LONDON MATHEMATICAL SOCIETY
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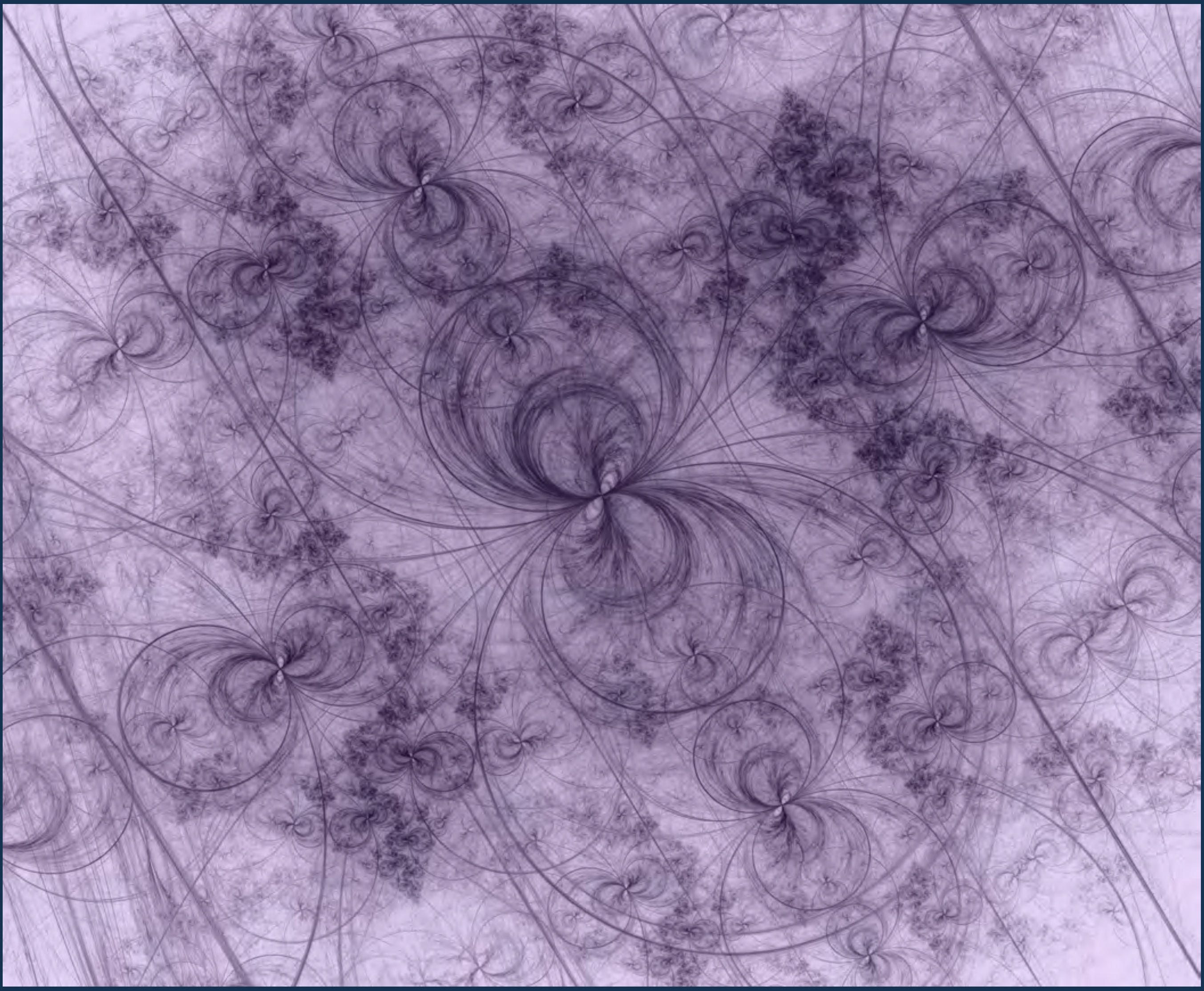
NEWSLETTER

Issue: 481 - March 2019



HILBERT'S SIXTH PROBLEM

FRACTALS AND GEOMETRY

CHANGING A-LEVEL STANDARDS

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COVER IMAGE

A fractal from conformal dynamics (see page 24).
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NEWSLETTER WEBSITE

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The Newsletter welcomes submissions of feature content, including mathematical articles, career related articles, and microtheses from members and non-members. Submission guidelines and LaTeX templates can be found at lms.ac.uk/publications/submit-to-the-lms-newsletter.

Feature content should be submitted to the editor-in-chief at iain.moffatt@rhul.ac.uk.

News items should be sent to newsletter@lms.ac.uk.

Notices of events should be prepared using the template at lms.ac.uk/publications/lms-newsletter and sent to calendar@lms.ac.uk.

For advertising rates and guidelines see lms.ac.uk/publications/advertise-in-the-lms-newsletter.

CONTENTS

Forthcoming LMS Events

The following events will take place in the next two months:

Christopher Zeeman Medal Presentation and Lecture: 5 March, Royal Society, London (tinyurl.com/y84gvbyk)

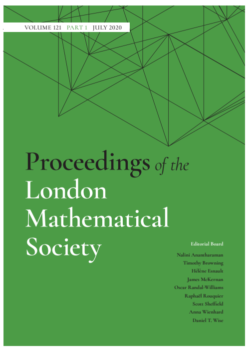
Hirst Lecture and Society Meeting: 21 March, De Morgan House, London (tinyurl.com/ybewfqz7)

Mary Cartwright Lecture: 5 April, ICMS, Edinburgh (tinyurl.com/ycv3ssfq)

LMS Meeting at the BMC: 10 April, Lancaster (tinyurl.com/yarpowdo)

A full listing of forthcoming LMS events can be found on page 50.

Changes to the *Proceedings of the London Mathematical Society*



Founded in 1865, the *Proceedings* is today seen by many as the flagship journal of the LMS. With the support of the LMS Publications Committee, we took on the task of elevating the position of the journal further and strengthening its position with relation to other top-flight mathematics journals.

The first step in this endeavour has been to appoint an Editorial Board of leading international experts, including Nalini Anantharaman, Hélène Ésnault, James McKernan, Bryna Kra, Manuel del Pino, Raphaël Rouquier, Scott Sheffield, Anna Wienhard and Daniel T. Wise.

The Editorial Board has been selected to attract articles of the highest quality and significance across a broad spectrum of mathematics, and the editorial procedures have been designed to help ensure submissions are dealt with in an efficient and timely manner.

From 1 March 2019, this new Editorial Board will begin handling submissions to the *Proceedings*, and will hold them to the highest standards. More details on the submission process and the link to submit can be found at tinyurl.com/y7cuaxbr.

The current joint Editorial Board of the LMS journals will continue to receive new submissions for the *Bulletin, Journal* and *Transactions*. Furthermore, papers which were submitted to the *Proceedings* prior to 1 March 2019 will continue to be handled by the existing Board and decisions for acceptance will be held against the same criteria that were in place at the time of submission. After 1 March, papers which would previously have been submitted to the *Proceedings* will now be welcomed by the *Journal*.

We ask the mathematical community to support this exciting new chapter by considering submitting to *Proceedings* any forthcoming papers that will help secure the uplift in quality for which we are striving.

Tim Browning and Oscar Randal-Williams
Managing Editors
Proceedings of the London Mathematical Society

Annual Elections to LMS Council

The LMS Nominating Committee is responsible for proposing slates of candidates for vacancies on Council and vacancies on its own membership. The Nominating Committee welcomes suggestions from the membership.

Anyone who wishes to suggest someone for a position as an Officer of the Society or as a Member-at-Large of Council (now or in the future) is invited to send their suggestions to Professor Kenneth Falconer, the current Chair of Nominating Committee (nominations@lms.ac.uk). Please provide the name and institution (if applicable) of the suggested nominee, their mathematical specialism(s), and a brief statement to explain what they could bring to Council/Nominating Committee.

It is to the benefit of the Society that Council is balanced and represents the full breadth of the mathematics community; to this end, Nominating Committee aims for a balance in subject area, geographical location and gender in its list of prospective nominees.

Nominations should be received by 19 April 2019 in order to be considered by the Nominating Committee.

In addition to the above, members may make direct nominations for election to Council or Nominating Committee. Direct nominations must be sent to the Executive Secretary's office (nominations@lms.ac.uk) before noon on 1 September 2019. For details on making a direct nomination, see lms.ac.uk/about/council/lms-elections.

The slate as proposed by Nominating Committee, together with any direct nominations received up to that time, will be posted on the LMS website in early August.

Compositio Prize

Every three years, the Foundation Compositio Mathematica awards a prize for the best paper published in *Compositio Mathematica* in a preceding three-year period. The winner of the Compositio Prize 2018 for the period 2014–2016 is the paper by James Maynard, 'Dense clusters of primes in subsets' (*Compositio Mathematica* 152 (2016) no. 7, 1517–1554).

A celebrated earlier result of Maynard (and discovered independently by Tao) established that for any natural number k one can find infinitely many intervals of bounded length $C(k)$ containing at least k primes. This paper gives a vast generalization of such a result to subsets of the primes meeting certain equidistribution criteria, and with special attention to the uniformity in k. For example, the paper establishes that there are intervals of length $(\log x)^\epsilon$ that contain at least $C(\epsilon)\log\log x$ primes for some positive $C(\epsilon)$. Since a typical interval of such length contains at most one prime, these intervals may be thought of as containing dense clusters of primes. Another striking example of the results in this paper: there are arbitrarily large intervals of length $\epsilon\log x$ containing $m \geq \epsilon\log\log x$ consecutive primes all lying in the same reduced residue class $a \pmod q$.

Compositio Mathematica is published by the London Mathematical Society on behalf of its owner, the Foundation Compositio Mathematica.

Gerard van der Geer
President
Foundation Compositio Mathematica

Abel Prize Announcement

The Norwegian Academy of Science and Letters will announce the winner of the 2019 Abel Prize at 11.00am (GMT) on Tuesday 19 March 2019. For the first time, the announcement will be transmitted live to an invited audience at the Science Gallery, King's College London, in collaboration with the London Mathematical Society. The transmission from Oslo will be followed by a panel, to include LMS President Professor Caroline Series FRS, which will discuss the winner's life and scientific work. Breakfast will be available in the Gallery from 10.00am and the event will begin at 10.45am. All LMS members have been invited to attend but registration is necessary. For more information about the event and to register, email Eirik Lislerud (eirik.lislerud@dnva.no).

MATHEMATICS POLICY DIGEST

Brexit, Science and Innovation: Preparations for 'No-Deal' inquiry

Building on its previous work on Brexit throughout 2018, the House of Commons Science and Technology Select Committee launched an inquiry in December 2018 on Preparations for 'No Deal' and what this will mean for science and innovation. The inquiry asked for written evidence by 23 January 2019 on the following issues:

- what a No Deal Brexit would mean for the science and innovation community;
- the adequacy of what the government and its non-departmental public bodies are doing to prepare for such an outcome.

An oral evidence session took place on 30 January. A report will be published in due course. More information is available at tinyurl.com/yclh2osp.

New DCMS Chief Scientific Adviser appointed

Professor Tom Rodden has been appointed as the Chief Scientific Adviser for the Department for Digital, Culture, Media and Sport (DCMS) and began his role in January 2019. Professor Rodden served as EPSRC's Deputy Chief Executive from April 2016 – April 2018 and subsequently as Deputy Executive Chair.

Knowledge Exchange Framework (KEF) consultation

In November 2017, the government asked the former HEFCE to lead on developing the knowledge exchange framework, to support the government's Industrial Strategy. Research England provided advice and recommendations to the Universities Minister and these recommendations were accepted in November 2018. Consultation responses on these recommendations are now being invited from any higher education institution, association, organisation or individual with an interest in knowledge exchange. The deadline for responses is midday on Thursday 14 March 2019. More information is available at tinyurl.com/yb4agv8h.

New Universities and Science Minister appointed

Chris Skidmore was appointed as a Minister of State jointly at the Department for Education and the Department for Business, Energy and Industrial Strategy on 5 December 2018. More information is available at tinyurl.com/y8nhruuq.

Digest prepared by Dr John Johnston
Society Communications Officer

Note: items included in the Mathematics Policy Digest are not necessarily endorsed by the Editorial Board or the LMS.

EUROPEAN MATHEMATICAL SOCIETY NEWS

Abel Prize

The winner of the Abel Prize 2019 will be announced on 19 March at 11:00 GMT (12 noon in Oslo, Norway). The announcement will be transmitted as a live webcast and can be watched on the Abel Prize website.

Betül Tanbay

The European Mathematical Society issued a statement on 16 November 2018 strongly protesting the detention by the Turkish authorities of its Vice-President Elect Professor Betül Tanbay. The EMS is pleased to report that Betül has subsequently been released from custody, but the Society continues to have grave concerns about her case and is monitoring the situation. The EMS is grateful for the many expressions of support and solidarity received from the community.

EMS Publishing House

Seventeen years after its foundation, the EMS-PH holds a firm position in the landscape of mathematical publishing. It was initially the work of Thomas Hintermann, to whom the whole society is immensely grateful. With Thomas now approaching his well-deserved retirement, a new chapter will start. At its recent meeting the EMS Executive Committee approved legal documents which would open the way for a transformation of the EMS-PH. Further details will be released in the coming months.

Hirzebruch Lecture

The German Mathematical Society (DMV) and the European Mathematical Society (EMS) have jointly agreed to establish a Hirzebruch Lecture given as a part of the European Congresses of Mathematics. Nominations for the Hirzebruch Lecture at 8ECM can be sent until 31 December 2019. See tinyurl.com/ybs3pca8.

International Day of Mathematics

The proposal that UNESCO proclaim 14 March (Pi Day 3.14) as the International Day of Mathematics (IDM) was adopted by the Executive Board of UNESCO at its October meeting. It had been proposed jointly by the Permanent Delegations at UNESCO of over thirty nations including Brazil, Canada, France, India, Iran, Pakistan, Portugal, Russia, Spain and Turkey (but not including the UK or USA). The proposal will now be on the agenda of the 40th General Conference of UNESCO in autumn 2019.

Taylor & Francis

The European Mathematical Society became aware last year of serious mistreatment of the Iranian mathematicians Abbas Fakhari and Mohammad Soufi by the publishing company Taylor & Francis. Their paper "Saturation of Generalized Partially Hyperbolic Attractors" was submitted to the journal *Dynamical Systems,* and after passing the peer review process was accepted by the journal and posted online on 3 December 2018. Subsequently the journal's managing editor informed the authors that the publishing house was unable to proceed with any further processing of their paper because of the authors' nationality, in "compliance with laws and regulations applied by the UK, US, European Union, and United Nations jurisdictions with respect to countries subject to trade restrictions".

The EMS regards such an attack on freedom of science as absolutely unacceptable, stating: *Actions of this type concern not only mathematics but every scientific and academic discipline, and threaten to return our society to the darkest moments of the last century.* Responding to wide protests from the scientific community, Taylor & Francis has since told the authors that their paper will be published.

EMS News prepared by David Chillingworth
LMS/EMS Correspondent

OPPORTUNITIES

PROMYS Europe 2019

PROMYS Europe, a challenging six-week residential mathematics summer programme at the University of Oxford, is seeking pre-university students from across Europe (including all countries adjacent to the Mediterranean) who show unusual readiness to think deeply about mathematics. PROMYS Europe is designed to encourage mathematically ambitious students who are at least 16 to explore the creative world of mathematics. Participants tackle fundamental mathematical questions within a richly stimulating and supportive community of fellow first-year students, returning students, undergraduate counsellors, research mentors, faculty, and visiting mathematicians.

First-year students focus primarily on a series of very challenging problem sets, daily lectures, and exploration projects in Number Theory. There will also be a programme of talks by guest mathematicians and the counsellors.

PROMYS Europe is a partnership of Wadham College and the Mathematical Institute at the University of Oxford, the Clay Mathematics Institute, and PROMYS (Program in Mathematics for Young Scientists, founded in Boston in 1989). The programme is dedicated to the principle that no one should be unable to attend for financial reasons. Full and partial needs-based financial aid is available, which can cover the fee and travel costs, in part or in full.

Download the application form and application problem set at promys-europe.org. The closing date for first-year student applications is 17 March. PROMYS Europe 2019 will run from 14 July to 24 August at the University of Oxford.

Call for EMS Prize Nominations

At the 8th European Congress of Mathematics in Portorož, Slovenia, in 2020, the traditional prizes will be awarded: ten EMS Prizes for young mathematicians, as well as the Felix Klein and Otto Neugebauer Prizes, will be awarded. The prize committees have been finalised and the community is encouraged to propose strong candidates for these distinctions. See tinyurl.com/ycyrzhmx.

VISITS

Visit of Zhibek Kadyrsizova

Dr Zhibek Kadyrsizova, Assistant Professor of Mathematics at Nazarbayev University, Astana, Kazakhstan, will visit Teesside University at Middlesbrough from 20 to 29 May 2019. As part of her visit she will talk on singularities in positive prime characteristic and varieties of commutative matrices. For further information email (s.wegner@tees.ac.uk). Supported by an LMS Scheme 4 Research in Pairs grant.

Visit of Stephen Pankavich

Professor Stephen Pankavich (Colorado School of Mines, USA) will visit Cardiff University from 11 February to 14 March 2019. He will talk on *Long time behavior of plasmas governed by the Vlasov-Poisson and Vlasov-Maxwell systems*. For further information email (Ben-ArtziJ@cardiff.ac.uk). Supported by an LMS Scheme 4 Research in Pairs grant.

Visit of Rahim Moosa

Professor Rahim Moosa (University of Waterloo, Canada) will visit the University of Manchester from 23 to 30 June 2019. During his visit he will give a seminar talk and undertake a research collaboration on model-theoretic applications to Poisson-algebraic groups. For further information email (omar.sanchez@manchester.ac.uk). Supported by an LMS Scheme 4 Research in Pairs grant.

Visit of Daniel Mitchell

Dr Daniel Mitchell (Postdoctoral Research Associate, RWTH Aachen) will visit the University of St Andrews from 17 to 24 April 2019, to work with Dr Isobel Falconer. During his visit he will give a public talk on 23 April. For further information email ijf3@st-andrews.ac.uk. Supported by an LMS Scheme 4 Research in Pairs grant.

Highlighting the Impact of LMS Education Grants

The LMS has a major commitment to mathematics education, and offers two grant schemes to teachers and other educators: the Small Grants for Education scheme and the Grants for Teacher CPD scheme. The Small Grants for Education are intended to fund activities that stimulate interest and enable involvement in mathematics from Key Stage 1 (age 5+) to undergraduate level and beyond, while the Grants for Teacher CPD are intended to facilitate mathematical development for teachers in the UK.

The grants are awarded by the LMS Education Committee, which considers four rounds of applications per year. The committee particularly encourages applications for activities that have a wide impact and that enhance and enrich mathematical study beyond the curriculum, and/or that stimulate interactions across the mathematical community. Some examples of recently funded activities are below.

