2018

7 LMS Meeting at the ICM, Rio de Janeiro

September 2018

11 Joint Society Meeting with IMA: Noether Celebration, London
19 LMS Popular Lectures, Birmingham

October 2018

9 Joint Society Meeting with the Fisher Trust, Galton Institute, Genetics Society and RSS; Royal College of Surgeons, Edinburgh

November 2018

9 Society and Annual General Meeting, London

December 2018

17 LMS South West & South Wales Regional Meeting, Exeter

Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website (www.lms.ac.uk/content/calendar). Please send updates and corrections to calendar@lms.ac.uk.

March 2018

2 Mary Cartwright Lecture, London (474)
15 David Crighton Award and Lecture, David Abrahams, Royal Society London (474)
19 Oxford Literary Festival, Oxford (475)
21–22 History of Computing Beyond the Computer, University of Oxford (475)
22 Teichmüller Dynamics, LMS Meeting in Honour of Maryam Mirzakhani, Warwick (475)
22–23 Frobenius Structures and Relations, University of Glasgow (475)
23 Fourth Symposium on the History and Philosophy of Programming, Oxford (475)

April 2018

3 Geometric and Topological Methods in Liquid Crystals, De Morgan House (475)
3-6 British Congress of Mathematics Education, Warwick (471)
4 Operator and Spectral Theory, University of Reading (475)
11–13 Ingrid Daubechies, Distinguished Lecture Series 2018, University of Bristol (475)
13 Functor Categories for Groups, University of Lincoln (475)
18 Young Mathematicians Colloquium 2018, University of Birmingham (475)
23–24 Stochastic Simulation, Uncertainty Quantification and Computational Imaging, ICMS Edinburgh (475)

May 2018

14–15 Collabor8.2, Lancaster University (475)
16–18 UK–Japan Workshop on Analysis of Nonlinear Partial Differential Equations, Swansea University (475)
17–19 Mathematics and Science: In Honour of Sir John Ball, University of Oxford (475)
21–23 Gregynog Welsh Maths Colloquium 2018, Gregynog Hall, Newtown (475)
22–24 Lancaster Probability Days 2018, Lancaster (475)
25 LMS Northern Regional Meeting, University of Northumbria (475)
28–31 Recent Advances in Nonlinear Analysis, Levico Terme, Italy (475)

June 2018

4 LMS Midlands Regional Meeting, Leicester (475)
5–8 CHAOS 2018, Rome, Italy (475)
11–14 British Mathematical Colloquium 2018, University of St Andrews (475)
13 Society Meeting at the BMC, St Andrews
18–22 Manifolds, Groups and Homotopy, Sabhal Mòr Ostaig, Isle of Skye (475)
25–29 An Analyst, a Geometer and a Probabilist Walk Into a Bar, Cardiff University (475)
25–29 Future Challenges in Statistical Scalability, Isaac Newton Institute, Cambridge (475)
27–29 Numerical Linear Algebra and Optimization, University of Birmingham (475)
29 General Meeting and Hardy Lecture (475)

July 2018

2–6 The Mathematics of Multiscale Biology, LMS Research School, Nottingham (475)
4 LMS Popular Lectures, London (475)
9–13 Homotopy Theory and Arithmetic Geometry: Motivic and Diophantine Aspects, LMS–CMI Research School, Imperial College London (475)
23–27 European Conference on Mathematical and Theoretical Biology, University of Lisbon (475)

August 2018

7 LMS Meeting at the ICM, Rio de Janeiro
13–17 New Trends in Analytic Number Theory, LMS–CMI Research School, Exeter (475)
13–17 Equivariant and Motivic Homotopy Theory, Isaac Newton Institute, Cambridge (475)

September 2018

2–4 Modern Mathematical Methods in Science and Technology, Kalamata, Greece (475)
11 Joint Society Meeting with IMA: Noether Celebration, London (475)
14–15 Theoretical and Computational Discrete Mathematics, University of Derby, (475)
19 LMS Popular Lectures, University of Birmingham (475)

October 2018

9 Joint Society Meeting with the Fisher Trust, Galton Institute, Genetics Society and RSS; Royal College of Surgeons, Edinburgh

November 2018

9 Society and Annual General Meeting, London

December 2018

17 LMS South West & South Wales Regional Meeting, Exeter

August 2019

4–9 Theory and Practice: an Interface or a Great Divide? Maynooth University