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NEWSLETTER

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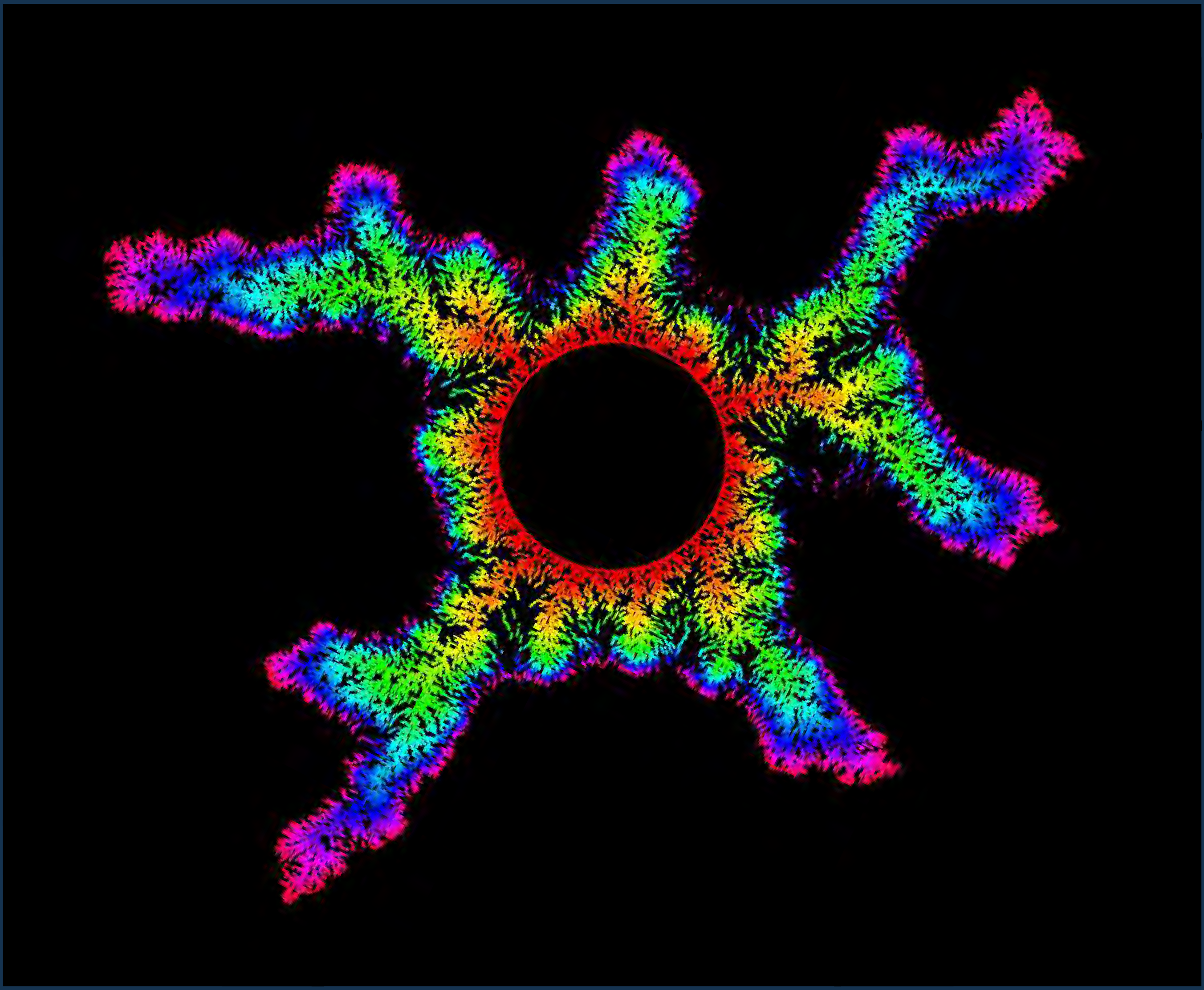

RANDOM GROWTH: FROM LICHEN TO LIGHTNING

FROM GAMBLING TO RANDOM MODELLING

COLOURING WITHOUT COLOURS

EDITORIAL OFFICE

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COVER IMAGE

Diffusion-limited aggregation grown by composing conformal mappings using the Hastings–Levitov construction. See page 19.

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The Newsletter welcomes submissions of feature content, including mathematical articles, career related articles, and microtheses from members and non-members. Submission guidelines and LaTeX templates can be found at lms.ac.uk/publications/submit-to-the-lms-newsletter.

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News items should be sent to newsletter@lms.ac.uk.

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CONTENTS

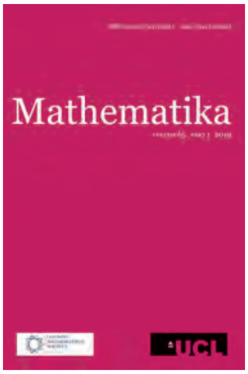
LMS NEWS

LMS General Meeting

There will be a General Meeting of the Society on 28 June 2019 at 3:30 pm, to be held at Mary Ward House, Tavistock Square, London. The business shall be: 1) the appointment of the Scrutineers; 2) announcement of Council's recommendation for election to Honorary Membership; 3) announcement of LMS prize winners for 2019.

The General Meeting, at which Professor Bakh Khoussainov (University of Auckland) will give the Aitken Lecture, will be followed by a Society Meeting. It is hoped that as many members as possible will be able to attend.