Small Grants for Education

1. Primary school mathematics club in Caerphilly
Funding was awarded to support a primary community school to create a mathematics after school club that taught mathematics to reception — year 6 students in an innovative way. The applicant was a senior lecturer at Cardiff University. Activities at the club were geared towards developing basic computational skills and an understanding of fractions; children took part in a 'fraction race', in which they were encouraged to find the biggest of a number of fractions, and played a 'pizza fraction board game' in teams. Many other maths board games were used to develop computational skills. At the time of application, the project was a partnership between two schools; however, following successful implementation, the applicant intends to expand the activity to other schools and local games clubs.

2. Mathematics camp in Kenya
The applicant, a UK postgraduate student, requested funding to work voluntarily at a mathematics camp in Kenya, which was set up by a Kenyan NGO with the aim of strengthening the mathematical community across Africa at all academic levels. The LMS grant contributed to the cost of flights to Nairobi. The camp took place over 2 weeks, the first of which was a planning week for the volunteers. The applicant assisted in lessons with such themes as 'maps, graphs and gluings' (exploring graph connectivity and surfaces) and 'maths in nature' (exploring waves, pendulums, predator-prey populations), among others.

Grants for Teacher CPD

The Grants for Teacher CPD scheme is divided into three elements: Element A, which supports teacher attendance at conferences; Element B, which assists providers of professional development to run conferences/courses; and Element C, which covers the costs of external trainers in schools. Thus, the scheme may be used in a number of different ways, as illustrated by the examples below.

1. Attendance at the 2018 MEI Conference
The applicant was one of a large number who applied for funding to attend the 2018 Mathematics Education Innovation (MEI) Conference, a three-day event run by the MEI for 11–19 mathematics teachers. The grant contributed to the cost of travel and accommodation. The applicant found the conference to be extremely productive and gained a variety of resources to support their teaching, including practical applications of sorting problems and ways to introduce the concept of algorithms to students in an engaging manner. All resources were shared with the department on the applicant's return.

2. Funding for a mathematics-based Inset day
The LMS grant was used to fund a guest trainer, Ben Sparks, at a mathematics Inset day at a school in Hampshire, attended by 50 maths teachers with a range of experience from across seven different schools within the trust. The theme of the day was 'Curiosity — Inspiring Minds', which opened with a session on 'Moving Mathematics' and was followed by cross-school working groups around such themes as 'Mastering Mastery', 'Problem Solving' and 'Teaching for Boys and Girls'. Prior to the session, two main needs had been identified from cross-school development plans: 'use of ICT in the classroom' and 'stretch and development'. The applicant reported excellent feedback from the session.

Further information on the LMS Education Grants can be found at tinyurl.com/ya26nhfo. You can read more about the work of the Education Committee at tinyurl.com/yax7nurx.

Dr Kevin Houston
LMS Education Secretary

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LMS Council Diary

The Council meeting on 9 November 2018 was the occasion for us to hear reports from three of the Society's committees. First was the annual report from Education Secretary, Kevin Houston. The Education Committee had had an active year, including submitting responses to a number of government consultations. Amongst other things, attendance at both the Gresham and Popular Lectures continues to be very good and there are plans afoot to extend the Popular Lectures from two to three venues. Kevin also gave us an update on the national education picture, highlighting the fact that mathematics is still the most popular subject for A-level. On the downside, as has been much discussed in the national press, we were reminded that teacher recruitment is still a major concern, and Kevin had recently attended a meeting at the Royal Society with the Department for Education to discuss this issue. Looking to the future, we learnt that the Committee intends to focus on teacher training and recruitment, and the possibility of implementing an 'English Maths Week' in line with Scotland, Wales and Northern Ireland.

Next we heard from the Publications Committee. The Publications Secretary, John Hunton, brought us up to date with developments around the proposed 'Plan S'. In short, the European research funders are proposing to implement a condition that any researcher funded (directly or indirectly) by them would be required to publish in a pure open access journal in future, a scenario which, if implemented, would have implications for almost every mathematician employed by a UK institution, as well as for the Society's publishing activities. Although it seems unlikely that the principles of Plan S will come to fruition exactly as currently articulated, Publications Committee will keep a careful eye on developments. An article on the matter also appeared in the January Newsletter.

Third, the Research Policy Committee's annual report was presented by the chair, Vice-President John Greenlees. In connection with the REF, John informed us that the Committee had submitted nominations for the mathematical sciences sub-panel and a response to the consultation on the REF guidance on submissions and criteria for working panels. The Committee had also undertaken a third survey on Doctoral Training Partnership funding in institutions, and had written a letter to Vice-Chancellors about this issue. The final matter John brought to our attention was the census carried out by the Committee on postdocs working within mathematical sciences departments in the UK. This turned out to have produced some very interesting results. Of particular note was the fact that the 52 departments who responded to the census between them identified a total of 756 postdocs, many more than anticipated. In the ensuing discussion it was agreed that it would be very useful to repeat the exercise in the future as there would be added value in holding longitudinal data. It was also suggested that it would be useful for the Women in Mathematics Committee if the data could be disaggregated by gender.

The President, Caroline Series, then closed the meeting by expressing very warm thanks on behalf of the Council for the work of those Officers and members-at-large who would be leaving Council: Francis Clarke, Iain Stewart, Sarah Zerbes and myself. She wished good luck to all those standing for re-election and thanked them for all the work they had done thus far.

As this was my last Council meeting after a period of 10 years as a member of Council and as the Society's Librarian, it gave me pause to reflect. When I was first elected I wasn't quite sure what to expect. As it turned out, I was rather thrown in the deep end as it was when the Society was in the middle of considering a merger with the IMA which was a somewhat testing time for Council. Despite the rather challenging start, I can honestly say that I have really enjoyed being on Council and, through various committees, being closely involved with several different aspects of the Society's business. Needless to say, I have learnt a lot about the workings of the mathematical community at large and I have been constantly impressed by the time and effort put in by so many colleagues in order to support and promote mathematics through the good offices of the Society. Of course, nothing would be possible without the staff at De Morgan House and I would like to take this opportunity to thank them all for their hard work and kindness to me, particularly Elizabeth Fisher and Fiona Nixon with whom I have worked most closely and who with great patience and gentle efficiency made sure that the things I had agreed to do got done!

June Barrow-Green

BJPT Editorial Board Meeting 2019

Attendees at the Editorial Board meeting

The Editorial Board of the *Bulletin, Journal, Proceedings* and *Transactions of the LMS* met at De Morgan House on 10 January 2019 for one of its triennial meetings. Twenty-two Editorial Board Members from the UK and Europe and two Managing Editors joined the Publications Secretary, LMS staff and representatives of Wiley for a day of discussions. The Editorial Board is crucial to the ongoing success of the LMS journals, ensuring that the Society delivers journals which publish a diverse range of good mathematics, across a broad range of subject areas from an international authorship. The meeting provided an opportunity for the group to look back on the journals' performance over recent years, and to discuss their development in support of the Society's publishing aims.

The rate of change within the scholarly publications landscape is currently faster than ever, so the group also considered changes connected to how readers access the Society's journals, including debates around Open Access and a decline in printed copies.

The Editorial Board also spent some time discussing how best to implement workflow changes which aim to deliver faster times from submission to first decision. These changes will be made alongside the ongoing updates to editorial policies and the online peer-review management system, EditFlow.

Another major change which the Board discussed at length was the move to differentiate the *Bulletin, Journal, Proceedings* and *Transactions* by individual aims and scopes rather than by length alone (see the article on page 4 for more on this topic). One motivation for doing this is to make clear the robust set of criteria each journal adheres to when judging whether a paper should be published, aiming to ensure as transparent and fair process for all parties as we can; another is to increase the diversity and quality of mathematics that the Society publishes.

The meeting was also a great opportunity for Board members and staff, who rarely see each other face to face, to meet up, while new members who started their terms this January had the chance to hear the thoughts and suggestions of the 'old hands'. After nearly seven hours and 190+ pages of supporting documentation, those who could stay repaired to an informal dinner around the corner from De Morgan House.

Professor John Hunton
LMS Publications Secretary

The Hardy Collection

The LMS's Special Collections includes The Hardy Collection, which is housed in the Verblunsky Members' Room at De Morgan House. The room is open to all members Monday to Friday, 9am–5pm.

The Hardy Collection contains over 300 volumes from G.H. Hardy's personal library of books, which were used by him at various points throughout his career. As such, one can get a glimpse of the authors who influenced his thinking or caught his attention. Many of these volumes contain Hardy's signature and in some cases they also contain a dedication.

According to Hardy's will, the primary recipient of his collection following his death in 1947 was his collaborator J.E. Littlewood, with the remainder going to New College Oxford. In 1971, a Cambridge bookdealer, Galloway & Porter, bought most of the books from the Littlewood collection to sell on to the public. Aston University obtained a large part of the collection and in 1998, the Society bought some 300 volumes from Aston University to house at De Morgan House.

Since then, the Hardy Collection has been extended through various generous donations from members and through further acquisitions. However, the collection is not complete and the Society would like to build an overall picture of the whereabouts of as much of it as is possible. We would therefore welcome any further information that members may have on the current location of any books from Hardy's library (please email Elizabeth Fisher librarian@lms.ac.uk).

To view the books when visiting De Morgan House, please ask Reception (Ground Floor) for the key to the Collection.

Elizabeth Fisher
Membership & Grants Manager

REPORTS OF THE LMS

Report: Model Sets and Aperiodic Order

Neil Manibo, Jamie Walton and Uwe Grimm trying to assemble patches of Sébastien Labbé's Wang tiles

The conference *Model Sets and Aperiodic Order* was held at Durham University from 3 to 7 September 2018. Thanks to the generous funding from an LMS Scheme 1 Conference grant and a grant from Bielefeld University, Germany, the meeting was able to support a strong international showing, with 21 of the 24 speakers coming from outside of the UK. There were around 40 attendees, with three UK-based PhD students whose travel was made possible by the funds from the LMS.

The central theme of the meeting was aperiodic order, the study of infinite patterns of space which lack global translational symmetry but nonetheless enjoy a high degree of structural order. Projection method patterns, sometimes called model sets, are a rich class of such patterns whose geometric properties are closely linked with number theoretic questions. They are obtained by cutting and then projecting a slice of a higher dimensional lattice. Penrose's rhomb tilings are well-known examples. The field benefits from a rich variety of overlapping perspectives and techniques. There were speakers specialising in topological aspects of aperiodic order, in dynamical systems, mathematical diffraction, combinatorics and number theory.

Sébastien Labbé's Wang tiles

The meeting had several highlights. One was the unique setting of the accommodation, in Durham Castle. The breaks between talks were enlivened by an additional activity to the usual tea, coffee and mathematical discussions: participants could try their hand at assembling patches of Sébastien Labbé's Wang tiles. Sébastien brought 627 of his laser-cut tiles to the conference. He left with at least 343 fewer: the first seven people to complete a 7-by-7 patch were allowed to keep their tiles. Whilst there are 152,244 possible 7-by-7 solutions, finding one of them proved to be a significant challenge. It is unlikely that anyone found one of the roughly 0.5% of those which extend to an infinite tiling.

James Walton, University of Glasgow
Dan Rust, Bielefeld University

CORRECTIONS AND CLARIFICATIONS

On page 4 of the January issue of the *Newsletter*, Vice-President John Greenlees' institution was incorrectly given as the University of Sheffield. He is in fact Head of Department of the Mathematics Institute at the University of Warwick.

LETTER TO THE EDITOR

Dear Editor,

You are perhaps aware of the situation of a number of our Turkish colleagues who have been summoned to court hearings for having signed a Peace Petition, bit.ly/2RYZcyw, which was widely circulated in Turkey. At this moment, there are at least 5 mathematicians/computer scientists who are already on trial, and this number could increase at any time. They are accused of "making propaganda for a terrorist organization".

Against this background, please read the speech (bit.ly/2RMUKnh) made by Professor Aye Berkman at her trial as one of Academics for Peace. Her speech is a remarkable personal and human document — but also a powerful statement about the social responsibility of a mathematician.

One of us (A.D.) attended Aye Berkman's court hearing, 10 January 2019, as an independent international observer; his report contains further details of the trial and its potential outcome. Our mathematician colleague, Professor David Pierce (Aye Berkman's husband), wrote an analysis of indictment (bit.ly/2RKFF5B) against Peace Academics.

In addition, two undergraduate mathematics students face prosecution for taking part in an anti-war demonstration (one of them spent over 2 months in jail, but was released on bail). U.K. has attended, as an international observer, the trial of students (3 October 2018), and offers for your attention her report (bit.ly/2UmtVmu). A very brief summary: the situation these students are facing has many very scary and sad aspects. In particular, they have no chance of continuing their normal life (planning their future or keeping up with their studies) during the very long process of the trial.

Adrien Deloro and Ulla Karhumäki

Dr Adrien Deloro is a lecturer at Sorbonne Université, and Ulla Karhumäki is a PhD student at University of Manchester. They write in their personal capacities. The views expressed in this letter and linked documents do not necessarily represent the positions of the Editorial Board or the LMS.

Records of Proceedings at LMS meetings
South West & South Wales Regional Meeting: 17 December 2018

The meeting was held at the Harrison Building, University of Exeter, as part of the 2nd Workshop on Number Theory and Function Fields at the Crossroads. Over 20 members and guests were present for all or part of the meeting. The meeting began at 2.00 pm, with the President, Professor Caroline Series, FRS, in the Chair.

There were no members elected to Membership at this Society Meeting. Four members signed the Members' Book and were admitted to the Society by the President during the meeting, and in addition the President extended an invitation to those present to look through the Members' Book during the tea break.

Records of Proceedings for the following six Society Meetings held in 2018, and published in the LMS Newsletter, were presented to the attending members. The Records of Proceedings were signed as a correct record by the President:

- **LMS Ordinary Meeting:** *In Honour of Maryam Mirzakhani: Teichmüller Dynamics,* University of Warwick, 22 March 2018
- **LMS Midlands Regional Meeting:** *Galois Covers, Grothendieck-Teichmüller Theory and Dessins D'enfants,* University of Leicester, 4 June 2018
- **LMS Ordinary Meeting at the British Mathematical Colloquium 2018,** St Andrews University, 13 June 2018
- **LMS General Society Meeting and Hardy Lecture,** BMA House, London, 29 June 2018
- **LMS Ordinary Meeting at the International Congress of Mathematics 2018,** Rio de Janeiro, Brazil, 7 August 2018
- **LMS Ordinary Meeting Joint with the Institute of Mathematics and its Applications,** De Morgan House, London, 11 September 2018

Dr Julio Andrade introduced the first lecture, given by Professor Keith Ball (Warwick) on *Rational Approximations to the Zeta Function*.

Dr Julio Andrade then introduced the second lecture, given by Dr Min Lee (Bristol) on *Applications of Trace Formulas for GL(2)*.

After the tea break, Professor Nigel Byott introduced the third and final lecture, given by Professor Jens Marklof (Bristol) on *Chaos and Randomness Modulo One: The Interplay of Number Theory and Dynamical Systems*.

The President thanked the speakers for their talks, and further extended her warm thanks to the local organisers, Dr Julio Andrade and Professor Nigel Byott, for organising and holding such an exemplary and interesting meeting.

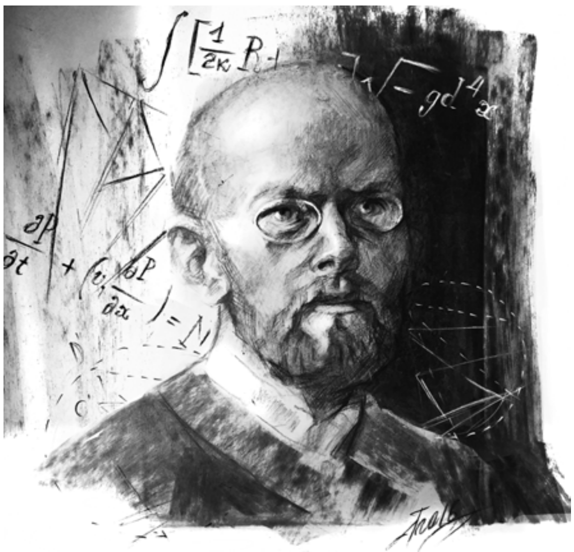
Professor Nigel Byott thanked the speakers, and invited all those present to attend a wine reception held in the foyer of the Harrison Building. A Society Dinner was held following the wine reception at the Queen's Court Hotel.

Hilbert's Sixth Problem: the Path to Mathematical Rigour in Science

ALEXANDER N. GORBAN

In his Sixth Problem, Hilbert proposed the expansion of the axiomatic method outside of mathematics. Its original challenge, the "Mathematical Treatment of the Axioms of Physics", remains beyond the horizon of modern mathematics. Nevertheless, progress has been made. We present a bird's-eye view of the history of expectations, efforts, and results inspired by this problem.

A hierarchy of questions hidden in the Sixth Problem



David Hilbert in the 1900s, artist Anna Gorban, (CC BY-SA 4.0)

In the year 1900, David Hilbert presented 10 problems to the International Congress of Mathematicians and published the full list of 23 problems a bit later [1]. This was a very successful attempt to direct future research in Mathematics through a list of problems. The problems were very diverse and many mathematicians responded to this challenge.

Even in this list of diverse problems there was an outlier. The Sixth Problem did not seem to be mathematical or physical. The title of this problem was mysterious: Mathematical Treatment of the Axioms of Physics. There is no such thing as an axiom of physics. Physics, in its essence, is a special activity for the creation, validation, and destruction of theories for real-world phenomena. In physics, "We never are definitely right, we can only be sure we are wrong." Moreover, "we are trying to prove ourselves wrong as quickly as possible, because only in that way can we find progress" (R. Feynman, *The character of physical law*). All physical truths are only temporary.

The axiomatization program was proposed by Hilbert as a solution to the crisis of foundations in mathematics. "The foundations of geometry" (1899) firmly implanted the axiomatic method not only in the field of geometry but also in other branches of mathematics. After the great success of this book, Hilbert proclaimed the expansion of the axiomatic method beyond existing mathematical disciplines, into physics and further on. He started his explanation of the Sixth Problem with direct reference to the foundations of geometry: "The investigations on the foundations of geometry suggest the problem: to treat in the same manner, by means of axioms, those physical sciences in which already today mathematics plays an important part; in the first rank are the theory of probabilities and mechanics." Here we see not only an explanation, but also a disclaimer: the axioms of physics are no longer mentioned, but the axiomatization of those physical sciences that are ready for this operation is proposed. This reduction of claims turned the mysterious "Mathematical Treatment of the Axioms of Physics" into a programmatic call: extract axiomatizable fragments of physics and transform them into axiomatized mathematical theories.

Two areas of physics were proposed as the first candidates for axiomatization: probability theory and mechanics (note that for Hilbert in 1890, probability theory was more physics than mathematics). Two other areas were proposed in several later paragraphs: "As to the axioms of the theory of probabilities, it seems to me desirable that their logical investigation should be accompanied by a rigorous and satisfactory development of the method of mean values in mathematical physics, and in particular in the kinetic theory of gases. ... Boltzmann's work on the principles of mechanics suggests the problem of developing mathematically the limiting processes, there merely indicated, which lead from the atomistic view to the laws of motion of continua." Equilibrium statistical mechanics was developed a bit later (by Gibbs, 1902, and Einstein, 1902–1903, independently). Now, we can interpret these two parts of the Sixth Problem as (i) a rigorous theory of equilibrium statistical physics and (ii) a rigorous path, which leads from the atomistic view to the laws of motion of continua. Surprisingly, the title of Einstein's 1902 paper can be read as an attempt to answer partially the Sixth Problem: "Kinetic Theory of Thermal Equilibrium and of the Second Law of Thermodynamics".

