NEWSLETTER


FROM THE ICOSAHEDRON TO E 8

EMMY NOETHER: AN APPOINTMENT TO KIEL?

THE ATTRACTION OF DYNAMICAL SYSTEMS

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lain Moffatt (Royal Holloway, University of London) iain.moffatt@rhul.ac.uk

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LMS/EMS Correspondent: David Chillingworth
Policy Roundup: John Johnston
Production: Katherine Wright
Printing: Holbrooks Printers Ltd

## EDITORIAL OFFICE

London Mathematical Society
De Morgan House
57-58 Russell Square
London, WC1B 4HS
newsletter@lms.ac.uk
Charity registration number: 252660

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## IN BRIEF

## Upcoming LMS Events

The following events will take place in the next two months:

Northern Regional Meeting: Northumbria, 25 May 2018 (tinyurl.com/ya27pwj8)

Midlands Regional Meeting: Leicester, 4 June 2018 (tinyurl.com/y7xxdnp3)

Society Meeting at the BMC: St Andrews, 13 June 2018 (tinyurl.com/yae88mts)

General Meeting and Hardy Lecture: London, 29 June 2018 (tinyurl.com/ybmyx6on)

A full listing of upcoming LMS events can be found on page 52.

## LMS General Meeting

There will be a General Meeting of the Society on Friday 29 June 2018 at 3:30 pm, to be held at BMA House, Tavistock Square, London WC1H 9JP. The business shall be: 1) the appointment of the Scrutineers; 2) announcement of Council's recommendation for election to Honorary Membership; 3) announcement of LMS prize winners for 2018.

The General Meeting, at which Professor Lauren Williams will give the Hardy Lecture, will be followed by a Society Meeting. It is hoped that as many members as possible will be able to attend.

## Vacancies on LMS Committees

The detailed business of the LMS is run by about 23 committees and working groups, each usually having around 10 people. Altogether this comes to a large number of people, to whom the Society is extremely grateful for this vital work.

It is Council's responsibility to make the appointments to all these committees and to turn their membership over regularly, so that (a) the broadest possible spectrum of our membership is represented, and (b) the committees remain fresh and energetic. Of course when forming a committee account has to be taken of many things, such as maintaining subject and demographic balance, which means that on a given occasion otherwise very strong candidates may not always be able to be appointed.

So we are always looking for new people! The list of committees can be found at Ims.ac.uk/about/committees.

If you are interested, or would like to recommend a colleague, please contact Katherine Wright at katherine.wright@lms.ac.uk in order that Council can maintain a good list of potential members of its various committees. It is not necessary to specify a particular committee. If you would like to know what is involved, you could in the first instance ask your LMS Departmental Representative.

In particular, Council will soon need to fill vacancies on the Women in Mathematics Committee and the Education Committee.

Please give this some thought.

Stephen Huggett
LMS General Secretary

## Faces of Women in Mathematics

Eugenie Hunsicker, Chair of the London Mathematical Society (LMS) Women in Mathematics Committee and her filmmaker sister, Irene Linke, have produced a short film for International Women's Day this year featuring cameos by women mathematicians from around the globe. The film was inspired by the work of the Committee for Women in Mathematics (CWM) of the International Mathematical Union (IMU) and comprises 146 clips of 243 women mathematicians from 36 different countries speaking 31 different languages.
"We didn't expect to get nearly as many people sending in their clips as we did - it's been phenomenal, but busy as we've done it all in just over a month", said Hunsicker.
"It also emphasises the international nature of mathematics", added Linke. "There are women in the film speaking Chinese in the US, Greek in the Netherlands. There's a clip of a Russian woman speaking Tatar in Germany; and in one single clip from the UK, Hebrew, Brazilian, German and English are spoken".

Hunsicker said she wanted the focus of the film to be on proud, strong women that are actively doing maths, but also wanted to emphasise that it's not unusual for women to work in mathematics. "There
are so many women everywhere that enjoy it and do it as a profession", she said. This is also an opportunity for people to see images of strength and pride from developing countries. Yes it's a film about women, but it's also about expanding people's ideas about countries that we often only hear about in the context of crisis. "I want that the next time people hear about Nigeria, Nepal or the Philippines, they think, Oh, yeah, that is the place with all of those fantastic women mathematicians".

You can watch the film at vimeo.com/259039018.

## Charlotte Scott Research Centre for Algebra opens in Lincoln



Sheriff's Consort, Sheriff and Mayor of Lincoln, Caroline Series, Andrew Hunter, Evgeny Khukhro, Andrei Zvelindovsky

On 7 February 2018, LMS President Professor Caroline Series, FRS, visited Lincoln, where she gave a public lecture, Indra's Pearls, a mathematical adventure. This was the second annual Charlotte Scott Lecture in Mathematics.

Charlotte Scott (1868-1931) was the first British woman to have a professional career as a mathematician. Born in Lincoln, she studied at Cambridge where her brilliant performance in the Tripos exams created a national stir. She gained a DSc from UCL under Cayley, and in 1885 became the first woman to join the London Mathematical Society. Subsequently she was recruited as head of mathematics in the newly founded Bryn Mawr College in the USA, where she remained until retirement, a respected and influential figure in the American mathematical community of the time.

During her visit to Lincoln, Professor Series inaugurated the new research centre in the University of Lincoln - the Charlotte Scott Research Centre for Algebra. The centre is based at the School of Mathematics and Physics, where the Algebra Research Group was active from the very beginning, when
the school opened in 2014. With Professor Evgeny Khukhro as the director of the new centre, the algebraists in Lincoln conduct research on finite and infinite groups and their automorphisms, Lie algebras, profinite and locally finite groups, topological groups, permutation groups, polynomial algebra, algebraic number theory.

Members of the centre have active research collaboration with colleagues in Australia, Brazil, France, Germany, Italy, Russia, Spain, Turkey, UK, USA. In 2016 the School of Mathematics and Physics conducted the first ever mathematics conference in Lincoln Groups, rings, and their automorphisms dedicated to Evgeny Khukhro's 60th birthday.

The creation of the Charlotte Scott Research Centre for Algebra is a testament to the strong point of mathematical research at the Lincoln School of Mathematics and Physics.

Professor Andrei Zvelindovsky Head of School of Mathematics and Physics University of Lincoln

"A historical and philosophical tour of major insights in the development of probability theory." —James Ryerson, The New York Times Book Review
"Mathematically rigorous, yet also reasonably accessible; informative, yet fun and entertaining to read. Both students and faculty should find reading this to be a rewarding experience."
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Mathematical Treasures in the Royal Society Collections


The formation of a collection including mathematical books, instruments and manuscripts was integral to the foundation of the Royal Society of London for Improving Natural Knowledge in 1660, whose first President, Viscount Brouncker, was a mathematician. Structured around its learned community, it quickly sought to create a place of knowledge to foster moments of discussions of contemporary methods and discoveries. A library containing mathematical references was crucial to fulfil this mission, as explicitly put forward by a proposal submitted to the Society in the 1670s:
> 'Further more let there be every yeare a summe of money laid aside out of the Society's stocke to be laid out at the discretion of the Council in books whose cheife scope is naturall Philosophy or mathematicks for the use of the Society'. (DM/5/11; f.3; attributed to William Neile by Michael Hunter, Establishing the New Science, 1989, p. 225)

The importance of mathematics is reaffirmed systematically through the history of the collection and whereas some subjects were deaccessioned following the evolution of scientific practices, mathematics remains at the core of the current collections, signifying powerfully the mathematisation of science.

It would be impossible to propose here an exhaustive list of the mathematical treasures contained in the Royal Society collections: they are both too numerous and scattered between the book collections, the institutional papers, the correspondences and personal papers of Fellows of the Royal Society. Instead, we hope to give readers of the LMS Newsletter a
taste of our holdings and encourage them to visit our collections to discover much more.

The library holds fundamental publications from the mathematical canon, including some on the history of mathematics. Euclid's Elements of Geometry (currently under study by the Oxford-based project 'Reading Euclid') is part of the collection, as are various texts on functions by Joseph-Louis Lagrange, some of which include fascinating marginal annotations by Fellows and benefactors of the Royal Society. The 1832 edition of the Hungarian textbook Tentamen, by Farkas Bolyai, of which there are few extant copies left, is a prime example of the rare mathematical books we hold.

As publisher of Isaac Newton's Principia Mathematica, the Royal Society holds the fair manuscript copy sent to the printer, dozens of editions of the printed work as well as relevant correspondence and notes by contemporary mathematicians (David Gregory, John Wallis...).

The symbolic algebraist George Peacock (1791-1858) was a very active contributor to the Royal Society, helping with the publication of a catalogue of its Library in the 1830 s, and acting as referee, communicator and author for the Society's periodical dedicated to mathematical studies, Philosophical Transactions A. His reports and those of other 19th century referees make for a fascinating read, as they unveil the beginnings of peer-review and include discussions of key mathematical discoveries by contemporary scientists.

Another treasure - still awaiting thorough study is the collection of George Boole's (1815-1864) mathematical and scientific papers (MS/782). Formed of 13 boxes, the collection was deposited in 1866 and completes the Boole papers kept at University College Cork.

The Royal Society collections are open to the public, and we hope to see LMS members visit soon to read correspondences, papers and admire portraits of various key mathematicians, from Thomas Bayes to Ronald Fisher.

Discover the digital exhibit on the statisticians at tinyurl.com/yby8wdj3 and Maurice Bartlett whose papers have just been catalogued at tinyurl.com/ybh2e7t7.

Louisiane Ferlier
Digital Resources Manager, The Royal Society

PEOPLE

## STEM for BRITAIN 2018: Maths Sciences Research Displayed in Parliament



STEM for BRITAIN poster winners, I to r: Dr Céline Maistret (gold), Ms Linda Irons (silver), Dr Yoann Altmann (bronze)

Early career mathematicians took part in the annual STEM for BRITAIN competition for the fifth time on Monday 12 March. Thirty posters presenting research in the Mathematical Sciences were presented at Parliament to politicians and a panel of expert judges, who awarded Gold, Silver and Bronze prizes to the top three exhibitors.

Dr Céline Maistret of the University of Bristol was awarded Gold for her research into the Birch and Swinnerton-Dyer conjecture, one of the Clay Mathematics Institute (CMI) Millennium problems.

Silver was awarded to Ms Linda Irons of the University of Nottingham, for her poster presenting the mathematical modelling of cell adhesion in asthma. The recipient of the Bronze award was Dr Yoann Altmann of Heriot-Watt University, for his research into new computational methods for low illumination imaging and sensing.

The Gold and Silver prizes (including $£ 2,000$ and $£ 1,250$ respectively) were sponsored by the CMI, and the Bronze prize ( $£ 750$ ) was sponsored by the Heilbronn Institute of Mathematical Research.

Professor Nick Woodhouse, President of the Clay Mathematics Institute, supporters of the Gold Award and Silver Awards said: "The inclusion of mathematics in STEM for BRITAIN recognises the vitality and strength of the discipline in the UK and the huge
part that all branches of mathematics play in underpinning science and technology."

Professor Jon Keating FRS, Chair of the Heilbronn Institute for Mathematical Research, supporters of the Bronze Award said: "As a Research Institute whose focus is on fundamental mathematics and its applications to UK national interests, and on supporting mathematical research across the country, HIMR is delighted to be associated with STEM for BRITAIN and offers its warmest congratulations to all of the Award winners."

The Council for the Mathematical Sciences continues to support the event in choosing the early career researchers who present their research in Parliament. It is paramount to encourage early-career research scientists, engineers, technologists and mathematicians and the STEM for BRITAIN event is a very effective way of doing this.

Sir Adrian Smith, Chair of the Council for the Mathematical Sciences (CMS), said: "The CMS is delighted that the mathematical sciences have been involved in this prestigious event once again, it is wonderful to showcase the importance of the mathematical sciences to a wider audience. We have been encouraged by the enthusiastic response from early-career researchers in the mathematical sciences and feel sure this will this continue in the future."

## Lifetime Achievement Award

## DR FAQIR BHATTI

Dr Faqir M. Bhatti, who is Professor and Director at the Riphah Institute of Computing and Applied Sciences, Lahore, Pakistan, received a Lifetime Achievement Award from the National Mathematical Society of Pakistan for contributions to Mathematics. The Award was given by Mr Khalid Maqbol, former governor of Punjab, at the ceremony which was held on 17 February 2018. The award is given to a Pakistani mathematician every two years. The award came with a certificate and cash prize. Dr Bhatti has been an LMS member since 1992.

## LMS Honorary Member Awarded 2018 Abel Prize



## PROFESSOR ROBERT LANGLANDS

The Norwegian Academy of Science and Letters has awarded the 2018 Abel Prize to Professor Robert Langlands, Institute of Advanced Studies, Princeton University, 'for his visionary program connecting representation theory to number theory'. Professor Langlands will receive his award from His Majesty King Harald V at the University Aula, Oslo, Norway on 22 May 2018. The Abel Prize carries a cash award of 6 million NOK (about €623,000)

Langlands' insights were so radical and so rich that the mechanisms he suggested to bridge these mathematical fields led to a project named the Langlands program. The program has enlisted hundreds of the world's best mathematicians over the last fifty years. Few other projects in modern mathematics have as wide a scope, have produced so many deep results, and have so many people working on it. Its depth and breadth have grown and the Langlands program has been described as a grand unified theory of mathematics.

Professor Langlands was elected a Fellow of the Royal Society in 1981 and was awarded Honorary Membership of the London Mathematical Society (LMS) in 2015, the Society's 150th Anniversary Year.

Professor Caroline Series, LMS President, will be giving a speech at the Abel Banquet in Professor Lang-
land's honour. She commented; ‘The Abel Prize, one of the highest honours in mathematics, is justly awarded to Professor Langlands for his pioneering insights which have transformed number theory and representation theory. One example of the Langlands correspondence is the modularity theorem which formed the basis of Andrew Wiles' proof of Fermat's last theorem'.

More information is available at abelprize.no/.

## 2017 Stanton Medal

## PROFESSOR ROBIN WILSON

The 2017 Stanton Medal has been awarded to Professor Robin Wilson (Open University and LSE). It is awarded every two years by the Institute of Combinatorics and its Applications (ICA) to honour significant lifetime contributions to promoting the discipline of combinatorics through advocacy, outreach, service, teaching and/or mentoring, and is named in recognition of Ralph Stanton, the founder of the ICA.

In the words of the ICA citation "Professor Robin Wilson has, for fifty years, been an outstanding ambassador for graph theory to the general public. He has lectured widely (giving some 1,500 public lectures), and extended the reach of his lectures through television, radio, and videotape. He has also published extensively on combinatorial ideas, written in a style that is engaging and accessible. He has provided direction, encouragement, and support to colleagues and students at all levels. His superb talents at conveying the beauty of graph-theoretic ideas, and inviting his readers and listeners to join in, have enthused many students, teachers, and researchers. Professor Wilson's advocacy and outreach for combinatorics continue to yield many positive impacts that are enjoyed by researchers and non-specialists alike."

## Clay Research Fellows 2018

DR ALEKSANDR LOGUNOV
DR WILL SAWIN
Aleksandr Logunov and Will Sawin have been awarded Clay Research Fellowships. Aleksandr Logunov obtained his PhD in 2015 under the supervision of Viktor Havin at the Chebyshev Laboratory, St Petersburg State University. After two years as a postdoctoral fellow at Tel Aviv University, he moved last year to the Institute for Advanced Study at Princeton. He will hold his Clay Research Fellowship at Princeton University. Together with Eugenia Malinnikova, he was given a Clay Research Award in 2017. The
award recognised Logunov's and Malinnikova's introduction of a novel geometric combinatorial method to study doubling properties of solutions to elliptic eigenvalue problems. Aleksandr has been invited to speak on his work at the 2018 International Congress of Mathematicians in Rio.

Will Sawin obtained his PhD in 2016 from Princeton University, under the supervision of Nicholas Katz. Since then he has worked with Emmanuel Kowalski as a Junior Fellow at ETH Zürich. Sawin's research is wide ranging, but focused on the interactions of analytic number theory and algebraic geometry. Amongst the many areas in which he has made ground-breaking contributions are the application of étale cohomology to estimates of exponential sums over finite fields and, with Tim Browning, the adaptation of classical counting arguments in analytic number theory to explore compactly supported cohomology in spaces of interest in algebraic geometry. He has also made many wider contributions to the mathematical community, not least through regular posts on diverse topics on the MathOverflow website.

For more information on Clay Research Fellowships see claymath.org.

## David Crighton Medal Presentation

PROFESSOR I. DAVID ABRAHAMS
The 2017 LMS-IMA David Crighton Medal was awarded to Professor I. David Abrahams, Director of the Isaac Newton Institute for Mathematical Sciences, at the Royal Society on 15 March 2018.

The audience was welcomed by the Vice-President of the LMS, Professor John Greenlees, who reminded the audience that the David Crighton Medal was instituted in 2002 in memory of Professor David Crighton, who was President of the IMA and PresidentDesignate of the LMS. Professor Alistair Fitt, IMA President, introduced David Abrahams as the 2017 IMA-LMS David Crighton Medallist and Professor Greenlees read the citation, which can be seen at tinyurl.com/abrahams-citation.

David then gave a lecture, Mathematics, Metamaterials and Meteorites, during which he discussed how applied mathematics can answer phenomenological questions, make new predictions, and assist industry in product design. In particular, he looked at how asymptotic techniques, developed by David Crighton and others, can be employed to reduce low-frequency noise from domestic appliances.

College Associate Lectureship and Fellowship in Pure Mathematics Salary: $£ 34,550-£ 39,992$ p.a. (depending on experience) plus benefits

St John's College is looking to appoint a College Associate Lecturer and Fellow in Pure Mathematics from 1 October 2018. This is an early career development post and is offered for a fixed-term period of five years.
The position offers the opportunity for progression towards an academic career in pure mathematics. The successful candidate will be expected to teach nine hours of small group teaching a week during the twenty teaching weeks of the year (amounting to 180 hours of contact time) and to assume a role as Director of Studies in Pure Mathematics (for which additional remuneration will be paid). He or she will also be expected to pursue scholarly research with a view to building up a high-quality publication record. The position provides the opportunity for collaboration and support from the world leading Cambridge mathematical research community.
Other duties include College examining, participating in the selection of candidates for admission, attendance at Open Days, and general support for the academic development of the Mathematics students in College.
The salary will be in the range of $£ 34,550-£ 39,992$ p.a., depending on experience; the post is pensionable under the Universities Superannuation Scheme (USS). The successful candidate will also be provided with the usual benefits of a Fellowship in the College, and membership of the Faculty of Mathematics in the University of Cambridge.
For further particulars, please visit www.joh.cam.ac.uk/vacancies or telephone 01223338794.
The closing date for the receipt of completed applications is 12 noon on Monday 14 May 2018.

## Stephen Hawking: 1942-2018

It is with great sadness that the London Mathematical Society has learned of the death of Professor Stephen Hawking on 14 March 2018. In the history of the development of general relativity, just a few individuals stand out as having contributed in such original, central and fundamental ways.

In 1999 Professor Hawking was awarded the LMS Naylor Prize and Lectureship. He presented his lecture titled Euclidean Quantum Gravity at the Society Meeting on Friday 20 October 2000. The citation for
his Naylor Prize is an eloquent description of some of his major contributions to the field.

## Sublime Symmetry

The exhibition Sublime Symmetry: The Mathematics behind William De Morgan's Ceramic Designs, which was shown in Burnley, Barnsley, Torbay and Leicester in 2016, is coming to London. It is going to be shown at the Guildhall Art Gallery from 11 May to 28 October 2018, where it will be augmented by two new displays, one about Augustus De Morgan and one about the London Mathematical Society. For more details see tinyurl.com/ybykqmwj.

## MATHEMATICS POLICY ROUNDUP

## Industrial Strategy Research Fund

Innovate UK on behalf of UKRI invited proposals on the potential future challenges to get support through the Industrial Challenges Research Fund.

UK industry and research were asked to submit proposals aligned clearly with at least one of the 4 grand challenges in government's Industrial Strategy. The set of grand challenges include artificial intelligence and the data economy. The deadline for submissions was 18 April 2018. More information is available at tinyurl.com/yaxhgkcc.

## Science and Technology Committee Summit

The House of Commons Science and Technology Committee recently held a Brexit science and innovation summit with representatives from across the STEM community. The transcript of the summit is available at tinyurl.com/y8kaavx8.