Mathematika partnering with Wiley



From January 2020, the LMS will partner with John Wiley & Sons Ltd for the production and distribution of the journal *Mathematika*. The decision to move to Wiley was taken jointly by the Managing Editors of *Mathematika* and the LMS Publications Committee and Council after a thorough review process. *Mathematika* is published by the London Mathematical Society on behalf of its owners University College London, and will be the sixth journal published in partnership with Wiley, along with the core journals of the LMS (the *Bulletin, Journal, Proceedings* and *Transactions*) and the *Journal of Topology*.

William Chen, Frank Smith, Alexander Sobolev
Managing Editors

Forthcoming LMS Events

The following events will take place in the next two months:

LMS Education Day: 13 May, De Morgan House, London (tinyurl.com/y65p23kk)

Invited Lecture Series: 20–24 May, ICMS, Edinburgh (tinyurl.com/yb7v47yu)

Northern Regional Meeting: 28–29 May, Newcastle University (tinyurl.com/y9d49s5t)

Popular Lectures: 26 June, Institute of Education, London (tinyurl.com/hu58wjk)

Graduate Student Meeting, General Meeting and Aitken Lecture: 28 June, Mary Ward House, London (tinyurl.com/y3umhyjl)

A full listing of upcoming LMS events can be found on page 50.

OTHER NEWS

Karen Uhlenbeck awarded the 2019 Abel Prize

This year's Abel Prize has been awarded to the US mathematician Professor Karen Uhlenbeck, Princeton University, for her 'pioneering achievements in geometric partial differential equations, gauge theory and integrable systems, and for the fundamental impact of her work on analysis, geometry and mathematical physics'. She is the first woman to be awarded the prestigious honour. Professor Uhlenbeck will be presented with the Prize by His Majesty King Harald V at an award ceremony in Oslo on 21 May 2019. The Prize carries a cash award of NOK 6 million (about €650,000 or about US$730,000).

Professor Uhlenbeck is a founder of modern Geometric Analysis. Her perspective has permeated the field and led to some of the most dramatic advances in mathematics in the past 40 years. Geometric analysis is a field of mathematics where techniques of analysis and differential equations are interwoven with the study of geometrical and topological problems, specifically, studies on objects such as curves, surfaces, connections and fields, which are critical points of functionals representing geometric quantities such as energy and volume. For example, minimal surfaces are critical points of the area and harmonic maps are critical points of the Dirichlet energy. The winner's major contributions include foundational results on minimal surfaces and harmonic maps, Yang-Mills theory, and integrable systems.

Professor Uhlenbeck has received many honours during a distinguished career and was elected an Honorary Member of the LMS in 2008.

For the first time the announcement was transmitted live to an audience at the Science Gallery in London, with the LMS working in collaboration with the Norwegian Academy of Science and Letters and the Norwegian Embassy in London. After the announcement, a panel including LMS President Professor Caroline Series FRS, past-LMS President Professor Nigel Hitchin FRS, Professor Mark Haskins and Professor Peter Topping discussed the winner's life and work. The panel was moderated by the Science Gallery's founding Director Daniel Glaser.

Professor Caroline Series FRS, President of the LMS, said: "I have always been a great admirer of Karen's. She is a powerful and distinguished mathematician whose foundational work in geometric analysis has had far reaching ramifications. Throughout her career she has also been a great leader and mentor, and I am sure that many others besides myself will be absolutely delighted that she has been recognised by the award of the Abel Prize, one of the highest awards in mathematics".

More information is available at abelprize.no/.

Turing Award announced

As the *Newsletter* was going to press, the Association for Computing Machinery (ACM) has announced the joint winners of the ACM A.M. Turing Award as Yoshua Bengio (Professor at the University of Montreal and Scientific Director at Mila, Quebec's Artificial Intelligence Institute), Geoffrey Hinton (VP and Engineering Fellow of Google, Chief Scientific Adviser of The Vector Institute, and University Professor Emeritus at the University of Toronto) and Yann LeCun (Silver Professor of the Courant Institute of Mathematical Sciences at New York University, and VP and Chief AI Scientist at Facebook). The award is "for conceptual and engineering breakthroughs that have made deep neural networks a critical component of computing," and recognises work carried out both independently and together by the three winners. Widely considered as the highest distinction in computer science, the award comes with a $1 million prize.

"Artificial neural networks" are inspired by the human brain, and enable computers to solve tasks such as object recognition, without explicit step-by-step instructions. This idea goes back to the early 1980s, when Hinton — who had previously studied at the Universities of Cambridge and Edinburgh — was an early advocate of this approach. The three winners were part of a small group of researchers who persevered with this "machine learning" approach to artificial intelligence during the next three decades; today this methodology has been incorporated into technology used by billions of people, as well as being dominant across most areas of scientific research.

ACM President Cherri M. Pancake said: "Artificial intelligence is now one of the fastest-growing areas in all of science and one of the most talked-about topics in society. The growth of and interest in AI is due, in no small part, to the recent advances in deep learning for which Bengio, Hinton and LeCun laid the foundation."

L'Oréal–UNESCO Prize for Women and Science

The 21st International Prize for Women in Science L'Oréal-UNESCO, on 11 February, awarded honours to five outstanding women scientists from different parts of the world, for excellence in their work in the fields of material science, mathematics and computer science. Among the five international prizes two are mathematicians, Ingrid Daubechies and Claire Voisin, both Honorary Members of the LMS.

Professor Daubechies was awarded for her "groundbreaking work on wavelet theory, which has transformed the numerical treatment of images and signal processing, providing standard and flexible algorithms for data compression. Her research has catalyzed multiple innovations, with new image processing and filtering methods used in technologies ranging from medical imaging to wireless communication."