Thus, Hilbert cascaded from the mysterious axiomatization of physics to the programmatic calls of axiomatic theories of those fragments of physics, which are already quite mathematical, and, finally, to four specific queries: axiomatic probability theory, mechanics, statistical physics and the theory of level jumps from atomic motion to continuum mechanics and thermodynamics (with special attention to the hydrodynamic limit of Boltzmann's equation).

The Big Sixth Problem

The question remains if the "axioms of physics" in the title of the Sixth Problem is a hint at issues that go beyond modern science, or if is it just a rhetorical device for attracting the attention of readers? The explanation that follows the headline reduces the shock of the title, and we should also take into account that Hilbert was a highly qualified scientist and a very experienced science writer. He must have understood the potential reaction of the readers of this title. The title must have been chosen with intent. It is a message, a mystery to future generations.

I would venture to suggest taking seriously the challenge contained in the title of the Sixth Problem. This can become a gateway to mathematical metaphysics, just as the axiomatic method led to the creation of mathematical metamathematics. Let us call this the "Big Sixth Problem". The main difficulty is that this mathematical metaphysics should not depend on a specific physical theory and should support the development of physics using logical tools and the atheoretical semantics of physical reality (following Feynman's maxim: "We never are definitely right, we can only be sure we are wrong"). Physicists operate by the concept of physical reality, experiment, and theory in a broader context than a particular physical theory. We should formalize these concepts, but we must bear in mind that this formalization is also temporary. It will also change over time.

If we accept this challenge then we should consider a hierarchy of problems:

- The Big Sixth Problem (mathematical metaphysics).
- The programmatic call for mathematization and axiomatization of the fields of physics which are ready for this approach.
- The specific calls for axiomatization of selected areas: probability, equilibrium statistical physics, kinetic theory with hydrodynamic limit, and mechanics.

It is also possible that any theoretical construction of physics will always include various internal contradictions and, instead of a single theory, we will have to permanently discuss the web of various standard models. The network of standard models, which contradict each other and have known mismatches with reality, is the everyday practice of applied science. The belief in the rational construction behind the Universe, the "ultimate rationalism which urges forward science and philosophy alike" (Whitehead) is a very powerful driving force for theoretical physics. Nevertheless, it may happen that our world will always be an "impossible possible world" (Hintikka) with contradictions, and the idea of nice rational construction behind the world will remain a beautiful dream. The mathematical tools for work with such a contradictory physical world are needed if we are to take the challenge of the Big Sixth Problem seriously.

Important results of the Hilbert program for physical disciplines

There are two opposing views on efforts to solve the Sixth Problem:

- Of all the problems on the Hilbert list, the Sixth Problem received the least attention and little effort has been directed towards solving it.

- The sixth problem inspired a huge flow of research and many important achievements.

This difference in positions is caused mostly by the different understandings of the problem. The Big Sixth Problem is still beyond the horizon and little effort has been directed towards solving it, indeed. The axiomatic quantum field theory [4] and various attempts of the "theory of everything" are just applications of the axiomatic method to important but yet particular physical theories (like axiomatization of mechanics, quantum mechanics, etc.). No atheoretical semantic has been developed and no tools for work in the world of dynamically developing physics have been created. Let us recall that "we are trying to prove ourselves wrong as quickly as possible". This is the life of physics. In this dynamically changing world any axiomatization is just temporary. The meta-knowledge will be, perhaps, also temporary but will be elaborated for a longer life in the developing physics.

The programmatic call for an axiomatic treatment of the physical disciplines which are ready for it has been fantastically successful. The first great achievement was the mathematical foundation of quantum mechanics. The famous book of von Neumann [5] was published in 1932. This project was inspired by Hilbert. Simultaneously with the birth of quantum theory (in 1925), Hilbert devoted a seminar to the description of its mathematical structure. The notes of this seminar were collected by two assistants, von Neumann and Nordheim.

The second great achievement is the axiomatic theory of classical probability. The Kolmogorov work on axiomatics of probability was inspired by Hilbert, at least, partially: in his book [6] from 1933, Kolmogorov did not mention the Sixth Problem, but did explicitly refer to Hilbert's Foundations of geometry as the prototype for "the purely mathematical development" of the theory. Moreover, Kolmogorov's trip abroad, from June 1930 to March 1931, had included a stay at Hilbert's department in Göttingen [7].

In 1943, another hero of the development of probability theory, Khinchin, published a book entitled *Mathematical foundations of statistical mechanics* [8]. He gave a "rigorous and satisfactory development of the method of mean values" in a branch of mathematical physics, equilibrium statistical mechanics. This was exactly a partial answer to the Sixth Problem, as Hilbert requested. Again, he did not refer to the Sixth Problem but the direct analogy to the title of von Neumann's book was obvious. It is worth mentioning that both Khinchin and Kolmogorov were from the same Moscow mathematical school, "Luzitania", named in the honour of their teacher, N.N. Luzin.

These three books give an impressive answer to the Sixth Problem's programmatic call. The axiomatics of mechanics, also requested by Hilbert, is now such a vast area that a separate and extensive overview is necessary.

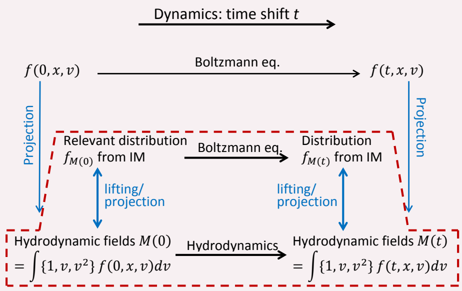
"From the atomistic view to the laws of motion of continua"

The fourth specific request of Hilbert in the sixth problem was the mathematical theory of "the limiting processes, … which lead from the atomistic view to the laws of motion of continua". The way from the mechanics of atoms to continuum mechanics includes at least two steps: (i) from mechanics to kinetics (from Newton to Boltzmann, in the classical world), and (ii) from kinetics to mechanics and nonequilibrium thermodynamics of continua (from Boltzmann to Euler and Navier-Stokes-Fourier equations).

Hilbert presumed the kinetic level of description (the "Boltzmann level") as the natural intermediate step between the microscopic mechanical description and the continuum mechanics. The first part of the problem (from Newton to Boltzmann) is still far from being a complete rigorous theory. The second part, (from Boltzmann to Euler and Navier-Stokes-Fourier), is ready for a mathematical treatment and a lot of effort has been put into this problem for a century. The first approach was invented by Hilbert in 1912 — Hilbert's method for solution of Boltzmann's equation. He started from the local equilibrium and solution of the corresponding Euler equation, then used this solution to find the first non-equilibrium correction, etc.

Enskog in 1916 decoupled the derivation of equations from their solutions. This method was developed further by Chapman, Burnett and many other researchers. The Chapman–Enskog approach has become the method of choice in the transition from kinetics to continuum mechanics. Enskog neglected mathematical rigour and the mathematical essence of his method remained unclear until 1969, when McKean explained it on a simple model. He gave the first clear explanation of the problem as a construction of a "nice submanifold" where "the hydrodynamical equations define the same flow as the (more complicated) Boltzmann equation does". He presented the problem using a partially commutative diagram (See "The McKean Diagram".)

The McKean Diagram



The Boltzmann equation describes the dynamics of a one-particle distribution function of positions x and velocities v of particles, $f(t, x, v)$. The hydrodynamic variables are moments of this distribution. The Chapman–Enskog method aims to create a lifting operation, from the hydrodynamic variables to the corresponding distributions on the dynamically invariant manifold (IM). The part of the diagram in the dashed polygon should be commutative. The classical Chapman–Enskog procedure creates an expansion for IM in powers of a small parameter, the Knudsen number. Commutativity of the diagram gives the invariance equation, as described in [9].

At the beginning of 1990s, Bardos, Golse, and Levermore proposed a program to obtain the theorem that finds the hydrodynamic limit of Bolzmann's equation. They did not require artificial conditions for the initial state, but only physically meaningful a priori estimates of density, energy, and entropy. A group of brilliant mathematicians joined the work of this program and the results of almost twenty years of intensive work are summarised in the book of Laure Saint-Raymond [10]. The main result is the incompressible Navier–Stokes–Fourier limit for the solutions of the "properly scaled" Boltzmann equation. This convergence result holds globally in time, and does not require any constraint on the initial hydrodynamic fields. The limiting form of the boundary conditions is found. The incompressive Euler limit (another scaling) is also described. The main remaining problem is the description of compressible limits when the Mach number does not tend to zero.

For some very simplified kinetic equations, the hydrodynamic limit problem can be solved explicitly [9]. Slemrod [11] analysed such an explicit solution and found the capillarity effects in a perfect gas. It seems that the proper compressible hydrodynamic limit of Boltzmann's equation (in McKean's sense) could include new terms and do not coincide with the usual compressible Euler or Navier–Stokes–Fourier equations.

A promising approach to this part of the Sixth Problem (from atomistic view to continuum mechanics) is analysis of direct limit transition, from particles to continuum mechanics, from Newton to Euler and Navier–Stokes–Fourier equations without such an intermediate element as kinetic equations. If the deviations from equilibrium are relatively small then such a transition could be possible [12].

A time to gather stones together

Results of work on the Sixth Problem can be found in various scientific areas. Let us gather these together, from bottom up.

The specific particular questions stated by Hilbert in the Sixth Problem seems to be largely solved:

- The Kolmogorov axiomatic framework of probability [6] is widely accepted as the truth.
- For quantum probabilities, discovered in physics early in the 20th Century, Hilbert's team found the appropriate mathematical form [5].
- The main ideas for the mathematical background of equilibrium statistical physics were proposed by Gibbs and Einstein, developed further in the spirit of the Sixth Problem on the basis of the limiting theorems of probability theory [8], and transformed now into the concentration of measure theory, the central chapter of geometric functional analysis.
- The "properly scaled" solutions of the Boltzmann equation have the incompressible Navier–Stokes–Fourier limit [10].

These results are not final. The road to rigour is long or even endless. Kolmogorov's foundations of probability are neither the first nor the last. In particular, Kolmogorov was not completely satisfied with the theory, returned to this problem in 1967, and proposed an algorithmic theory of randomness and complexity (simultaneously with Solomonoff). The study of the compressible hydrodynamic limit of the Boltzmann equation and analysis of recently found exact solutions of the "kinetics to hydrodynamics" reduction problem [9] lead to new hypotheses and equations [11]. Quantum probability was developed further and a big flow of new applications of this non-classical probability theory goes now back, from mathematics to science and engineering, including even such areas as psychology and economics.

This endless movement is a clear manifestation of the fact that the programmatic call, announced by Hilbert in his Sixth Problem, is relevant now and will be relevant in the future, as far as we can see. The work goes both in depth and in breadth. New areas are and will be involved in this quest for mathematical rigour.

But what about the Big Sixth Problem? It is my dream that I shall read the first solution of the Big Sixth Problem in this life.

Acknowledgements

The conference "Hilbert's Sixth Problem" (Leicester, 2016) and the preparation of the special issue [13] was supported by LMS (Ref 11503), EPSRC (EP/N022653/1), and IMA.

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Measuring Changing Standards in Mathematics Exams

IAN JONES, CHRIS WHEADON, SARA HUMPHRIES AND MATTHEW INGLIS

Have A level mathematics standards been maintained over time? This question is of obvious interest to universities, employers, students and the government, but is difficult to answer using traditional methods. We used *comparative judgement*, a method that relies upon combining very many pairwise judgements of 'quality', to tackle this question, finding mixed results. [1]

The majority of entrants on BSc Mathematics programmes in the UK arrive having taken an A level mathematics examination of one form or another. As is the case in many countries, concerns have been expressed that the standards of A level mathematics exams have declined over time, and that school leavers therefore enter university mathematics degrees without the required level of knowledge and skills. Unfortunately, accurately assessing examination standards over time is a difficult task, and therefore claims of dropping standards have tended to be based on anecdote and personal belief rather than robust research evidence.

We recently reported, in the *British Educational Research Journal*, a study that sought to address this problem by comparing standards over the last five decades of A level mathematics exams [1]. We used a novel method based on experts' comparative judgement, which we think avoids many of the problems of existing work in this area.

Our goal was to address the following question. A level mathematics has been offered as a qualification since the 1950s, but have standards been maintained over this period? In other words, is a candidate who obtained an A grade in the 1950s comparable to a candidate who obtains an A grade today? Unfortunately, this question is not straightforward to answer.

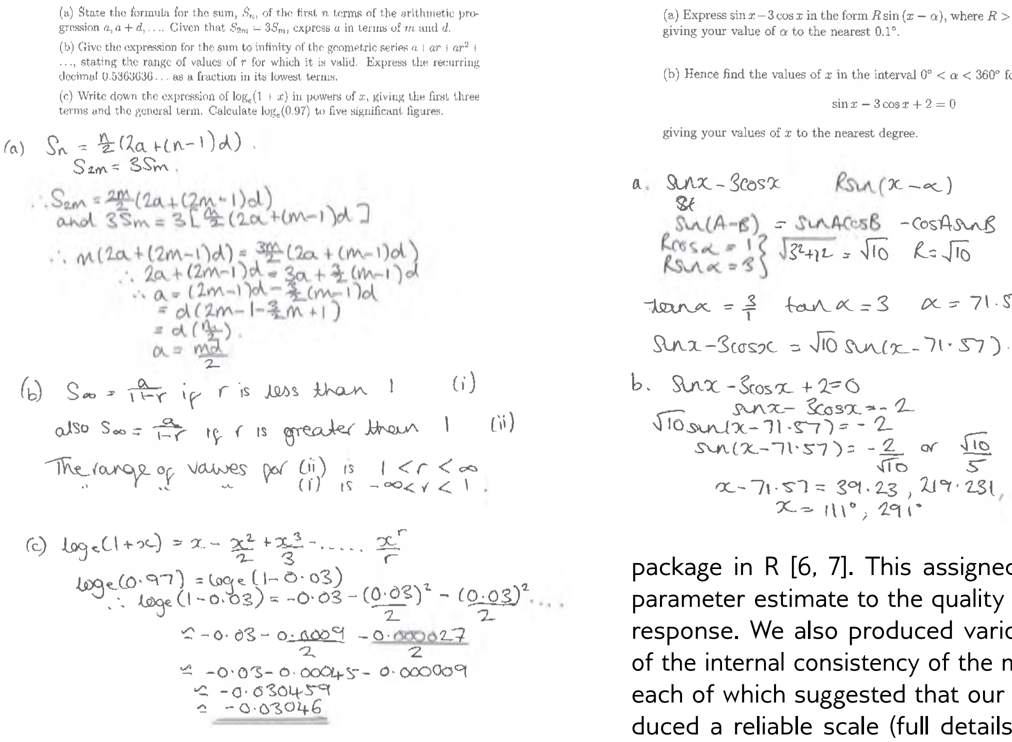
One obvious approach is to administer a common test to candidates from different years: if the A level grades given to candidates who achieved the same scores on the common test differ, then this provides some evidence that standards have changed [2, 4]. However, given that course syllabuses change on a regular basis it is unclear if this method is merely measuring the extent to which the examinations in different years are similar to the common test. Another method is to give old examination papers to modern candidates. But again, this approach is vulnerable to changes in syllabuses and the style of examination questions.

For our investigation we adopted a different strategy, based on the method of comparative judgement. The rationale for this approach is that, across a whole range of stimuli, human judgement is more accurate when it compares two objects than when it judges a single object in isolation [5]. For example, it is much easier to state which of two rooms is the warmer than it is to estimate the temperature of a single room. Psychophysicists, led by L. L. Thurstone in the early 20th century, now have good models that account for comparative judgements across a whole host of stimuli.

In a comparative judgement exercise mathematicians are presented with two candidates' work, side by side, and are simply asked to assess which is better. The decision is based on a simple, high-level criterion such as 'who is the better mathematician?' The outcomes of many such decisions from many experts can then be statistically analysed, using the so-called Bradley-Terry model [6], to produce a relative parameter estimate of the 'quality' of each script. Between-judge consistency can be estimated to determine the extent to which judges have assessed the same construct. In other words, through this process we are able to evaluate the extent to which mathematicians tend to agree on what makes a good mathematician.

To conduct our study we obtained 66 student A level examination scripts from 1964, 1968, 1996, and 2012 from the national archive held by Ofqual, the government's examinations regulator. Ideally, we would have been able to include scripts from many more years, but unfortunately the archive is rather sparse.

[1]An earlier version of this article appeared in volume 24 of *Mitteilungen der Deutschen Mathematiker-Vereinigung*.



(a) State the formula for the sum, S_n, of the first n terms of the arithmetic progression $a, a+d, \ldots$. Given that $S_{2m} = 3S_m$, express a in terms of m and d.

(b) Give the expression for the sum to infinity of the geometric series $a + ar + ar^2 + \ldots$, stating the range of values of r for which it is valid. Express the recurring decimal 0.5363636... as a fraction in its lowest terms.

(c) Write down the expression of $\log_e(1+x)$ in powers of x, giving the first three terms and the general term. Calculate $\log_e(0.97)$ to five significant figures.

(a) $S_n = \frac{n}{2}(2a+(n-1)d)$. $S_{2m} = 3S_m$.
$\therefore S_{2m} = \frac{2m}{2}(2a+(2m-1)d)$ and $3S_m = 3[\frac{m}{2}(2a+(m-1)d]$
$\therefore m(2a+(2m-1)d) = \frac{3m}{2}(2a+(m-1)d)$
$\therefore 2a+(2m-1)d = 3a+\frac{3}{2}(m-1)d$
$\therefore a = (2m-1)d - \frac{3}{2}(m-1)d$
$= d(2m-1-\frac{3}{2}m+1)$
$= d(\frac{m}{2})$.
$a = \frac{md}{2}$

(b) $S_\infty = \frac{a}{1-r}$ if r is less than 1 (i)
also $S_\infty = \frac{a}{1-r}$ if r is greater than 1 (ii)
The range of values for (ii) is $1 < r < \infty$
(i) is $-\infty < r < 1$.

(c) $\log_e(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \ldots \frac{x^r}{r}$
$\log_e(0.97) = \log_e(1-0.03)$
$\therefore \log_e(1-0.03) = -0.03 - \frac{(0.03)^2}{2} - \frac{(0.03)^2}{2} \ldots$
$\simeq -0.03 - \frac{0.0009}{2} - \frac{0.000027}{2}$
$\simeq -0.03 - 0.00045 - 0.000009$
$\simeq -0.030459$
$\simeq -0.03046$

(a) Express $\sin x - 3\cos x$ in the form $R\sin(x-\alpha)$, where $R > 0$ and $0° < \alpha < 90°$, giving your value of α to the nearest 0.1°.