## Subject-level TEF: Government consultation

The Department for Education is inviting responses to a technical consultation to inform the development of the Teaching Excellence and Student Outcomes Framework (TEF) at subject level. It is keen to hear from higher education providers, students, employers and others with an interest in learning,
teaching and student information. The consultation closes on 21 May 2018. More information is available at tinyurl.com/y7ps23ou.

## Exiting the EU: Challenges and Opportunities for Higher Education

The government has responded to the House of Commons Education Select Committee report Exiting the EU: challenges and opportunities for higher education. The report is available at tinyurl.com/y958aoon.

## New Advanced Maths Premium

A new fund to help schools and colleges increase the number of students studying maths after GCSE was announced recently by Schools Standards Minister Nick Gibb and Chief Secretary to the Treasury Elizabeth Truss. It follows a commitment from the Education Secretary to 'continue improving academic standards'.

From September 2018, schools and colleges will receive an extra $£ 600$ premium for each additional student taking the one-year AS mathematics or the Core Mathematics qualification. This could mean $£ 1,200$ for each additional student who takes the two-year A-level in mathematics or further mathematics. More information is available at tinyurl.com/yct6rbv3.

John Johnston
Joint Promotion of Mathematics

## Call for Satellite Conferences for ICIAM 2019

The organizers of ICIAM2019-Valencia Congress are pleased to launch the call for satellite meetings. A satellite meeting takes place within a few weeks of ICIAM 2019 on a topic of interest to ICIAM (International Congress on Industrial and Applied Mathematics) in a location that makes it convenient for ICIAM participants to combine the events into a single trip. If interested in organizing a satellite meeting, please contact Mari Paz Calvo, the chair of the Satellite and Embedded Meetings Committee, at maripaz@mac.uva.es. Closing date for submissions is 1 October 2018. The final list of invited speakers for the congress is already online at the Congress website.

## Call for Nominations for Gordin Prize 2018

The EMS Gordin Prize is established to honour the memory of Mikhail Gordin. It will be awarded to a junior mathematician from an Eastern European country working in probability or dynamical systems, and presented at the International Vilnius Conference on Probability Theory and Mathematical Statistics from 2 to 6 July 2018. For further details see imsvilnius2018.com.

## Nouzha El Yacoubi Elected New President of AMU

Nouzha El Yacoubi, professor at the Faculty of Sciences of Mohammed $V$ University of Rabat, Morocco, has been elected president of the African Mathematical Union (AMU), for the period 2017-21. She is the first woman in this position.

## Public Consultation on EU Funds

In 2018 the EU Commission will make comprehensive proposals for the next generation of financial programmes for the post-2020 Multiannual Financial Framework, which is the EU's long-term budget. Consultations are taking place in the context of evaluations of existing EU financial programmes covering several policy areas including research and
innovation. Anyone can contribute to this public consultation by filling in the online questionnaire. The Commission will summarise the replies after the end of the consultation period. For further details see tinyurl.com/y7x5u3es.

## Bergman Prize

Bo Berndtsson, Chalmers University and University of Gothenburg, and Nessim Sibony, Université Paris-Sud Orsay, have been awarded the 2017 Stefan Bergman Prize for their many fundamental contributions to several complex variables, complex potential theory and complex geometry. For further details see tinyurl.com/ycqp9kqy.

## International Day of Mathematics

The International Mathematical Union has prepared a proposal to have 14 March declared as the International Day of Mathematics (IDM) by UNESCO. Every year, all countries will be invited to celebrate the IDM in the schools and with the public, under a theme proposed by the Committee of the IDM. This initiative is supported by the European Mathematical Society.

## Royal Spanish Mathematical <br> Society (RSME)

The second meeting of mathematical societies from Brazil (SBM and SBMAC) and Spain (RSME and SEMA) will take place at the University of Cádiz, 11-14 December 2018. The main goal is to strengthen partnerships and collaborations between researchers and institutions in Brazil and Spain.

## EMS Council

The EMS's highest body, the Council, will convene 23-24 June 2018 in Prague. Among other tasks, the Council meeting will renew the Executive Committee, as well as elect the new President and one VicePresident.

David Chillingworth LMS/EMS Correspondent

## OPPORTUNITIES

New funding for PhD places at the University of Leeds

The University of Leeds is set to receive $£ 6$ million in funding for additional PhD places, including in the field of mathematics-based research, as part of a major investment in science and engineering. The funds to support the University's extensive postgraduate provision will come from the EPSRC through its doctoral training partnerships.

As part of its allocation of 62-plus places, the university will create at least seven PhD positions in mathematical sciences for researchers seeking to begin their studies in 2018 or 2019. Leeds encourages industrial partnerships between PhD researchers and
businesses and further places will be available in the mathematics area for these type of places.

Details about how to apply for postgraduate study at the University are available at physicalsciences.leeds.ac.uk/.

## Book Reviews Section

The Editorial Board of the LMS Newsletter would like to expand its pool of reviewers for the Book Reviews section. Reviews are normally 600 to 1000 words in length, and it is expected that they will be completed in a timely fashion (typically within three months of the review being commissioned). If you would like to be added to the list of potential reviewers, please contact the Reviews Editor at m.mccartney@ulster.ac.uk.

## CAMBRIDGE

## LMS Members Save 25\%

## London Mathematical Society Series from Cambridge University Press

## Topological Methods in Group Theory <br> Edited by <br> N. Broaddus, M. Davis, <br> J.-F. Lafont, and I. J. Oritz <br> Part of London Mathematical Society Lecture Note Series Paperback | 9781108437622 | July 2018

## Permutation Groups and Cartesian

Decompositions
Cheryl E. Praeger
and Csaba Schneider
Part of London Mathematical Society Lecture Note Series
Paperback | 9780521675062 | May 2018


## Synthetic Differential Topology

Marta Bunge, Felipe Gago, and Ana María San Luis

Part of London Mathematical Society Lecture Note Series
Paperback | 9781108447232 | March 2018

## The Block Theory of Finite Group Algebras

Markus Linckelmann
Part of London Mathematical Society Student Texts 91 and 92
Multiple copy pack | 9781108441902 | April 2018 Also available as separate volumes


## Visit of Alexander Gamburd

Professor Alexander Gamburd (City University of New York Graduate Center) will visit the UK from 1 to 30 June 2018. He will visit:

- University of Oxford, 5-8 June (contact Ehud Hrushovski: ehud.hrushovski@maths.ox.ac.uk)
- University of Cambridge, 12 and 15 June (contact Emmanuel Breuillard: efjb2@dpmms.cam.ac.uk)
- University of Bristol, 18 June (contact Tim Browning: timdanielbrowning@google.com)
- Durham University, minicourse 25-27 June (contact Michael Magee:
michael.r.magee@durham.ac.uk)
For further details contact Michael Magee. The visit is supported by an LMS Scheme 2 grant.


## Visit of Eric Swenson

Professor Eric Swenson (Brigham Young University) will be visiting the UK between 7 June and 7 July 2018. He will give the following lectures:

- University of Southampton, 8 June (contact lan Leary: i.j.leary@soton.ac.uk)
- Newcastle University, 12 June (contact Alina Vdovina: Alina.Vdovina@newcastle.ac.uk)
- Oxford University, 18 June (contact Panos Papazoglou: papazoglou@maths.ox.ac.uk)

For further details contact Panos Papazoglou. The visit is supported by an LMS Scheme 2 grant.

## Visit of Nikolai Kolev

Professor Nikolai Kolev (Institute of Mathematics and Statistics, University of São Paulo) will visit the UK from 11 to 23 June 2018. He will visit:

- The York Management School, University of York, 13 June (contact Alexander McNeil: alexander.mcneil@york.ac.uk)
- School of Mathematics, University of Manchester, 15 June (contact Georgi Boshnakov: georgi.boshnakov@manchester.ac.uk)
- Department of Mathematical Sciences, Durham University, 18 June (contact Jochen Einbeck: jochen.einbeck@durham.ac.uk)

Dates are provisional. In addition, he will give a postgraduate lecture on Copulas at Durham on June 20. The visit is supported by an LMS Scheme 2 grant.

## Visit of Oleg Kuzenkov

Dr Oleg Kuzenkov (Lobachevsky State University of Nizhni Novgorod, Russia) will be visiting the Department of Mathematics, University of Leicester from 12 June to 2 July 2018. He will give lectures at:

- University of Leicester, 15 June (contact Andrew Morozov: am379@leicester.ac.uk)
- University of Birmingham, 18 June (contact Natalia Petrovskaya: n.b.petrovskaya@bham.ac.uk)
- City, University of London, 26 June (contact Mark Broom: Mark.Broom.1@city.ac.uk)

For further details contact Andrew Morozov (am379@leicester.ac.uk). The visit is supported by an LMS Scheme 2 grant.

## Visit of Oleg Smolyanov

Professor Oleg G. Smolyanov (Professor of the Faculty of Mechanics and Mathematics at Lomonosov Moscow State University, Russia) will visit the UK between 29 May and 24 June 2018. He will give talks at:

- Durham University, 4 June (contact Anton Savostianov: anton.savostianov@durham.ac.uk)
- University of Manchester, 7 June (contact James Montaldi: j.montaldi@manchester.ac.uk)
- Aberystwyth University, 15 June (contact John Gough: jug@aber.ac.uk)
- University of Surrey, 21 June (contact Sergey Zelik: s.zelik@surrey.ac.uk)

For further details contact Anton Savostianov (anton.savostianov@durham.ac.uk). The visit is supported by an LMS Scheme 2 grant and the Department of Mathematical Sciences at Durham University.

# LMS Council Diary: A Personal View 

The meeting on 2 February 2018 was this new diarist's first meeting of Council. Let me begin by thanking, on behalf of all Council members, the outgoing diarist, Tara Brendle, for her excellent service.

This Council meeting also marked the first meeting as Chair for our new President, Caroline Series. She began by reporting her activities since the last Council meeting, and informed Council that Marta SanzSolé had accepted the invitation to be the speaker at the Society's meeting at the 2018 International Congress of Mathematicians (ICM). She also reported that Alison Etheridge had been appointed Chair of the Research Excellence Framework (REF) 2020 Mathematical Sciences Sub-Panel.

Council then agreed to appoint Martin Mathieu to the Society's delegation to the European Mathematical Society meeting in 2018. It was also noted that the President, General Secretary, Publications Secretary, Jonathan Pila and Mark Roberts would form the Society's delegation to the ICM and International Mathematical Union (IMU) General Assembly. In addition, June Barrow-Green would be attending the ICM in her capacity as Chair of the International Commission on the History of Mathematics (ICHM).

In the absence of Gwyneth Stallard and on behalf of the Early Career Research Committee the Executive Secretary reported that the Clay Mathematics Institute (CMI) was withdrawing its funding from the LMSCMI Research Schools partnership. Council agreed that the Research School budget for 2018-19 would be increased to allow for funding up to three Research Schools in 2019. There followed a lively discussion on the Early Career Research Committee's proposal that the Postdoctoral Mobility Grant scheme be reinstated from 2019. It was agreed that the Committee should give further consideration to the exact aims of the scheme and on how to address various aspects of diversity within the scheme.

The report from the Education Committee included a report on the Department for Education's consultation on Accelerated Degrees, which focuses solely on how funding should be set, rather than the principle of Accelerated Degrees. In discussion it was suggested that a letter from the Council of Mathe-
matical Sciences (CMS) might be sent to the Minister for Science to indicate disapproval of the principle of Accelerated Degrees. Council agreed on funding for the Library Committee to arrange a complementary exhibition and a joint reception to be held as part of the Sublime Symmetry exhibition at the Guildhall Art Gallery in 2018. The Chair of the Research Grants Committee provided the Committee's annual report to Council. In 2016-17 the Committee had awarded grants to 53 different institutions as well as running three schemes that involved international collaboration. He noted that amounts on individual grants had not increased in a number of years and that the Committee was intending to review these.

Vice-President Greenlees provided an update on Research Policy Committee activities and reported that 51 nominations had been made to the Research Excellence Framework (REF) Mathematical Sciences SubPanel. Returns had been received for the LMS census of postdoctoral fellows and research assistants, and the Committee is currently considering how best to analyse and represent the data collected. He also reported that, in conjunction with the Women in Mathematics Committee, the Committee was in the process of purchasing data from the Higher Education Statistics Agency (HESA) to possibly track the shift of students and staff between institutions.

The President introduced proposals for amendments to the Standing Orders. Points of discussion included the question of the size of Council and the number of Officers relative to the number of Members-at-Large, as well as the term of office for both Members-atLarge and for Officers. We also heard a first quarter financial review and the revised budgets for 2017-18 from the Treasurer, and a report by Rodney Sharp, one of the Scrutineers for the 2017 Council and Nominating Committee elections. He noted that the voting turnout of $30 \%$ had been relatively high compared to other learned societies.

Finally, Council agreed on a list of 48 applications for membership to be proposed to the Society meeting to be held on 2 March 2018.

## AIMS Visit

In the March issue of the LMS Newsletter was an informative article, Initiatives for the Mathematical Sciences in Africa (pages 25-28). By coincidence I was in South Africa at that time, and so took the opportunity to contact the African Institute for Mathematical Sciences (AIMS) in Muizenberg. Barry Green, the AIMS South Africa Director, informed me that 10.30 am was the time for my visit. On arrival it was immediately obvious why the timing was so precise. This was when everyone, including academics and administrative members of AIMS, met for coffee in the central hall. What was different from the normal 'senior common' room approach? It was this intermingling and networking of people from all areas within AIMS: students were playing musical instruments whilst discussing mathematics and other matters with a range of people.

Several people mentioned that AIMS students are highly sought after and then able to further spread the word about the AIMS project. Institutions with an AIMS person, e.g. a researcher or school teacher, on their staff, rate very highly; this encourages more people to go to AIMS, thus spreading the network.

This is a most impressive place with their commitment to supporting young mathematicians in shaping their future. I met some of the external lecturers, including Dr Wilson Lamb from Strathclyde University and Dr Syamala Krishnannair from the University of Zululand.

It was good to see the contribution the LMS continues to make in assisting the objectives of AIMS, particularly through the use of the Scheme 5 International Short Visits grant award, which encourages lengthy collaborative research visits between UKbased and international mathematicians. Professor Alan Beardon, an LMS member, has been involved with AIMS since its inception as a visiting lecturer and researcher and is doing a sterling job at AIMS. For instance he compiled the book AIMS First Decade. I was pleased to see it includes the LMS in the list of donors. AIMS is Africa's first network of centres of excellence in mathematical sciences, with centres located in South Africa, Senegal, Cameroon, Ghana, Tanzania and Rwanda. It enables Africa's youth to shape the continent's future through STEM education. It provides world-class education for Africa's next generation, and its academic networks and the


Dr Syamala Krishnannair; Barry Green, AIMS Director; and Susan Oakes
knowledge and skills of its members are a valuable resource. The core AIMS programme is a Master's in Mathematical Science, which can be completed in a year. The Research Centre hosts Research Masters students, PhD students and visiting researchers.

The AIMS Next Einstein Initiative has an office at De Morgan House in London. From my own experience, the office in Muizenberg is always happy to receive visitors from overseas, and I am told the same is true for other AIMS centres. Those UK-based mathematicians interested in conducting collaborative research with mathematicians abroad should see the Scheme 5 International Short Visits grant on the LMS website at Ims.ac.uk/grants/international-short-visits-scheme-5.

For more information about AIMS visit their website at aims.ac.za/ or nexteinstein.org/.

Susan Oakes

## The Aims of the LMS Scholarly Publications

LMS Council, in conjunction with Publications Committee, has updated the Society's statement of aims for its books and journals programme. Last year's review of publications observed that the existing aims (essentially "to disseminate mathematics and to raise money for the Society") had ceased to be a particularly useful statement. For example, unlike the 19th and most of the 20th century, in an internet world dissemination is no longer an issue; on the other hand, the opportunities and service offered to authors and readers, in particular the stamp of quality implied by the LMS insignia, are much more to the point.

Council approved the following statement, framed around the service the LMS provides to the mathematical community through its scholarly publishing activities.

## LMS Publications Aims

The London Mathematical Society has been a leading academic publisher of Mathematics for over 150 years. It continues to serve the global mathematics community by publishing and endorsing high quality research and exposition, made available worldwide and selected irrespective of the nationality or geographic location of its authorship.

As the Society's overarching mission concerns the advancement of mathematics, so its publishing activities aim to contribute to this goal by providing:
(1) Services to authors, through a broad and developing portfolio of high quality international titles, which are peer reviewed, managed and produced in a transparent, timely and professional manner.
(2) Services to readers through publishing content of wide interest and high quality, circulated globally and accessible through a comprehensive range of reader platforms.
(3) A reliable source of funding to give ongoing support for mathematics, delivered through the Society's national and international grants schemes, research schools and workshops, international research networks, support for early career researchers, the development of the mathematics people pipeline, shaping of mathematics education, and representation at national policy discussions on mathematics.
(4) Leadership and representation of UK mathematics publishing through advocacy, communication and education.

The Society aims to ensure that its publishing activities continue on a sustainable basis, embrace the use of new technology to improve author and reader service, and uphold the Society's principles of diversity. The Society takes care to maintain reasonable and ethical pricing, including free availability to members and developing countries.

John Hunton
LMS Publications Secretary

## Member Benefit: Using the LMS Library at UCL

Members of the London Mathematical Society can register as users of the UCL Library, where the LMS Library is held. The LMS Library contains a collection of periodicals published by other mathematical societies, copies of books and journals published by the Society, and items acquired by the Society as review copies or gifts.

## UCL Library Privileges

LMS Members may use all the material available in the reading rooms and stores of the UCL family of libraries and borrow up to ten items at any one time, among other privileges. Members also have access to MathSciNet and specific electronic journals from designated terminals in the Science Library. A full list of Member privileges can be found at Ims.ac.uk/library/lms-library. To check the listings of electronic journals available to visitors, use Explore (sfx.ucl.ac.uk/sfx_local/az/walkin).

## Registering and Renewing

For details on how to register/renew your UCL Library card, visit Ims.ac.uk/library/how-register. No charge is made for the initial registration or for renewing expired library cards or cards which are within one calendar month of expiring. Library cards are valid for 12 months from date of issue and should be renewed each year.

## Visiting the Library

See ucl.ac.uk/library/opening for opening hours. For all other queries, please contact a member of staff during office hours (ucl.ac.uk/library/help).

## LMS Prospects: Call for Expressions of Interest

UK departments are invited to submit expressions of interest to host the LMS Prospects in Mathematics Meeting 2019 to the Prospects in Mathematics Meeting Steering Group. The deadline for submission is 1 June 2018.

Up to $£ 7,000$ is available to support the annual twoday events (usually taking place in September) for finalist mathematics undergraduates who are considering apply for a PhD after they have completed their current studies.

LMS Prospects in Mathematics Meetings should feature speakers from a wide range of mathematical fields across the UK who discuss their current research and what opportunities are available to prospective PhD students. Prospective organisers should send an expression of interest (max. one A4 side in length) to Imsmeetings@lms.ac.uk by 1 June 2018. The following should be included:

- department's confirmation of support to host the LMS Prospects in Mathematics Meeting;
- reasons to host the LMS Prospects in Mathematics Meeting;
- a provisional list of speakers who are representative of the UK research landscape both geographically and scientifically;
- speakers from under-represented groups should be included and women speakers should account for at least $40 \%$ of the invited speakers;
- confirmation that prospective organisers have read and understood the terms and conditions in the Guidelines for Organisers (available from tinyurl.com/y9yn2ryo);
- willingness to attend an upcoming LMS Prospects in Mathematics Meeting in Warwick from 7-8 September 2018 to get an idea of the event.

For further details about the LMS Prospects in Mathematics Meetings, please visit: Ims.ac.uk/events/lms-prospects-mathematics-meeting.

## Report: YTF 10 Conference



YTF attendees. © Matheus Hostert
The YTF (Young Theorists' Forum) 10 conference took place at Durham University from 10 to 12 January 2018. YTF is a long running conference organised by a collaboration of PhD researchers from the University's Departments of Physics and Mathematical Sciences, with further support from PhD students at the University of Glasgow. The purpose of the conference is to bring together postgraduate students working in theoretical physics, providing them with the opportunity to present their work to a friendly
audience. To celebrate the tenth anniversary of the conference, it was, for the first time ever, extended to two days, with two invited plenary speakers.

With 86 registered attendees - a $20 \%$ increase compared to last year - this was a vibrant event. Topics included Beyond the Standard Model Theory and Phenomenology, Supersymmetry, The LHC and QCD, Non-Perturbative Physics, Asymptotic Safety, Gravity, Cosmology, and Neutrinos.