Professor Voisin was "rewarded for her outstanding work in algebraic geometry. Her pioneering discoveries have answered fundamental questions on the geometry, the topology and Hodge structures of complex algebraic varieties."

Adams Prize 2019

Dr Heather Harrington (University of Oxford), an LMS member and a 2018 LMS Whitehead Prize winner, has been awarded the 2019 Adams Prize jointly with Dr Luitgard Veraart (LSE). The subject of the prize was the Mathematics of Networks. The Adams Prize is awarded each year by the Faculty of Mathematics and St John's College, University of Cambridge, to a young researcher based in the UK who is doing first-class international research in the Mathematical Sciences.

Launch of the Newton Gateway to Mathematics

The Newton Gateway to Mathematics was launched in January 2019, having rebranded from the Turing Gateway to Mathematics. The Gateway is the impact initiative of the Isaac Newton Institute for Mathematical Sciences (INI). Supported by INI and the University of Cambridge, it reaches out to and engages with the users of mathematics — in industry, the public sector and other scientific disciplines. It helps to bridge the gap between those engaged in frontier mathematical research and those working in more applied areas, facilitating knowledge exchange between the mathematical sciences and potential users of mathematics, both in the UK and internationally. It does this by bringing academic researchers involved with INI research programmes into contact with industrial, commercial and governmental organisations and individuals. For further information on the Newton Gateway to Mathematics visit gateway.newton.ac.uk/.

Survey on the transition from school to university mathematics

Student transition from school-level mathematics to university-level mathematics, often referred to as the secondary-tertiary transition (STT) is an enduring, complicated and multi-faceted process. STT is a long-standing issue of concern, which has merited significant attention in mathematics education research and practice. At its 2018 meeting in Cyprus, the European Mathematical Society Education Committee recognized that our knowledge about successful ways of dealing with STT is still insufficient and that moving forward requires a large-scope effort on the part of all parties involved, including mathematics lecturers, school teachers, education researchers, policymakers and students in transition. As part of this effort, the Committee is conducting a survey among mathematicians. The goal of the survey is to collect and report to the mathematics community information needed in order to devise national and international actions that can essentially improve the state of the art with respect to STT.

The survey takes about 15 minutes. It is open until 15 September 2019 (tinyurl.com/y22e9o6o). For more background information about STT see: tinyurl.com/yyeehkvg.

Celebrating Caucher Birkar

Ivan Fesenko, Caucher Birkar, Shearer West, Paul Houston. (Photo: University of Nottingham)

The University of Nottingham celebrated Professor Caucher Birkar's remarkable achievements in mathematics, and his recent award of the Fields Medal, with a special reception in Nottingham.

Caucher studied at Nottingham under the supervision of Ivan Fesenko. Professor Fesenko said: "Caucher's story is quite extraordinary. He came from Kurdistan to live in in Nottingham whilst he waited for a decision about his asylum application from the Home Office. He was so passionate about striving to learn cutting edge mathematics that he almost immediately came to our maths department and asked me for a supervision of his work. I recommended some books for him to read and set an ambitious research project for his PhD years which I thought would take quite a time for him to implement. He came with its solution in three months, instead of the usual three years!" Birkar played the most fundamental role in two waves of major advances in birational geometry in the last 15 years. In August 2018 he was awarded the Fields Medal (see Issue 479, page 20, of the *LMS Newsletter*, for background on Caucher's work).

During the celebration, held on 26 February, Ivan Cheltsov (Edinburgh) gave a talk on mathematicians contributing to the statement of the BAB conjecture solved by Birkar. Birkar also answered questions and described his plans to help to promote scientific work and activities in the Middle East.

Final guidance and criteria for REF 2021 published

In January the UK's four higher education funding bodies published the key documents that provide guidance to UK universities when submitting their research to the next Research Excellence Framework (REF) 2021. The documents have been developed by the REF's expert panels and the four funding bodies following consultation with the sector in summer 2018. The changes from the last REF in 2014 to REF 2021 largely follow the recommendations set out in 2016 in an independent review led by Lord Stern.

Members may be reassured to see clarity on preprints and the arXiv through Rule 258 of the Guidance on Submissions: "An output first published in its final form during the REF 2021 publication period that was 'pre-published' in the previous publication period — whether in full in a different form (for example, as a pre-print), or as a preliminary version or working paper — is eligible for submission to the REF, provided that the 'pre-published' output was not submitted to REF 2014". More information is available here tinyurl.com/y266j4mm.

Review of TEF

In late 2018 the Secretary of State for Education appointed Dame Shirley Pearce to conduct an independent review of the Teaching Excellence and Student Outcomes Framework (TEF). A report is expected to be submitted to the Secretary of State in Summer 2019 and the recommendations will be considered before the implementation of subject-level TEF. The LMS response to the review consultation is available at tinyurl.com/yyvuakba.

LMS responds to Open Access consultations

UK Research and Innovation (UKRI) are currently undertaking a review of their Open Access policies and as part of the process invited evidence from learned societies on how they provide benefit to the research community using funds generated from publishing activities.

Plan S is a position document from a group of European national funders, cOALition S, with the support of the European Commission and the European Research Council aiming to accelerate the transition to Open Access publishing. Over 600 individuals and organisations, including the LMS, provided feedback to cOAlition S on the implementation guidance of Plan S (tinyurl.com/y44dt76z).The Society expressed concerns with various aspects of the plan, particularly the short timeline proposed for the transition and the fact that the policy is not yet supported by countries outside Europe who are large producers of research.