(b) Hence find the values of x in the interval $0° < \alpha < 360°$ for which

$$\sin x - 3\cos x + 2 = 0$$

giving your values of x to the nearest degree.

a. $\sin x - 3\cos x$ $R\sin(x-\alpha)$
$\sin(A-B) = \sin A\cos B - \cos A\sin B$
$R\cos\alpha = 1$, $R\sin\alpha = 3$ $\sqrt{3^2+1^2} = \sqrt{10}$ $R = \sqrt{10}$
$\tan\alpha = \frac{3}{1}$ $\tan\alpha = 3$ $\alpha = 71.56 = 71.57$ 2d.p.
$\sin x - 3\cos x = \sqrt{10}\sin(x - 71.57)$.

b. $\sin x - 3\cos x + 2 = 0$
$\sin x - 3\cos x = -2$
$\sqrt{10}\sin(x-71.57) = -2$
$\sin(x-71.57) = -\frac{2}{\sqrt{10}}$ or $\frac{\sqrt{10}}{5}$
$x - 71.57 = 39.23, 219.231$,
$x = 111°; 291°$

Figure 1. Two responses: which candidate is the better mathematician?

In particular, no scripts at all have been retained from examinations conducted in the 1970s, 80s or 2000s.

Each of the 66 scripts had been awarded a grade A, B or E. Only papers graded unambiguously (not near the grade boundaries) were included. These 66 scripts were split into 546 individual questions, which formed the input to our comparative judgement exercise. We were concerned that candidates' handwriting would reveal the age of the script, so we re-typeset all questions and then re-wrote each of the 546 answers by hand using the same handwriting.

Our twenty judges were all PhD mathematics students. Critically, they were not told the purpose of the study, and a post-study questionnaire revealed that none guessed that we were interested in standards over time. The judging took place using an online comparative judgement engine (nomoremarking.com). For each judgement the judge saw two pieces of work, as in Figure 1, and simply had to indicate which candidate was the better mathematician.

We obtained a total of 5000 judgements of this sort, and fitted these data using the Bradley-Terry 2 package in R [6, 7]. This assigned a standardised parameter estimate to the quality of each question response. We also produced various assessments of the internal consistency of the modelling process, each of which suggested that our method had produced a reliable scale (full details are given in [1]). The mean parameter estimates of all the questions attempted by each candidate were calculated to produce an overall 'ability' measure for that candidate.

The mean scores for each grade at each year are shown in Figure 2. We conducted a multiple regression analysis using these scores as the dependent variable and grade (A=5, B=4, E=1) and year as predictors. The model predicted 74.8% of the variance in mean parameter estimates. Both grade, $\beta = 0.69, p < .001$, and year, $\beta = -0.51, p < .001$, were significant predictors. As can be seen in Figure 2, this substantial decline in standards appeared to take place between the 1960s and 1990s: roughly speaking we found that a candidate who achieved a grade B in the 1990s or today exhibited a similar level of achievement to a candidate who achieved an E in the 1960s. Contrary to some claims made by the current UK government, we found no evidence for a decline in standards between the 1990s and today.

Overall, our comparative judgement approach appeared to work well. Judges had no difficulty with our high-level criterion, and we were able to fit the data to the Bradley–Terry model with no problems. We therefore believe that comparative judgement offers a productive way of assessing educational standards across different curricula.

This issue is particularly timely for university-level educators given recent concerns about grade inflation in undergraduate degrees. For instance, in October

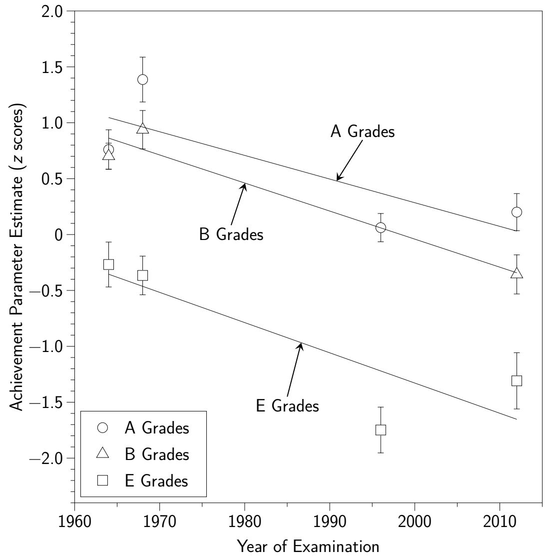


Figure 2. Mean standardised parameter estimates for the scripts at each grade and year. Error bars show ±1 SE of the mean.

a report in *The Guardian* [3] announced that the government planned to "crack down" on universities that awarded too many first class degrees. The then minister, Sam Gyimah, was quoted as saying:

> When you look at what makes our universities so prestigious, it comes down to the value of our degrees. The value of those degrees is threatened by grade inflation and that is a problem for students, employers and the universities themselves.

Without conducting an exercise analogous to the sort reported here it is hard to know whether these concerns are merited. If your institution has access to historical undergraduate examination scripts and would like to be involved in a project to examine undergraduate mathematics standards over time please get in contact with us. If your institution (like ours) does not keep historical scripts, you may wish to consider whether a change in policy is merited.

Futher resources:

- You can judge some A level examination scripts for yourself by visiting our site at: tinyurl.com/judgeAlevel
- If you would like to conduct a comparative judgement exercise yourself, then the NoMoreMarking engine is free to use for non-commercial purposes: www.nomoremarking.com.

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Fractal Geometry and Dynamics: One semester at Institut Mittag-Leffler

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The study of rough irregular objects (fractals) and the evolution of systems over time (dynamics) have attracted much attention in recent years, both in mathematics and physics. In this feature, we motivate and present a selection of open problems and stimulating advancements made in these fields, with a focus on research stemming from a 2017 Autumn research semester at Institut Mittag-Leffler[2].

Introduction

(a) Fractal geometry — Modelling irregular sets

Fractal geometry is a part of modern mathematical analysis. The fundamental research theme is to study and model geometric objects which are often so irregular that the methods of classical analysis are unsuitable or inefficient. Such objects occur, for instance, in nature (coast-lines, human anatomy, mountain ranges and volcanic rock), as invariant sets of dynamical systems, for example, attractors of systems of differential equations or group actions (Hénon attractors and limit sets), and as implicitly and explicitly defined objects in number theory, physics and other areas of the natural sciences (badly approximable numbers, quantum gravity and Brownian motion).

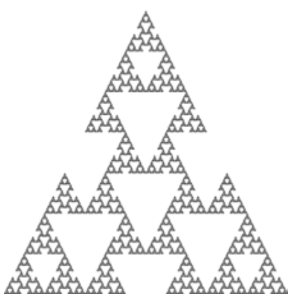

Figure 1. A modified (self-similar) Sierpiński gasket

The most prominent, simplest and widely studied type of fractal model assumes self-similarity, meaning, geometrically, if one zooms in on the structure, then one sees an exact scaled down replica of the original structure, see for instance Figure 1. Since in nature and in dynamically and implicitly defined irregular objects, often such precise self-similarity does not occur, there is an abundance of interest and need to investigate more general models as well as tools which can be used to characterise and study such irregular objects.

Among these more general models are self-affine and -conformal sets, where, when zooming in, one respectively sees an affinely or conformally distorted copy of the original set, see for instance Figure 2 and "Some models for irregular sets". Long standing and developing theories that are used to analyse and understand such sets are dimension theory, potential theory and thermodynamics to name but a few. In the present feature, we will pay particular attention to dimension theory.

Some models for irregular sets

Self-similar, -affine and -conformal sets arise as attractors of iterated function systems. Given a compact nonempty set $X \subseteq \mathbb{R}^n$ equipped with the Euclidean norm $|\cdot|$ an *iterated function system* (*IFS*) is a finite family $\Phi := \{\phi_1, \ldots, \phi_m\}$ of contracting maps $\phi_i \colon X \circlearrowleft$. For each IFS there exists a unique nonempty compact set $F \subseteq X$ satisfying $F = \bigcup_{i=1}^{m} \phi_i(F)$, called the *attractor* of Φ.

If all the maps are similarities, i.e. there exist $r_1, \ldots, r_m \in (0,1)$ with $|\phi_i(x) - \phi_i(y)| = r_i |x-y|$ for all $x, y \in X$, then F is called *self-similar*; if all the maps are affine, then F is called *self-affine*; if all the maps are conformal (preserve angles), then F is called *self-conformal*.

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[2]Many of the advancements mentioned in this feature were initiated and substantiated during the semester at Institut Mittag-Leffler. Details of it can be found at tinyurl.com/IMLfgd2017.

Cantor and Mittag-Leffler

Cantor is famous for his fundamental work in set theory. Initially, Cantor's achievements were controversially received. His, at first sight, counter-intuitive ideas were discarded by many influential mathematicians. However, Cantor also had prominent supporters, among them Hilbert and Mittag-Leffler in whose journal, *Acta Math.*, Cantor published a significant number of articles.

Cantor and Mittag-Leffler were in regular correspondence and numerous letters are displayed in the library of Institut Mittag-Leffler. In his letters, Cantor did not hide his disappointment at the reactions he received to his work. In 1885 the correspondence ended, after Mittag-Leffler rejected one of Cantor's articles from being published in *Acta Math.* providing the reason that his work would be 100 years too early.

Nowadays, Cantor's achievements are well-received and are fundamental to many areas of mathematics, in particular, to fractal geometry and dynamical systems.

(b) Dimension theory — Historical remarks

The desire for introducing concepts of dimension has a long standing history. For instance, in Euclid's Elements (~ 300 b.c.) it states the following.

I. A point is that which has no parts.
II. A line is length without breadth.
III. The extremities of a line are points.
IV. A straight line is that which lies evenly between its extreme points.
V. A surface is that which has only length and breadth.
VI. The extremities of a surface are lines.

A fruitful interpretation of Euclid's concept was given by Henri Poincaré in his 1913 essay *Dernières pensées*, to which Benoît Mandelbrot gave the following free translation:

> When we say that space has the dimension three, what do we mean? If to divide a continuum $\mathscr{C}$ it suffices to consider as cuts a certain number of distinguishable elements, we say that this continuum is of *dimension one* [...]. If, on the contrary, [...] to divide a continuum it suffices to use cuts which form one or several continua of dimension one, we say that $\mathscr{C}$ is a continuum of *dimension two*. If cuts which form one or several continua of at most dimension two suffice, we say that $\mathscr{C}$ is a continuum of *dimension three*.

At the end of the 19th century, works of Georg Cantor and Giuseppe Peano appeared, which puzzled the community (see "Cantor and Mittag-Leffler"). Cantor showed that the cardinalities of the unit interval $[0,1]$ and the unit square $[0,1]^2$ coincide by providing an injection from $[0,1]^2$ to $[0,1]$. Subsequently, Peano constructed a continuous surjection $\gamma\colon [0,1] \to [0,1]^2$. Now, $[0,1]^2 = \gamma([0,1])$ can be cut at the single element $\gamma(1/2)$ into $\gamma([0,1/2])$ and $[0,1]^2 \setminus \gamma([0,1/2])$. Thus, by the previous definition, $[0,1]^2$ could be thought to be one-dimensional, but clearly we wish to assign dimension two to $[0,1]^2$.

Thus, new concepts of dimension were sought. Nowadays, irregular objects are often classified by their "fractal dimensions", for instance Hausdorff, Minkowski, similarity, affinity or Assouad dimension. All these dimensions, although different in general, have in common that they take non-negative values in $\mathbb{R}$ which are not necessarily integers. For the modified Sierpiński gasket, displayed in Figure 1, all above-mentioned "fractal dimensions" coincide, taking the value of approximately 1.56. Intuitively, this means that the modified Sierpiński gasket has 'more substance' than a one-dimensional object (whose length we can measure), but 'less substance' than a two-dimensional object (which has positive area). Three of the above-mentioned dimensions are introduced in "Similarity dimension", "Hausdorff dimension" and "Affinity dimension". Often it is challenging to determine the dimension of a set, and it is an active area of research to gain formulas for these dimensions for classes of fractal sets, as we will shortly see.

(c) Dynamical Systems

In dynamical systems the fundamental research theme is to study self-maps f on (compact) topological spaces (X,τ); the pair (X,f) is referred to as a *dynamical system*. As f is iterated (repeatedly applied to subsets of X), asymptotic structure appears, for example, eventually periodic orbits, strange attractors, recurrent and non-wandering points, as well as minimal sets. Results and methods from fractal geometry and dimension theory can be used to develop an understanding of such structures, points and sets. On the other hand, classes of self-similar, -affine and -conformal sets arise as forward invariant sets of dynamical systems, where a set $A \subseteq X$ is called *forward invariant* if $f(A) \subseteq A$.

Similarity dimension

Let $\Phi = \{\phi_1, \ldots, \phi_m\}$ be an IFS of similarities acting on $X \subseteq \mathbb{R}^n$ with attractor F. Let r_i denote the similarity ratio of ϕ_i and assume that $\phi_i(F) \cap \phi_j(F) = \emptyset$ for all $i \neq j$. The *similarity dimension* of F is the unique real solution of the equation $\sum_{i=1}^m r_i^s = 1$. It is a well-defined notion of the attractor F, i.e. it is independent of the choice of IFS.

The condition $\phi_i(F) \cap \phi_j(F) = \emptyset$ can be relaxed to the *open set condition* (*OSC*), i.e. existence of an open set $O \subseteq X$ satisfying $\phi_i(O) \cap \phi_j(O) = \emptyset$ for all $i \neq j$.

The gasket S given in Figure 1 is the attractor of an IFS consisting of 21 similarities, each having similarity ratio $1/7$, acting on X, the convex hull of S, an equilateral triangle. The OSC is satisfied with O the interior of X. Thus, the similarity dimension is the unique real solution of $1 = 21/7^s$, i.e. $s = \log(21)/\log(7)$.

Hausdorff dimension

For $U \subseteq \mathbb{R}^n$, let diam(U) denote the diameter of U. For $\delta > 0$ and $E \subseteq \mathbb{R}^n$, we call a family $\{U_k\}_{k\in\mathbb{N}}$ of subsets of $\mathbb{R}^n$ with diam$(U_k) \leq \delta$, satisfying $E \subseteq \bigcup_{k\in\mathbb{N}} U_k$, a δ-*cover* of E; the collection of all δ-covers of E is denoted by $\mathcal{U}_\delta(E)$. For $s \geq 0$ the s-*dimensional* (*outer*) *Hausdorff-measure* of E is defined as

$$\mathcal{H}^s(E) := \sup_{\delta>0} \inf_{\{U_k\}\in\mathcal{U}_\delta(E)} \sum_{k\in\mathbb{N}} \mathrm{diam}(U_k)^s.$$

One can show that $\mathcal{H}^s$ is a metric outer measure implying that every Borel subset of $\mathbb{R}^n$ is $\mathcal{H}^s$-measurable. In fact, $\mathcal{H}^s$ defines a measure on the Borel σ-algebra.

The *Hausdorff dimension* of a Borel subset E of $\mathbb{R}^n$ is defined to be

$$\dim_{\mathrm{H}}(E) := \inf\{s \geq 0 \colon \mathcal{H}^s(E) = 0\}.$$

If the similarity dimension of E is defined (for which E in particular needs to be a self-similar set and the OSC needs to be satisfied), then the Hausdorff dimension of E coincides with its similarity dimension, this is commonly known as the Moran–Hutchinson formula.

Further insight to asymptotic structures of dynamical systems can be gained by taking a probabilistic viewpoint to investigate the average statistical behaviour of the system. This naturally leads to applications and developments of multifractals, the thermodynamic formalism (as pioneered by David Ruelle and Rufus Bowen) and ergodic theory. Taking things one step further, one can begin to study the speed at which the system begins to look random, a key signature of what is known as chaotic behaviour.

We return to the connection between fractal geometry and dynamical systems, as well as applications of the thermodynamic formalism in parts (c) and (d) of the next section.

Mathematical advancements

Below we give a flavour of some recent advancements made in the above vibrant areas of mathematical research. We highlight three topics, which are linked via dimension theory. In particular, we discuss recent progresses in

(a) Falconer's distance set conjecture,
(b) fractal dimensions of self-affine sets, and
(c) characterising irregular sets beyond dimension.

These topics have at their heart determining invariants which describe the complexity of the underlying rough, irregular and complex sets. In addition to these three topics, we discuss, in (d), an interesting related open problem, namely Littlewood's conjecture, and how dynamical systems and dimension theory are being utilised to approach this challenging open conjecture in multiplicative Diophantine approximation.

(a) Falconer's distance set conjecture

Given $A \subseteq \mathbb{R}^n$, its *distance set*

$$\Delta(A) = \{|x - y| \colon x, y \in A\}$$

is the set of all distances between points in A, where $|x-y|$ denotes the Euclidean distance between x and y. An interesting question, which has been studied by many leading mathematicians, including Paul Erdős and Terence Tao, is the following. Given the "size" of A, how "big" is $\Delta(A)$?

In the situation that $n = 2$ and A is of finite cardinality, this is Erdős' famous 1946 *distinct distances problem*. A solution was given in 2015 by Larry Guth and Nets Katz, who showed $|\Delta(A)| \geq c|A|/\log|A|$, where c is some global constant and $|A|$ denotes the cardinality of A. But what can be said when A is of infinite cardinality?

In the situation that $n \geq 2$ and $A \subseteq \mathbb{R}^n$ is a Borel (or even analytic) set, Kenneth Falconer proved in 1985,

$$\dim_{\mathrm{H}}(\Delta(A)) \geq \min\{1, \dim_{\mathrm{H}}(A) - (n-1)/2\}.$$

Moreover, Falconer's 1985 article led to the conjecture that $\dim_{\mathrm{H}}(\Delta(A)) = 1$ if $\dim_{\mathrm{H}}(A) \geq n/2$, which is now known as *Falconer's distance set conjecture* and still remains open.

There has been considerable interest in resolving this conjecture. The best understood and first non-trivial case is the case $n = 2$. In 1999, Thomas Wolff proved, for Borel sets $A \subseteq \mathbb{R}^2$,

$$\dim_{\mathrm{H}}(A) \geq 4/3 \Rightarrow \dim_{\mathrm{H}}(\Delta(A)) = 1.$$

He also gave a lower bound on $\dim_{\mathrm{H}}(\Delta(A))$ in the situation that $s := \dim_{\mathrm{H}}(A) \in (1, 4/3)$. Building on work of Katz and Tao (2001), Jean Bourgain (2003) obtained the lower bound $0.5 + \delta$ (with some $\delta > 0$) in the case $s \geq 1$, which improves Wolff's result for values of s slightly larger than one and which is currently the best known bound when $s = 1$.

During the 2017 semester at Institut Mittag-Leffler Tamás Keleti and Pablo Shmerkin worked on improving the previously known bounds. By using conceptually different methods of proof to the aforementioned works, they obtained the, at the time, best bound for $s > 1$ close to one and Borel sets $A \subseteq \mathbb{R}^2$. Namely, they showed that if $s \in (1, 4/3)$, then

$$\dim_{\mathrm{H}}(\Delta(A)) \geq \frac{s(147 - 170s + 60s^2)}{18(12 - 14s + 5s^2)} \geq \frac{37}{54} \approx 0.685.$$

Subsequently, this result has been improved by various authors, with the current best bound for $s > 1$ close to one given by Shmerkin in [7]. We refer the interested reader to Keleti's and Shmerkin's article [3] as well as [7] and references therein for a more detailed exposition on the history of the problem, relations of their result to previous ones, as well as results on dimensions of planar pinned distance sets.