Overall, 35 PhD students had the opportunity to present either a 10 or 20 minute talk on their research, with each talk being followed by a brief discussion session. Attendees also had a chance to present their research at a pizza and poster session, taking place at the end of the first day. The evening of the second day featured a social evening in Durham city.

The first plenary talk was given by Professor John Ellis of King's College London. He spoke on the topic of Particle Physics Today, Tomorrow and Beyond discussing how experiments at the Large Hadron

Collider at CERN are being used to probe physics beyond the Standard Model.

The second plenary talk was given by Dr Silke Weinfurtner of Nottingham University and titled Black hole physics in the laboratory: Rotational superradiant scattering in a vortex flow. She spoke about her research into analogue gravity systems, in particular the ongoing work at her Quantum Gravity Laboratory into modelling black holes using experimental water vortex systems.

The conference was supported by an LMS Postgraduate Research Conference grant (Scheme 8); the Institute of Physics; the Scottish Universities Physics Alliance; the Durham Centre for Academic, Researcher and Organisation Development; and the Durham University Institute for Particle Physics Phenomenology.

Maciej Matuszewski Durham University

## Report: Mary Cartwright Lecture

In the bleak midwinter, also known as Friday 2 March, the 2018 Mary Cartwright Lecture was held at De Morgan House, London. The Mary Cartwright Lecture is an annual Women in Mathematics event organised by the London Mathematical Society and forms part of the annual programme of Society Meetings. Despite the snow lying round about and the icy winds making moan, a lively group attended the meeting and talks.

Professor Caroline Series, President of the LMS, opened the meeting and invited two new members in attendance to sign the Members' Book, then welcomed them to the Society. Dr Eugenie Hunsicker, Chair of the LMS Women in Mathematics Committee, then spoke briefly about Mary Cartwright before introducing the first speaker (all the way from sunny Los Angeles, but excited to have an occasion to wear her new warm boots), Professor Andrea Bertozzi. Professor Bertozzi is the Betsy Wood Knapp Chair for Innovation and Creativity and Director of Applied Mathematics from the University of California, Los Angeles. She gave the opening lecture, Geometric graph-based methods for high dimensional data.

Professor Bertozzi introduced her talk by speaking about how brief discussions can lead to interesting and fruitful collaborations. Such a discussion with

Arjuna Flenner of China Lake was the inspiration for the research she spoke about, which uses methods from PDE to solve problems in machine learning. She then showed us a picture of some farm animals to introduce this work, which applies methods related to the Graph Laplacian to segmentation of high dimensional data, including image data. The main idea of this work is to categorise points in a dataset into two groups using a subset of labelled data by solving the graph cut problem to a weighted proximity graph formed by the data. The graph cut problem asks for a "minimal" set of edges to cut in order to break the graph into two pieces containing the marked points for the two groups of labelled data. Professor Bertozzi talked about how this problem can be solved using ideas from analysis on metric graphs that are analogous to motion by mean curvature in Euclidean space. She then discussed how these ideas can be used not only to find cows in pictures, but also to analyse videos from police body cams or infrared footage of gas plumes.

The opening lecture was followed by a break with tea and coffee which gave the opportunity for discussions among participants.

After the break, Dr Hunsicker introduced this year's Mary Cartwright Lecturer, Dr Carola BibianeSchönlieb, who is a Reader in Applied and Computational Mathematics at the University of Cambridge, the head of the Cambridge Image Analysis (CIA) group (suitable pictures of the international woman of mystery and her crew were shown) and Director of the Cantab Capital Institute for the Mathematics of Information. Dr Bibiane-Schönlieb has been awarded an LMS Whitehead Prize and a Philip Leverhulme Prize. She is also active in women in mathematics work, and is the Convenor of the European Women in Mathematics network. Her talk was titled Model-based learning in imaging.

In her talk, Dr Bibiane-Schönlieb showed how mathematics can make you fly, or at least permit you to do a better job restoring old master paintings than the earnest but painfully bad failed Ecce Homo "restoration" of internet fame. These two examples both relate to methods for manipulating images, including segmentation, denoising and infilling. Mathematical inverse problem methods for image manipulation use models, called variational models, that minimise some cost function. Variational models give a mathematically rigorous approach with stability and approximation guarantees as well as a control on qualitative and physical properties of the solution. After introducing the mathematics of these models, Dr

Bibiane-Schönlieb spoke about her work to combine these models with flexible approaches from machine learning using methods of bilevel optimisation and quotient minimisation. The goal of this work is to create methods that learn the optimal parameters in the cost function to employ with a given dataset. She discussed existence results for optimal parameters as well as results about convergence for numerical schemes to obtain these, before concluding with a philosophical overview of mathematics and machine learning, and a call for further work to combine these
approaches, which have complementary strengths and weaknesses.

After the Mary Cartwright Lecture there was a wine reception at De Morgan House which was followed by a dinner at the Montague Hotel, after which Dr. Hunsicker was able to make off with a large number of leftover shortbread cookies and macaroons.

Eugenie Hunsicker
Loughborough University

## Records and Proceedings at LMS meetings Mary Cartwright Lecture and Society Meeting, 2 March 2018

Approximately 30 members and visitors were present for all or part of the meeting.
The meeting began at 3.30 pm with the President, Professor Caroline Series FRS, in the Chair.
23 members were elected to Ordinary membership: Mrs Jehan Alswaihli, Professor Christopher Beem, Dr Dmitry Belyaev, Dr Siri Chongchitnan, Dr Chunrong Feng, Professor Alain Goriely, Dr John Harvey, Mr Renaud Lambiotte, Dr Christie Marr, Dr Alexandre Martin, Mr Carlo Mercuri, Professor Michael Monoyios, Professor Karen Page, Dr Christopher Penrose, Dr David Proemel, Dr Inayatur Rehman, Professor Endre Süli, Dr Paul Shafer, Dr Raphael Stuhlmeier, Dr Priya Subramanian, Professor Bálint Tóth, Dr lan Vernon and Dr Alex Watson.
17 members were elected to Associate membership: Mr Joe Allen, Mr Antoine Beraud, Mr Stuart Burell, Mr Mriganka Shekhar Chaki, Dr Radu Cimpeanu, Mr Tee-Jay Dack, Ms Pratiksha Devshali, Mr Rhys Evans, Mr Azhir Mahmood, Mr Lee McManus, Mr Samuel Ogunjo, Miss Christina Pospisil, Mr Leonardo Ripoli, Ms Bopitiye Indunil Niroshini, Ms Jayarathna Sikurajapathi, Ms Brigitte Stenhouse and Mr Ross Strachan. Six members were elected to Associate membership for Teacher Training Scholars: Mrs Ayan Ali, Miss Amanda Beddows, Mr Samuel Clark, Ms Sabah Malik, Ms Joanna Reeve and Miss Chloe Williams.
Four members were elected to Reciprocity membership: Mr Kai Bollmann, Mr K.G. Sreekumar, Dr Henrik Rasmussen and Dr Dominic Thorrington.
Two members signed the Members' Book and were admitted to the Society.
Dr Eugenie Hunsicker introduced a lecture given by Professor Andrea Bertozzi (UCLA) on Geometric graph-based methods for high dimensional data.
After tea, Dr Hunsicker introduced the Mary Cartwright Lecture given by Dr Carola Bibiane-Schönlieb (University of Cambridge) on Model-based learning in imaging.
The President, Professor Series, expressed the thanks of the Society to the Women in Mathematics Committee for putting on such a successful meeting. Afterwards, a reception was held at De Morgan House, followed by dinner hosted at the Montague on the Gardens Hotel.

## CORRECTIONS AND CLARIFICATIONS

In the March issue (475) of the Newsletter, the image on the lower-right of page 25 had an incorrect caption. The photo shows an Algebraic Geometry

Summer School in Mombasa in 2013. The caption has been changed in the online edition of the Newsletter.

# From the Icosahedron to $\mathrm{E}_{8}$ 

JOHN C. BAEZ

The regular icosahedron is connected to many exceptional objects in mathematics. Here we describe two constructions of the $\mathrm{E}_{8}$ lattice from the icosahedron. One uses a subring of the quaternions called the "icosians", while the other uses du Val's work on the resolution of Kleinian singularities. Together they link the golden ratio, the quaternions, the quintic equation, the 600-cell, and the Poincaré homology 3-sphere. We leave it as a challenge to the reader to find the connection between these two constructions.

In mathematics, every sufficiently beautiful object is connected to all others. Many exciting adventures, of various levels of difficulty, can be had by following these connections. Take, for example, the icosahedron - that is, the regular icosahedron, one of the five Platonic solids. Starting from this it is just a hop, skip and a jump to the $\mathrm{E}_{8}$ lattice, a wonderful pattern of points in 8 dimensions! As we explore this connection we shall see that it also ties together many other remarkable entities: the golden ratio, the quaternions, the quintic equation, a highly symmetrical 4-dimensional shape called the 600-cell, and a manifold called the Poincaré homology 3-sphere.

Indeed, the main problem with these adventures is knowing where to stop. The story we shall tell is just a snippet of a longer one involving the McKay correspondence and quiver representations. It would be easy to bring in the octonions, exceptional Lie groups, and more. But it can be enjoyed without these digressions, so let us introduce the protagonists without further ado.

The icosahedron has a long history. According to a comment in Euclid's Elements it was discovered by Plato's friend Theaetetus, a geometer who lived from roughly 415 to 369 BC. Since Theaetetus is believed to have classified the Platonic solids, he may have found the icosahedron as part of this project. If so, it is one of the earliest mathematical objects discovered as part of a classification theorem. In any event, it was known to Plato: in his Timaeus, he argued that water comes in atoms of this shape.

The icosahedron has 20 triangular faces, 30 edges, and 12 vertices. We can take the vertices to be the four points

$$
(0, \pm 1, \pm \Phi)
$$

and all those obtained from these by cyclic permutations of the coordinates, where

$$
\Phi=\frac{\sqrt{5}+1}{2}
$$

is the golden ratio. Thus, we can group the vertices into three orthogonal "golden rectangles": rectangles whose proportions are $\Phi$ to 1 .


Icosahedron with three golden rectangles

In fact, there are five ways to do this. The rotational symmetries of the icosahedron permute these five ways, and any nontrivial rotation gives a nontrivial permutation. The rotational symmetry group of the icosahedron is thus a subgroup of $\mathrm{S}_{5}$. Moreover, this subgroup has 60 elements. After all, any rotation is determined by what it does to a chosen face of the icosahedron: it can map this face to any of the 20 faces, and it can do so in 3 ways. The rotational symmetry group of the icosahedron is thus a 60-element subgroup of $\mathrm{S}_{5}$. Group theory therefore tells us that it must be the alternating group $\mathrm{A}_{5}$.

The $\mathrm{E}_{8}$ lattice is harder to visualize than the icosahedron, but still easy to characterize. Take a bunch of equal-sized spheres in 8 dimensions. Get as many of these spheres to touch a single sphere as you possibly can. Then, get as many to touch those spheres as
you possibly can, and so on. Unlike in 3 dimensions, where there is "wiggle room", you have no choice about how to proceed, except for an overall rotation and translation. The balls will inevitably be centered at points of the $\mathrm{E}_{8}$ lattice!

We can also characterize the $\mathrm{E}_{8}$ lattice as the one giving the densest packing of spheres among all lattices in 8 dimensions. This packing was long suspected to be optimal even among those that do not arise from lattices - but this fact was proved only in 2016, by Maryna Viazovska [12].

We can also describe the $\mathrm{E}_{8}$ lattice more explicitly. In suitable coordinates, it consists of vectors for which:
(1) the components are either all integers or all integers plus $\frac{1}{2}$, and
(2) the components sum to an even number.

This lattice consists of all integral linear combinations of the 8 rows of this matrix:

$$
\left(\begin{array}{rrrrrrrr}
1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
-\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2} & -\frac{1}{2}
\end{array}\right)
$$

The inner product of any row vector with itself is 2 , while the inner product of distinct row vectors is either 0 or -1 . Thus, any two of these vectors lie at an angle of either $90^{\circ}$ or $120^{\circ}$. If we draw a dot for each vector, and connect two dots by an edge when the angle between their vectors is $120^{\circ}$, we get this pattern:


This is called the $\mathrm{E}_{8}$ Dynkin diagram. In the first part of our story we shall find the $\mathrm{E}_{8}$ lattice hiding in the icosahedron; in the second part, we shall find this diagram. The two parts of this story must be related - but the relation remains mysterious, at least to this author.

## The icosians

The quickest route from the icosahedron to $\mathrm{E}_{8}$ goes through the fourth dimension. The symmetries of the icosahedron can be described using certain quaternions; the integer linear combinations of these form a subring of the quaternions called the "icosians", but the icosians can be reinterpreted as a lattice in 8 dimensions, and this is the $\mathrm{E}_{8}$ lattice. Let us see how this works, following the treatment in Conway and Sloane's Sphere Packings, Lattices and Groups [1, 2].

The quaternions, discovered by Hamilton, are a 4dimensional algebra

$$
\mathbb{H}=\{a+b i+c j+d k: a, b, c, d \in \mathbb{R}\}
$$

with multiplication given as follows:

$$
i^{2}=j^{2}=k^{2}=-1
$$

$$
i j=k=-j i \text { and cyclic permutations. }
$$

It is a normed division algebra, meaning that the norm

$$
|a+b i+c j+d k|=\sqrt{a^{2}+b^{2}+c^{2}+d^{2}}
$$

obeys $\left|q q^{\prime}\right|=\left|q \| q^{\prime}\right|$ for all $q, q^{\prime} \in \mathbb{H}$. The unit sphere in $\mathbb{H}$, a 3 -sphere, is therefore a group. We shall sloppily call this group $\mathrm{SU}(2)$, since its elements can be identified with $2 \times 2$ unitary complex matrices with determinant 1. A more correct name for this group would be $\operatorname{Sp}(1)$. But whatever we call it, this group acts as rotations of 3-dimensional Euclidean space, since we can see any point in $\mathbb{R}^{3}$ as a "purely imaginary" quaternion $x=b i+c j+d k$, and $q x q^{-1}$ is then a purely imaginary quaternion of the same norm for any $q \in \mathrm{SU}(2)$. Indeed, this action gives a homomorphism

$$
\alpha: \mathrm{SU}(2) \rightarrow \mathrm{SO}(3)
$$

where $\mathrm{SO}(3)$ is the group of rotations of $\mathbb{R}^{3}$. This homomorphism is two-to-one, since $\pm 1 \in \mathrm{SU}(2)$ both act trivially. So, we say $\mathrm{SU}(2)$ is a "double cover" of the rotation group $\mathrm{SO}(3)$.

We can thus take any Platonic solid, look at its group of rotational symmetries, get a subgroup of $\mathrm{SO}(3)$, and take its double cover in $\mathrm{SU}(2)$. If we do this starting with the icosahedron, we see that the 60 -element group $\mathrm{A}_{5} \subset \mathrm{SO}(3)$ is covered by a 120 -element group $\Gamma \subset \mathrm{SU}(2)$, called the "binary icosahedral group". Of course the elements of $\Gamma$ lie on the unit sphere in the quaternions - but it turns out that they are the vertices of a 4-dimensional regular polytope: a

4-dimensional cousin of the Platonic solids! This polytope looks like a "hypericosahedron", but it is usually called the " 600 -cell", since it has 600 tetrahedral faces.


The 600-cell projected down to 3 dimensions, drawn using Robert Webb's Stella software

Explicitly, if we identify $\mathbb{H}$ with $\mathbb{R}^{4}$, the elements of $\Gamma$ are the points

$$
\left( \pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2}, \pm \frac{1}{2}\right), \quad( \pm 1,0,0,0), \quad \frac{1}{2}\left( \pm \Phi, \pm 1, \pm \frac{1}{\Phi}, 0\right)
$$

and those obtained from these by even permutations of the coordinates. Since these points are closed under multiplication, if we take integral linear combinations of them we get a subring of the quaternions:

$$
\mathbb{I}=\left\{\sum_{q \in \Gamma} a_{q} q: a_{q} \in \mathbb{Z}\right\} \subset \mathbb{H} .
$$

Conway and Sloane [1] call this the ring of "icosians". The icosians are not a lattice in the quaternions: they are dense. However, any icosian is of the form $a+b i+c j+d k$ where $a, b, c$, and $d$ live in the "golden field"

$$
\mathbb{Q}(\sqrt{5})=\{x+\sqrt{5} y: x, y \in \mathbb{Q}\} .
$$

Thus we can think of an icosian as an 8-tuple of rational numbers. Such 8 -tuples form a lattice in 8 dimensions.

In fact we can put a norm on the icosians as follows. For $q \in \mathbb{\square}$ the usual quaternionic norm has

$$
|q|^{2}=x+\sqrt{5} y
$$

for some rational numbers $x$ and $y$, but we can define a new norm on $\mathbb{\square}$ by setting

$$
\|q\|^{2}=x+y .
$$

With respect to this new norm, the icosians form a lattice that fits isometrically in 8-dimensional Euclidean space. This is none other than $\mathrm{E}_{8}$ !

## Klein's icosahedral function

Not only is the $\mathrm{E}_{8}$ lattice hiding in the icosahedron; so is the $\mathrm{E}_{8}$ Dynkin diagram. The space of all regular icosahedra of arbitrary size centered at the origin has a singularity, which corresponds to a degenerate special case: the icosahedron of zero size. If we resolve this singularity in a minimal way we get eight Riemann spheres, intersecting in a pattern described by the $\mathrm{E}_{8}$ Dynkin diagram!

This part of our story starts around 1884, with Felix Klein's Lectures on the Icosahedron [6]. In this work he inscribed an icosahedron in the Riemann sphere, $\mathbb{C} \mathrm{P}^{1}$. He thus got the icosahedron's symmetry group, $\mathrm{A}_{5}$, to act as conformal transformations of $\mathbb{C} \mathrm{P}^{1}$ indeed, rotations. He then found a rational function of one complex variable that is invariant under all these transformations. This function equals 0 at the centres of the icosahedron's faces, 1 at the midpoints of its edges, and $\infty$ at its vertices.


Klein's icosahedral function drawn by Abdelaziz Nait Merzouk. The color shows its phase, while the contour lines show its magnitude

We can think of Klein's icosahedral function as a branched cover of the Riemann sphere by itself with 60 sheets:

$$
\mathcal{J}: \mathbb{C} \mathrm{P}^{1} \rightarrow \mathbb{C} \mathrm{P}^{1}
$$

Indeed, $\mathrm{A}_{5}$ acts on $\mathbb{C} \mathrm{P}^{1}$, and the quotient space $\mathbb{C} \mathrm{P}^{1} / \mathrm{A}_{5}$ is isomorphic to $\mathbb{C} \mathrm{P}^{1}$ again. The function J gives an explicit formula for the quotient map $\mathbb{C} \mathrm{P}^{1} \rightarrow \mathbb{C} \mathrm{P}^{1} / \mathrm{A}_{5} \cong \mathbb{C} \mathrm{P}^{1}$.

Klein managed to reduce solving the quintic to the problem of solving the equation $\mathcal{J}(z)=w$ for $z$. A modern exposition of this result is Shurman's Geometry of the Quintic [9]. For a more high-powered approach, see Nash [8]. Unfortunately, neither of these treatments avoids complicated calculations. But our interest in Klein's icosahedral function here does not come from its connection to the quintic: instead, we want to see its connection to $\mathrm{E}_{8}$.

For this we should actually construct Klein's icosahedral function. To do this, recall that the Riemann sphere $\mathbb{C} \mathrm{P}^{1}$ is the space of 1 -dimensional linear subspaces of $\mathbb{C}^{2}$. Let us work directly with $\mathbb{C}^{2}$. While $\mathrm{SO}(3)$ acts on $\mathbb{C} \mathrm{P}^{1}$, this comes from an action of this group's double cover $\mathrm{SU}(2)$ on $\mathbb{C}^{2}$. As we have seen, the rotational symmetry group of the icosahedron, $\mathrm{A}_{5} \subset \mathrm{SO}(3)$, is double covered by the binary icosahedral group $\Gamma \subset \mathrm{SU}(2)$. To build an $\mathrm{A}_{5}$-invariant rational function on $\mathbb{C} P^{1}$, we should thus look for $\Gamma$-invariant homogeneous polynomials on $\mathbb{C}^{2}$.

It is easy to construct three such polynomials:

- $V$, of degree 12 , vanishing on the 1d subspaces corresponding to icosahedron vertices.
- $E$, of degree 30 , vanishing on the 1d subspaces corresponding to icosahedron edge midpoints.
- $F$, of degree 20 , vanishing on the 1d subspaces corresponding to icosahedron face centres.