LMS becomes a signatory to DORA

The London Mathematical Society has signed the San Francisco Declaration on Research Assessment (DORA) in support of the movement towards the responsible use of research metrics within mathematics.

DORA is a worldwide initiative covering all scholarly disciplines which recognizes the need to improve the ways in which the outputs of scholarly research are evaluated and seeks to develop and promote best practice. To date it has been signed by over 1,200 organizations and around 13,000 individuals. Its main themes are the need to eliminate the use of journal-based metrics, such as Journal Impact Factors, in funding, appointment, and promotion considerations; the need to assess research on its own merits rather than on the basis of the journal in which the research is published, and exploring new indicators of significance and impact. More information is available at sfdora.org/.

Digest prepared by Dr John Johnston
Society Communications Officer

Note: items included in the Mathematics Policy Digest are not necessarily endorsed by the Editorial Board or the LMS.

EUROPEAN MATHEMATICAL SOCIETY NEWS

Open access: Plan S feedback

The mathematical community has long been at the forefront of establishing an accessible, efficient, fair and transparent scholarly publishing system. The EMS and its Publications and Electronic Dissemination Committee are in favour of Open Access to the scientific literature. They have given detailed feedback to the Science Europe Office on the Plan S as devised within the European Political Strategy Centre at the European Commission. The text can be found at tinyurl.com/y45rbetn.

Potential changes for ERC

A reorganisation of the Research and Innovation Directorate General of the European Commission is scheduled to take place this year with the aim of revising reporting procedures and coordination between the different agencies. The EMS is concerned that such reorganisation may have a negative impact on the European Research Council (ERC), and consequently on scientific research within Europe.

ICM Structure Committee

In the last few years there has been discussion regarding ways in which the process for structuring the International Congress of Mathematicians (ICM) and inviting lecturers could be improved. At the 2018 ICM General Assembly meeting in Rio de Janeiro, a resolution was adopted to create a new Structure Committee. As one of the first actions, this committee is soliciting input from the mathematical community. See tinyurl.com/yd8dhn8f.

EMS News prepared by David Chillingworth
LMS/EMS Correspondent

VISITS

Visit of Javad Asadollahi

Professor Javad Asadollahi (University of Isfahan and IPM-Isfahan, Iran) will visit the University of Leicester from 1 June to 15 July 2019 to work with Professor Sibylle Schroll. During his visit he will give a seminar on *Gorenstein Cluster Tilting Subcategories*. For further information email (schroll@leicester.ac.uk). Supported by an LMS Scheme 5 Collaborations with Developing Countries grant.

Visit of Boris Bychkov

Dr Boris Bychkov (Higher School of Economics, Moscow) is visiting the University of Leeds from 28 April to 18 May. Boris is an early career researcher working at the interface of enumerative geometry and theory of integrable systems. He is particularly interested in possible connections of generating functions for graph invariants and solutions of integrable hierarchies. For further information email Alexander Mikhailov (a.v.mikhailov@leeds.ac.uk). Supported by an LMS Scheme 4 Research in Pairs grant.

Visit of Paolo Caldiroli

Professor Paolo Caldiroli (Department of Mathematics, University of Torino, Italy) will visit the University of Bath from 3 to 14 June 2019 to collaborate with Professor Monica Musso on some problems of surfaces with prescribed mean curvature. He works in the field of Geometric Analysis, with variational and perturbative methods. He will also give talks at the University of Warwick and Swansea University. For further information contact m.musso@bath.ac.uk. The visit is supported by an LMS Scheme 2 grant.

Visit of Christian Müller

Professor Christian Müller (Wien University of Technology) will visit the Department of Mathematical Sciences, University of Liverpool, from 1 to 14 June 2019. He is particularly known for his contributions to discrete and semidiscrete differential geometry and its applications. He will give lectures at the University of Bath (4 June), the University of Liverpool (5 June) and Lancaster University (7 June). For further details contact karpenk@liv.ac.uk. The visit is supported by an LMS Scheme 2 grant

Visit of Chimere Anabanti

Dr Chimere Anabanti (University of Nigeria, Nsukka), will visit the University of Essex from 10 to 28 June 2019. He works in computational group theory. During his visit he will give talks at Essex (13 June) and Birkbeck, University of London (19 June). For further details contact gerald.williams@essex.ac.uk. The visit is supported by an LMS Scheme 5 grant.

Visit of Maria Joao Oliveira

Dr Maria Joao Oliveira (Universidade Aberta, Lisbon) will visit the UK from 2 to 15 June 2019. Her research involves stochastic analysis and infinite dimensional analysis, including its algebraic and combinatorial aspects. She will give lectures at York (3 June), Swansea (7 June) and Reading (14 June). For further details contact e.lytvynov@swansea.ac.uk. The visit is supported by an LMS Scheme 2 grant.

Visit of Michael Overton

Professor Michael Overton (New York University) will be visiting Northumbria University from 7 to 19 July 2019. His research interests are at the interface of optimization and linear algebra, especially nonsmooth optimization problems involving eigenvalues, pseudospectra, stability and robust control. For further information contact oleg.kirillov@northumbria.ac.uk . The visit is supported by an LMS Scheme 4 Research in Pairs grant.

Visit of Pedro Tradacete Perez

Professor Pedro Tradacete Perez (ICMAT, Madrid) will be visiting the UK during May 2019. He will give talks at the University of Cambridge *Free Banach lattices*, University of Oxford *The least doubling constant of a metric space*, Lancaster University *Valuations on Banach lattices* and Queen's University Belfast *Strictly singular multiplication operators on L(X)*. For further details contact Dr Martin Mathieu (M.M@qub.ac.uk).The visit is supported by an LMS Scheme 2 grant.