Figure 2. The Barnsley fern — a self-affine set

Affinity dimension

A *planar affine transform* is a map $\phi\colon \mathbb{R}^2 \circlearrowleft$ of the form $\phi(x) = A \cdot x + b$, where $A \in \mathbb{R}^{2\times 2}$ is a (2×2)-matrix and $b \in \mathbb{R}^2$ is a vector. A planar affine transform ϕ maps circles to ellipses when it is *non-singular*, i.e. when $(0,0)$ is the only vector mapped to $(0,0)$ by ϕ.

Naturally associated to a non-singular affine transform ϕ are two values $\alpha(\phi) \geq \beta(\phi)$: the lengths of the major and minor axes of the ellipse $\phi(\mathbb{S}^1)$, where $\mathbb{S}^1$ denotes the unit circle in $\mathbb{R}^2$. These two values are called the *singular values* of ϕ and the *singular value function* $T^s(\phi)$ of ϕ is defined to be

$$T^s(\phi) := \begin{cases} \alpha(\phi)^s & \text{if } 0 \leq s \leq 1, \\ \alpha(\phi)\beta(\phi)^{s-1} & \text{if } 1 \leq s \leq 2. \end{cases}$$

Given an IFS $\Phi := \{\phi_1, \ldots, \phi_m\}$ consisting of non-singular planar affine transforms,

$$\inf\left\{ s : \sum_{n\in\mathbb{N}} \sum_{(i_1,\ldots,i_n)\in\Sigma^n} T^s(\phi_{i_1} \circ \cdots \circ \phi_{i_n}) < \infty \right\}$$

is called the *affinity dimension* of Φ, where $\Sigma := \{1, \ldots, m\}$. One often denotes this value by $\dim_{\mathrm{A}}(F)$, where F is the attractor of Φ.

(b) Dimension of self-affine sets

Computing the Hausdorff dimension of self-affine sets, such as that depicted in Figure 2, is one of the major open problems in fractal geometry. Falconer pioneered these studies and established a general upper bound on the Hausdorff dimension in terms of the affinity dimension $\dim_{\mathrm{A}}$, see "Affinity dimension". In special cases, $\dim_{\mathrm{A}}$ has also been shown to be a lower bound. However, the Hausdorff dimension is not always equal to the affinity dimension, as Bedford–McMullen carpet-like constructions show. Thus, the challenging task is to find mild conditions under which equality holds.

In [1], Balázs Bárány, Michael Hochman and Ariel Rapaport found such general conditions for planar self-affine sets. They are expressed in terms of properties of the group which the underlying affine transforms generate. Let us be more precise. A planar self-affine set $F \subseteq \mathbb{R}^2$ is made up of small affine copies of itself. We let $\phi_1, \ldots, \phi_m\colon \mathbb{R}^2 \circlearrowleft$ denote the planar affine transforms for which $F = \bigcup_{i=1}^{m} \phi_i(F)$. In other words, F is the attractor of the IFS $\Phi := \{\phi_1, \dots, \phi_m\}$, see "Some models for irregular sets". Let A_i be the matrix and b_i be the vector so that $\phi_i(x) = A_i \cdot x + b_i$ and let $\overline{A_i} := A_i / \sqrt{|\det(A_i)|}$ denote the respective normalised matrices. Notice, each $\overline{A_i}$ is an element of $GL_2(\mathbb{R})$.

Bárány's, Hochman's and Rapaport's remarkable result is the following. If the open set condition (OSC) is satisfied with an open set O that additionally satisfies $O \cap F \neq \emptyset$ and if $\overline{A_1}, \dots, \overline{A_m}$ generate a non-compact, totally irreducible group in $GL_2(\mathbb{R})$, then $\dim_{\mathrm{H}}(F) = \dim_{\mathrm{M}}(F) = \dim_{\mathrm{A}}(F)$. Here, a group of matrices is called *totally irreducible* if it does not preserve any finite union of non-trivial linear spaces, OSC is defined in "Similarity dimension" and $\dim_{\mathrm{M}}$ denotes the Minkowski dimension, which we will shortly introduce. We refer the interested reader to [1] for further details, historical remarks.

(c) Characterising beyond dimension

It is well known that for $n \in \mathbb{N}$ there is an abundance of n-dimensional sets of different forms, sizes, etc. Likewise, there is a whole zoo of irregular sets of a given "fractal dimension". In Euclidean geometry, important and commonly used characteristics for describing the geometry of a set beyond its dimension are volume, surface area, curvatures and Euler characteristic. Unfortunately, for sets of a non-integer "fractal dimension" these quantities generally do not yield any meaningful geometric information. For instance, the area of the gasket displayed in Figure 1 is zero, whereas its length, when viewed as a curve, is infinite. An important question is, how can we distinguish such sets? How can we characterise their geometry beyond dimension? To present an approach to these questions, let us first look at how the above-mentioned geometric characteristics can be obtained for sufficiently nice sets.

Given a compact set $A \subseteq \mathbb{R}^n$ and $\epsilon > 0$ the *ϵ-parallel set* of A is defined as

$$A_\epsilon := \{x \in \mathbb{R}^n : \mathrm{dist}(x, A) \leq \epsilon\},$$

where $\mathrm{dist}(x, A) := \inf_{a \in A} |x - a|$. Let P denote a point; L a straight line of length ℓ; R a rectangle with side-lengths a and b; B a ball of radius r, all embedded in $\mathbb{R}^3$. Further, let vol_3 denote 3-dimensional volume. Then, we have the following.

A	d	$\mathrm{vol}_3(A_\epsilon)$
P	0	$\kappa_3 \epsilon^3$
L	1	$\ell \kappa_2 \epsilon^2 + \kappa_3 \epsilon^3$
R	2	$ab\kappa_1 \epsilon + 2(a+b)(\kappa_2/2)\epsilon^2 + \kappa_3 \epsilon^3$
B	3	$(4/3)\pi r^3 \kappa_0 + 4\pi r^2 (\kappa_1/2)\epsilon + 4r\kappa_2 \epsilon^2 + \kappa_3 \epsilon^3$

Here, d denotes the Hausdorff dimension of A and κ_k denotes the k-dimensional volume of the k-dimensional unit ball. In all of the above four examples, the coefficients of $\mathrm{vol}_3(A_\epsilon)$ in ϵ correspond to the above noted geometric quantities; this is the statement of the famous Steiner formula. Moreover,

$$\lim_{\epsilon \to 0} \epsilon^{d-3}\, \mathrm{vol}_3(A_\epsilon) = c\, \kappa_{3-d},$$

where c is the d-dimensional volume of A.

For general Borel sets $F \subseteq \mathbb{R}^n$, we first need to find an appropriate value for d. Observe, if δ is too large, then $\lim_{\epsilon \to 0} \epsilon^{\delta - n} \mathrm{vol}_n(F_\epsilon) = 0$, and if δ is too small, then $\lim_{\epsilon \to 0} \epsilon^{\delta - n} \mathrm{vol}_n(F_\epsilon) = \infty$. In certain situations there exists a unique critical value for δ, denoted by $\dim_{\mathrm{M}}(F)$, at which the behaviour changes and this value is defined to be the *Minkowski dimension* of F. For instance, if F is a self-similar or -conformal set and the OSC is satisfied, then $\dim_{\mathrm{M}}(F)$ is well defined and coincides with $\dim_{\mathrm{H}}(F)$. However, this is not necessarily the case for arbitrary Borel sets.

The limit $\lim_{\epsilon \to 0} \epsilon^{\dim_{\mathrm{M}}(F) - n} \mathrm{vol}_n(F_\epsilon)$, known as the *Minkowski content* of F, has an interpretation as "fractal volume", or as the "$\dim_{\mathrm{M}}(F)$-dimensional volume" of F, whenever the limit exists. Indeed, the Minkowski content can distinguish sets of the same dimension and can be viewed as a refinement of dimension. Further, in the mid 1990s, Michel Lapidus and Helmut Maier provided an interesting connection between the existence of Minkowski content of subsets of $\mathbb{R}$ and the Riemann Hypothesis.

Showing existence or non-existence of the Minkowski content and determining its value is a delicate task for which tools from dynamical systems (ergodic theory and the thermodynamical formalism) and probability theory have been and are being applied. During the research semester at Institut Mittag-Leffler, such dynamical and probabilistic tools were extended and generalised so that larger classes of irregular sets (among them self-affine and generalisations of self-conformal sets) can be treated, see [4]. Moreover, under OSC, Steffen Winter and the first author obtained the last puzzle-piece for completely classifying those self-similar sets of $\mathbb{R}$ whose Minkowski content exists, see [5] for further details.

(d) A note on Littlewood's conjecture

We end with a beautiful connection between dynamical systems, fractal geometry and Littlewood's conjecture; a famous open problem in multiplicative Diophantine approximation.

Let $G\colon [0,1] \circlearrowleft$ denote the *Gauss map*, defined by $G(x) := 1/x - [1/x]$; here $[y]$ denotes the integer part of $y \in \mathbb{R}$. Further, for $N \in \mathbb{N}$, let

$$F_N := \{x \in [0,1] \colon G^n(x) \geq \tfrac{1}{N+1} \text{ for all } n \in \mathbb{N}_0\}.$$

By construction each F_N satisfies $G(F_N) \subseteq F_N$, thus it is a forward invariant set. On the other hand, restricted to $[1/(N+1), 1]$ the Gauss map has N inverse branches $\phi_1, \ldots, \phi_N$, which are contractive conformal self-maps on $[1/(N+1), 1]$. As stated in "Some models of irregular sets" there exists a unique nonempty compact set F satisfying $F = \bigcup_{i=1}^{N} \phi_i(F)$, the self-conformal set associated with $\{\phi_1, \ldots, \phi_N\}$. This set precisely coincides with F_N.

A natural and interesting question which one may ask is, what is $\dim_{\mathrm{H}}(F_N)$? The Hausdorff dimension of self-conformal sets, and in particular of F_N, can be determined by Bowen's formula; a central result within the thermodynamic formalism which can be seen as a generalisation of the Moran–Hutchinson formula, see "Hausdorff dimension".

In Diophantine approximation the set F_N naturally appears as the set of points in $[0,1]$ whose continued fraction expansions consist of entries in the digit set $\{1, \ldots, N\}$. Moreover, $\mathrm{Bad} := \bigcup_{N \in \mathbb{N}} F_N$ is known as the set of *badly approximable numbers* and plays an important role in Littlewood's conjecture, which states that for any α and $\beta \in \mathbb{R}$ and $\varepsilon > 0$ there are infinitely many $q \in \mathbb{N}$ such that

$$q \, \|q\alpha\| \, \|q\beta\| < \varepsilon,$$

where $\|\cdot\|$ denotes the distance to the nearest integer. The conjecture is trivially true if at least one of α and β does not belong to the set Bad. Hence, the exceptional values for Littlewood's conjecture lie in Bad. Results of Aleksandr Khintchine show that, in terms of Lebesgue measure, the set Bad is small, i.e. has Lebesgue measure equal to zero; on the other hand, in terms of Hausdorff dimension, it is as large as it can be, i.e. $\dim_{\mathrm{H}}(\mathrm{Bad}) = 1$.

In [6], Andrew Pollington and Sanju Velani made a significant breakthrough, by showing that a stronger 'logarithmic' form, namely

$$q \, \ln(q) \, \|q\alpha\| \, \|q\beta\| < \varepsilon,$$

for infinitely many $q \in \mathbb{N}$, holds on a set of badly approximable pairs (α, β) of full Hausdorff dimension. More recently, in [2], Manfred Einsiedler, Anatole Katok and Elon Lindenstrauss proved the measure rigidity conjecture of Grigory Margulis (under the assumption of positive entropy), and as a consequence, it is now known that the set of possible exceptions to Littlewood's conjecture is of Hausdorff dimension equal to zero. However, the conjecture remains open for pairs of numbers such as $\alpha = \sqrt{2}$ and $\beta = \sqrt{3}$.

FURTHER READING

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Reciprocal Societies: The Singapore Mathematical Society

The Singapore Mathematical Society (SMS) was founded in June 1952. The founding president was Alexander Oppenheim, Professor of Mathematics and later Vice Chancellor of the University of Malaya.

The objective of the Society was "to effect improvements in the teaching of elementary mathematics and to provide means of intercourse between students, teachers and others interested in mathematics and its teaching". Later this objective was enlarged to include the phrase "to maintain the status and advance the interests of the profession of mathematics". To this end, the society organises activities to cater to the needs of the school community as well as professional mathematicians.

Alexander Oppenheim

The Society organises the annual Singapore Mathematical Olympiad (SMO), the largest and oldest mathematics competition in Singapore, in which thousands of students take part nationwide. The Singapore Mathematics Project Festival (SMPF) was inaugurated by the Singapore Mathematical Society in 2001 in recognition of the fact that creative and innovative work in mathematics may best be reflected in project work. The SMS Essay Competition began in 2007 and has a different theme every year. The objective of the competition is to expose students to various topics related to mathematics, including to the history, applications or other aspects of mathematics.

The Society also organises a wide variety of lectures and workshops every year. The talks are delivered by international and local experts in mathematics and its applications, and mathematics education. Most of the talks are aimed at a general audience and are suitable for schools students and teachers. The aims are to raise awareness of aspects of mathematics aside from the normal school curriculum, and to promote interest in mathematics among students, teachers, and the general public.

The Distinguished Visitor Programme was launched by the Singapore Mathematical Society in 1998. Through the visit of a distinguished mathematician/mathematics educator, who will interact with both mathematicians/mathematics educators at the universities as well as teachers and pupils at the schools here, the aim of the Programme is to expose as large and diverse an audience as possible to the excitement and relevance of mathematics, thereby enhancing the awareness of mathematics in our society. Recent speakers of the programme include Phillip A. Griffiths of the Institute of Advanced Study, Imre Leader of the University of Cambridge and Ron Aharoni of Technion, Israel Institute of Technology.

The annual Singapore Mathematics Symposium, inaugurated in 2010, is another initiative of the Society to promote interaction within the mathematical community and to showcase some of the exciting developments originating from Singapore. This annual one-day event will include several invited lectures by mathematical scientists from various institutions and a poster exhibition and competition for graduate students.

Public Lecture by Professor Ron Aharoni

In 2014, the Singapore Mathematical Society established a reciprocity agreement with the London Mathematical Society. More information about the SMS can be found at asms.math.nus.edu.sg/.

Victor Tan
President, Singapore Mathematical Society

Editor's note: the LMS and SMS have a reciprocity agreement, meaning members of either society may benefit from discounted membership of the other.

PWSAfrica

BENJAMIN MERLIN BUMPUS

PWSAfrica is an initiative run by five PhD students in the School of Computer Science at the University of Glasgow, which aims to teach students and academics in mathematics in Africa how to program. This article gives a brief explanation of our experience and of what we learned during our two-week adventure in Nigeria.

Last August 5 tutors from the University of Glasgow travelled to Nigeria for two weeks to teach over 100 students how to program in Python: this was the start of the PWSAfrica project.

It was in February 2018 that my PhD office-mate Sofiat Olaosebikan first approached me with the idea of going to her home country to teach computer science to mathematics students at the University of Ibadan, Nigeria. I was immediately onboard: having first experienced computer science through programming, I knew how it felt to go from being immersed in mathematical theory to suddenly creating something interactive and practical. It was that feeling that I wanted to share. It took a team of five PhD students from the School of Computing in the University of Glasgow to get the "Programming Workshop for Scientists in Africa" project (PWSAfrica) off the ground. Apart from obvious challenges such as finding funding (which our department and the Scottish Informatics and Computer Science Alliance provided), planning our trip and applying for visas, we were faced with the task of designing a curriculum tailored to the occasion which could be delivered in two weeks. It was a non-trivial challenge!

Planning the Workshop

There were three considerations we needed to make: computer science education is not readily available in Nigeria, so it was vital for us to make sure that our students would be able to continue learning even after the workshop; we needed to stimulate the varied interests of over one hundred students in a very short time; and, finally, we needed to tailor the material to an audience consisting predominantly of undergraduate and postgraduate mathematics students with no prior programming experience.

I believe that making the material appealing and relevant is the best way to ensure that students will continue learning. With this in mind, we began by selecting a choice of four different projects for the students to solve at the end of the workshop so that it would culminate in a challenge involving planning and implementing a program of some significance (unlike learning syntax, this is a difficult task to learn through self-study). These projects also acted as a guideline for us when we chose the material because we simply had to include all the concepts which were necessary building blocks for the completion of the tasks.

The next challenge was to design the course in such a way that it could leverage the strengths of mathematics students when dealing with unfamiliar material. This resulted in a course which initially stressed the use of functional programming. Although Python has weak support for functional programming, we thought that mathematics students would be more familiar with notions of higher order functions and mappings than they would be with concepts such as stateful variables and memory allocation.

Nothing goes according to plan

It came as no surprise that, when travelling roughly 7000km to another continent, most things are not as expected. First of all, many more students attended the workshop than we anticipated. This was dealt with by running two sessions a day which meant we had to work at least 12-hours a day. It was even more challenging to overcome the technical difficulties we were faced with. Some of these were mechanical: for example, there were often power cuts because our generator would either malfunction or run out of gas (or both!). Other problems had to do with our teaching style: we quickly learned that our tutorial-style classes were not appreciated and that the students tended to encounter difficulties with syntax and some computing fundamentals. We tried to resolve these last issues by switching to a more traditional lecture-driven format and by running an extra problem session to give the students more time to consolidate their learning.

The funniest hiccup happened when we handed out pen-drives containing full installation packages of the materials and necessary dependencies. Since we had fewer drives than there were students, we asked the students to pass the drives to one another. Rather than solve the problem, we had unknowingly prepared the perfect Petri dish for an experiment in computer epidemiology because some students forgot to safely eject the devices after they finished their installations. This caused a rapid spread of computer malware: another problem to deal with!

The team, left to right: Fionnuala Johnson, Fatma Elsafoury, Sofiat Olaosebikan, me and Tom Wallis

What we learned for next time

We can definitely pat ourselves on the back about a few things PWSAfrica did well. For example, we were right in thinking that students with a mathematical background would find functional programming a natural concept. More importantly, at the end of the workshop, the quality of the students' projects was consistently very impressive! It is difficult to congratulate them enough on the hard work that they put into their studies.

We were also able to improve our teaching by iteratively revising our methods after each session. We learned that the students generally seemed to prefer it when we solved problems by coding "live" with a projector, rather than working through examples on the board. Given our audience, it was also very helpful to draw explicit comparisons between the syntax of Python and that of mathematics. For example, a subtle notion which confused our students was that of equality. In mathematics, when one says x is equal to 2, one means that x is a pseudonym for the name 2; in Python (and most programming languages) this is not the case since $x = 2$ means that we are letting the variable x "*hold*" the value 2. The mathematician is rightly confused when she is told to make x "drop" the 2 and "pick up" a 3! Interestingly, we found that the programming language "BrainFuck", with its minimalistic six-symbol syntax, was actually a very good language for the students to learn about computational thinking. This language is a close replica of a Turing Machine and writing small elegant programs in it can be challenging. After a long day of teaching, some students joined us in some games of code-golf (i.e. minimizing code length) in this language and we found this to be such a successful method of teaching that we are considering including this sort of activity in next year's school.