Remember, we have embedded the icosahedron in $\mathbb{C} \mathrm{P}^{1}$. Each point in $\mathbb{C} \mathrm{P}^{1}$ is a 1 -dimensional subspace of $\mathbb{C}^{2}$, so each icosahedron vertex determines such a subspace, and there is a nonzero linear function on $\mathbb{C}^{2}$, unique up to a constant factor, that vanishes on this subspace. The icosahedron has 12 vertices, so we get 12 linear functions this way. Multiplying them gives $V$, a homogeneous polynomial of degree 12 on $\mathbb{C}^{2}$ that vanishes on all the subspaces corresponding to icosahedron vertices. The same trick gives $E$, which has degree 30 because the icosahedron has 30 edges, and $F$, which has degree 20 because the icosahedron has 20 faces.


240 points closest to the origin in the $\mathrm{E}_{8}$ lattice, projected to a plane, drawn by J. Gregory Moxness

A bit of work is required to check that $V, E$ and $F$ are invariant under $\Gamma$, instead of changing by constant factors under group transformations. Indeed, if we had copied this construction using a tetrahedron or octahedron, this would not be the case. For details, see Shurman's free book [9] or van Hoboken's nice thesis [11].
Since both $F^{3}$ and $V^{5}$ have degree $60, F^{3} / V^{5}$ is homogeneous of degree zero, so it defines a rational function $\mathcal{J}: \mathbb{C} P^{1} \rightarrow \mathbb{C} P^{1}$. This function is invariant under $\mathrm{A}_{5}$ because $F$ and $V$ are invariant under $\Gamma$. Since $F$ vanishes at face centres of the icosahedron while $V$ vanishes at vertices, $\mathcal{J}=F^{3} / V^{5}$ equals 0 at face centres and $\infty$ at vertices. Finally, thanks to its invariance property, J takes the same value at every edge center, so we can normalize $V$ or $F$ to make this value 1 .

Thus, $\mathcal{J}$ has precisely the properties required of Klein's icosahedral function! And indeed, these properties uniquely characterize that function, so that function is $J$.

## The appearance of $\mathbf{E}_{\mathbf{8}}$

Now comes the really interesting part. Three polynomials on a 2-dimensional space must obey a relation, and $V, E$, and $F$ obey a very pretty one, at least after we normalize them correctly:

$$
V^{5}+E^{2}+F^{3}=0
$$

We could guess this relation simply by noting that each term must have the same degree. Every $\Gamma$ invariant polynomial on $\mathbb{C}^{2}$ is a polynomial in $V, E$ and $F$, and indeed

$$
\mathbb{C}^{2} / \Gamma \cong\left\{(V, E, F) \in \mathbb{C}^{3}: V^{5}+E^{2}+F^{3}=0\right\}
$$

This complex surface is smooth except at $V=E=$ $F=0$, where it has a singularity. And hiding in this singularity is $\mathrm{E}_{8}$ !

To see this, we need to "resolve" the singularity. Roughly, this means that we find a smooth complex surface $S$ and an onto map

$$
\begin{gathered}
\stackrel{S}{\downarrow} \begin{array}{c}
\| \\
\mathbb{C}^{2} / \Gamma
\end{array} .
\end{gathered}
$$

that is one-to-one away from the singularity'. There are many ways to do this, but one "minimal" resolution, meaning that all others factor uniquely through this one is:


What sits above the singularity in this minimal resolution? Eight copies of the Riemann sphere $\mathbb{C}{ }^{1}$, one for each dot here:


Two of these $\mathbb{C} P^{1}$ s intersect in a point if their dots are connected by an edge: otherwise they are disjoint.

This amazing fact was discovered by Patrick du Val in 1934 [3]. Why is it true? Alas, there is not enough room in the margin, or even the entire page, for a full explanation. The books by Kirillov [5] and Lamotke [7] fill in the details. But here is a clue. The $\mathrm{E}_{8}$ Dynkin diagram has "legs" of lengths 5,2 and 3 :


On the other hand,

$$
\mathrm{A}_{5} \cong\left\langle v, e, f \mid v^{5}=e^{2}=f^{3}=v e f=1\right\rangle
$$

where in terms of the icosahedon's symmetries:

- $v$ is a $1 / 5$ turn around some vertex of the icosahedron,
- $e$ is a $1 / 2$ turn around the center of an edge touching that vertex,
- $f$ is a $1 / 3$ turn around the center of a face touching that vertex,
and we must choose the sense of these rotations correctly to obtain vef $=1$. To get a presentation of the binary icosahedral group we drop one relation:

$$
\Gamma \cong\left\langle v, e, f \mid v^{5}=e^{2}=f^{3}=v e f\right\rangle .
$$

The dots in the $\mathrm{E}_{8}$ Dynkin diagram correspond naturally to conjugacy classes in $\Gamma$, not counting the conjugacy class of the central element -1 . Each of these conjugacy classes, in turn, gives a copy of $\mathbb{C} \mathrm{P}^{1}$ in the minimal resolution of $\mathbb{C}^{2} / \Gamma$.

Not only the $\mathrm{E}_{8}$ Dynkin diagram, but also the $\mathrm{E}_{8}$ lattice, can be found in the minimal resolution of $\mathbb{C}^{2} / \Gamma$. Topologically, this space is a 4-dimensional manifold. Its real second homology group is an 8 -dimensional vector space with an inner product given by the intersection pairing. The integral second homology is a lattice in this vector space spanned by the 8 copies of $\mathbb{C} \mathrm{P}^{1}$ we have just seen - and it is a copy of the $\mathrm{E}_{8}$ lattice [4].

But let us turn to a more basic question: what is $\mathbb{C}^{2} / \Gamma$ like as a topological space? To tackle this, first note that we can identify a pair of complex numbers with a single quaternion, and this gives a homeomorphism

$$
\mathbb{C}^{2} / \Gamma \cong \mathbb{H} / \Gamma
$$

where we let $\Gamma$ act by right multiplication on $\mathbb{H}$. So, it suffices to understand $\mathbb{H} / \Gamma$.

Next, note that sitting inside $\mathbb{H} / \Gamma$ are the points coming from the unit sphere in $\mathbb{H}$. These points form the 3 -dimensional manifold $\mathrm{SU}(2) / \Gamma$, which is called the "Poincaré homology 3 -sphere" [4]. This is a wonderful thing in its own right: Poincaré discovered it as a counterexample to his guess that any compact 3 -manifold with the same homology as a 3 -sphere is actually diffeomorphic to the 3-sphere, and it is

[^0]deeply connected to $\mathrm{E}_{8}$. But for our purposes, what matters is that we can think of this manifold in another way, since we have a diffeomorphism
$$
\mathrm{SU}(2) / \Gamma \cong \mathrm{SO}(3) / \mathrm{A}_{5} .
$$

The latter is just the space of all icosahedra inscribed in the unit sphere in 3d space, where we count two as the same if they differ by a rotational symmetry.

This is a nice description of the points of $\mathbb{H} / \Gamma$ coming from points in the unit sphere of $\mathbb{H}$. But every quaternion lies in some sphere centred at the origin of $\mathbb{H}$, of possibly zero radius. It follows that $\mathbb{C}^{2} / \Gamma \cong \mathbb{H} / \Gamma$ is the space of all icosahedra centred at the origin of 3d space - of arbitrary size, including a degenerate icosahedron of zero size. This degenerate icosahedron is the singular point in $\mathbb{C}^{2} / \Gamma$. This is where $\mathrm{E}_{8}$ is hiding.

Clearly much has been left unexplained in this brief account. Most of the missing details can be found in the references. But it remains unknown - at least to this author - how the two constructions of $\mathrm{E}_{8}$ from the icosahedron fit together in a unified picture.

Recall what we did. First we took the binary icosahedral group $\Gamma \subset \mathbb{H}$, took integer linear combinations of its elements, thought of these as forming a lattice in an 8-dimensional rational vector space with a natural norm, and discovered that this lattice is a copy of the $\mathrm{E}_{8}$ lattice. Then we took $\mathbb{C}^{2} / \Gamma \cong \mathbb{H} / \Gamma$, took its minimal resolution, and found that the integral 2nd homology of this space, equipped with its natural inner product, is a copy of the $\mathrm{E}_{8}$ lattice. From the same ingredients we built the same lattice in two very different ways! How are these constructions connected? This puzzle deserves a nice solution.

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## John Baez

John Baez is a mathematical physicist who works at U. C. Riverside and also at the Centre for Quantum Technologies, which is in Singapore. He used to work on quantum gravity and $n$-categories, but now he studies the mathematics of networks that appear in natural and engineered systems. He blogs copiously on Azimuth and The n-Category Café.

# Emmy Noether: "The Experiment to Promote a Woman to a Full Professorship" 

REINHARD SIEGMUND-SCHULTZE

The article, which is a translation of [11], investigates the arguments used by Noether's colleagues Adolf Fraenkel and Helmut Hasse to rule out Noether's possible appointment to Kiel in 1928. Apart from partly justified but nevertheless hypocritical doubts about Noether's suitability for elementary teaching, one finds a feeling of inferiority from Fraenkel and a general resentment against women in academia from Hasse.

## Preliminary remarks

It is October 1928 and Ernst Steinitz (1871-1928), the founder of the modern theory of abstract fields has just died in Kiel, in Northern Germany. Adolf (Abraham Halevi) Fraenkel (1891-1965), Steinitz's colleague at the University of Kiel until the latter's death, prepares in the name of the Philosophical Faculty (which included mathematics) a list of candidates for Steinitz's successor. This was the usual procedure for proposing a successor who was then as a rule appointed by the (Prussian) ministry in Berlin.

Emmy Noether (1882-1935), the founder of modern structural algebra, was mathematically a strong candidate. Her work belonged clearly to Steinitz's tradition. She occupied only an "extraordinary" professorship in Göttingen, basically a "Privatdozentur" with an additional meagre stipend.

In this article, which is based on unpublished archival material, I will discuss why Noether was briefly mentioned in the Faculty's proposal but did not, in the end, have any chance of an appointment. More precisely it is about why Fraenkel and Helmut Hasse (1898-1979), the former who felt the injustice which had been done to Noether in her career and the latter Noether's close collaborator and co-author, ${ }^{2}$ did not proactively support her appointment to Kiel.

Helmut Hasse, the creator of the local-global principle in number theory, was a close friend of Fraenkel, and the latter brought him into the appointment negotiations in Kiel. Hasse was at the time full professor in Halle. Both Hasse and


Emmy Noether, photographed in September 1930 by Helmut Hasse on the boat trip through the Baltic Sea to the DMV meeting in Königsberg (Archives Peter Roquette)

Fraenkel were students of Kurt Hensel (1861-1941) in Marburg. It is generally agreed, by both mathematicians and historians, that Noether and Hasse are among the most influential mathematicians of the 20th century. Both were invited to give plenary talks at the International Congresses of Mathematicians (ICM): Noether in 1932 (Zurich), Hasse in 1936 (Oslo). Due to his work in set theory, particularly the Zermelo-Fraenkel axioms, Fraenkel was probably as well known at the time as the other two. However, he was clearly less prolific and less influential, both with his own work and in the number and importance of his students.

[^1]

Left: Ernst Steinitz (Bildarchiv des Mathematischen Forschungsinstituts Oberwolfach/Archives Peter Roquette). Middle: Adolf Fraenkel (The David B. Keidan Collection of Digital Images from the Central Zionist Archives). Right: Helmut Hasse (standing) and Emil Artin (Bildarchiv Mathematisches Forschungsinstitut Oberwolfach/Archives Peter Roquette)

## "We are human beings first and mathematicians second"

In order to better understand Fraenkel's and Hasse's positions with regard to political questions, including issues regarding women, some remarks are necessary. In the case of Fraenkel, they are partly based on his autobiography from 1967, ${ }^{3}$ partly on his correspondence with Hasse. Coming from an orthodox Jewish home, Fraenkel - under the influence of the First World War - had become more and more open to Zionist thinking. Only the foundation of a Jewish homeland seemed to him to offer remedy for the sufferings of many of his Jewish contemporaries in Europe, some of whom even lacked full rights as citizens.

Fraenkel's social empathy with the underprivileged, which set him apart from many of his professorial colleagues in the Weimar Republic, is palpable also in his correspondence with Hasse. When Fraenkel in a time of mass unemployment in Germany, during the emergency decrees (Notverordnungen) and cuts in salaries even for professors - was invited to Marburg ( 500 km from Kiel) to celebrate his teacher

Hensel's 70th birthday, the father of four wrote to Hasse on 13 December 1931:
"I simply cannot make it. The salary is now lower here than when I was appointed. I say this only to explain my absence, because in general terms, we are still much better off than most of the people." ${ }^{4}$

Several passages in the correspondence with Hasse reveal that Fraenkel did not share the extremely nationalistic feelings that were typical of many German professors after the the war who were opposed to the stipulations of the Versailles treaty. A Nazi questionnaire, which Fraenkel had to complete before he was finally dismissed in Kiel in 1933, shows that he temporarily belonged to the "Republican Teachers' Association" (Republikanischer Lehrerbund), an organization despised by the Nazis. ${ }^{5}$ In connection with the so-called Bologna affair concerning the participation of German mathematicians at the ICM in Bologna 1928, Fraenkel wrote to Hasse on 14 June 1928:
"Re Bologna I do not completely share your opinion, although I am impressed by your arguments, partly influenced by them and grateful for them. Originally I was exactly of the same opinion. Meanwhile, however,

[^2]I have been impressed by an as usual brilliant plea on the part of Hilbert. He told me in detail why he deems - in spite of his opposition to the Entente and to pacifist ideas - his own participation in Bologna in a leading role the right thing, and that he asks the younger not to leave the old guard alone in this respect. Nevertheless, your main argument 'We are human beings first and mathematicians second' I fully support and I use it often as my own." ${ }^{6}$

I leave it undiscussed whether Fraenkel for tactical reasons somewhat distorted Hilbert's utterances or whether Hilbert actually paid tribute to the overwhelming anti-Versailles emotions among German professors and students. The latter would surprise the Hilbert scholar. In any case, Hasse's indirect quote is remarkable, as it seems to express that Hasse felt he could not go to Bologna in 1928 as a decent "human being" even though mathematical arguments may have told in favour of going. Although by 1928 the boycott of the Western allies against German science had long begun to wane, Hasse seemed to feel that Germany needed full international recognition and equal rights in all respects before normal scientific contact was possible. The 37 -year-old Fraenkel writes in the same letter from 14 June 1928 to the not yet 30 -year-old Hasse that he felt "attracted to you with admiration due to your rare combination of human being and researcher." Such confession may strike us as unusual today; it seems to convey something of the personal charm and charisma which Hasse must have possessed and radiated. This might have caused Fraenkel to downplay their political differences.

Hasse was shaped by the war too, though differently from Fraenkel. ${ }^{7}$ He had volunteered for the navy in 1914, and politically he was at the extreme right end of the spectrum of the Weimar Republic. Hasse described his position best in an interview with Constance Reid, when, in a sense, he located himself politically to the right of the Nazi Party:
"My political feelings have never been NationalSocialistic but rather 'national' in the sense of the

Deutschnationale Partei, which succeeded the Conservative Party of the Second Empire [under Wilhelm III. I had strong feelings for Germany as it was created by Bismarck in 1871. When this was heavily damaged by the Treaty of Versailles in 1919, I resented that very much. I approved with all my heart and soul of Hitler's endeavors to remove the injustices done to Germany in that treaty." ${ }^{8}$

It is this political and human perspective which helps us better understand the discussions about the succession to Steinitz in 1928 Kiel, to which I now return.

## The Kiel list for Steinitz's successor

Fraenkel wanted Hasse - whom he admired mathematically and who was his good friend despite some political differences of opinion - to win the succession. He sent a 7-page letter to him on 8 October 1928 with proposals for the list to which Hasse replied with 16 pages two days later. The two friends paid much attention to all the candidates but placed Hasse as first nominee in both the original list proposed by Fraenkel and the strongly revised one by Hasse. As a matter of fact, Hasse in his letter revealed that he was "intent to become Hensel's successor in Marburg, a prospect which in all probability will arise 1 April 1930."9 Apparently Hasse was hoping to improve his chances for the appointment in Marburg by being placed first on the list for Kiel. As it turned out, Hasse was appointed to Marburg earlier than expected in October 1929, after the Kiel list had been turned upside down by the ministry in Berlin. The applied mathematician Theodor Kaluza (1885-1954) was appointed to Kiel on 27 February 1929. ${ }^{10}$

Although Fraenkel had gained a good reputation as a mathematician due to his set theoretic research, he remained very self-critical and knew his limits, as transpires from his 1967 autobiography [2]. This is visible also in his proposals for the succession to Steinitz. He desired a pleasant future colleague who would not show his or her mathematical superiority and would tolerate his own, in some respects controversial, limited research area. It is in this light that one must see Fraenkel's resentment against

[^3]the possible appointment of Emmy Noether in Kiel. Unlike Fraenkel and Hasse, who were much younger, Noether had not acquired a permanent and remunerated position in the German university system. Fraenkel knew her work was closely related to the tradition of Steinitz's abstract theory of fields. The great Hermann Weyl (1885-1955), Hilbert's successor in Göttingen, said about Emmy Noether in his obituary of 1935:
"I was ashamed to occupy such a preferred position beside her whom I knew to be my superior as a mathematician in many respects." [13, p.208]

Fraenkel was concerned about fairness and objectivity, including for women. In his letter to Hasse from 8 October 1928, to which I was alerted by Mechthild Koreuber's recent and interesting book on Emmy Noether [4], Fraenkel says the following:
"3. Miss Noether. Here the situation is almost opposite. There is no doubt that as a man she would have received a call a long time ago and that she would be successful in Kiel as a researcher, although she has but little talent for elementary teaching. Personally I imagine a collaboration with her quite unpleasant [unerquicklich]. This would not be relevant if utterances from herself (seriously meant?) and from others had not represented it as such ... that she much prefers her huge influence in Göttingen over a professorship at a small university with only few students. Is it justified - as I actually would prefer - not to put her on the list?"11

Fraenkel's remark "the situation is almost opposite" was related to the possibility of putting the logician Paul Bernays (1888-1977) on the list. Fraenkel deemed Bernays' appointment impossible (although "injustice has been done to him") because "he and I would together make a too one-sided representation of mathematics in Kiel." One can therefore conclude that Fraenkel was less concerned about the closeness of Noether's research to his own and did not object to the lack of an analyst in the faculty. Rather he was concerned about insufficient teaching for the beginners; the pedagogical aspects of mathematics always mattered to him [2]. It is not clear to what extent male prejudice influenced Fraenkel's choice of the word "unpleasant". It is probably an allusion to her personality as a somewhat eccentric and very
focused genius. There is no explicit discrimination against women in Fraenkel's letter. However, rather to the contrary, the admission that "as a man she would have received a call a long time ago" expresses some pangs of conscience.

But Fraenkel's bad conscience was quickly relieved by Hasse's reaction two days later. In another letter to Hasse, Fraenkel wrote on 14 October 1928:
"As to Miss Noether, your opinions, which I share at least with respect to big universities, encourage me to abstain with good conscience from her nomination." ${ }^{12}$

Koreuber says at this point that one has to speculate about Hasse's encouraging remarks because his letter is not available. Her position as the ombudsperson for women's rights [Frauenbeauftragte] at the Free University in Berlin enables Koreuber to make the following apt remark: "Analyses of today's appointment procedures show that quite often assumptions are being made about female scholars which may lead and do lead to rejections, without even asking the woman involved." [4, p.53]

This remark fits rather well to Fraenkel's letter, but one also has Hasse's reply of 10 October 1928 which is available in the Fraenkel Papers in the National Library in Jerusalem. There one can read the following opinions, which were probably shared by many of Hasse's colleagues during the Weimar Republic. Nevertheless they appear as shocking to us today, not least against the backdrop of Hasse's close collaboration with Noether:
"3. Miss Noether: I am astonished that you even seriously consider this possibility. [lch staune, daß Sie diese Möglichkeit im Ernst erwägen.] Although I regard her highly in scientific matters I deem her totally unfit to fill a regular teaching position, even less so at a small university like Kiel, where there is only one besides her who could improve what she has spoiled. She fits only in a university where she is never obliged to give main and introductory courses, i.e. on a big scale such as Göttingen etc. I am, in addition, of the opinion that one should not make the experiment to promote a woman to a full professorship at such a solid place as Kiel. This experiment should be tried on a bigger scale where an unsuccessful outcome would not do so much harm."13

[^4]

Extract from Hasse's letter to Fraenkel, Halle 10 October 1928 (courtesy National Library of Israel, Jerusalem, Fraenkel Archives, File correspondence Hasse-Fraenkel)

Neither Fraenkel or Hasse admit that Emmy Noether had had to overcome many obstacles in her academic career and that her long delayed habilitation (teaching permit) [4, 19ff], prevented her from systematically and gradually developing a teaching practice.