Visit of Dominik Francoeur

Mr Dominik Francoeur (University of Geneva, Switzerland) will visit the University of Lincoln from 26 May to 9 June 2019. His research involves groups acting on trees and automata semigroups. During his visit he will give a talk *On Free Subsemigroups in Automata Groups*. For further details contact AThillaisundaram@lincoln.ac.uk. The visit is supported by an LMS Scheme 4 Research in Pairs grant.

Visit of Ricardo Nochetto

Professor Ricardo Nochetto (University of Maryland) will visit the UK from 12 to 30 June 2019. He will give a plenary lecture on fractional partial differential equations at the sixteenth MAFELAP conference (brunel.ac.uk/mafelap) 18–21 June, as well as lectures at the University of Oxford (12–15 June), Imperial College London (17 June) and University of Sussex (22–30 June). The visit is supported by an LMS Scheme 2 grant.

Visit of Rachel Skipper

Dr Rachel Skipper (University of Göttingen, Germany, and ENS Lyon, France) will visit the UK from 2 to 26 June 2019. Her research involves groups acting on trees, self-similar and automata groups, branch groups and groups in the extended Thompson family. She will give lectures at St Andrews (6 June), Oxford (18 June) and Lincoln (20 June). For further details contact AThillaisundaram@lincoln.ac.uk. The visit is supported by an LMS Scheme 2 grant.

Visit of Zenebe Wogderesegn

Zenebe Wogderesegn (PhD student, Addis Ababa University) will visit Oxford Brookes University from 1 June to 31 July. He works in analysis of existence and uniqueness of solutions of boundary-domain integral equations. He will give a talk on *On united boundary-domain integro-differential equations for variable coefficient Neumann problem with general right hand side*. For further information email c.portillo@brookes.ac.uk. The visit is supported by an LMS Scheme 5 grant.

UNIVERSITY OF CAMBRIDGE

ADAMS PRIZE

Algebra

The University of Cambridge has announced the subject for one of its oldest and most prestigious prizes. The Adams Prize is named after the mathematician John Couch Adams and was endowed by members of St John's College. It commemorates Adams's role in the discovery of the planet Neptune, through calculation of the discrepancies in the orbit of Uranus.

The Chairman of the Adjudicators for the Adams Prize invites applications for the 2019-2020 prize which will be awarded this year for achievements in the field of Algebra.

The prize is open to any person who, on 31st October 2019, will hold an appointment in the UK, either in a university or in some other institution; and who is under 40 (in exceptional circumstances the Adjudicators may relax this age limit). The value of the prize is expected to be approximately £15,000, of which one third is awarded to the prize-winner on announcement of the prize, one third is provided to the prize-winner's institution (for research expenses of the prize-winner) and one third is awarded to the prize-winner on acceptance for publication in an internationally recognised journal of a substantial (normally at least 25 printed pages) original article, of which the prize-winner is an author, surveying a significant part of the winner's field.

Applications, comprising a CV, a list of publications, the body of work (published or unpublished) to be considered, and a brief non-technical summary of the most significant new results of this work (designed for mathematicians not working in the subject area) should be sent to the Secretary of the Adams Prize Adjudicators via email to adamsprize@maths.cam.ac.uk.

The deadline for receipt of applications is 31st October 2019.

More information is available at www.maths.cam.ac.uk/adamsprize

Council Diary and Strategic Retreat

Strategic Retreat

Council met for its 2019 Strategic Retreat on 1st and 2nd February at Woodland Grange in Leamington Spa. The four main areas of discussion were the Standing Orders Review, our response to the Bond Review, the LMS's grant portfolio, and a more general discussion of Mathematics: The State of the Nation.

The Standing Orders Review Group had been working on revisions to the Standing Orders for the past few years, and these had been sent out to the membership for consultation last autumn. Some of the points raised in the consultation could be incorporated directly, and some others will need more discussion by the Standing Orders Review Group. There were, however, some specific issues that needed discussion during the retreat: Charter 7 and Statute 31, which deal with removal of members and trustees respectively, as well as Statute 14 regarding reciprocity membership. After long deliberation it was felt that the current Charter 7, modulo some minor revisions, should be retained so that a member can only be removed at a General Meeting. Statute 31, on the other hand, was referred back to the Review Group requiring it to take into consideration that it should not replace legal proceedings, should ensure that the Statute is as required by law and allows Council to act within the law, and that it is transparent and leaves power with the membership where possible. It was agreed to change the wording of Statute 14 with regard to reciprocity agreements to allow more flexibility.

The Society had set up a Working Group to discuss the implications of the Bond Review for the LMS and to put forward the viewpoint of the LMS and its members. We discussed an initial response document drafted by the LMS Working Group, taking into account feedback from members and from Council. It was welcomed that the Bond Review suggests additional resources for the Mathematical Sciences, and that it emphasises the importance of Pure Mathematics in its introduction. However, views differed on how the emphasis on Knowledge Exchange would affect funding for Pure Mathematics. We discussed the proposed formation of an Academy for the Mathematical Sciences as a single voice for Mathematics. It was felt by some that the case for such an Academy is not convincing, as it would sit between LMS and Government, and there were worries about its perceived focus on Knowledge Exchange. The discussions will be fed back to the Working Group and incorporated into the response document.