Reflections

Nigeria was in many ways completely alien to me and being there often challenged my understanding of my role and the role of my community in the world. Saying it was a "cultural exchange" is a gross understatement. It was a humbling experience which turned my reality on its head and has grown my desire to live and work as an academic in environments that can challenge me on a cultural level.

The impact we had is difficult to convey in a few lines, but we can proudly say that many students stayed in touch and continue to learn programming in their spare time! I think that we can all relate to the feeling of wanting to go back where we have come from to inspire students to venture down a similar path to our own. For many of us that might not involve travelling to such a geographically and culturally distant location, but the satisfaction of spreading some mathematical enthusiasm is the same even if we do it on our own doorstep.

Benjamin Merlin Bumpus

Benjamin is a PhD student studying Graph Theory and Parameterized Algorithms at the University of Glasgow. His love for agronomy would often get him in trouble because he would spend too long staring at the breathtaking Nigerian flora.

Microtheses and Nanotheses provide space in the Newsletter for current and recent research students to communicate their research findings with the community. We welcome submissions for this section from current and recent research students. See newsletter.lms.ac.uk for preparation and submission guidance.

Microthesis: Dynamical Systems Methods for Waves in Fluids: Stability, Breaking and Mixing

FRANCISCO DE MELO VIRÍSSIMO

Waves are everywhere in nature and play a key role in a range of natural phenomena, from ocean circulation to weather forecast. Mathematical models for these are helpful in understanding the real world and my PhD thesis focuses on studying different aspects of waves in fluids using dynamical systems techniques.

Waves in fluids

When compared to mechanical (e.g. sound waves) or electromagnetic waves, waves in fluids present an additional and substantial difference: while, for instance, both sound and light waves normally do not interact with each other, waves in fluids do interact with each other, usually in a very complex way. This behaviour is reflected by the mathematics of the problem: sound and light waves are linear phenomena, modelled by linear equations, which satisfy the principle of superposition; on the other hand, waves in fluids are usually highly nonlinear. Their fundamental model is the Navier–Stokes equations. All these complications motivated the famous quote of Richard Feymann, made in 1963 in one of his lectures at Caltech:

> [Water waves] that are easily seen by everyone and which are usually used as an example of waves in elementary courses [...] are the worst possible example [...]; they have all the complications that waves can have.

Stratified flows and internal waves

It turns out that most waves in geophysical fluid mechanics are internal, and their existence is linked to stratified flows. These are ubiquitous in nature, with the ocean and the atmosphere as prime examples. Even though modelled by incompressible equations, most geophysical flows are density-stratified, meaning that the density can change due to the concentration of sediments, substances or differences in the temperature in various parts of the flow.

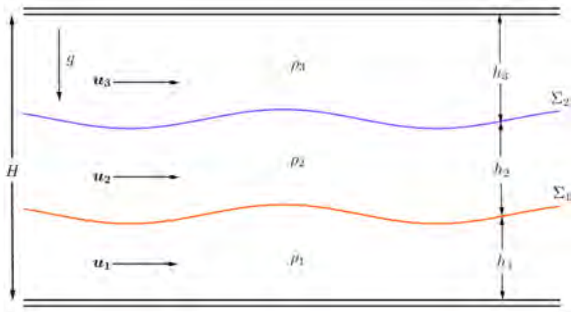
Ocean internal waves in Rosario Strait, Washington State, USA. Source: TAF Lab at University of California (taflab.berkeley.edu/)

The stratification is the main mechanism behind the existence of internal waves in stratified flows. The simplest example is of waves propagating on the interface between two layers of fluid (salty and fresh water for example).

Three-layer shallow water flows

Since stratification is intrinsically linked to internal waves, there needs to be at least one interface for a flow to generate and propagate such waves. Most mathematical studies so far considered the particular case of a two-layer flow, in which there is exactly one interface.

My PhD work focussed on the strongly nonlinear non-dispersive setting of three layers [2], and thus two interfaces, in a channel bounded by horizontal rigid walls, and also on the study of a model with the same setting but without the upper rigid lid [3].



Schematic representation of a three-layer channel.

Both of these are important cases as they capture a class of slow (so-called mode 2) internal waves, in addition to a class of fast (mode 1) waves. Mode 2 waves, although less common than mode 1 waves, have now been observed in the ocean and have attracted a lot of interest from the scientific community.

Dynamics and stability

As expected, the dynamics of waves in a three-layer setting is much richer than in a two-layer flow. In [2], it was shown that a particular set of pure mode 2 solutions forms an invariant subspace in the phase space. This means that if a solution is initially a pure mode 2, then it will remains a mode 2 until it either breaks or loses stability, which is not the case for pure mode 1 solutions.

A question of crucial interest concerns the long term well-posedness or nonlinear stability of the model. In other words, does an initially wave-like (so-called hyperbolic) solution remain hyperbolic until it breaks? The answer is yes for a two-layer system, but for three-layer systems it still an open question. Partial results could be proven for a three-layer flows. In particular, on the set of pure mode 2 solutions, the three-layer system is equivalent to the two-layer system, meaning that for each solution of the two-layer problem, there exists a unique three-layer equivalent solution and vice-versa. Therefore, all non-linear stability results in [1] apply to pure mode 2 solutions. It was also shown that the full hyperbolic region is not invariant under the flow.

Mixing and entrainment

Layered-stratified flows, although driven primarily by the density differences between the layers, have the capacity to alter the underlying stratification. Most studies assume that the flow remains with the original density stratification over time. Although this can be the case in many applications, shallow water waves tend to break and can change the stratification. At this point, the movement of particles in the flow can become quite turbulent, leading to mixing and entrainment processes (think of a breaking wave on the beach, for example). These are usually guided by small scale motions that are difficult to model in detail. Part of my work was to model breaking waves using carefully selected conservation laws which avoid small-scale dynamics [4].

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Francisco de Melo Viríssimo

Francisco has just completed his PhD in Mathematics at the University of Bath, under the supervision of Paul Milewski. He is now a postdoctoral researcher at the National Oceanography Centre in Southampton, UK. His main research interests are in mathematical and numerical aspects of fluid mechanics, but he recently has shifted his interests to environmental sciences. Francisco was born in the Brazilian state of Minas Gerais and has found in pottery a back up plan for the (perhaps not so far-distant...) future.

His PhD research was supported by CNPq (Brazil), under the grant number 249770/2013-0.

Is That a Big Number?

by Andrew C. A. Elliot, Oxford University Press, 2018, £18.99, US$24.95, ISBN: 978-0-19-882122-9

Review by Ian Flood

Addressing the great divide between “expert” and “citizen” shows serious motivation but this is a very difficult thing to get right. This is a book that attempts to bridge the divide. In the front matter, the question is posed: “Bewildered by Big Numbers?” followed by specific examples such as “There are 7.6 billion people in the world. *Is that a big number?*”. This sets a didactic tone based on a particular view of the reader’s abilities given in the Introduction: “Whether we’re faced with immigrant numbers, national budgets or deficits, costs of space programmes, health services, or defence budgets, we lack the ability to easily put these into context. In large part, this is simply because we cannot fully grasp the large numbers themselves.” So the reader requires help but the remedial teaching effort is a little inconsistent. The author explains what an integer is but uses “infinite set” in relation to counting numbers earlier in the text with no explanation of sets or infinity.

Andrew Elliot’s “guiding principle” is “cross-comparison”. That is, numbers are compared with other numbers via five techniques:

- Landmark numbers
- Visualisation
- Divide and conquer
- Rates and ratios
- Log scales

With the first technique, the reader is invited to think about numbers that will help place other numbers into some context. Thus, for example, there are more than 1.3 billion people in China; the length of a football field is 100 metres; Israel is about the same size as Wales.

Technique number two, Visualisation, is all about creating “dataviz” images in the mind. A Volkswagen Beetle is 4.08 metres long which is around the same length as 1000 imaginary ants, each 4mm long. I found it easier to imagine 1020 ants for this particular visualisation.

Divide and conquer breaks problems down but often in a kooky way: the world’s population has increased from around 3 billion in 1960 to more than 7 billion at the present time. Elliot suggests that we place “billion” on one shoulder and “the 3 and the 7” on the other. I don’t really know what this means but there are other examples that make more sense such as breaking down the population of a theatre into sections and rows.

The fourth technique develops rates and ratios in order to “reduce the monster to a number that is well within our comfort zone...”. Examples such as birth and death rates are given and that a mouse is roughly 1/10 of the mass of a rat which is roughly 1/10 of the mass of a rabbit which is roughly 1/10 of the mass of a dog which is roughly 1/10 of the mass of a donkey. You get the idea.

With Log scales, Elliot gives a simple enough explanation and some examples such as Moore’s law and the Richter scale.

The book goes on to examine “The Numbers of Science” and “Numbers in Public Life” covering such topics as computer memory and inflation. The technical subject matter is embroidered with various philosophical wanderings which do not make it any easier to digest. For example “Recently, a particular image has come repeatedly to mind. The image is of a body of water, a vast lake or river. The surface of this water is a jumble of flotsam: human litter, leaves, seeds and pollens, iridescent contours of oily pollution. The wind blows across the water and whips up waves chaotically. There are whirlpools, some tiny, some vast. On the shore there are inlets and backwaters." The author uses this image to comment on the complex nature of mathematical modelling, but this reader at least, was left unclear as to the point being made, or how the illustration helped.

I did not find this to be an engaging read and, although the book is linked to an established project of the author's, it's not easy to see how it could be successful in reaching out to people wanting to make progress with numeracy. It's one of those books where you immediately get an impression of the author downloading a great deal of what they know rather than that of a disciplined and well-structured endeavour.

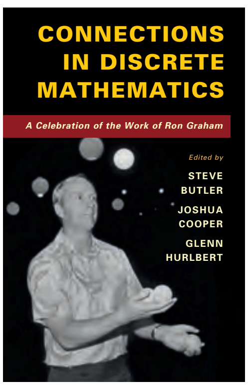
Ian Flood

Ian Flood is a consultant with Transfinite Systems, London. His work involves modelling spectrum sharing problems in the radio interference environment. He is a Chartered Engineer and holds a PhD in graph-theoretic studies.

Connections in Discrete Mathematics
A celebration of the work of Ron Graham

edited by Steve Butler, Joshua Cooper and Glenn Hurlbert,
Cambridge University Press, 2018, £44.99, US$59.99, ISBN: 978-1316607886

Review by David Bevan



In October 2015, Ron Graham celebrated his 80th birthday. To mark this milestone, a special conference, *Connections in Discrete Mathematics*, was held. The book under review came out of the conference. Ten years previously, a similar conference took place on the occasion of Graham's 70th birthday, and a similar book was produced [1].

So, who is Ron Graham, and why is he considered to be deserving of these accolades? For the past fifty years or so, he has been one of the central figures behind the growth of combinatorics and its connections with other areas of mathematics such as number theory, geometry, algorithms, and probability. Here are a couple of his results: In 1972, he constructed the first algorithm that could find the convex hull of n points in the plane in $O(n \log n)$ time, an achievement that was later proved to be optimal. In 1975, he proved that there is a unique hexagon of unit diameter with maximal area. Its area exceeds that of a *regular* hexagon by about 4%.

Many of his contributions have been to the area known as *Ramsey theory*, which, broadly speaking, concerns the study of properties that must necessarily hold in "sufficiently large" structures. A result which gained a degree of popular attention concerns the following question: consider colouring either red or blue each of the line segments between all possible pairs of vertices of the n-dimensional hypercube. What is the least value of n such that in *every* such colouring there are four *coplanar* vertices joined by six line segments *of the same colour*? In 1971, Graham and Bruce Rothschild established an enormous upper bound on the answer. Martin Gardner subsequently described this as "so vast that it holds the record for the largest number ever used in a serious mathematical proof".

The book under review consists of twenty papers of differing styles covering a spectrum of discrete mathematics and its connections. They vary in length from four to thirty-two pages. Sixty percent of the articles cite work by Ron Graham, and Graham himself is a co-author of two of them. He is still extraordinarily productive: there are more than a dozen of his papers on *MathSciNet* published since 2015, the year he turned 80.

The first paper in the book is by Persi Diaconis, on the properties of a "typical" Fibonacci number, and reflects the tension, often present in Graham's work, between recreational and "real" mathematics. It also yields some insight into the author's close friendship with Graham.

Other papers address questions concerning cellular automata, graph polynomials, the Collatz conjecture, Pick's Theorem, Apollonian packings, and various perspectives on and applications of Ramsey theory, among other topics. There are also two articles concerned with juggling. As evidenced by the photo on the cover of the book, Graham is an accomplished juggler; indeed, he served as president of the International Jugglers' Association. Several of his own papers concern the mathematics of juggling.

Most of the papers are quite technical, though two just present short proofs. A number of them provide some historical context, one explicitly surveying progress over the ten years since the conference for Graham's 70th birthday.

The final article in the book, which is also the longest, is by Graham himself, in collaboration with Joe Buhler, Anthony Gamst and Alfred Hales. It is motivated by the following, somewhat unexpected, behaviour of dice: suppose $A = [2, 6, 7]$ and $B = [1, 5, 9]$ are dice, where $[a, b, c]$ denotes that each of the outcomes a, b and c is equally likely (a fair cubical dice would have each of the three values on two of its faces). If A and B are each thrown once, A is more likely to win, since five of the nine possible outcomes favour A.

Now suppose we throw both dice twice and take, for each dice, the sum of the two throws. Surprisingly, if we do this, B is more likely to win, since 42 of the 81 possibilities favour B.

What happens if we take the sums of $k > 2$ throws of each dice? Here is a table showing, for small values of k, which of the dice is more likely to win:

1	2	3	4	5	6	7	8	9	10
A	B	A	A	B	A	A	B	A	A

With this data, it would be natural to conjecture that B wins whenever $k \equiv 2 \pmod 3$, and A wins otherwise. However, the pattern breaks down and in fact A wins for all $k \geqslant 9$, a "steady state" having been attained. Other pairs of dice exhibit similar periodic behaviour followed by stasis.

Graham and his coauthors study this general phenomenon in detail, explaining the source of the periodicity and investigating how soon the steady state is reached. They have recently produced a follow-up paper [2], in which they construct, for each n, sets of n dice that exhibit all possible pairwise "more likely to win" relationships when the dice are thrown different numbers of times.

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[2] J. Buhler, R. Graham, A. Hales, Maximally non-transitive dice, Am. Math. Mon., 125 (2018) 387–399.

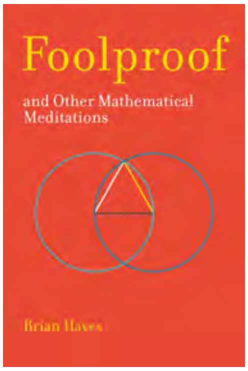
David Bevan

David Bevan is a lecturer in combinatorics at the University of Strathclyde in Glasgow — though he lives in Shropshire. His research interests primarily concern the combinatorics of permutations. David had a long career as a software developer before becoming an academic. This included several years living and working in Papua New Guinea.

Foolproof and Other Mathematical Meditations

by Brian Hayes, MIT Press, 2017, £20, US$24.99, ISBN: 978-0-126-203686-3

Review by Noel-Ann Bradshaw



Brian Hayes is an American scientist and this book, *Foolproof and Other Mathematical Meditations*, is a collection of his computer science columns that have previously been published in *American Scientist*. Considering this, I feel that the blurb on the back of the book describing it as "accessible", and Hayes as a non-mathematician who "makes math seem fun", could be a tad misleading for those in the UK not in the know.

Given Hayes' background, it is understandable that this is not a book designed to convince the average member of the public of the delights and usefulness of mathematics but rather a book that has been written to engage with scientists and highly numerate readers, showing them the beauty and breadth of mathematics and how it connects with scientific topics. Having said that, I am still not convinced that the medical doctor who once described himself to me as mathematically illiterate would feel entirely confident reading this. Hayes assumes his readership's knowledge of scientific and mathematical terminology. He refers to his work on protein folding as if this is something widely understood, and mentions terms such as "dyadic rationals" and "eigenvalues" without a glimmer of explanation. Although this won't trouble LMS members it might baffle some readers. But once one accepts that Hayes is writing for a specific audience, then it helps make sense of the level of detail he uses in telling these mathematical anecdotes.

The highlight of this book for me was not so much the range of subject material of the individual chapters but rather Hayes' delightful turn of phrase. He has a wonderfully dry and often self-deprecating sense of humour which, when it comes to the fore, is enough to make the most straight-faced reader chuckle aloud in a crowded commuter train. I particularly liked the imaginary dialogue that a computer programme named Hilbert might have with itself when enacting a programme designed to reach all the points in a square.

Each chapter begins with a highly entertaining and colourful description of a situation. This includes a quasi-film script introducing the distribution of zeros of the Riemann zeta function, a description of the day the US National Debt Clock ran out of digits and the problems of the regular Sunday morning walker who actively ensures that they never retrace their steps or cross their own path. Hayes then proceeds to show how he himself has investigated similar phenomena, frequently including examples of his own computer code. Hayes' explanations are regularly quite detailed and involved, sometimes leaving the reader a little in awe as to what one can "easily" programme a computer to do.

One of my favourite chapters is on Sudoku puzzles. As a recovering Sudoku addict, I had forgotten some of the heuristic reasoning procedures, such as "x-wing" and "swordfish", that I used to employ on a daily basis. Hayes describes these and, after delving into the history of the puzzles and investigating how many Sudoku solutions exist for puzzles of order n (note the standard 81 cell grid is order 3), he concludes with a brief but interesting discussion on the distinction between solving a puzzle using pure logic and solving by backtracking.

Noel-Ann Bradshaw

Noel-Ann Bradshaw was until recently a principal lecturer of mathematics and operational research at the University of Greenwich. She is now a Data Scientist for Sainsbury's Argos. She is never happier than when explaining, in costume, how Florence Nightingale used her mathematical knowledge to gather and communicate data to a non-numerate audience.

Obituaries of Members

Professor Sir Michael Atiyah: 1929 – 2019

Professor Sir Michael Atiyah, OM, FRS, who was elected a member of the London Mathematical Society on 27 January 1955, died on 11 January 2019, aged 89. Sir Michael was LMS Vice-President (1973–74) and LMS President (1974–76).

Caroline Series writes: Atiyah's work spanned many fields. Together with Friedrich Hirzebruch, he laid the foundations for topological K-theory. The Atiyah–Singer index theorem, proved with Isidore Singer in 1963, not only vastly generalised classical results from the 19th century, but also provided an entirely new bridge between analysis and topology. Regarded as one of the great landmarks of 20th century mathematics, it has profoundly influenced many of the most important later developments in topology, differential geometry and quantum field theory. His more recent work was inspired by theoretical physics, in particular instantons and monopoles.

In 1966 he was awarded a Fields Medal for his work in developing K-theory, a generalised Lefschetz fixed-point theorem and the Atiyah–Singer theorem. In 2004 he was awarded the Abel Prize jointly with Singer "for their discovery and proof of the index theorem, bringing together topology, geometry and analysis, and their outstanding role in building new bridges between mathematics and theoretical physics".