However, Hasse goes a decisive step further than Fraenkel by expressing resentment against women in general with his proposal to reserve "the experiment to promote a woman to full professorship" for bigger universities. Fraenkel's reply "at least with respect to big universities" should also rather be read as being related to the problem of introductory courses and the qualification "at least" maybe even interpreted as criticism of Hasse's general antifeminist position.

In light of Hasse's later behaviour, it is worth observing that Hasse's remarks do not contain anti-Semitic sentiment, which would of course be barely imaginable in a letter to the orthodox Jew Fraenkel. ${ }^{14}$ In a later letter to Fraenkel, dated 6 November 1928, Hasse showed his openness for male Jewish colleagues when he underlined the mathematical kinship between Steinitz and the well-known topologist Max Dehn (1878-1952), who was also a personal friend of Steinitz. Hasse brought Dehn into the discussion for the Kiel list: "As a successor to Steinitz I deem also a man as M. Dehn ideally suited." ${ }^{15}$

Fraenkel's reaction to this seems to indicate he was afraid of the sometimes scathing, polemical style of Dehn, or maybe it was expressing his uneasiness with the unbridled spirit of competition which the top-mathematician Dehn, who came - like Noether - from the Göttingen school, seemed to personify. Fraenkel wrote to Hasse on 15 November 1928:
"Dehn, whom you propose is not an option (even ignoring his age and the aversion here against having two Jews on the list). ${ }^{16} \ldots$ With his somewhat arrogant classification not only of mathematicians but also of mathematical research tendencies Dehn would not appeal to me as a very pleasant neighbour." ${ }^{17}$

On the same page of his letter Fraenkel revealed certain inferiority feelings in comparison to Hasse which could influence possible collaboration between them. He said "Research is not my strongest side" and he felt a "weakening of my memory."

In the same letter to Hasse, Fraenkel reported on the now completed list:
"In the preamble [to the list], which has caused much discussion ... Bernays and E. Noether have been mentioned expressly, but declared as unsuitable for Kiel."

In the official list of the Philosophical Faculty, which is kept at the Geheimes Staatsarchiv in Berlin, one reads:
"She [the Faculty] had to leave aside the names of two personalities who should be plausible in this context and who have been deserving full professorships for a long time. These are the non-tenured extraordinary professors at the University of Göttingen Dr. P. Bernays and Miss Dr. E. Noether. By unanimous vote of the Faculty both are not appropriate for the conditions here; the first because his research is too closely related to the research of the other mathematician here, the second because her exclusive focus on extraordinarily talented students

[^5]would constitute a danger to teaching at a smaller university such as ours." 18

One may safely assume - particularly in the light of Fraenkel's remarks about Dehn - that the "discussion" in the faculty was rather anti-Semitic and directed against mentioning Bernays and Noether as "deserving" candidates at all.

At least Fraenkel did manage to secure a mention of Bernays and Noether in the preamble to the list. Both would be dismissed five years later by the Nazis, along with Fraenkel himself. But unlike Fraenkel they would never, not even in exile, obtain academic positions commensurate with their abilities and merits.

## Final remarks

It is highly likely that other faculty commissions in the Weimar Republic "solved the Noether-problem" in the same way as the one in Kiel. It is undisputed that doubts about Noether's ability to teach elementary courses were widespread and partially justified. However, this example shows that male prejudices and anti-Semitic feelings (Kiel faculty) could play a role in the background. The traditional dominance of research and publications in the academic career system is well known, and this makes the emphasis on pedagogy problematic, even hypocritical, in the case of an outstanding scholar such as Noether. Proactive attitudes among male German professors for the involvement of women in academic teaching were rare, as is testified by the experiences of female academics of the time ([9], [12]).

It is therefore tempting to rephrase Fraenkel's and Hasse's credo in the following words:
"We are men first, and mathematicians second."

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## Reinhard SiegmundSchultze

Reinhard SiegmundSchultze is a professor in history of mathematics at the University of Agder, Kristiansand, Norway. His main research interests are the history of applied mathematics and probability around 1900, and the biography of Richard von Mises. He spends much of his spare time singing (bass) in several choirs, swimming and playing table tennis.

# The Attraction of Dynamical Systems 

HOLLY KRIEGER
We discuss the notion of attraction for a complex dynamical system, and how the properties of local attractors generalize to the $p$-adic setting. Inspired by the work of Benedetto, Ingram, Jones, and Levy, we describe how these properties lead to an understanding of special dynamical systems known as post-critically finite maps.

## Newton's method

Your first meeting with attraction in a complex dynamical system was almost certainly in a calculus course: remember Newton's method? To the credulous calculus student, this is a rather mysterious strategy to find the root of a polynomial $p(z)$. The method proceeds by taking an initial guess $z_{0}$ and defining a sequence

$$
z_{k+1}=z_{k}-\frac{p\left(z_{k}\right)}{p^{\prime}\left(z_{k}\right)}
$$

and hoping that after a few iterations the values $z_{k}$ become very close to a root of $p(z)$. The usual pedagogical justification for this hope seems to be: Newton's method works a lot of the time, so hey if it works, great! If not, try something else. But what is Newton's method really doing?

We can rephrase the success of the method in the following way: if Newton's method works, then

$$
\lim _{k \rightarrow \infty} z_{k}=\alpha
$$

for some root $\alpha$ of $p(z)$. In dynamical terminology, Newton's method succeeds if $\alpha$ attracts $z_{0}$ under iteration of the rational function

$$
f(z):=z-\frac{p^{\prime}(z)}{p(z)},
$$

meaning that

$$
\lim _{n \rightarrow \infty} f^{\circ n}\left(z_{0}\right)=\alpha .
$$

## Complex attraction

If we apply Newton's method to a polynomial, then we become interested in the long-term behaviour under iteration of the function

$$
f(z)=z-\frac{p(z)}{p^{\prime}(z)} .
$$

This is a self-map of the Riemann sphere $\hat{\mathbb{C}}:=\mathbb{C} \cup$ $\{\infty\}$ which is given by a quotient of polynomials;
that is, a rational map $f(z) \in \mathbb{C}(z)$. The study of the iteration of such functions becomes interesting when the degree $d$ of the map - here $d$ is the maximal degree of the polynomials in the quotient - satisfies $d \geq 2$. In that case, some of the points of the Riemann sphere will be chaotic for the map, meaning that their long-term iterative behaviour is unstable under perturbation. In this setting, points with small derivative for the map become very important to the dynamics, because the map is generally expanding - after all, it must map the sphere to $d$ copies of itself! - and points with small derivative are locally contracting. This produces the tension that leads to chaotic dynamics.

A point $\alpha \in \hat{\mathbb{C}}$ satisfying $f(\alpha)=\alpha$ and $\left|f^{\prime}(\alpha)\right|<1$ is known as an attracting fixed point, so called because in this case there is a neighbourhood $A$ of $\alpha$ with the property that for all $z \in A, f^{\circ n}(z) \rightarrow \alpha$ as $n \rightarrow \infty$. That is, $\alpha$ attracts the nearby points under iteration. We can make a similar definition for any point satisfying $f^{\circ k}(\alpha)=\alpha$ for some $k \in \mathbb{N}$ (known as a periodic point) by passing to the iterate $f^{\circ k}(z)$. Returning to the example of Newton's method, notice that if $\alpha$ is a root of $p(z)$ but not of $p^{\prime}(z)$, then the rational map

$$
f(z):=z-\frac{p(z)}{p^{\prime}(z)}
$$

will satisfy $f(\alpha)=\alpha$ and $f^{\prime}(\alpha)=0$, so that $\alpha$ is an attracting fixed point for $f$. Newton's method hinges on the hope that the starting point $z_{0}$ chosen for the method is one of those attracted to $\alpha$; not an unreasonable hope in small degree, as shown in Figure 1; see McMullen [8] for a general discussion about the success of iterative root-finding methods.

One of the most interesting features of attracting cycles is that though they have a local definition, they have an impact on the global dynamics of the function, thanks to a theorem proved by both Fatou [4], [5] and Julia [6] stating that any attracting cycle must attract a critical point of the map; that is, a point where the map is not locally conformal. Since a degree $d$ map has $2 d-2$ critical points, this means that despite the fact that there are cycles of arbitrary length, only $2 d-2$ of those cycles can be attracting.


Figure 1. Attraction to the root 1 of $z^{3}-1$ in the complex plane: for all points coloured red (including most of the real axis), Newton's method for $z^{3}-1$ will converge to 1 . Iteration of a sample starting point is shown in white. (Image created using FractalStream)

## $p$-adic attraction

The concepts of attraction and small derivative use the standard notion of distance on the plane, which is the absolute value given by the complex magnitude. Sometimes, though, a rational map is defined over a field with other notions of distance, such as the rational numbers $\mathbb{Q}$. Over $\mathbb{Q}$, we have for each prime number $p$ a $p$-adic valuation: if $a \in \mathbb{Z}$ is not zero, define the $p$-adic valuation $v_{p}(a)$ to be the exponent of $p$ in the prime factorization of $a$, and extend $v_{p}$ to $\mathbb{Q}$ by

$$
v_{p}(a / b):=v_{p}(a)-v_{p}(b)
$$

setting $v_{p}(0)=\infty$. We can then define the $p$-adic norm $|\cdot|_{p}$ on $\mathbb{Q}$ by

$$
\left|\frac{a}{b}\right|_{p}:=p^{-v_{p}(a / b)}
$$

Thus two rational numbers are $p$-adically close if a large power of $p$ divides their difference, giving us a new notion of distance on $\mathbb{Q}$. For any prime $p$ this norm is in fact non-archimedean, as it satisfies the non-archimedean triangle inequality

$$
|x+y|_{p} \leq \max \left\{|x|_{p},|y|_{p}\right\}
$$

In general this inequality - which is stronger than the standard triangle inequality - makes computation easier in the $p$-adic world than in the usual world of archimedean norms. For example, in the $p$-adic setting a series converges if and only if the terms of the series $p$-adically converge to 0 . Just as in the case of the usual absolute value, the algebraic closure of $\mathbb{Q}$ has a completion with respect to the $p$-adic norm, denoted $\mathbb{C}_{p}$.

Given that we now have for each prime $p$ a new notion of distance on $\mathbb{Q}$, we can ask to what extent they give us information about iteration of maps for which they make sense; namely, for rational maps $f(z)$ which are quotients of polynomials with coefficients in $\mathbb{Q}$ (or in $\mathbb{C}_{p}$, if we wish to focus on a single prime). The notion of attraction obviously generalizes: a fixed point $\alpha \in \mathbb{Q}$ is said to be $p$-adically attracting if $\left|f^{\prime}(\alpha)\right|_{p}<1$; as before, we can extend this definition to cycles. Using extensions of the $p$-adic norm, we can also extend this notion to any algebraic $\alpha$ and so to all periodic points of $f$. But immediately we see that some very strange dynamical behaviour can occur, quite unlike the Fatou-Julia bound of $2 d-2$ attracting cycles in the complex setting. As an example, consider the map $f(z)=z^{p}$, which has the rather bizarre property that all cycles are $p$-adically attracting! Indeed, if $|\alpha|_{p}=1$ (as must be true in all non-zero cycles), the derivative $\left(f^{\circ n}\right)^{\prime}(\alpha)=p^{n} \alpha^{p^{n}-1}$ satisfies

$$
\left|\left(f^{\circ n}\right)^{\prime}(\alpha)\right|_{p}=\left|p^{n}\right|_{p}=\frac{1}{p^{n}}<1
$$

Fortunately, as we will see, there are limitations on when this pathological behaviour can occur in $p$-adic dynamics.

## Newton polygons

As in the $z^{p}$ example, to understand the possible attractive behaviour of a fixed point (or cycle), we first need to understand the $p$-adic norm of the fixed point. In the case when $f(z) \in \mathbb{Q}[z]$ is a polynomial, any fixed point will be a root of the polynomial $f(z)-z$, so we can use a combinatorial construction known as a Newton polygon to determine the $p$-adic size of the fixed points of $f(z)$. In general, consider a polynomial $q(z)=a_{0}+a_{1} z+\cdots+a_{d} z^{d}$ with $a_{i} \in \mathbb{Q}$ and $a_{0} a_{d} \neq 0$. For $0 \leq i \leq d$, plot the points $\left(i, v_{p}\left(a_{i}\right)\right)$ in the plane. The Newton polygon of $q(z)$ is defined to be the lower convex hull of this set of points (see Figure 2 for an example). Note this is not a true polygon, as it is not a closed figure.

If $\left(x_{0}, y_{0}\right), \ldots,\left(x_{k}, y_{k}\right)$ are the vertices of the Newton polygon of $q(z)$ with $x_{0}<\cdots<x_{k}$, define

$$
m_{i}:=\frac{y_{i}-y_{i-1}}{x_{i}-x_{i-1}}
$$

to be the slope of the $i$ th line segment for $i=$ $1, \ldots, k$. The theorem of the Newton polygon asserts that in $\mathbb{C}_{p}$, there are precisely $x_{i}-x_{i-1}$ roots of $q(z)$ of valuation $-m_{i}$. This simple statement turns out to
have some beautiful applications; see for instance Baker [1], which gives an excellent exposition of the consequences of the theorem of Newton polygons to the truncated exponential (as in Figure 2 above). The Eisenstein criterion for irreducibility of polynomials over $\mathbb{Q}$ is also an immediate consequence of this theorem: if a polynomial of degree $d$ satisfies Eisenstein's criterion, then the Newton polygon consists of a single segment of slope $-1 / d$, so all roots have valuation $1 / d$, and so no product of less than $d$ of them can have integer valuation (and so possibly be rational).


Figure 2. The 2-adic Newton polygon of $1+x+\frac{x^{2}}{2!}+\frac{x^{3}}{3!}+$ $\frac{x^{4}}{4!}+\frac{x^{5}}{5!}+\frac{x^{6}}{6!}$

As an example, consider the dynamical function $f(z)=z^{p}$ we discussed earlier. A point $\alpha$ of period $n$ for $f$ will be a root of the polynomial $f^{\circ n}(z)-z=$ $z^{p^{n}}-z=z\left(z p^{p^{n}-1}-1\right)$, so if $\alpha \neq 0$ then $\alpha$ is a root of $q(z):=z^{p^{n}-1}-1$. The Newton polygon for this polynomial is simply the line segment of slope zero, since all nonzero coefficients of $q(z)$ have $p$-adic valuation zero. Thus the theorem of the Newton polygon immediately yields that all roots of $q$ are of valuation zero. This proves our assertion that $|\alpha|_{p}=1$ for all periodic points $\alpha$, and hence the infinitude of attracting cycles for $f(z)=z^{p}$.

## P -adic attraction

As we've seen, when the degree $d$ of the map $f(z)$ is equal to $p$, there can be 'false' attracting cycles, in the sense that though the derivative has $p$-adic norm less than 1 , the cycles do not attract any critical points of the map (of which there are only $2 d-2$ ). Since the possible attraction of a cycle is determined by the derivative of $f(z)$ along the cycle, one might imagine that similar issues can arise whenever $p \leq d$ (and they do!). But when $p>d$, the elegant tool of
the Newton polygon can be used to rule out such wild dynamical behaviour. More concretely, following an argument of Benedetto, Ingram, Jones, and Levy [2], we will see that if $f(z) \in \mathbb{Q}[z]$ is a polynomial of degree $d$ and $p$ is a prime satisfying $p>d$, then any fixed point must attract a critical point of $f$.

To prove this critical point attraction, we first make simplifying assumptions. If $\phi(z)$ is a linear polynomial, then conjugation by $\phi$ commutes with iteration of $f$ :

$$
\phi^{-1} \circ f^{\circ n} \circ \phi=\left(\phi^{-1} \circ f \circ \phi\right)^{\circ n}
$$

Thus notions like attraction and iteration are preserved by conjugation, and we can assume that the fixed point under consideration is 0 (by a translation conjugation), and that the minimum of $|c|_{p}$ for a nonzero critical point $c$ of $f(z)$ is equal to 1 (by a scaling conjugation). We can then write

$$
f(z)=a_{1} z+a_{2} z^{2}+\cdots+a_{d} z^{d}
$$

where $\left|a_{1}\right|_{p}<1$, so that in terms of the valuation, we have $v_{p}\left(a_{1}\right)>0$. Since $p>d$, the Newton polygon of the derivative $f^{\prime}(z)$ will be the same as that of $f(z) / z=a_{1}+a_{2} z+\cdots+a_{d} z^{d-1}$, since the derivative introduces new factors to the coefficients which are coprime to $p$, so of valuation 1 . Now, if any coefficient $a_{i}$ satisfies

$$
v_{p}\left(a_{i}\right)<v_{p}\left(a_{1}\right)
$$

then that Newton polygon will have a negative slope, and we would obtain a root of $f^{\prime}$ with positive valuation, contradicting our assumption that $|c|_{p} \geq 1$ for all critical points $c$ of $f$. Therefore all coefficients have valuation at least $v_{p}\left(a_{1}\right)>0$, so norm $\left|a_{i}\right|_{p}<$ $\left|a_{1}\right|_{p}<1$. Therefore by the non-archimedean triangle inequality, for any point $z$ with $|z|_{p} \leq 1$,

$$
\begin{aligned}
|f(z)|_{p} & \leq \max \left\{\left|a_{1} z\right|,\left|a_{2} z^{2}\right|, \ldots,\left|a_{d} z^{d}\right|\right\} \\
& \leq \max \left\{\left|a_{1}\right|_{p},\left|a_{2}\right|_{p}, \ldots,\left|a_{d}\right|_{p}\right\} \\
& \leq\left|a_{1}\right|_{p}
\end{aligned}
$$

Applying this repeatedly, we see that if $|z|_{p} \leq 1$ then

$$
\left|f^{\circ n}(z)\right|_{p} \rightarrow 0 \text { as } n \rightarrow \infty
$$

so 0 attracts all points $|z|_{p} \leq 1$, including the critical point satisfying $|c|_{p}=1$.

Actually, the attraction is strict for some critical point $c$ with $|c|_{p}=1$, meaning that $f^{\circ n}(c) \neq 0$ for all $n \in \mathbb{N}$. For if $f^{\circ n}(c)=0$ for some $n$, then we must have $n=$ 1: we showed above that $f(z)$ strictly decreases the $p$-adic norm for those $z$ with $|z|_{p}=1$, and because the Newton polygons of $f(z) / z$ and $f^{\prime}(z)$ are the same, the non-zero roots of $f$ all have $p$-adic norm at least 1 . But any critical point which is also a root
of $f$ must be a repeated root of $f$, and so the number of critical points that are roots of $p$-adic norm 1, counted with multiplicity, is always strictly less than the number of roots of $p$-adic norm 1 , also counted with multiplicity. Thus we have at least one critical point of norm 1 which is not a root of $f$.

Thanks to this counting argument, we see that when $p>d$, a $p$-adically attracting cycle really does attract a critical point, and we have the same restriction on the number of possible $p$-adically attracting cycles as on cycles which are attracting in the usual complex norm!

In fact, Benedetto, Ingram, Jones, and Levy proved something much stronger, showing that even with $p \leq d$, any sufficiently attracting cycle will strictly attract a critical point, and they were able to show this for all rational maps, not just polynomials. In this case, the preimage of the maximal disk not containing critical points is more complicated, and so their argument relies on more advanced $p$-adic analytic techniques, particularly the induced dynamics on the Berkovich line.

## Post-critically finite maps

The result on the $p$-adic attraction of critical points is satisfying in its own right, but Benedetto, Ingram, Jones, and Levy used it to prove a remarkable theorem. Among all rational maps of a given degree $d$, there is a special subset of post-critically finite maps, which are those maps where all critical points have finite forward orbit. These maps have additional structure - for example, by the attraction of critical points we've discussed, there can be no attracting cycles except those containing a critical point, as in the example of $f(z)=z^{2}$. To an extent, these maps can be described simply by understanding the combinatorial information of the orbits of the critical points. Indeed, a beautiful result of Thurston [3] implies that apart from a well-understood family of post-critically finite maps associated to elliptic curve multiplication known as the Lattès maps, there are only finitely many conjugacy classes of rational maps realizing a given combinatorial critical orbit portrait. It immediately follows that there are only countably many (non-Lattès) post-critically finite maps; moreover, they all are defined over $\overline{\mathbb{Q}}$, meaning that any post-critically finite map is conjugate to one with algebraic coefficients.