We then moved on to consider the results of a survey of the community with respect to the Society's grant schemes. We were split into small groups to consider some specific questions that had arisen. The overriding opinion was that the Society's grants were extremely valuable, yet there seemed to be a need to better fund early career researchers. It was decided to allow committees more flexibility, and to make some other small changes to the portfolio.

On Saturday we reconvened to discuss the State of the Nation, in particular to consider common issues currently affecting UK mathematical sciences departments. In small groups we thought about how the Society might provide support to address any issues of concern. The topics discussed were infrastructure, recruitment, teaching and research. It was felt that many department face similar issues with top down, one size fits all policies regarding infrastructure and teaching. Recruitment of female and minority members of staff remains low, and there is uncertainty caused by Brexit. The Society might have a role in trying to influence policy. As for research, currently more funding is being channelled into large departments. There was a general understanding that excellent research was widespread and not just concentrated into these departments. It was suggested that a model operating in a similar way to the joint Centres for Doctoral Training (CDTs) might be beneficial by allowing for joint funding for smaller and larger departments. The Society might have a role in bringing this model together.

We also discussed which form a fitting memorial for Sir Michael Atiyah organised by the Society could take. Many ideas were floated, and this will be further discussed at the Council Meeting in April. Finally, we spent some time talking about a possible name-change, seeing that there is a wide-spread misconception that the LMS is a local rather than a national society. It was noted that it would not be possible or wise to change our Journal titles, and that there is a long and distinguished history associated with the name LMS. The discussion remained inconclusive, and I expect that this topic will resurface regularly.

Council Diary

The Strategic Retreat was followed on Saturday afternoon by a shorter than usual Council meeting. The President started with an oral update on her activities, and reported that the Abel Prize Committee had approached the Society to be involved in the live-screening of the 2019 Prize announcement which would be held at the Science Gallery at King's College. Members have received their invitations by now. Most of Council's business had already been discussed at the Retreat, hence we were left with more or less routine business. We heard a report from the Publications Secretary, and agreed the 2019–2021 LMS Publications Strategic Plan. Council also agreed that the Society should become a signatory to DORA, the San Francisco Declaration on Research Assessment, which originated from a recognition of the misuse of journal impact factors. After signing off minutes from previous meetings, we discussed funding for the Isaac Newton Institute, Maths Week Scotland, and Talking Maths in Public. We heard the Scrutineers' report, and decided that the Society should become an Associate Member of the Heads of Departments of Mathematical Sciences (HoDoMS). We agreed to implement a reciprocity agreement with the Irish Mathematical Society, and approved the list of 43 members to be elected at the Society meeting to be held on 21 March 2019.

Brita Nucinkis
Council Member-at-Large

REPORTS OF THE LMS

Report: LMS SW&SW Regional Meeting and Workshop

Keith Ball

The South West and South Wales Regional Meeting was held on Monday 17 December 2018 at the University of Exeter. The invited speakers were Professor Keith Ball (University of Warwick), Dr Min Lee (University of Bristol) and Professor Jens Marklof (University of Bristol).

Keith Ball's talk was on *Rational approximations to the Riemann zeta function*. He explained how to construct a sequence of rational functions which interpolate the zeta function at negative integers and converge to it locally uniformly to the right of the critical line. The numerators and denominators of these can be expressed as characteristic polynomials of certain matrices. This allows the Riemann Hypothesis to be stated as what appears to be a fairly conventional spectral problem involving functions that may be susceptible to quantitative estimation.

Min Lee

Min Lee spoke about *Twist-minimal trace formulas and applications*. She first reviewed the work of Selberg on the existence of Maass forms and the present state of knowledge on his Eigenvalue Conjecture, which predicts that the corresponding eigenvalues of the Laplace-Beltrami operator are at least ¼. She then explained an explicit form of the trace formula for Maass forms associated to a Dirichlet character whose conductor cannot be reduced by a twist. This enables the eigenvalues of Maass forms to be calculated numerically. Using a list of 2-dimensional Artin representations, she identified a specific icosahedral Galois representation for which the minimal eigenvalue is expected to attain the value ¼ of Selberg's Eigenvalue Conjecture.

Jens Marklof

Jens Marklof gave a talk on *Chaos and randomness modulo 1: the interplay of number theory and dynamical systems*. He gave many examples of deterministic or random sequences which are uniformly distributed mod 1, and for which the gap distribution and two-point correlation are known. Sources of such examples include random matrices, lacunary sequences, polynomials and fractional powers. The limiting statistical properties of sequences coming from square roots mod 1 and from directions of shifted lattice points are the same. This is explained by the fact that both give rise to the same invariant measure.

After these talks, a brief LMS business meeting was held, chaired by the President, Professor Caroline Series, at which several new members signed the Members' Book. The evening then continued with a wine reception and ended with a dinner at the Queens Court Hotel.

Following on from the Regional Meeting, there was a workshop on the theme of *Number Theory and Function Fields at the Crossroad,* 18–20 December. This comprised a variety of talks on wide-ranging aspects of analytic number theory and function fields, covering topics including moments of L-functions, the prime number theorem, character sums, random matrix theory and quantum chaos. Among the speakers were several early career researchers, including PhD students Emma Bailey (University of Bristol), Dan Carmon (Tel Aviv University) and Allysa Lumley (York University, Toronto).

Julio Andrade and Nigel Byott (organisers)
University of Exeter

From Lichen to Lightning: Understanding Random Growth

AMANDA TURNER

Random growth arises in physical and industrial settings, from cancer to polymer creation. Despite considerable effort, mathematicians and physicists have been unable to answer fundamental questions such as "What do typical clusters look like?". This article explores how combining probability and complex analysis can provide mathematical descriptions of random growth.