Besides the brilliance of his mathematics, Atiyah was a visionary leader. His former students include no less than three LMS Presidents, as well as Fields medallist Simon Donaldson. He was involved in the creation of the Isaac Newton Institute in Cambridge, becoming its first Director in 1990. He was also closely involved in the foundation of the European Mathematical Society, having chaired the meetings which led up to its inception, also in 1990.

Born in London of Lebanese and Scottish descent, Atiyah's early education was in Sudan and Egypt. He came back to England immediately after the war, finishing his final school years at Manchester Grammar School. He did his undergraduate and doctoral work in Cambridge, studying under William Hodge, and in 1954 became a fellow of Trinity College, Cambridge. In 1961 he moved to Oxford where in 1963 he became the Savilian Professor of Geometry, having been elected to the Royal Society in 1962 at the early age of 32. In 1969 he left Oxford for the Institute for Advanced Study in Princeton, returning to Oxford three years later to take up a Royal Society Research Professorship. In 1990 he returned to Cambridge to take up the Directorship of the Newton Institute, becoming at the same time Master of Trinity College and President of the Royal Society (1990–95).

Following his retirement, he moved to Edinburgh where he was an honorary professor at the University of Edinburgh and President of the Royal Society of Edinburgh (2005–2008).

Atiyah received awards and honours far too numerous to list here. He had honorary degrees and memberships of Academies of Science from around the world. His medals include the LMS's highest award, the De Morgan Medal (1980), as well as the Berwick Prize (1961), and the Royal Society's Royal Medal (1968) and its highest award, the Copley Medal (1988). He was knighted in 1983 and became a member of the Order of Merit in 1992. He was a Grand Officier of the French Légion d'honneur. Some of his medals and certificates are on display in the Newton Institute.

Atiyah was the dominant figure in British mathematics during my entire career. Even to those far from his subject, he was an inspirational lecturer who had the gift of elucidating complicated ideas and taking his listeners with him on a journey which created the illusion that one understood far more than one really did. His life and work must have touched all those who came into contact with him and British mathematics would be very different now without him. Despite increasing frailty in the past few years, he retained his enormous energy and enthusiasm, travelling to Rio in August 2018 to deliver the Abel lecture at the ICM. Only very recently, he wrote to the LMS expressing his strong support for an Academy of Mathematical Sciences, believing it essential to have such a body to argue the case for the broad mathematical community. Michael would have been the person to make it happen. He will be sorely missed.

More information about Atiyah's life and works can be found in *Celebratio Mathematica* (tinyurl.com/ybcx28hf), edited by Nigel Hitchin, Andrew Ranicki and Rob Kirby.

Michael Walker, OBE: 1947 – 2018

Professor Michael Walker, who was elected a member of the London Mathematical Society on 21 October 1977, died on 27 September 2018, aged 71.

Keith Mayes and Fred Piper write: It is with regret and sadness that we marked the passing of Professor Michael Walker OBE FREng FIET CMath FIMA, on 27 September 2018, following a long battle with cancer. Throughout his illness, "Mike" conducted himself with dignity and determination, rarely complaining and remaining very active and supportive to his family, friends and colleagues. Mike achieved a great deal during his life, and had a career which included major contributions to the fields of telecommunications, information security, international standardisation, and outstanding achievements working in industry, and with academic research and innovation.

One of Mike's enduring academic connections began in 1966 when he joined what is now Royal Holloway, University of London. He first successfully completed his BSc in Mathematics, and then went on to become the second PhD student to be supervised by Professor Fred Piper, who founded the Information Security Group (ISG); with which Mike was to have a lifelong association. With the PhD obtained in 1973, Mike was awarded a Royal Society Research Grant and moved to the Technical University of Kaiserslauten, before progressing to an academic staff position at the University of Tübingen in 1974. Having reached the rank of Reader, the award of Dr rer. nat. habil., and with various visiting positions, Mike's path looked set for a successful academic career. However, in 1984 he returned to the UK and switched to an industry career, becoming the head of the Mathematics Department at Racal Research, in Reading. This gave Mike the opportunity to turn his mathematical skills to applied real-world problems in the areas of telecommunications, engineering and security; a role for which he was perfectly suited. He remained at Racal Research until 1991, when he was spirited away by one of Racal's risky new spin-off ventures. Today this successful global giant is known as Vodafone, but in those early days its success was not certain, as there were serious security problems with the cloning of mobile phones. Mike and his team helped Vodafone by introducing an effective interim method of authentication to combat this. This experience took Mike into the world of telecommunications standards, where he was to have a major impact. This was not just embedding security into the design of future networks, but rising to the most senior position in the European Telecommunications Standard Institute, Chairman of its Board.

As his reputation and activities grew, along with the rapid expansion of Vodafone, Mike set up the UK-based Communications Security and Advanced Development team, which later evolved into Vodafone Global Research and Development, with Mike as the Director. Mike was greatly respected within Vodafone, being also appointed as a Director of Vodafone Ventures and of the Vodafone Pension Scheme, and eventually becoming the first Fellow of the Vodafone Group.

Despite his many industry activities, Mike never lost his keen interest for academic research and for many years he held the Vodafone chair as Professor of Telecommunications at Royal Holloway; as part of the ISG. In 2002, Mike was instrumental in founding the ISG Smart Card Centre which continues today.

He retired from Vodafone in 2009, but was certainly not idle. Notable activities included a period as Head of the School for Natural and Mathematical Sciences at King's College London. He was also a member of the UK Technology Strategy Board, and a member of the UK Government's OFCOM Spectrum Advisory Board. He was also an advisory board member of

Surrey University, where he had earlier been a visiting professor. When he could find time to spare, he loved to spend it with his family, or wood-working, or watching “the cricket”.

Mike had so many recognition awards. If we begin with Royal accolades, he received his OBE in 2009 for his services to telecommunications, and he was also a Fellow of the Royal Academy of Engineering. He was a chartered mathematician and a Fellow of the Institute of Mathematics and Applications, as well as a Fellow of the Wireless World Research Forum.

Perhaps the best accolade is that Mike achieved so much on merit, from following his curiosity rather than from ambition, and remaining a thoroughly fair and decent person. There was no better example of a “Scholar and a Gentleman”, and he will be sadly missed.

Ronald Harrop: 1926 – 2017

Professor Ronald Harrop, who was elected a member of the London Mathematical Society on 12 May 1955, died on 22 October 2017, aged 91.

John Harrop writes: Ronald Harrop was born in 1926 on the first day of the General Strike, which he would enjoy pointing out. His parents owned a bakery in Manchester and were of a generation that left the coal mining and cotton milling towns around Manchester for other work in the cities. When he was eight years old the family moved to Southport but he was always clear that he never lost his fondness for Manchester.

In 1943 Ronald graduated from King George V School a year earlier than normal and with a full scholarship to Cambridge. This pivotal event brought him to Gonville and Caius where he earned a BA (1946) and MA (1950) before going on to a PhD (1953) under the supervision of S.W.P. Steen. During his Cambridge years he received the 1951 Rayleigh Prize and was nominated by Ronald Fisher as Unofficial Fellow of Gonville and Caius, elected 1951.

Ronald was called up to the Royal Aircraft Establishment at Farnborough between 1945 and 1948, where he worked on developing the first continuous flow, supersonic wind tunnel. Some of the papers he authored on this work became early standard references on supersonic airflow. The applied and interdisciplinary nature of this early work became characteristic of his career and how he would later apply computing. Another significant part of his time at Farnborough was meeting Olive, who became his wife in 1952.

In 1951, shortly before finishing his PhD at Cambridge, Ronald took a position as Lecturer at King’s College in Newcastle-upon-Tyne. Some of his work in intuitionistic logic during the Newcastle years led to the Hereditary Harrop Formulae, which became foundational in the theory of logic programming. In 1957 Ronald was awarded a Fulbright Scholarship, which enabled him to spend two semesters at Penn State University working with Haskell Currie, followed by a summer semester at Berkeley.

In the particularly snowy winter of 1964 Ronald visited the construction site of Simon Fraser University in Vancouver. Appointed as charter head of the Department of Mathematics, in December 1964, Ronald immigrated with his family to Vancouver in March 1965. Ronald worked tirelessly as head, building the new department until November 1968, when due to political unrest of that time the positions of department heads were abolished. He continued at SFU as professor for the rest of his career. Although he had already started teaching what would become computing courses while head, Computing Science did not become an independent department until 1973. Then, directed in part by Ronald’s sense of the importance of interdisciplinary studies, the department was established under Interdepartmental Studies, which allowed it to offer combined programmes with subjects such as kinesiology, biology, chemistry or mathematics even at the undergraduate level. In 1974 Ronald was one of the first two professors to receive joint professorships — both of these individuals combining computing with another discipline. His imprint in the early leadership of maths and computing science built the foundations for their ongoing success.

During the late 1970s Ronald’s research activities started to include medical applications of computing and appointments as Visiting, Adjunct and Honorary Professor in Radiology and Pharmaceutical Sciences at the University of Southern California and the University of British Columbia followed. Ronald joined the Positron Emission Tomography Project at TRIUMF (UBC) in 1980. The team built the second operational PET system in Canada with Ronald specifically contributing to the development of the image reconstruction component.

Ronald's research interests continued to focus on medical imaging until (and after) his retirement in 1991, primarily continuing with PET but also including SQUID and MRI. In 2005 Ronald was recognized as a Canadian Pioneer of Computing in the Pioneers of Computing project of the IBM Centre for Advanced Studies. Major achievements cited included: As charter head of the math department quickly spreading the department's involvement in computing; after the formation of Computing Science, establishing a joint honours programme that facilitated graduate work in either discipline; founding the BC Committee on Computing in Education; research in medical imaging and dosimetry; introduction of many groups of high school students to the maths/computing/medicine borderline.

Ronald is survived by his beloved wife Olive, two sons and five grandchildren.

Alan James Douglas: 1932 – 2018

Alan Douglas, who was elected a member of the London Mathematical Society on 20 December 1962, died on 14 October 2018, aged 86.

Rodney Sharp writes: Alan was born in 1932 and educated at Hanley High School in Stoke-on-Trent. After National Service, he took up a place to read chemistry at Keble College, Oxford University. However, university chemistry was not to his liking, and after one year he switched to mathematics; he graduated with a mathematics degree after three further years.

Alan spent the academic year 1957–58 on a Fulbright Scholarship at Indiana University in Bloomington. Actually, his studies in that year were adversely affected by a health condition; after that had been treated, Alan was told by an American consultant that a good way to avoid a recurrence would be to drink lots of beer. Alan enjoyed following this advice, and telling the story!

During 1958–61, Alan Douglas studied towards a PhD degree at Sheffield University under the supervision of Professor Douglas G. Northcott. Northcott was one of the world's leaders in commutative algebra, and at that time was very interested in applications to commutative algebra of the recently-developed subject of homological algebra. So Alan soon found himself working in homological algebra.

After his PhD, Alan became a Lecturer (later Senior Lecturer) in Pure Mathematics at Sheffield University. Highlights of his subsequent research activities included a series of joint papers with H.K. Farahat in which they studied the projective dimension of an Abelian group when it is considered as a left module over its own endomorphism ring, and a research visit to a former student that resulted in Alan's being caught up in a revolution in Algeria.

Alan excelled in the areas of teaching and administration. His lectures were always clear and effective, and well-liked by the students. Over the years, he, along with his great friend and colleague Barry Cruddis, built up a reputation as "safe pairs of hands" that could be relied upon to carry out efficiently and successfully a wide variety of administrative tasks in the department. Alan became responsible for the department's relationships with the Social Science Faculty, organising and overseeing the whole teaching programme in that area; and for many years before computers made the task easier, he, together with a colleague Glen Vickers, constructed a teaching timetable that worked for the whole of mathematics. Alan was very welcoming and helpful to new members of staff. All this was accomplished with a positive, optimistic and cheerful outlook. That the department was such a congenial place in which to work owed a lot to his equanimity and attitude.

Alan Douglas took early retirement following heart surgery in 1988. He was then able to devote more time to his family, and his hobbies of gardening, listening to music and railways (both model and full-scale); a highlight was a session as a fireman on a real steam locomotive. He died peacefully at home; he is survived by his wife Joyce, his children Hilary and Bill, and two grandchildren Elizabeth and Jonny.

Michael Atiyah and the Mediterranean

By Minhyong Kim

When I was an undergraduate at Seoul National University, Professor Dong Pyo Chi tried to inspire students with stories about the Atiyah–Singer Index Theorem, laying particular emphasis on the "local" proof, a topic on which he was then writing a book. He was a soft-spoken and modest man, not prone to exaggeration, although his easy style of presentation could make even the inverse function theorem look elegant and appealing. So it was an impressive matter to hear him quoting Hirzebruch to describe the Atiyah–Singer theorem as "one of the deepest and hardest theorems in mathematics". By nature, I wasn't one of those energetic young people who feel themselves compelled to pursue depth and difficulty, but it was hard not to be intrigued by the challenge of coming to terms with such a result. Indeed, quite a few years elapsed before I attained more than superficial familiarity with the prerequisites involved: geometry, topology, differential equations, and a smattering of intuition from physics. Efforts to understand the specific theorem led to many other things, and I was for years under the overwhelming spell of Atiyah's scientific perspective. It was during my first year in graduate school in America that I bought his collected works, which at that point already comprised five volumes. When I (ridiculously) compare the works to my own poor opus, it's hard even now not to be amazed that essentially all papers I've ever looked into there have numerous enlightening insights and very little repetition. Then there was the more obvious fact that each volume contained a substantial number of papers that were truly ground-breaking. For me, it would have been natural to try to be Atiyah's Ph.D. student. However, surveyed from Korea, the Oxford landscape of graduate admissions looked rather forbidding. Nevertheless, even though I eventually went into number theory, Atiyah's mathematics wasn't too far from my concerns most of the time in my early years. For example, my Ph.D. dissertation ended up in arithmetic intersection theory, which incorporated refined versions of the index theorem within the geometry of the "fibre over infinity" of arithmetic schemes.

In November 2015, I received with some excitement the following email:

> Minhyong Kim, Would you be interested in coming to Edinburgh for a few days say in February? I hope you might be prepared to give us a seminar on a mutually convenient date. I will explore possibilities at this end and see how it would fit in with your schedule. Best wishes, Michael Atiyah

It turned out he wanted mostly to discuss his ideas on number theory and physics. This invitation led to substantial communication and conversation over the last three years, about which I hope to write at a later date from a mathematical perspective[1]. One of his last emails, on 25 December, 2018, closes with the sentences: "Your message provided Xmas cheer, since I have not been feeling well right now and can only work spasmodically. Do Koreans celebrate Xmas?"

Even when ill health was making Atiyah's mathematics progressively difficult to comprehend, his charm, wit, and insight on non-mathematical matters continued unabated, and his ideas about the world were almost always clear and engaging. After a long dinner at our flat in May, 2018, my wife was amazed to learn his age on account of the mental acuity and conversational energy (as well as excellent appetite!) he had displayed all evening. While we were indulging in some gossip about Grothendieck, I found using my phone a photo of the cowled figure publicised by the Heidelberg forum, just to give the non-mathematicians present a sense of the mystique that surrounded him. We were interrupted by a deep voice coming out of a video advert that was on the same page, at which point Atiyah put forward a finger and exclaimed "That's Grothendieck, scolding us from beyond the grave!", resulting in great jollity all around.

As indicated by the title, my main intention here is to comment briefly on Michael Atiyah's roots in the Mediterranean. Prior to meeting him, I had thought of him as prototypically British. It would be hard to think of any mathematician as highly honoured by the British scientific establishment. He was the head of two genuinely venerable academic institutions of the country, the Royal Society of London and Trinity College, Cambridge. His lectures were consistently delivered in an impeccably elegant British

[1] In brief, I agree with skepticism about his claims to have proved difficult conjectures. Nevertheless, there is much of interest to be found in the unified vision of mathematics and physics that informed his thoughts in this connection.

manner. However, during the period I came to know him personally, the sense of his Mediterranean heritage was omnipresent, not least because of his own frequent reference to it. When I visited Atiyah in Edinburgh, we went to his favourite Lebanese restaurant following the colloquium, where the serving staff pampered him and treated him like a prophet and an old friend. All members of our party were happy to have him order for us, which he did with gusto, going on numerous explanatory digressions along the way about the items on the menu. In short, I don't think I've ever had a better middle-Eastern dinner, nor learned so much about the cuisine. Prior to this meeting, I had also been unaware that his father had written an autobiography with the title *An Arab Tells His Story* in 1946 and had held a position as secretary of the Arab League's London office. The Wikipedia entry on Edward Atiyah says he "died in 1964 at the age of 61 while taking part in a debate on Arab-Israeli relations at the Oxford Union."

During the dinner in Oxford, Atiyah spoke with great fondness about the time he spent in Victoria College, Alexandria. The actor Omar Sharif inevitably comes up in any talk about its alumni, and Atiyah remembered him as a boy a few years below him and something a ruffian. I couldn't resist asking him about Edward Said, another celebrity alumnus. Atiyah seemed to avoid touching on matters that were overtly political, but he did express his disapproval of Said's description of the school essentially as an institution of imperialist indoctrination. Atiyah felt that Said had not given proper tribute to its positive aspects including, but not confined to, the teachers of mathematics. Atiyah and Said appear to have agreed on Sharif, since Said described him as a "supercilious head boy" and a great bully.

Atiyah took the opportunity of this query to expound on his theory of civilisation, characteristically grand and sweeping. Sadly, I've lost the details of what he said, but retain the distinct sense that he was conversant in the complex and rich tradition of Mediterranean science from the time of the Rhind papyrus onwards. He spoke of problems of Babylonian and Egyptian origin that would be classified nowadays as algebra, the discovery of incommensurables by the Pythagoreans, and the subsequent suspicion of numbers leading to the reliance on geometry. It was at Atiyah's 85$^{\text{th}}$ birthday conference in Oxford that I heard the following conversation between two leading physicists (the precise wording may have been distorted by my memory). Sergei Gukov: "Do you think the universe is geometric or algebraic? If you had to bet, what would you say?" Robbert Dijkgraaf (after much hesitation): "I think it's algebraic. Geometry seems to be a statistical thing." Atiyah unequivocally answered the other way when I recounted this exchange to him in 2016. Indeed, some of his last papers try to construct "geometric models of matter" in the spirit of Kelvin and Skyrme. In my own small way, I tried to argue for a while on the side of Dijkgraaf.

When I heard of Atiyah's death, it was in the midst of exchanging emails with the classicist Jonathan Prag, with whom I am working on a book about Archimedes. It struck me that Archimedes was himself a Mediterranean geometer in the truest sense. He lived mostly in Syracuse, which is said to have been founded by settlers from the Peloponnese, and had studied in Alexandria, probably within a few kilometres of the current site of Victoria College. His lifetime was marked by several stages of the Punic wars. Carthage started in North Africa as a colony of Phoenicians originating in present day Lebanon. Roman mythology maintained that its own founders were immigrants from Asia Minor. Part of the reason for Rome's Siege of Syracuse, leading to the death of Archimedes, was the switch of Syracusan alliance from Rome to Carthage following Hannibal's decisive victory in Cannae. The depth and breadth of cultural currents that ran through the region at the height of Alexandrian influence in mathematics is hard now to imagine, as is the inspiration Archimedes might have received among scholars at the Library. I think Archimedes would have agreed with Atiyah about the universe, influenced as he was by Euclid and Eudoxus, who had systematically established geometry as an appropriate foundation for rigorous argumentation. Going beyond rigour, the heuristic reasoning presented in Archimedes's "mechanical method" of proof is very much in the spirit of quantum field theoretic geometry, the formal use of path integrals in particular, pioneered by Atiyah more than two millennia later. When the obituary for Atiyah in the New York Times drew the comparison to Newton, my own preoccupations immediately conjured up an equally plausible link to the ancient thinker, and the warm expansive climes that must have nurtured his soul.