It is not immediately clear, though, just how large the collection of non-Lattès post-critically finite maps really is in the set of maps with algebraic coefficients
(which is, after all, a countable set!!). Using the fact that a post-critically finite map cannot have very $p$-adically attracting cycles, together with the theory of heights and McMullen's results [8] on the geometry of the moduli space of rational maps of a given degree, they prove that there are only finitely many post-critically finite maps of a given degree defined over a given number field! In fact, they give an effective height bound for such maps, so that one could (as Lukas, Manes, and Yap [7] have done in degree 2) compute all non-Lattès post-critically finite maps of given degree over $\mathbb{Q}$, for example. This important work towards understanding the arithmetic geometry of families of dynamical systems is possible thanks to our improved understanding of non-archimedean dynamics and $p$-adic attraction.

## FURTHER READING

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## Holly Krieger

Holly Krieger is the Corfield Lecturer at the University of Cambridge. She studies the interaction between number theory and families of complex dynamical systems, and is well known for her appearances on the popular YouTube maths channel Numberphile.

# Reciprocal Societies: The Southeast Asian Mathematical Society 




#### Abstract

The Southeast Asian Mathematical Society (SEAMS) is a regional society composed of national mathematical societies in Southeast Asia. It seeks to promote the advancement of math-


 ematics and facilitate exchange and collaboration among mathematicians in the Southeast Asian region, and Asia in general. Its inaugural meeting was held in July 1972 in Singapore with Wong Yung Chow of the University of Hong Kong as founding president.The member-societies and their foundation dates are: Singapore Mathematical Society (founded 1952), Vietnam Mathematical Society (1966), Malaysian Mathematical Sciences Society (1970), Mathematical Society of the Philippines (1973), Indonesian Mathematical Society (1976), Mathematical Association of Thailand (1978), Hong Kong Mathematical Society (1979), Cambodian Mathematical Society (2005) and Mathematical Society of Myanmar (2013). In 2010, SEAMS welcomed the Nepal Mathematical Society as affiliate member. The SEAMS Council governs the Society. The presidents and representatives of the membersocieties comprise the Council.


SEAMS Council meeting in Bandung, Indonesia, in August 2017

The International Mathematical Union approved the affiliate membership status of SEAMS in the Union in 2010.

Supported by the mathematical community of the region, the Society's objectives involve advancing mathematical sciences in Southeast Asia and facilitating the exchange of information on current research work and teaching methods among mathematicians in the region. Since it was formed, SEAMS has played a key role in the region by stimulating the creation of national mathematical societies, supporting mathematical activities, and assisting in developing mathematical sciences.

SEAMS has held a number of regional conferences on mathematics and mathematics education over the years, including the Southeast Asian Conference on Mathematics Education (SEACME), which began in 1978 and was held every three years until 1999. SEACME has since merged with the East Asia Regional Conference on Mathematics Education (ICMIEARCOME).

A focal activity of SEAMS is the Asian Mathematical Conference (AMC) series, held every four or five years and hosted by countries in Asia. The latest AMC took place on 25-29 July 2016 in Bali, Indonesia with over 600 participants from 29 countries. The next AMC will be held in Vietnam in 2020.

SEAMS also conducts the SEAMS School Programme (seams.maths.web.id), a series of intensive study programs aimed at providing opportunities for advanced learning experiences in mathematics.

Since 1976, SEAMS has published the Southeast Asian Bulletin of Mathematics (seams-bullmath.ynu.edu.cn/), a bimonthly journal featuring research papers in all areas of mathematics. More information on the Society may be found in the SEAMS website seams-math.org/.

> José Maria P. Balmaceda President of the Southeast Asian Mathematical Society

Editor's note: LMS and SEAMS have a reciprocity agreement meaning members of either society may benefit from discounted membership of the other.

## LMS Hardy Lecture Tour 2018

The Hardy Lectureship, founded in 1967 in memory of G.H. Hardy in recognition of outstanding contributions to mathematics and the Society, is a lecture tour of the UK by a mathematician with a high reputation in research.

G.H. Hardy, LMS President 1926-1928 and 1939-194I Photo Courtesy of Master and Fellows of Trinity College Cambridge


Lauren Williams (UC Berkeley) Hardy Lecturer 2018

The 2018 LMS Hardy Fellow is Professor Lauren Williams (UC Berkeley). Professor Williams' research is highly interdisciplinary and has had a broad impact on several areas of mathematics, ranging from combinatorics and algebra to probability and mathematical physics, with applications to particle processes and shallow water waves. ProfessorWilliams delivers excellent talks in which she complements clarity with really advanced inspiring mathematics. As a down-to-earth and engaging speaker,Williams' lectures will illustrate how combinatorial tools can be very useful in algebra, probability, integrable systems, and mathematical physics.

Professor Williams will visit the following locations:

- 29 June ( 3.30 pm): BMA House,Tavistock Square London (organiser: LMS)
- 2 July, ( 4.30 pm): Oxford (organiser: Lionel Mason)
- 4 July: Cambridge (organiser: Mark Gross)
- 5 July: Kent (organisers: Clelia Pech, Stephane Launois)
- 9 July: Edinburgh (organiser: Milena Hering)
- IO July: Leeds (organiser: Robert Marsh)
- I I July: Birminham (organiser: Marta Mazzocco)

Contact the local organisers for further information on attending each lecture. For general enquiries about the Hardy Lectures, contact Elizabeth Fisher (Imsmeetings@Ims.ac.uk).

Microtheses and Nanotheses provide space in the Newsletter for current and recent research students to communicate their research findings with the community. We welcome submissions of micro and nanotheses from current and recent research students. See newsletter.Ims.ac.uk for preparation and submission guidance.

# Microthesis: Mathematicians in Early British Aeronautics through Contemporary Literature 

TONY ROYLE

The advent of fixed-wing, powered flying machines at the dawn of the 20th century set new challenges for mathematics. In response, a number of brave mathematicians took to the air to conduct experiments whilst others wrestled with the related theory on the ground. News of progress was delivered to an expectant public via an array of contemporary literature. Here I report on my findings on mathematicians in early British aeronautics viewed through the lens of contemporary literature.

## Engineering revolution

The turn of the twentieth century witnessed a revolution in humankind's attempts to master the air, and mathematics became an intrinsic part of a genre of engineering that had to be created to deal with the rather bespoke requirements of the new craft, much of its detail naturally evolving from known concepts in fields such as fluid dynamics and structures.

## E.T.Busk (1886-1914) and Stability



Edward Busk in the R.E.1, the first inherently stable aircraft - precursor of the B.E.2c, the workhorse of the Royal Flying Corps in WW1
An aircraft is considered 'stable' if it returns naturally to a state of equilibrium following a perturbation. Busk combined the work of Bangor University's G.H. Bryan [1], and his own flying skills to solve the problem in 1913.

## Flying mathematicians

Aeronautical research was centred at the Royal Aircraft Factory in Farnborough, and it would be here where some of Britain's finest mathematicians and scientists gathered during WW1 to develop the new field of aeronautical engineering. Frustrated by the use of test pilots bereft of formal mathematical training, a number of the academics, including Edward Teshmaker Busk, became test pilots or experimental observers, a move that certainly accelerated progress, but at a cost - a number would perish in flying accidents, Busk himself a casualty soon after solving the central issue of aircraft stability. I examined the impact these individuals had through study of their academic papers (collated in the Technical Reports of the Advisory Committee for Aeronautics, a body formed in 1909 to orchestrate aeronautical research in Britain), their personal archives (some, newly discovered), and their interactions with the general public via a large and varied selection of contemporary newspapers, journals, and magazines. These provided a source of record and reference, a forum for debate and exchange, and a medium where new ideas could be exposed to a wider audience to inform, update, and often attract or invite critical comment. Inevitably, the War dampened international exchange and caused much cutting-edge knowledge to be redacted, but there was still enough visible in the public domain to allow a rudimentary technical understanding and a tracking of progress.

## Women mathematicians

Structural failures in early aircraft were a persistent and commonplace problem. Poorly understood aerodynamic forces, particularly aeroelastic forces, meant that the design of early aircraft rarely compensated for the stresses they would experience in flight.


Hilda Hudson; Letitia Chitty; Beatrice Cave-Browne-Cave
In London during WW1, forces gathered in the Technical Section of the Admiralty with the aim of mitigating this scourge and minimizing the numbers of aircrew and airframes being lost. At its core were a select group of women mathematicians, Hilda Hudson, Letitia Chitty, and Beatrice Cave-Browne-Cave. In [4], I examined their individual roles and the pathways that brought them together. Their stories add a different dimension to the narrative, one that reveals as much about the contemporary issues in society in the early 1900s as it does about their mathematical prowess and performance under pressure.


Hudson's wartime work on stresses in laterally loaded aircraft wing struts is made public in 1920 in an edition of Aeronautical Engineering [2]

## Archival work

Research in History of Mathematics often includes time spent exploring existing archives or trying to discover new ones. Interesting archival material has already emerged during this project that gives deeper insight into the mathematical and practical contributions in aeronautics of characters such as David

Hume Pinsent, Keith Lucas, Roderick Hill, Albert Thurston, and the mathematicians at the Admiralty, and there is surely more out there that will inspire and direct our future research and interest.

## Hilda Hudson (1881-1965)

Hudson's obituary appeared in a Bulletin of the London Mathematical Society in 1969 [5]. It highlights her prowess in the field of Cremona transformations, and makes clear the strong mathematical influences present in her immediate family, all of whom attended Cambridge; Hilda herself was ranked equal to the 7th wrangler in 1903. She received an OBE for her work during the War, which included applying Castigliano's method of least strain energy to determine the strength of the wires required to brace aircraft wings [3], and a modification to extant analysis regarding the strength of struts under compression and a simultaneous lateral force.

## FURTHER READING

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[3] H.P. Hudson, Incidence Wires, Aeronautical Journal 24 (1920) 505-516.
[4] A.P. Royle, The impact of the women of the Technical Section of the Admiralty Air Department on the structural integrity of aircraft during World War One, Historia Mathematica 44 (2017) 342-366. [5] J.G. Semple, Hilda Phoebe Hudson, Bull. London Math. Soc. 3 (1969) 357-359.


## Tony Royle

Tony Royle is a PhD candidate in History of Mathematics at the Open University, with specific interests in aeronautics having previously enjoyed a 34-year career as a pilot with the Royal Air Force and Virgin Atlantic Airways. He enjoys competing in triathlons, singing tenor with the local choral society, and watching sport, particularly Rugby Union.

## Success Stories in Mathematics

What does it mean to be a successful mathematician? What is involved in a successful mathematical career? The LMS Success Stories project aims to celebrate the diversity of successful careers and mathematicians. We are always interested in new profiles! If you have an idea, or would like to submit your own profile, please email Success.Stories@lms.ac.uk.

Name: Neil McIvor
Job: (Acting) Chief Data Officer, Department for Work and Pensions Substantive: Chief Statistician, Department for Work and Pensions


During a period of unemployment, I decided to start a degree course in mathematics with the Open University. It started out as a hobby as I had always been interested in maths, and frankly, just wanted something to do. Indeed, in my first year I forgot even to mention it on my CV.

I successfully finished both a first degree and a masters in maths and stats with the OU, taking 11 years in total from start to finish. And this was a life changing experience for me.

While studying for my first degree I held down a variety of different jobs, some of which were enhanced by my studies. After obtaining a first class honours degree, I was able to meet the minimum requirements to join the Government Statistical Service's Fast Stream programme. I successfully passed the recruitment process in 2003 Since then I have enjoyed a range of roles in the civil service, not only as a professional statistician, but also as a professional policy developer. I rose quickly through the grades to enter the Senior Civil Service in late 2012, starting my current role in early 2013.

Now I am the Deputy Director, Statistical Services Division and Head of Profession for Statistics at the Department for Work and Pensions - the Department's Chief Statistician - and just coming to the
end of a six month stint as the Department's Chief Data Officer, which included building the Department's new Data Science function.

As DWP's Chief Statistician, I am mainly responsible and accountable for the production of the Department's official and national statistics. We publish around 60 series of official and national statistics to the highest standards of integrity, free from political interference, as laid out in the Codes of Practice for Official Statistics. This ensures that all my statistics are trustworthy, and of the highest quality. These range from the UK's income statistics, overseas nationals registering for a National Insurance number, to caseload statistics on the UK's welfare system.

Between May 2010 and January 2017 I was responsible for over 1,000 separate publications. I am also responsible for advising ministers and senior officials around the appropriate use of statistics, and ensure statisticians play a major role in influencing the development of current and future policy.

In May 2016 I appeared before Parliament alongside the UK's National Statistician, giving evidence to the Public Administration and Constitutional Affairs Committee on the UK's migration statistics, just prior to the referendum. This session was live recorded, and soon reports were appearing online and in mainstream media, each with a different perspective on what we had said. This was a great development opportunity, and whilst challenging at the time, I look back at the experience with pride.

# Kathleen Hyndman - A Retrospective 

at Zuleika Gallery, 6 Mason's Yard, Duke Street, St James's London SW1Y 6BU

Review by Diane Reade and Chris Reade

Much of Kathleen Hyndman's art has been inspired by mathematical sequences and geometric figures. She has been exhibiting widely since the 1970s, including at the Oxford Museum of Modern Art 1972, the Hayward Gallery in 1982, in Germany, Poland, France and South Africa. She was awarded a Millennium Fellowship in 2000 and her work has featured in exhibitions of abstract art in the Netherlands. She originally trained at Kingston School of Art, and is now in her $91^{\text {st }}$ year. This retrospective at the Zuleika Gallery, London, which ran from 7 to 24 February this year, was a celebration of her artistic career.

The exhibition consists of 10 pieces of work displaying a variety of styles from monochrome patterns to subtly transforming combinations of vibrant colours. There are some similarities in style to Mondrian, Bridget Riley and the Delaunays; Mondrian for the search of a perfect balance in line, form and colour; Bridget Riley for the use of repeating patterns and optical illusion; and the Delaunays ('Orphism phase') with their explorations into pure abstraction based on bright colour and shape. These comparisons with other artists serve to illustrate the range of work by Kathleen, who is clearly a masterful practitioner in all these areas.

At a private viewing, Kathleen explained to us that her pieces did not always start out with a mathematical concept, but this could be introduced along the way. Thus, the mathematical inspiration may appear enigmatic.


Sometimes the source of the inspiration is easy to spot when you know what you are looking for. For example in India (2001), there is an easily discernible sequence, although that may be hard to spot in the (previous) thumbnail image.


In contrast Tangent: Op posing Flows (1992) gives an illusion of movement across the canvas which was inspired by "...the pattern made by water from a dripping tap hitting stuck crumbs in a sink". It was constructed from tangents to two circles and parallel lines but leaving only short segments at intersection points after much of the construction has been elided.


As an example of subtle colour transformation, the piece entitled 25 Colour combinations (1986) consists of spectrum sequenced stripes through spectrum sequenced squares to give various colour variations. "One different colour in each sequence of squares is doubled so giving a shift and a little unpredictability".


Chris Reade is a mathematician, computer scientist and LMS member.

Diane Reade is an artist specialising in paper casting.

Chris and Diane are a couple from Thames Ditton with a keen interest in art. Reviewing mathematically inspired art together was very rewarding.

# The Best Writing on Mathematics 2017 edited by Mircea Pitici, Princeton University Press, 2018, $£ 19.95$, US\$ 24.95, ISBN: 978-0-691-17863-9 <br> Review by Robin Wilson 

This anthology, the eighth annual collection of The Best Writing on Mathematics, aims to present a collection of 'diverse, surprising, and well-written pieces, all published originally during 2016 in academic journals, scientific magazines, or mass media'. Also included is an extensive and impressive reference list of about 500 other 'notable writings' published in the same year, mainly in North America and Europe.

The 19 selected articles, described in the introduction as 'expository and interpretative pieces on mathematics and aspects of life in the mathematical community, historical and contemporary', cover a wide range of topics - from the search for prime numbers and the career of the centenarian Richard Guy, via mathematical education and the use of linear algebra to create beautiful fractal images, to the tiling patterns of Iranian domes, a geometric-algebraic construction of Leibniz, and a discussion of whether our brains are Bayesian. Jeremy Gray's article on 'Who would have won the Fields Medal 150 years ago' is modelled on a classic historical article he wrote in 1985 [1], and there are entertaining articles by Viktor Blåsjö on how to find the logarithm of any number using nothing but a piece of string, and by Carlo $P$ Séquin and Raymond Shiau on recreating Luca Pacioli's rhombicuboctahedron. Most of the selected articles are well written and (to me) interesting, and several of the authors are well-known figures, while there is a healthy sprinkling of newcomers to the world of popular mathematics writing.

I have mixed feelings about the book's title. Selecting just 19 of the 500 listed pieces as 'the best writing on mathematics' tends to place these particular articles on a pedestal and downgrade the rest. The editor has generally made a good job in his selections, but we could all produce similar lists of well-written pieces that could equally have been included.

There are many well-drawn diagrams and attractive colour pictures, but all the latter have been collected into a single section in the centre of the book. This has the bizarre consequence that several pictures appear twice - once in black-and-white in the body of the article, and again in colour in this central section, so that one is expected to flip forwards and backwards while reading the article. Moreover, while
some colour images are indeed beautiful, a few are no clearer than before. Many mathematics publications now feature colour images throughout, even those by the same publisher (see, for example, [2]).

In spite of the above misgivings, a book like this that presents the world of mathematics to a wider audience is surely to be welcomed, and the continuing popularity of this series shows that such books are needed. Inevitably the level expected of the reader varies from piece to piece, but on the whole the editor has succeeded in provided a good variety of articles that can be read with benefit by an interested UK sixth former or US college student.

Finally, a quibble. When the book's publication date is given as 2018, and the articles were all originally published in 2016, how can the articles be presented as the best mathematical writing of 2017?

## FURTHER READING

[1] J. Gray, Who would have won the Fields Medals a hundred years ago?, The Mathematical Intelligencer 7(3) (Summer 1985), 10-19.
[2] R. Wilson, Four Colors Suffice (revised color edition), Princeton University Press, 2013.


Robin Wilson

Robin Wilson is an Emeritus Professor of Pure Mathematics at the Open University, Emeritus Professor of Geometry at Gresham College, London, and a former Fellow of Keble College, Oxford University. He is currently a Visiting Professor at LSE. A former President of the British Society for the History of Mathematics, he has written and edited over 40 books on graph theory and the history of mathematics. Involved with the popularization and communication of mathematics and its history, he has received awards from the Mathematical Association of America for his 'outstanding expository writing'. He has Erdớs Number 1.

# Priest of Nature: The Religious Worlds of Isaac Newton 

by Rob lliffe, Oxford University Press, 2017, $£ 22.99$, US $\$ 34.95$, ISBN: 978-0-19-999535-6

Review by Allan Chapman


As Professor lliffe tells us in his Introduction to Priest of Nature, it was long the fashion to assume that Newton's voluminous manuscript writings on religion, alchemy, prophecy, and Biblical studies were fruits of his intellectual decline. Largely pointless studies, only undertaken after his stupendous mathematical intellect had passed its peak of creativity. But as Rob lliffe makes abundantly plain, this was most definitely not the case, for Newton's theological studies suffused the whole of his creative life, from his boyhood and undergraduate jottings down to his old age. Indeed, when Robert Hooke and other colleagues were trying to draw the reclusive Newton back into natural philosophy in the 1670s and 1680s (which culminated in Principia in 1687), the 'other studies' which he was so reluctant to abandon were mystical alchemy, prophecy, and Biblical interpretation.

Of course, as lliffe himself makes clear, he is by no means the first scholar to examine Newton's theological writings, for he had been preceded by Frank Manuel and others back into the twentieth century: scholars, indeed, who were interested in seeing Newton in his wider historical and intellectual context rather than focussing exclusively on Newton the deified genius.

What emerges from lliffe's magisterial study is a much more comprehensive picture of the intellect and achievement of Sir Isaac Newton. He is presented as a man for whom the elucidation of a pure, authentic, and uncorrupted Christian faith was paramount,
and for whom those truths were seamlessly integrated into his mathematics and natural philosophy.

To this end, Rob lliffe, who is Editorial Director of the Newton Project, has meticulously read and analysed a mass of manuscript documents located in Cambridge, Jerusalem, and elsewhere in the world. Much of this enterprise began when some sources in distant libraries were available only in poor microfilm copies, though part of the Newton Project's wider aim has been to make the Newton documents globally available via the internet.