Random growth in nature

Random growth processes occur widely in nature. They appear as collections of particles (often cells or molecules), called clusters, to which new particles are attached according to some stochastic rule. The physical mechanism by which the growth occurs determines the precise stochastic rule, and different types of growth can produce very different clusters.

Some of the most commonly seen examples of random growth in nature arise through biological growth, such as lichen on a rock or cancer tumours grown in a lab. In these examples, growth occurs due to a reproductive event taking place on the boundary of the cluster and so (in the absence of environmental influences) the next particle is equally likely to be added anywhere on the cluster boundary. Many lichens tend to grow as clusters that are round with roughness at the edges. Other biological growth processes also commonly produce this kind of shape.

Another way in which random growth can occur is through mineral aggregation. An example of this is soot deposition in a diesel engine. Fine particles of carbon diffuse around the engine until they either hit the surface of the engine or the soot aggregate which has started to line the engine. At this point they stick, becoming part of the aggregate. Similar kinds of clusters arise as frost flowers, which appear on car windows on some winter mornings, and in industrial processes such as electro-deposition. In all of these examples, the position at which each successive particle is attached is given by the hitting distribution of a diffusion process (such as Brownian motion) on the cluster boundary. It is no longer the case that all positions on the cluster boundary are equally likely as diffusion processes have a greater chance of hitting protrusions than indentations. As a result, clusters tend to be much more irregular than those formed from biological growth processes, often exhibiting long fingers, deep fjords and fractal branching structures (see Figure 1).

Figure 1. A "pseudo fossil" formed by the deposition of manganese oxide in a sandy substrate. This illustrates the typical features present in clusters formed through mineral aggregation. (Photo: Alan Dickinson.)

Although lightning may not look like an obvious example of random growth, it also fits into this framework. The density, humidity, and conductivity of air is very inhomogeneous. Lightning surveys all possible paths through this medium and then strikes along those paths with the least electrical resistance. The molecules through which the lightning strike passes can be viewed as a cluster to which molecules are successively added in a way which minimises the total electrical resistance at each time-step. Surface discharges of lightning (also called Lichtenberg figures) and polymer formation are examples of random growing clusters which also arise in this way. The shape of the cluster formed depends on the strength of the local electric field and can range from a solid disk if the electric field is very weak to a one-dimensional path if the electric field is very strong.

Discrete models

The earliest mathematical models for random growth were discrete in nature. In discrete models, growing clusters $K_0 \subset K_1 \subset \cdots$ are constructed as increasing sets of connected vertices on an underlying (infinite) connected graph $G = (V, E)$. At each time step a new vertex is chosen from the neighbours of the cluster according to some stochastic rule and added to the cluster. To be precise, for any $\emptyset \subset A \subset V$, let

$$\partial A = \{v \in V \setminus A : (u, v) \in E \text{ for some } u \in A\}$$

denote the boundary of the subgraph A and suppose $p_A : \partial A \to [0, 1]$ is some function which satisfies

$$\sum_{v \in \partial A} p_A(v) = 1.$$

A random growth process $K_0, K_1, \ldots$, of sets of connected vertices, can be constructed recursively by starting from some connected set $\emptyset \subset K_0 \subset V$ and taking $K_{n+1} = K_n \cup \{v\}$ with probability $p_{K_n}(v)$. It is usual to start with $K_0 = \{v_0\}$ where v_0 is some distinguished vertex called the seed particle. Typically G is a lattice such as $\mathbb{Z}^d$ and $v_0 = 0$.

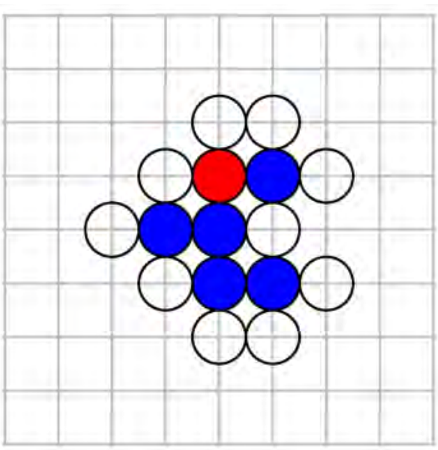


Figure 2. A representation of a random cluster grown on $\mathbb{Z}^2$. The red disk represents the seed particle which was placed at the origin. The blue disks are the first five particles to be attached to the cluster. The empty black circles represent the possible locations at which the next particle might be attached.

In order to model the physical growth processes discussed in the previous section, we need to show how to construct probability distribution functions p_A which correspond to the physical attachment rules described above. We begin by considering biological growth. One possible way to model uniformly random reproductive events on the cluster boundary is by choosing each successive particle uniformly from all possible locations on the boundary i.e. $p_A(v) = 1/|\partial A|$ for each $v \in \partial A$. This model is called the Eden model and was first proposed in 1961 [1]. This is not the only way in which biological growth can be modelled on a lattice. Observe that in Figure 2, the vertex at position $(1, -1)$ has three possible parents whereas the vertex at position $(2, 0)$ has only one possible parent. If the new particles are actually arising as offspring of the parent particles, one would expect the next particle to be more likely to be added at $(1, -1)$ than $(2, 0)$. A more realistic variation on the Eden model is to pick each vertex with a probability that is proportional to the number of edges which connect that vertex to the cluster, i.e., if