I am not a very political person, but it is hard not to hear (*and* hard to hear) about the supposedly sharp divisions of the globe to which some politicians and journalists attach enormous significance in recent years. So the passing of one of the world's great mathematicians seems a fitting occasion to celebrate the heritage of the Mediterranean, whose variegated shores circumscribe a rich and timeless cultural zone unifying Africa, Asia, and Europe.

Lancaster Undergraduate Mathematical Society Meeting

Location: Lancaster University
Date: 4 March 2019

Anna Felikson (Durham) will be one guest speaker in a series to give talks in areas of mathematics to an audience of first year undergraduates and above. This event is supported by an LMS ECR grant. For further details email Ben at b.redhead@lancaster.ac.uk.

2018 Christopher Zeeman Medal presentation and lecture

Location: Royal Society, London
Date: 5 March 2019, 5:30pm
Website: tinyurl.com/zeeman19

The 2018 Christopher Zeeman Medal will be presented to Dr Hannah Fry. The presentation will be followed by a talk by Dr Fry, and a wine reception. Email prizes@lms.ac.uk to register your attendance.

Women in Mathematics Day 2019

Location: Queen Mary, University of London
Date: 12 March 2019
Website: tinyurl.com/y8lzsowu

A day of talks and workshops celebrating the range of opportunities open to women in maths. Open to all undergraduate and postgraduate students, and early-career mathematicians.

Ergodic Theory Meeting

Location: University of Manchester
Date: 20 March 2019
Website: tinyurl.com/y9jdy6xh

This is part of a series of meetings in ergodic theory and related topics, supported by a Scheme 3 grant from the LMS. Speakers are Ai-Hua Fan (Picardie), De-Jun Feng (CUHK) and Natalia Jurga (Surrey).

LMS Meeting

Hirst Lecture & Society Meeting

21 March 2019, De Morgan House, 57-58 Russell Square, London WC1B 4HS

Website: tinyurl.com/ybewfqz7

The meeting features the Hirst Lecture, given by the winner of the Hirst Prize and Lectureship 2018, Jeremy Gray (OU and Warwick). The second speaker will be June Barrow-Green (OU). It will be followed by a reception at De Morgan House. For further details and to register for a place, see the website. A Society Dinner will be held after the meeting at a nearby venue, at a cost of £35.00, including drinks. To reserve a place at the dinner, email lmsmeetings@lms.ac.uk.

Analysis of and Analysis on Networks

Location: Teesside University
Date: 22 March 2019
Website: tinyurl.com/yddbzn5e

A one-day meeting will take place at Middlesbrough and bring together researchers from analysis and computer science with an interest in networks. Slots for contributed talks are available. The meeting is supported by an LMS Scheme 9 grant. Please contact Sven-Ake Wegner (s.wegner@tees.ac.uk).

Mirrors in the Midlands

Location: University of Birmingham
Date: 1–3 April 2019
Website: tinyurl.com/ybkconv8

This conference will focus on recent work in algebraic geometry and mathematical physics surrounding mirror symmetry. It is supported by two LMS Scheme 9 grants to celebrate the lectureships of Andrea Brini and Tyler Kelly. Support for PhD students is available.

Young Functional Analysts' Workshop

Location: University of Leeds
Date: 3–5 April 2019
Website: tinyurl.com/ybn72vnx

Aimed at early-stage researchers in functional analysis and related areas, YFAW offers participants an opportunity both to present their own work and to hear talks by invited speakers on their current research. Supported by an LMS Scheme 8 Postgraduate Research Conference Grant.

Mary Cartwright Lecture

Location: ICMS, Edinburgh
Date: 5 April 2019
Website: tinyurl.com/ycv3ssfq

The 2019 Mary Cartwright Lecture will be presented by Professor Beatrice Pelloni (Heriot-Watt University) with an accompanying presentation by Professor Vassilios Dougalis (University of Athens). To register, contact womeninmaths@lms.ac.uk.

LMS Meeting at BMC 2019

Location: Lancaster
Date: 10 April 2019, 4:40 pm
Website: tinyurl.com/yarpowdo

The lecture is aimed at a general mathematical audience. The speaker is Kathryn Hess (EPFL, Lausanne, Switzerland). All interested, whether LMS members or not, are most welcome to attend this event. This day meeting takes place during the British Mathematical Colloquium 2019 from 8 to 11 April 2019.

North British Functional Analysis Seminar

Location: Lancaster University
Date: 11–12 April 2019
Website: tinyurl.com/y8qk2ryd

NBFAS was founded in 1968 by Frank Bonsall and John Ringrose. Since then, almost 200 meetings have taken place, with the purpose of bringing to the UK leading figures in functional analysis. Current members: Aberdeen, Queen's University Belfast, Birmingham, Cambridge, Edinburgh, Glasgow, Heriot-Watt, Lancaster, Leeds, Newcastle-upon-Tyne, Nottingham, Oxford. Speakers are Piotr Koszmider and Lyudmilla Turowska.

EiM2: The second workshop on Ethics in Mathematics

Location: University of Cambridge
Date: 3—5 April 2019
Website: tinyurl.com/ya4gw8ut

A workshop bringing together the rare few mathematicians addressing ethics in mathematics. Speakers are drawn from a variety of disciplines: mathematics, law, psychology, computer science. Email mcc56@cam.ac.uk to request an invitation.

Dynamics, Number Theory, and Geometry @ Exeter

Location: University of Exeter
Date: 5 April 2019
Website: tinyurl.com/ycqmwagb

This meeting aims to bring together researchers in homogeneous dynamics, number theory and geometry and to enhance interactions between these fields. Registration is required; email j.tseng@exeter.ac.uk. Supported by an LMS Scheme 9 grant.

Stable Categories

Location: Lancaster University
Date: 11 April 2019
Website: tinyurl.com/y9pp7ong

An afternoon meeting around the topic of combinatorics and computation in infinite groups, with talks given by Sarah Rees (Newcastle), Derek Holt (Warwick) and Laura Ciobanu (Heriot-Watt). Some support for early-career participants is available. Supported by an LMS Scheme 3 grant.

British Young Mathematicians' Colloquium

Location: University of Birmingham
Date: 17 April 2019
Website: tinyurl.com/y8crze2g

The colloquium is aimed at mathematicians from all areas at early stages in their careers (postgraduate students, postdocs, etc.). There will be six invited speakers but the majority of the conference will be made up of talks contributed by participants. If you have any questions, please email us at bymc@contacts.bham.ac.uk.

Ergodic Theory Meeting

Location: Loughborough
Date: 17 April 2019
Website: tinyurl.com/y9jdy6xh

This is part of a series of meetings in ergodic theory and related topics, supported by a Scheme 3 grant from the LMS. Speakers are Henna Koivusalo (Vienna), Anke Pohl (Bremen), Mark Pollicott (Warwick). Run in conjunction with a school on smooth ergodic theory and partially hyperbolic systems.

Scottish Combinatorics Meeting

Location: ICMS, University of Edinburgh
Date: 25–26 April 2019
Website: tinyurl.com/ybdjkevl

Members of the Combinatorics community are encouraged to attend, especially researchers at PhD and postdoc level. To register email mcryan@inf.ed.ac.uk. The meeting is supported by an LMS Conference grant and the Glasgow Mathematical Journal Trust.

British Applied Mathematics Colloquium

Location: University of Bath
Date: 24–26 April 2019
Website: go.bath.ac.uk/bamc

UK's largest applied mathematics conference bringing together students, academics and industry professionals to discuss recent advances within the field. The conference is an excellent forum for exploring collaboration opportunities, networking and in-depth discussions about research projects.

Wales Mathematics Colloquium 2019

Location: Gregynog Hall, Tregynon
Date: 20–22 May 2019
Website: tinyurl.com/yaw9esak

The Colloquium is a forum for the promotion and discussion of current research in Mathematics in Wales. The principal themes are mathematical biology, partial differential equations and mathematical modelling. Supported by an LMS Conference grant.

LMS Meeting

LMS Invited Lecture Series 2019

20–24 May 2019: ICMS,The Bayes Centre, 47 Potterrow, Edinburgh EH8 9BT,

Website: https://tinyurl.com/yb7v47yu

Søren Asmussen (Aarhus University): *Advanced topics in life insurance mathematics*. Accompanying lecturers: Dr Corina Constantinescu (Liverpool University), *Applications of fractional calculus in insurance/risk theory*; Professor Pauline Barrieu (London School of Economics).

For further details about the LMS Invited Lecture Series and to register for a place, visit the website.

Funds are available for partial support to attend the LMS Invited Lecture Series. Requests for support with an estimate of expenses, email the organiser Fraser Daly (f.daly@hw.ac.uk).

LMS Meeting

Northern Regional Meeting & Workshop

28 and 29 May 2019, Newcastle University

Website: tinyurl.com/y9d49s5t

The Northern Regional Meeting on 28 May will include lectures by Karin Baur (Universities of Graz and Leeds) and Sibylle Schroll (University of Leicester). All interested are welcome to attend. The Workshop on Higher Homological Algebra will take place on 29 May. There is a registration fee of £25, to be waived for PhD students.

Funds are available for partial support to attend the meeting and workshop. For further details and to register, email peter.jorgensen@ncl.ac.uk. Deadline: 1 May.

Combinatorics One-Day Meeting

Location: University of Oxford
Date: 29 May 2019
Website: tinyurl.com/32yrf2d

This year's speakers include Vida Dujmovic (Ottawa), Louis Esperet (Grenoble) and Martin Grohe (Aachen). Supported by an LMS Conference grant and the British Combinatorial Committee.

Dense Granular Flows

Location: University of Cambridge
Date: 1–4 July 2019
Website: tinyurl.com/y9jtvkkb

Flows involving solid particulates are ubiquitous in nature and industry alike; in order to be able to solve problems, granular flows need to be understood. Abstracts for presentations may be submitted.

LMS Meeting

General Society Meeting & Aitken Lecture

28 June 2019, Mary Ward House, London

Website: tinyurl.com/ybmyx6on

The Aitken Lecture will be given by Bakh Khoussainov (University of Auckland). Funds will be available for partial support to attend the meeting and workshop.

A Society Dinner will be held after the meeting at a nearby venue, at a cost of £35.00 each, including drinks. To reserve a place, email Elizabeth Fisher (lmsmeetings@lms.ac.uk).

Current Status and Key Questions in Landscape Decision Making

Location: INI, Cambridge
Date: 3–5 July 2019
Website: tinyurl.com/y9emsysr

This workshop focuses on reviewing the state-of-the-art in modelling land systems and identifying key knowledge gaps where collaboration between different environmental, mathematical and social science disciplines may lead to new insights, methods and tools. Closing date for applications: 31 March 2019.

Postgraduate Group Theory Conference

Location: University of Birmingham
Date: 22–26 July 2019
Website: tinyurl.com/yawz4b6g

This postgraduate conference is organised by and for postgraduate students in group theory and related areas. The bulk of the conference will be made up of contributed postgraduate talks, with plenary talks by Martin Liebeck and Colva Roney-Dougal. Supported by an LMS Scheme 8 grant.

Geometry, Compatibility and Structure Preservation in Computational Differential Equations

Location: INI, Cambridge
Date: 8–12 July 2019
Website: tinyurl.com/y8gfjze7

The tutorial workshop for the programme *Geometry, Compatibility and Structure Preservation in Computational Differential Equations* will feature five series of introductory lectures. Closing date for applications: 7 April 2019.

Progress on Novel Mathematics and Statistics for Landscape Decisions

Location: INI, Cambridge
Date: 31 July – 2 August 2019
Website: tinyurl.com/y83ejouv

This three-day workshop closes the *Mathematical and Statistical Challenges in Landscape Decision Making* programme via a series of research roadmaps that synthesize new research frontiers and synergies. Closing date for applications: 28 April 2019.

Factorisation of Matrix Functions

Location: INI, Cambridge
Date: 12–16 August 2019
Website: tinyurl.com/y7u3unaq

The aims of this workshop include sharing new tools for the solution of matrix Wiener–Hopf equation and the generalisations of the Wiener–Hopf method. Deadline for applications: 12 May 2019.

IWOTA 2020

Location: IWOTA 2020
Date: 17–21 August 2020
Website: tinyurl.com/y5fb3l28

This annual workshop brings together mathematicians and engineers who work in functional analysis and its application to related areas. Email g.blower@lancaster.ac.uk for details.

LMS Meeting

LMS Prospects in Mathematics Meeting

Lancaster University, 6–7 September 2019

Website: tinyurl.com/yysds39f

All Finalists Maths Undergraduates who are considering applying for a Maths PhD in 2020 are invited to attend the 2019 LMS Prospects in Mathematics Meeting. There are 50 places are available. See website for details on how to apply. Application deadline: 19 July 2019.

Society Meetings and Events

March

5 Christopher Zeeman Medal Presentation and Lecture, Royal Society, London (481)
21 Hirst Lecture and Society Meeting, De Morgan House, London (481)

April

5 Mary Cartwright Lecture, ICMS, Edinburgh (481)
10 LMS Meeting at the BMC, Lancaster (481)
29–3 May LMS-CMI Research School, *PDEs in Mathematical Biology—Modelling and Analysis,* ICMS, Edinburgh

May

20–24 Invited Lecture Series, London (481)
28–29 Northern Regional Meeting, Newcastle (481)

June

28 Graduate Student Meeting, London
28 General Meeting of the Society and Aitken Lecture, London (481)

July

1–5 LMS Research School: *Random Structures: from the Discrete to the Continuous,* Bath
8–12 LMS Research School, *Mathematics of Climate,* Reading

September

6–7 Prospects in Mathematics Meeting
11 Midlands Regional Meeting, Nottingham

November

15 Graduate Student Meeting, London
15 Society Meeting and AGM, London

January 2020

15 South West & South Wales Regional Meeting, Bristol

Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website (www.lms.ac.uk/content/calendar). Please send updates and corrections to calendar@lms.ac.uk.

March

4 Lancaster Undergraduate Mathematical Society Meeting, Lancaster University (481)
5 Christopher Zeeman Medal Presentation and Lecture, Royal Society, London (481)
12 Women in Mathematics Day 2019, Queen Mary, University of London (481)
20 Ergodic Theory Meeting, University of Manchester (481)
18-22 Electricity Systems of the Future: Incentives, Regulation and Analysis for Efficient Investment, INI, Cambridge (479)
21 Hirst Lecture and Society Meeting, De Morgan House, London (481)
22 Analysis of and Analysis on Networks, Teesside University (481)
27–30 Geometry and Mathematical Physics Workshop, Loughborough University (480)
29–30 Integrability and Conformal Field Theory 2019, University of Oxford (480)

April

1-3 Mirrors in the Midlands, University of Birmingham (481)
3-5 Young Functional Analysts' Workshop, University of Leeds (481)
3-5 EiM2: Ethics in Mathematics Workshop, University of Cambridge (481)
5 Mary Cartwright Lecture, ICMS, Edinburgh (481)
5 Dynamics, Number Theory, and Geometry @ Exeter, University of Exeter (481)
8-11 British Mathematical Colloquium 2019, Lancaster University (479)
8-12 Tropical Geometry meets Representation Theory II, University of Leicester (480)
10 LMS Meeting at the BMC, Lancaster (481)
11 Stable Categories, Lancaster University (481)
11-12 North British Functional Analysis Seminar, Lancaster University (481)
17 British Young Mathematicians' Colloquium, University of Birmingham (481)
17 Ergodic Theory Meeting, Loughborough University (481)
24-26 British Applied Mathematics Colloquium, University of Bath (481)
24-26 British Algebraic Geometry Meeting, University of Liverpool (480)
25-26 Scottish Combinatorics Meeting, ICMS, University of Edinburgh (481)
25-26 Mathematics of Operational Research, Aston University, Birmingham (479)
29-3 May Looking Forward to 2050: Closing Workshop, INI, Cambridge (480)
29-3 May LMS-CMI Research School, *PDEs in Mathematical Biology—Modelling and Analysis,* ICMS, Edinburgh

May

7-10 Interactions between Representation Theory and Homological Mirror Symmetry, University of Leicester (480)
13-17 Optimal Design of Soft Matter, INI, Cambridge (480)
15-17 Modern Applied Probability Workshop, ICMS, Edinburgh (480)
16-17 Idealised Models of Numerical Weather Prediction for Data Assimilation Research, University of Leeds (480)
20-22 Wales Mathematics Colloquium 2019, Gregynog Hall, Tregynon (481)
20-24 LMS Invited Lecture Series 2019, Professor Soren Asmussen (Aarhus University), ICMS, Edinburgh (481)
20-24 Women in Noncommutative Algebra and Representation Theory Workshop, University of Leeds (480)
28-29 LMS Northern Regional Meeting & Higher Homological Algebra Workshop, Newcastle (481)
29 Combinatorics One-Day Meeting, University of Oxford (481)

June

10-14 New Trends and Challenges in the Mathematics of Optimal Design, INI, Cambridge
17-21 MAFELAP 2019, Brunel University London (479)
17-21 Approximation, Sampling, and Compression in High Dimensional Problems, INI, Cambridge
24-26 Mathematical Neuroscience International Conference, Copenhagen, Denmark
28 LMS Graduate Student Meeting, London
28 LMS General Meeting and Aitken Lecture, London (481)

July

1-4 Dense Granular Flows IMA Conference, Cambridge (481)
1-5 LMS Research School: *Random Structures: from the Discrete to the Continuous,* Bath
3-5 Current Status and Key Questions in Landscape Decision Making, INI, Cambridge (481)
8-12 LMS Research School, *Mathematics of Climate,* Reading
8-12 Measurability, Ergodic Theory and Combinatorics, University of Warwick (479)
8-12 Geometry, Compatibility and Structure Preservation in Computational Differential Equations, INI, Cambridge (481)
22-26 Postgraduate Group Theory Conference 2019, University of Birmingham (481)
28-3 Aug International Mathematics Competition for University Students, Blagoevgrad, Bulgaria (481)
29-2 Aug British Combinatorial Conference 2019, University of Birmingham (479)
31-2 Aug Progress on Novel Mathematics and Statistics for Landscape Decisions, INI, Cambridge (481)

August

12-16 Factorisation of Matrix Functions, INI, Cambridge (481)

September

6-7 Prospects in Mathematics Meeting, Lancaster (481)
11 Midlands Regional Meeting, Nottingham

November

15 Graduate Student Meeting, London
15 Society Meeting and AGM, London

January 2020

15 South West & South Wales Regional Meeting, Bristol

August 2020

17-21 IWOTA 2020, Lancaster University (481)