But what were the theological issues that came to dominate Newton's entire understanding? Central here was what he saw as the Satan-induced corruption of the early Christian Church and Gospel writings. And the true villain of the piece, as Newton saw it, was the massively influential early-fourth-century AD theologian, Athanasius. For in Newton's considered opinion, it was Athanasius who had enshrined the doctrine of the Holy Trinity at the heart of Christian thinking, via the Church Council of Nicaea in AD 325 and the Creed that bears his name. For while Newton had no especial problem in accepting the Three Persons of Father, Son, and Holy Ghost as a foundational Christian principle (for they appear in the Gospels and in St Paul's concluding Benediction in his Second Epistle to the Corinthians of the mid first century AD), he fervently rejected their seamless unity as three co-equal parts of the same, singular, Divine Godhead. This doctrine, primarily formulated by Athanasius, was for him Christianity's supreme heresy, plunging subsequent centuries of Christian theology and Church practice, including that of the Roman Catholic, Orthodox, and Reformation Protestant churches, into a species of pagan superstition, and alienating them from the True Godhead and Christ.

Much attention is paid to the Homoousion 'heresy', in which God the Father, the Son (Jesus), and the Holy Ghost came to be seen, post-Athanasius, as 'consubstantial', 'co-eternal' and 'co-equal', in what would become orthodox Trinitarian theology. It was Newton's argument in his voluminous manuscript (but never published) writings, founded upon many decades of combing early Church sources, that while Jesus was indeed divine and partook of God the Father, He was not His whole and integrated equal. And likewise for the Holy Ghost.

Newton's researches into Biblical prophecy also related to his concern with the Homoousion and Athanasian 'heresy'. In particular, the mystical, deeply dramatic, and powerful prophecies of the Bible's concluding Book of Revelation, written by St John the Divine, probably towards the end of the first century AD, were seen by Newton as a prefiguring of the Athanasian and Trinitarian horrors to come.

For in Revelation, we encounter the Beast and the Whore of Babylon and their corrupting wiles, the First and Second Judgments, the blessed saved (who would not follow the as yet unborn Athanasius), and the deluded damned, destined to suffer eternally in the Lake of Fire. Indeed, it all fits together beautifully, once one has the key to the cosmic drama of Salvation and Redemption. In his thorough-going quest for purity and logic, however, Newton condemned imaginative literature and poetry, and even jokes, as essentially delusional and false.

One wonders what inner struggles Newton may have experienced in holding such views secretly while a respected Fellow of the College of the 'Holy and Undivided Trinity' in Cambridge, even when he could obtain permission to avoid taking Anglican Holy Orders, and Oaths. Had they been made public, they would have cost him his academic and social reputation
and appointments, just as they would his professorial successor, William Whiston, when he himself 'came clean' about his own views. Newton's opinions on the Holy Trinity, however, only came to light when his manuscript remains came to be examined after his death.

Of course, all this is not the Newton of the Enlightenment, the 'deity' with which most people are familiar. But as Rob lliffe's meticulous scholarship demonstrates, the man who emerges from Priest of Nature is a much more complex and much more interesting figure than the stereotype. And in that seamless web of thinking which aspired to the purest and highest truth in all things, Newton's religious and scientific perceptions are shown to constitute one coherent whole. For pure theology, pure logic, and pure mathematical truth were really all aspects of the same Divine perfection.

Rob lliffe's book is, without doubt, the authoritative statement on Newton's religion and its relationship to his natural philosophy. It is meticulously researched from a large collection of manuscript sources, and his interpretations are convincing and lucidly conveyed, demonstrating a mastery of the detailed historical theology involved. And while Priest of Nature is neither a quick nor an easy read, it is a deeply rewarding one.


## Allan Chapman

Allan Chapman in a historian of science based at Wadham College, Oxford. He has published widely, particularly on the history of astronomy, and is a founding member of the Society for the History of Astronomy.

## Obituaries of Members

Alan Baker: 1939-2018


© jetphotographic Roger Heath-Brown writes: Alan Baker, who was elected a member of the London Mathematical Society on 4th April 1966, died on 4th February 2018 following a severe stroke. He was one of the most prominent British number theorists of his generation, and the founder of modern transcendental number theory.

Alan was born in Finsbury, London, on 19th August 1939, the only child of Barnet and Bessie Baker (née Sohn). After studying at Stratford Grammar School he won a state scholarship to do a degree in mathematics at UCL. He then moved to Cambridge, completing a PhD in 1964 under Harold Davenport, with a thesis entitled "Some Aspects of Diophantine Approximation". In 1965 he was elected a Fellow of Trinity College Cambridge, where he was based for the rest of his career. He was awarded a Fields Medal at the 1970 ICM in Nice, became a Fellow of the Royal Society in 1973, and was appointed to a chair by the University of Cambridge in 1974. He was fond of remarking on the unusual chronological order of these events!

Baker had 8 PhD students: Coates, Odoni, Masser, Stewart, Flicker, Heath-Brown, Mason, and Coleman. Of these, only Masser and Stewart ended up working in the same area as Baker, the others pursuing different interests.

Baker's most important work concerns "linear forms in logarithms". For example, if $\alpha_{1}, \ldots, \alpha_{n}, \beta_{1}, \ldots, \beta_{n}$ are algebraic numbers, then Baker showed that the linear form

$$
\sigma:=\sum_{i=1}^{n} \beta_{i} \log \alpha_{i}
$$

must be non-vanishing, except in the obvious trivial cases. Moreover, and crucially, he gave an explicit positive lower bound for $|\sigma|$. This produced a whole new class of transcendental numbers, a very simple example being $\pi+\log 2$. His work also had very important consequences for Diophantine equations. For example, if $F(x, y) \in \mathbb{Z}[x, y]$ is a binary form of degree at least 3 , it had been known since the work of Thue (1909) that the equation $F(x, y)=1$ has
finitely many solutions. However the unusual logical structure of Thue's argument meant that it was not necessarily possible to find all the solutions. In contrast, Baker's work led to an explicit bound $B_{F}$ such that any solution must lie in the box $\max (|x|,|y|) \leq$ $B_{F}$. Although the bound $B_{F}$ produced by the method is very large, Baker and his followers were able to make the techniques viable for practical examples. Thus, in joint work with Davenport, he showed that there is only one positive integer $N$ for which each of $N+1,3 N+1$ and $8 N+1$ is a square, namely $N=120$.

Baker produced a flurry of important papers up to 1980, by which time his work had led to the new field of transcendental number theory, with a worldwide following. This is evidenced by the MSC classification 11 J86 "Linear forms in logarithms; Baker's method". His treatise Transcendental number theory (CUP, 1975) gives a comprehensive account of the early developments. Subsequently he was less involved in the subject, but returned to it in 2007 with the book Logarithmic forms and Diophantine geometry (CUP, 2007), written jointly with Wustholz. Baker also produced two number theory textbooks, A concise introduction to the theory of numbers (CUP, 1984) and A comprehensive course in number theory (CUP, 2012). Overall it is clear that Baker's work made a contribution of enduring significance to the development of mathematics.

Béla Bollobás writes: In spite of his great mathematical achievements, Baker was not often happy: he saw himself as snubbed by what he viewed as the mathematical establishment because he was born 'on the wrong side of the tracks'. He spent most of his time in Trinity College, with now and then a trip to London. He had some outstanding research students, and he remained proud of what they achieved. He liked to go for long walks, and was a keen player of Trinity Bowls.

He loved his occasional trips abroad, to the States, China and various European countries; in particular, he was happy to collaborate with Gisbert Wüstholz at ETH, and he enjoyed being pampered by Vera T. Sós and Kálmán Győry in Hungary.

He was a very private person, but in the company of his friends he was talkative and lively: he was a generous dinner guest, who happily contributed to the conversation, and he had an unexpected penchant for dancing. Sadly, though, as his hearing failed towards the end of his life, he slowly became more reclusive.

## Andrew Ranicki: 1948-2018



Professor Andrew Ranicki FRSE, who was elected a member of the London Mathematical Society on 18 May 1972, died on 20 February 2018, aged 69.

Clark Barwick and lain Gordon write: Andrew Ranicki was born in London in 1948, moving later to Warsaw, Frankfurt and Hamburg. He was the only child of Marcel and Teofila Reich-Ranicki. Polish was his first language, but as his father was the preeminent post-war literary critic in Germany, Andrew grew up in a remarkable environment, meeting some of the most famous artists and public figures of the time in Germany. He moved to England for school aged 16, and then studied Mathematics at Cambridge, gaining his doctorate in 1973 under the supervision of Andrew Casson and John Frank Adams. He was a Fellow of Trinity College between 1972 and 1977, and then an Assistant Professor at Princeton University from 1977 until 1982. It was in Princeton that he met his wife Ida Thompson. They were married in Fine Hall in 1979. Andrew joined the University of Edinburgh 1982, extending its traditions in 1995 by becoming the first Chair of Algebraic Surgery.

The majority of Andrew's work was dedicated to surgery theory. This began as a technique - originally due to Milnor - to replace a compact manifold with another of the same dimension in a carefully prescribed manner. In the 1960s, Browder, Novikov, Sullivan, and Wall used sequences of surgeries in a systematic way in the classification of high-dimensional manifolds. (Andrew's now classic book, Algebraic and Geometric Surgery, is certainly the reference for this beautiful story.) Perhaps the most well-known of Andrew's monumental insights was that in high dimensions, the homotopy theory of compact manifolds admits a purely algebraic incarnation: he showed that, in dimensions more than or equal to 5 , the moduli space of compact manifolds modelling a fixed ho-
motopy type can be identified with a corresponding space of 'local algebraic Poincaré complexes'. This result is typical of Andrew's remarkable ability to capture subtle aspects of geometry through intricate - but still computable! - algebraic structures.

Over the course of Andrew's career, he wrote around 80 articles, authored seven books, and edited around a dozen proceedings. His final book, The Geometric Hopf Invariant and Surgery Theory, joint with Michael Crabb, is the result of a project they began almost 20 years ago to relate algebraic surgery to $\mathbb{Z}_{2}$-equivariant stable homotopy theory. Dedicated to Andrew's grandson Nico Marcel, it was published this year. Just a week before his death, Andrew proudly shared a photo of Nico holding a paper copy of the book.

Andrew had an irrepressible spirit, embodied by his ever-ready laughter and his passion for literature and music, always having a quote ready to capture the essence of any situation. His extraordinary appreciation of people and immense hospitality led to him being well-known and loved across the entire mathematical world. Greatest of all was his profound love for his family, apparent to everyone who was lucky enough to visit his house after a topology seminar or School colloquium, to eat and to drink and to talk.

There will be a memorial service for Andrew on 16 June 2018 at the University of Edinburgh.

## Mark Joshi: 1969-2017



Mark Joshi, who was elected a member of the London Mathematical Society on 18 November 1994, died suddenly on 8 October 2017, aged 48.

David Dickson writes: Mark grew up in Dunblane, Scotland. After schooling in Scotland, he studied mathematics at the University of Oxford, graduating at the top of his year in 1990, then moved to Massachusetts Institute of Technology where he completed his PhD in pure mathematics in 1994. Mark then took up a fixed-term appointment in the Department of Pure Mathematics and Mathematical Statistics at the University of Cambridge where he remained until 1999. During this time he established his reputation as a mathematician with a series of publications in top journals, but with a dearth of
permanent positions in pure mathematics in the UK at that time, Mark decided to reinvent himself as a financial mathematician and moved to RBS as a quant, where he spent six years. It was during this period that Mark produced his first two books, one of which, The Concepts and Practice of Mathematical Finance, has become a standard text that is used in leading universities across the globe.

The second phase of Mark's academic career began in November 2005 when he moved to his wife Jane's hometown, Melbourne. He was appointed at the University of Melbourne as Associate Professor in the Centre for Actuarial Studies, perhaps somewhat ironically as Mark had internships in the actuarial department of an insurance company as a student, and this experience had put him off a professional career as an actuary. Mark quickly established himself in Melbourne as a popular teacher and a sought-after supervisor. Inspired by the many practical issues he faced at RBS, Mark and his PhD students produced a stream of papers, many of which aimed at bridging the gap between the academic and practitioner worlds. Mark was highly productive in his 12 years in Melbourne: over 50 publications, many in very highly rated journals, three new books plus a second edition of Concepts, and seven PhD graduates, with an eighth student close to submission. His performance in research was recognised by the award of the Dean's Prize for Exceptional Distinction in Research and Research Training in 2011, whilst the
award for his overall performance was promotion to professor.

Mark also took on leadership roles in Melbourne. Perhaps his most noteworthy contribution was his two-year term on University Council as an elected staff member while he was an Associate Professor. Mark was also an active participant in Academic Board, serving at appeals hearings as well as regularly contributing to discussions at board meetings. He was always prepared to speak for what he believed in. Within the Faculty of Business and Economics Mark had been Director of the Centre for Actuarial Studies since the beginning of 2015.

Mark was also very engaged with the broader financial community. He ran a personal webpage which offered plenty of constructive advice to wouldbe quants, he was involved in open source coding projects, and he regularly provided consultation to banks and insurers and ran training courses.

Away from work, Mark was very much a family man who took a keen interest in the activities of his five sons. He had an unconventional view of some mainstream activities; for example, he viewed the AFL Grand Final afternoon as the perfect opportunity to go shopping in the deserted city centre.

Mark was many things to people at Melbourne: scholar, teacher, supervisor, colleague, mentor, an inspiration, and a friend.

## Groups in Galway

Location: National University of Ireland, Galway Date: 18-19 May 2018
Website: tinyurl.com/y91j9odo
Groups in Galway has been running on an annual basis since 1978. The scope of the conference covers all areas of group theory, applications and related fields. There is no registration fee. However, if you intend to participate, email the organizers Kevin Jennings (kevin.jennings@nuigalway.ie) and Dane Flannery (dane.flannery@nuigalway.ie).

## One-Day Meeting in Combinatorics <br> Location: Mathematical Institute, Oxford <br> Date: 23 May 2018 <br> Website: tinyurl.com/ycs8434s

The speakers are Maria Chudnovsky (Princeton), Daniel Kral (Warwick), Benny Sudakov (Zurich), Kristina Vušckovi (Leeds) and Paul Wollan (Rome). Anyone interested is welcome to attend. Funds may be available to research students. The meeting is supported by an LMS Conference grant and the British Combinatorial Committee.

## 6th Scottish PDE Colloquium

Location: University of Edinburgh
Date: $\quad 31$ May - 1 June 2018
Website: tinyurl.com/ybrcqegd
Partial differential equations are ubiquitous in several branches of modern science. This two-day workshop follows the tradition of the previous Scottish PDE Colloquia and will gather established leading experts together with young mathematicians. Supported by the EMS, GMJT, MIGSAA and an LMS Conference grant.

## Integrable Models, Conformal Field Theory and Related Topics

| Location: | Cardiff University |
| :--- | :--- |
| Date: | 1-2 June 2018 |
| Website: | Iqp2.org/icft22 |

This is part of a workshop series promoting collaboration between researchers working on topics related to integrable and conformal field theories. An essential aim is to give early career researchers in the UK the opportunity to present their work; apply online to give a talk. Supported by an LMS Conference grant.

## Joint LMS/Gresham College Lecture

Location: Museum of London
Date:
Website: 22 May 2018 tinyurl.com/yc7293h5

The 2018 Joint London Mathematical Society/Gresham College Annual Lecture, titled Mathematical Research from Toy Models, will be given by Professor Tadashi Tokieda (Stanford University). No reservations are required for this lecture. It will be run on a 'first come, first served' basis. Doors will open 30 minutes before the start of the lecture.

## LMS Northern Regional Meeting

Location: University of Northumbria
Date: 25 May 2018
Website: tinyurl.com/ya27pwj8
Speakers are Tamara Grava (Bristol \& SISSA), Noel Smyth (Edinburgh) and Sara Lombardo (Loughborough). These lectures are aimed at a general mathematical audience. All interested, whether LMS members or not, are most welcome to attend this event. The meeting forms part of a workshop on Advances in the Theory of Nonlinear Waves.

## British and Irish Geometry Meeting 2018

Location: Queen's University Belfast
Date: 1-2 June 2018
Website: go.qub.ac.uk/BI-GEM18
Speakers include Barbara Baumeister (Bielefeld, Germany), Jürgen Berndt (UCL London, UK), and Tom Brady (DCU Dublin, Ireland). The meeting is supported by conference grants from the LMS and IMS. Financial support for PhD students is available; please contact the organisers for details.

## Midlands Regional Meeting

| Location: | University of Leicester |
| :--- | :--- |
| Date: | 4 June 2018 |
| Website: | tinyurl.com/y $7 x x d n p 3$ |

The meeting forms part of a workshop on Galois Covers, Grothendieck-Teichmüller Theory and Dessins D'enfants on 5-7 June 2018. Requests for support and queries about the two events may be addressed to the organisers: Frank Neumann and Sibylle Schroll (Imsmrm2018@le.ac.uk). See the website for details of speakers.

## LMS Meeting at the BMC 2018

13 June 2018; 4.30-6.00 pm, University of St Andrews
Website: tinyurl.com/yae88mts

This meeting is open to all, including non-LMS members. The opening of the meeting and Society Business section will include an opportunity to sign the Members' Book.

The LMS lecturer will be Laura DeMarco (Northwestern University), who will give a talk on Complex dy-
namics and arithmetic equidistribution. The lecture will be aimed at a general mathematical audience.

The meeting will be followed by the Colloquium Dinner (cost: $£ 42$ ). To register, visit tinyurl.com/yae88mts.

## LMS General Meeting and Hardy Lecture

## 29 June 2018, BMA House, London

Website: tinyurl.com/ycwz86yq

This meeting is open to all, including non-LMS members. Members' Book signing will take place during the opening of the meeting and Society Business section.

The speakers will be Konstanze Rietsch (KCL) and Hardy Lecturer Lauren Williams (UC Berkeley).

The meeting will be followed by a reception, which will be held at De Morgan House.

A Society Dinner will be held after the meeting at a location to be confirmed. The cost of the dinner will be $£ 35.00$, including drinks. To reserve a place at the dinner, email Elizabeth Fisher (Imsmeetings@lms.ac.uk).

## Bond-node Structures: Rigidity, Combinatorics and Chemistry

Location: Lancaster University
Date: 4-6 June 2018
Website: tinyurl.com/y7ygo2fm/
This meeting will include Entangled Crystal Frameworks as a theme. Plenary speakers: Igor Baburin (Dresden), Davide Proserpio (Milan), lleana Streinu (Smith College) and Shin-ichi Tanigawa (Tokyo).

## Preservers: Modern Aspects and New Directions

Location: Queen's University Belfast
Date: 18-21 June 2018
Website: tinyurl.com/y8uy22sg
This conference will present recent applications and links in the field of Preservers with other scientific areas at a comprehensive level, and provide a discussion forum for researchers. Supported by an LMS Conference grant.

## Quantum Integrability and Quantum Schubert Calculus

| Location: | Kavli Royal Society |
| :--- | :--- |
| Date: | $11-13$ June 2018 |
| Website: | tinyurl.com/yc9j76oa |

This meeting will bring together scientists of different career stages working in algebra, geometry, topology and mathematical physics to build interdisciplinary contacts. Advance registration essential.

## Geometry in Stochastic Analysis and Statistics

Location: Imperial College London
Date: 21-22 June 2018
Website: tinyurl.com/y8joa6wk
This conference explores the use of geometric techniques in stochastic analysis in statistics. Topics include rough paths, regularity structures, information geometry and the variational approach to FokkerPlanck. Supported by an LMS Conference grant.

## Calculus of Variations and Geometric Measure Theory at Sussex

| Location: | University of Sussex |
| :--- | :--- |
| Date: | 2-4 July 2018 |
| Website: | tinyurl.com/ycqvemks |

The aim of the conference is to bring together leading international experts in calculus of variations and geometric measure theory. Registration is free but mandatory (deadline midnight 15 May 2018). We will be holding a conference dinner on Tuesday 3 July. The conference is supported by the EPSRC Grant "Symmetry of Minimisers in Calculus of Variations".

## International Statistical Ecology Conference 2018

| Location: | University of St Andrews |
| :--- | :--- |
| Date: | $2-6$ July 2018 |
| Website: | isec2018.org |

ISEC is the premier research conference for those working at the interface of statistics and ecology. A series of research workshops will also be held; see website for details. Supported by an LMS Conference grant to fund participation of researchers from developing countries, UK students, and researchers with young children.

## LMS Popular Lectures

Location: London and Birmingham
Date: $\quad 4$ July and 19 September 2018
Website: Ims.ac.uk/events/popular-lectures
The LMS Popular Lectures are free annual events, open to all, which present exciting topics in mathematics and its applications to a wide audience. The speakers for the 2018 Popular Lectures are Katie Steckles (Maths's Greatest Unsolved Puzzles) and Jennifer Rogers (University of Oxford; Risky Business).

Young Researchers in Mathematics 2018

| Location: | University of Southampton |
| :--- | :--- |
| Date: | 23-26 July 2018 |
| Website: | yrm2018.wordpress.com |

YRM is the UK's biggest conference run for young research students, by young research students. We are pleased to invite contributed talks and posters. Financial support is available. Visit the website or email yrminfo2018@gmail.com. Supported by an LMS Postgraduate Research Conference Scheme 8 grant.

## Low Energy Effective Dynamics of Skyrmions

| Location: | University of Leeds |
| :--- | :--- |
| Date: | 2-5 July 2018 |
| Website: | tinyurl.com/ycj6uyfg |

The subject of this conference is the Skyrme model, a geometric model of nuclear matter in which nuclei are represented by topological solitons. The aim is to review recent developments in the model and highlight future challenges. Funding is available to support students and early career researchers. Supported by an LMS Conference grant.

## K-theory, Representation Theory and Hecke Algebras <br> Location: University of Sheffield <br> Date: $\quad 3-6$ July 2018 <br> Website: tinyurl.com/ybf779y5

The conference will focus on the links between operator $K$-theory and representation theory and bring together experts in representation theory, harmonic analysis, automorphic forms, noncommutative geometry and operator $K$-theory. It will mark the 75th birthday of Professor Roger Plymen. Supported by an LMS Conference grant.

## LMS Invited Lecture Series 2018

Location: University of Warwick
Date: 9-13 July 2018
Website: tinyurl.com/ybz5oq7s
The annual Invited Lecture Series bring a distinguished overseas mathematician to the UK to present a course of ten lectures over a week. The 2018 Invited Lecturer is Art Owen (Stanford). Supplementary lectures by Nicolas Chopin (ENSAE), Mark Huber (Claremont-McKenna) and Jeff Rosenthal (Toronto).

## Probability and Statistics: 41st Research Students Conference

Location: University of Sheffield
Date: 24-27 July 2018
Website: rsc2018.co.uk
The RSC brings together research students from any area of probability and statistics to present their work and network with their peers. The RSC is supported by an LMS Scheme 8 grant and other generous donations. Enquiries to rsc2018@sheffield.ac.uk.

## LMS Meeting at the ICM 2018

7 August 2018; 6.00-7.00 pm, ICM, Rio de Janeiro

Website: tinyurl.com/LMSRec2018

This meeting is open to all, including non-LMS members. Members' Book signing will take place during the opening of the meeting and Society Business section.

The LMS lecturer will be Marta Sanz-Solé (Barcelona University), who will give a talk on From gambling to random modelling.

The meeting will be followed by the LMS reception for members and guests. LMS members can register for a ticket online.

For further details about the ICM 2018 and to register for a place, visit the ICM website at icm2018.org tinyurl.com/y8yxplq.

## Differential Algebra and Related Topics

Location: University of Leeds
Date: $\quad 30$ July-2 August 2018
Website: tinyurl.com/ya3xs7x7
This ninth DART conference will focus on connections between model theory, differential-difference algebra and Galois theory with integrable systems and applications. Registration is open until 13 June. DART-IX is supported by an LMS Conference grant and the School of Mathematics, University of Leeds.

## Simple Groups: New Perspectives and Applications

Location: University of Bristol
Date: 29-31 August 2018
Website: tinyurl.com/y9nkmyue
This conference will bring together leading experts in the study of simple groups and related areas. Supported by an LMS Conference grant and the Heilbronn Institute for Mathematical Research.

## Model Sets and Aperiodic Order

Location: Durham University
Date: 3-7 September 2018
Website: tinyurl.com/yctmathw
This meeting will bring together researchers interested in exploring new techniques in aperiodic order with a focus on model sets. The meeting welcomes researchers from a range of mathematical viewpoints. Supported by an LMS Conference grant.

New Trends in Analytic Number Theory: LMS-CMI Research School

| Location: | Exeter University |
| :--- | :--- |
| Date: | $13-17$ August 2018 |
| Website: | tinyurl.com/LMSRS2 |

The main lecture course topics will be: Topics in Classical Analytic Number Theory; Hardy and Littlewood Circle Method and Diophantine Geometry; and AnaIytic Number Theory in Function Fields. Apply by 14 May.

## Dynamics Days Europe 2018

Location: Loughborough University
Date: 3-7 September 2018
Website: tinyurl.com/y8g4ls37
Dynamics Days Europe is a series of annual international conferences that provides a European forum for developments in the theory and applications of nonlinear dynamics. This year's conference includes nine world-leading experts as plenary speakers.

## Renormalisation in Quantum Field Theory and in Stochastic PDEs <br> Location: Isaac Newton Institute, Cambridge <br> Date: 3-7 September 2018 <br> Website: tinyurl.com/yd9bcm5f

This workshop will introduce participants to problems that exemplify renormalisation in quantum field theory and stochastic partial differential equations. Apply by 3 June.

## 2nd IMA Conference on Theoretical and Computational Discrete Mathematics

Location: University of Derby<br>Date: 14-15 September 2018<br>Website: tinyurl.com/ycywrt5t

This conference will showcase theoretical and computational advances in the general field of discrete mathematics. It is open to researchers working with mathematical structures and abstract constructs, and to those involved in the theory and practise of discrete algorithmic computing. The purpose of this event is to highlight progress in the field through the development of novel theories, methodologies and applications accordingly, and to inspire future work.

## Clay Research Conference 2018

Location: Mathematical Institute, Oxford Date: 24-26 September 2018 Website: tinyurl.com/yaxen19a

The conference will celebrate the Clay Mathematics Institute's contributions to the international mathematical community over the past 20 years and highlight the outstanding work of some of the mathematicians whose research it has supported. Registration is free but required; email Naomi Kraker (admin@claymath.org). See website for full details, including the schedule, titles and abstracts when they become available.

# LMS-IMA Joint Meeting: Noether Celebration 

11 September 2018, De Morgan House, London

Website: tinyurl.com/LMS-IMANoether2018
The speakers will be Katherine Brading (Duke University), Elizabeth Mansfield (University of Kent), Cheryl Praeger (University of Western Australia), Norbert Schappacher (I.R.M.A. / U.F.R. de mathématique et d'informatique) and Reinhard Siegmund-Schultze (University of Agder).

The meeting is free to attend. Please register for your place online.

The meeting includes lunch and will be followed by a reception.

After the reception, the LMS and IMA will host a Joint Society Dinner at a nearby venue tbc. The cost to attend the dinner, including drinks, is $£ 30.00$ per person. If you would like to attend the dinner, please email Elizabeth Fisher (Imsmeetings@lms.ac.uk).

## CONFERENCE FACILITIES

De Morgan House offers a 40\% discount on room hire to all mathematical charities and $20 \%$ to all not-for-profit organisations. Call 02079270800 or email roombookings@demorganhouse.co.uk to check availability, receive a quote or arrange a visit to our venue.


## Derived Algebraic Geometry and Chromatic Homotopy Theory

Location: Isaac Newton Institute, Cambridge
Date: 24-28 September 2018
Website: tinyurl.com/ybvdpyt9
Chromatic homotopy theory is the classical subfield of algebraic topology which uses the theory of smooth one-parameter formal groups to organize calculations and the search for large-scale phenomena in stable homotopy theory. The application closing date is 24 June 2018.

## MathTech2018

Location: Hotel Equatorial Penang, Malaysia
Date: 10-12 December 2018
Website: tinyurl.com/yahxvzay
The international conference Mathematical Sciences and Technology 2018 provides a platform to share knowledge, ideas and experiences. The conference covers pure and applied mathematics, statistics, operations research, technology and interdisciplinary mathematics. Submit abstracts by 31 May.

## Quantum Field Theory, Renormalisation and Stochastic Partial Differential Equations

| Location: | Isaac Newton Institute, Cambridge |
| :--- | :--- |
| Date: | $22-26$ October 2018 |
| Website: | tinyurl.com/ychzvesv |

This workshop will focus on some of the latest developments in the fields of quantum field theory, critical phenomena in statistical mechanics, and stochastic PDEs. At the interface of these topics is renormalisation; this will provide the focus of the workshop. Application deadline: 15 July 2018.

## Theory and Practice: an Interface or a Great Divide?

Location: Maynooth University, Kildare, Ireland Date: 4-9 August 2019
Website: tinyurl.com/yagguvt3
This is the 15th international conference of The Mathematics Education for the Future Project, which was founded to encourage innovation in mathematics, science, statistics and computer education. For further details email alan@cdnalma.poznan.pl.


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## ADVERTISE INTHE LMS NEWSLETTER

The LMS Newsletter appears six times a year (September, November, January, March, May and July).

The Newsletter is distributed to just under 3,000 individual members, as well as reciprocal societies and other academic bodies such as the British Library, and is published on the LMS website at Ims.ac.uk/publications/lms-newsletter.

Information on advertising rates, formats and deadlines are at newsletter.lms.ac.uk/rate-card.
Examples in this issue can be found on pages 3,7 and 10 , and on the back page.
To advertise contact Susan Oakes (susan.oakes@lms.ac.uk).

# Society Meetings and Events 

May 2018
22 Joint LMS/Gresham College Annual Lecture, Museum of London
25 Northern Regional Meeting, Northumbria

## June 2018

4 Midlands Regional Meeting, Leicester
13 Society Meeting at the BMC, St Andrews
29 Gen. Meeting \& Hardy Lecture, London

## July 2018

4 LMS Popular Lecture, London
9-13 LMS Invited Lecture Series 2018, University of Warwick

## August 2018

7 LMS Meeting at the ICM, Rio de Janeiro

## September 2018

11 Joint Society Meeting with IMA: Noether Celebration, London
19 LMS Popular Lecture, Birmingham

## October 2018

9 Joint Society Meeting with the Fisher Trust, Galton Institute, Genetics Society and RSS; Royal College of Surgeons, Edinburgh

## November 2018

9 Society and Annual General Meeting, London

## December 2018

17 LMS South West \& South Wales Regional Meeting, Exeter

## Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website (www.Ims.ac.uk/content/calendar). Please send updates and corrections to calendar@lms.ac.uk.

May 2018

$$
\begin{aligned}
\text { 9-10 } & \begin{array}{l}
\text { Celebrating Algebraic Geometry in Lough- } \\
\text { borough }
\end{array} \\
\text { 9-10 } & \begin{array}{l}
\text { Colloquia in Combinatorics, Queen Mary, } \\
\text { University of London and LSE }
\end{array} \\
\text { 14-15 } & \text { Collabor8.2, Lancaster University (475) } \\
\text { 16-18 } & \begin{array}{l}
\text { UK-Japan Workshop on Analysis of } \\
\text { Nonlinear Partial Differential Equations, } \\
\text { Swansea University (475) }
\end{array} \\
\text { 17-19 } & \begin{array}{l}
\text { Mathematics and Science: In Honour of } \\
\text { Sir John Ball, University of Oxford (475) }
\end{array} \\
\text { 18-19 } & \begin{array}{l}
\text { Groups in Galway, National University of } \\
\text { Ireland, Galway (476) }
\end{array} \\
\text { 21-23 } & \begin{array}{l}
\text { Gregynog Welsh Maths Colloquium 2018, } \\
\text { Gregynog Hall, Newtown (475) }
\end{array} \\
22 & \begin{array}{l}
\text { Joint LMS/Gresham College Annual Lec- } \\
\text { ture, Museum of London (476) }
\end{array}
\end{aligned}
$$

22-24 Lancaster Probability Days 2018, Lancaster (475)
23 Combinatorics, Mathematical Institute, Oxford (476)
25 LMS Northern Regional Meeting, University of Northumbria (476)
28-31 Recent Advances in Nonlinear Analysis, Levico Terme, Italy (475)
31-1 June 6th Scottish PDE Colloquium, University of Edinburgh (476)

## June 2018

[^6]4 LMS Midlands Regional Meeting, Leicester (476)

4-6 25th Postgraduate Combinatorial Conference, LSE
4-6 Bond-node Structures: Rigidity, Combinatorics and Chemistry, Lancaster University (476)
5-8 CHAOS 2018, Rome, Italy (475)
11-13 Quantum Integrability and Quantum Schubert Calculus, Kavli Royal Society, Chicheley Hall, Buckinghamshire (476)
11-14 British Mathematical Colloquium 2018, University of St Andrews (475)
13 Society Meeting at the BMC, St Andrews (476)

18-21 Preservers: Modern Aspects and New Directions, Queen's University Belfast (476)
18-22 Manifolds, Groups and Homotopy, Sabhal Mòr Ostaig, Isle of Skye (475)
21-22 Geometry in Stochastic Analysis and Statistics, Imperial College London (476)
25-29 An Analyst, a Geometer and a Probabilist Walk Into a Bar, Cardiff University (475)
25-29 Future Challenges in Statistical Scalability, INI, Cambridge (475)
27-29 Numerical Linear Algebra and Optimization, University of Birmingham (475)
29 LMS General Meeting and Hardy Lecture, London (476)

## July 2018

2-4 Calculus of Variations and Geometric Measure Theory at Sussex, University of Sussex (476)
2-5 Low Energy Effective Dynamics of Skyrmions, University of Leeds (476)
2-6 The Mathematics of Multiscale Biology, LMS Research School, Nottingham (475)
2-6 International Statistical Ecology Conference 2018, University of St Andrews (476)
3-6 K-theory, Representation Theory and Hecke Algebras, University of Sheffield (476)

4 LMS Popular Lectures, London (476)
9-13 Homotopy Theory and Arithmetic Geometry: Motivic and Diophantine Aspects, LMS-CMI Research School, Imperial College London (475)
9-13 LMS Invited Lecture Series 2018, Art Owen (Stanford University), University of Warwick (476)

23-26 Young Researchers in Mathematics 2018, University of Southampton (476)
23-27 European Conference on Mathematical and Theoretical Biology, University of Lisbon (475)
24-27 Probability and Statistics: 41st Research Students Conference, University of Sheffield (476)
30-2 Differential Algebra and Related Topics, University of Leeds (476)

## August 2018

7 LMS Meeting at the ICM 2018, Rio de Janeiro (476)
13-17 New Trends in Analytic Number Theory, LMS-CMI Research School, Exeter (476)
13-17 Equivariant and Motivic Homotopy Theory, INI, Cambridge (475)
29-31 Simple Groups: New Perspectives and Applications, Bristol (476)

## September 2018

2-4 Modern Mathematical Methods in Science and Technology, Kalamata, Greece (475)

3-7 Dynamics Days Europe 2018, Loughborough University (476)
3-7 Model Sets and Aperiodic Order, Durham University (476)
3-7 Renormalisation in Quantum Field Theory and in Stochastic Partial Differential Equations, INI, Cambridge (476)
11 Joint Society Meeting with IMA: Noether Celebration, London (476)
14-15 Theoretical and Computational Discrete Mathematics, University of Derby (476)
19 LMS Popular Lectures, University of Birmingham (476)
24-26 Clay Research Conference 2018, Mathematical Institute, Oxford (476)
24-28 Derived Algebraic Geometry and Chromatic Homotopy Theory, INI, Cambridge (476)

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[^0]:    ${ }^{1}$ More precisely, if $X$ is an algebraic variety with singular points $X_{\text {sing }} \subset X, \pi: S \rightarrow X$ is a "resolution" of $X$ if $S$ is smooth, $\pi$ is proper, $\pi^{-1}\left(X-X_{\text {sing }}\right)$ is dense in $S$, and $\pi$ is an isomorphism between $\pi^{-1}\left(X-X_{\text {sing }}\right)$ and $X-X_{\text {sing }}$. For more details see [7].

[^1]:    ${ }^{1}$ This material is from the Niedersächsische Staats- und Universitätsbibliothek Göttingen, department of manuscripts and rare prints (briefly quoted as Göttinger Handschriftenabteilung), from the Geheimes Staatsarchiv Preußischer Kulturbesitz, Berlin (quoted as GStA PK), from the University Archives Marburg, and from The National Library of Israel, Jerusalem, where the Fraenkel Archives are located. I thank all archives for providing the materials and for giving permission to quote from them. I also thank June Barrow-Green for revising the English.
    ${ }^{2}$ One has to think for example of the fundamental joint work on algebras by Hasse, Noether, and Richard Brauer [1] and of the correspondence between Hasse and Noether [5].

[^2]:    ${ }^{3}$ Fraenkel's interesting autobiography has recently appeared in English translation [2]. Unfortunately, errors based on slips of memory on the part of Fraenkel have not been removed or commented upon by the editor, who is the granddaughter of Fraenkel. She did not use literature on the history of mathematics or Fraenkel's correspondence. See my review in Historia Mathematica 44 (2017), 288-292.
    ${ }^{4}$ Fraenkel to Hasse, Kiel 13.12.31. Göttinger Handschriftenabteilung, cod. H.Hasse, 32-2, fol. 421.
    ${ }^{5}$ GStAPK, Rep. 76Va, Sekt. 9, Tit.4, no. 22 "Durchführung des BBG an der Universität Kiel", fol. 57.

[^3]:    ${ }^{6}$ Fraenkel to Hasse, Kiel 14.6.28. Göttinger Handschriftenabteilung, cod. H.Hasse, 1-474, letter no. 7, pp.4/5. CF. Facsimile Fraenkel. For the Bologna affair and Hilbert's standpoint on international collaboration see my recent [10].
    ${ }^{7}$ Fraenkel had volunteered during the war to become a Jewish army chaplain. In the Nazi questionnaire of 1933, mentioned above, Fraenkel insisted that he did not "wish to draw advantages from my following the path of duty during the war." Same source as footnote 5 , fol.. 55. This would appear to be a protest against the arbitrary dismissal of Jewish civil servants from which those who had been "front fighters" during the war were exempted by the Nazis for - as it turned out later - a period of two years.
    ${ }^{8}$ [6], p. 250, also quoted in [7], p. 47.
    ${ }^{9}$ Hasse to Fraenkel, Halle, 10 October 1928. The National and University Library, Jerusalem, Abraham Fraenkel Archive.
    ${ }^{10}$ GStA PK, I. HA Rep. 76 Kultusministerium, Va Sekt. 9 Tit. IV Nr. 1 vol. 21, fol. 392-393. Hensel had retired from Marburg on 12 February 1929, one year earlier than anticipated, maybe on the basis of the Emergency decrees. University Archives Marburg, 310 Kuratorium Acc 195176, No. 319.

[^4]:    ${ }^{11}$ [4, pp.52-3]. I have corrected a few slips in the transcription comparing it with the original in the Göttinger Handschriftenabteilung. Only the word "unbearable" ("unerträglich"), instead of (correctly) "unpleasant' ("unerquicklich"), distorts the meaning. See the facsimile in [11].
    ${ }^{12}$ [4, p.53], where the letter is inadvertently dated 10.10.28. The original is in the Göttinger Handschriftenabteilung, cod. H.Hasse,1-474, letter no.9. The emphasis (underlining) in this letter, and in subsequent letters, is by the author of the letter.
    ${ }^{13}$ The National Library of Israel, Jerusalem, Fraenkel archives, file correspondence Hasse-Fraenkel, Hasse to Fraenkel, Halle 10 October 1928, 16 pages, page 8. See facsimile in the original publication [11].

[^5]:    ${ }^{14}$ The German-National People's Party (DNVP), to which Hasse was close politically at time, had strong tendencies to anti-Semitism. During the years of Nazi dictatorship Hasse shocked his American colleagues by claiming that there was a "war between the Germans and the Jews", thus supporting the anti-Semitic measures at the German Zentralblatt around 1938. See [8], p. 164.
    ${ }^{15}$ Source as in footnote 14.
    ${ }^{16}$ The one Jewish mathematician who would finally make it on the list was A. Rosenthal from Heidelberg, nominated in second position.
    ${ }^{17}$ Fraenkel to Hasse, Kiel 15.11.1928. Göttinger Handschriftenabteilung, cod. H.Hasse,1-474, letter no. 12. fol. 12/2. Fraenkel seems to allude to some resentment against abstract set theory on Dehn's part.
    ${ }^{18}$ GStA PK), I. HA Rep. 76 Kultusministerium, Va Sekt. 9 Tit. IV Nr. 1 vol. 21, fol. 380.

[^6]:    1-2 British and Irish Geometry Meeting 2018, Queen's University Belfast (476)
    1-2 Integrable Models, Conformal Field Theory and Related Topics, Cardiff University (476)