$$N_A(v) = \{u \in A : (u, v) \in E\},$$

then

$$p_A(v) = \frac{|N_A(v)|}{\sum_{u \in \partial A} |N_A(u)|}.$$

In 1981, Witten and Sander proposed the following model for mineral aggregation which they called diffusion-limited aggregation or DLA [6]. As before, initialise with a seed at the origin or some other distinguished vertex. At each time-step place a particle on a random site at a large distance from the seed. Allow the particle to perform a simple random walk on the graph until it visits a site adjacent to the cluster, at which point attach the particle to the cluster, i.e., add the vertex on which the particle stopped. The distribution function p_A can therefore be obtained by computing the hitting probabilities of the cluster boundary ∂A by a simple random walk started at "infinity". In order to guarantee that the particle eventually reaches the cluster we need the random walk to be a recurrent process on the underlying graph. This means that if $G = \mathbb{Z}^d$, we must take $d \leq 2$. The case $d = 1$ is not very interesting as particles will always just attach to the left or right of the cluster with equal probability, so it is usual to take $d = 2$ when talking about DLA. A variant of this model is called multi-particle DLA in which infinitely many particles are released from random positions simultaneously, rather than one at a time, and they perform independent random walks until they hit the cluster. Computing p_A explicitly for larger and larger sets A gets complicated very quickly and the long-term behaviour of DLA is notoriously difficult to understand.

In order to model clusters formed by electric discharge, the notion of electric potential is needed. For simplicity, take $G = \mathbb{Z}^2$ and $v_0 = 0$. Suppose there are two electrodes in the system: one placed at 0 and the second modelled as a large circle of radius R. Let $B_R(0) \subset \mathbb{R}^2$ denote the open ball with centre 0 and radius R. Given a conducting cluster $\{0\} \subseteq A \subset B_R(0) \cap \mathbb{Z}^2$, the electric potential is a discrete harmonic function ϕ_A (see the aside on "Harmonic functions") defined on the lattice with $\phi_A(v) = 0$ if $v \in A$ and $\phi_A(v) = 1$ if $|v| \geq R$. (It follows from the discussion on harmonic functions that $\phi_A(v)$ is equal to the probability that a simple random walk started at v exits the ball $B_R(0)$ before hitting the set A.) The probability of attaching a particle at a vertex $v \in \partial A$ depends on the local electric field, or potential difference, $\phi_A(v) - \phi_A(u) = \phi_A(v)$ for each $u \in N_A(v)$. Fix $\eta \in \mathbb{R}$. For each $v \in \partial A$, let

$$p_A(v) = \frac{|N_A(v)|\phi_A(v)^\eta}{\sum_{u \in \partial A} |N_A(u)|\phi_A(u)^\eta}.$$

This model is called dielectric-breakdown or DBM(η) and it was proposed in 1984 by Niemeyer, Pietronero and Wiesmann [3]. The parameter η determines the extent to which the attachment locations are influenced by the electric potential. In the case when $\eta = 0$, this model is just the variation of the Eden model described above; when $\eta = 1$ it produces clusters which are believed to behave qualitatively like DLA; as $\eta \to \infty$ growth concentrates at the point of maximal potential difference. It is conjectured that the limit in this case is a simple path. Similarly to DLA, computing ϕ_A (and hence p_A) explicitly becomes increasingly complicated as the cluster grows.

Conformal models for planar random growth

The discrete models defined above are challenging to study, in part due to a lack of available mathematical tools. In 1998 Hastings and Levitov [2] formulated an approach to modelling planar growth, which included versions of the physical models described above. The idea was to represent growing clusters as compositions of conformal mappings. This approach provided a way in which techniques from complex analysis could be used to study planar random growth.

Let K_0 denote the unit disk in the complex plane $\mathbb{C}$ and let $D_0 = \mathbb{C} \setminus K_0$. A particle is any compact set $P \subset D_0$ such that $K_0 \cup P$ is simply connected. By the Riemann mapping theorem (see the aside) there exists a unique conformal bijection $f_P : D_0 \to D_0 \setminus P$ which fixes infinity. We use this mapping as a mathematical description of a particle attached to the unit disk. We say that the particle is attached at 1 if 1 lies in the closure of P. Examples of allowable particle shapes include a bump, as shown in Figure 3, a disk tangent to the unit circle at 1, or a slit (line segment) of the form $(1, 1+d]$. If P represents a particle attached at 1, then $e^{i\theta}P$ represents a particle of the same shape attached at $e^{i\theta}$. It has corresponding conformal mapping

$$f_{e^{i\theta}P}(z) = e^{i\theta} f_P(e^{-i\theta}z).$$

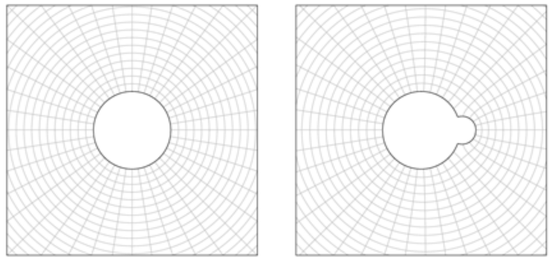


Figure 3. The exterior unit disk and its image under the conformal mapping corresponding to a single particle attached at 1.

The Riemann mapping theorem

Let $K \subset \mathbb{C}$ be a connected compact subset of $\mathbb{C}$, larger than a single point, such that $\mathbb{C} \setminus K$ is simply connected. There exists a unique conformal bijection $f : D_0 \to \mathbb{C} \setminus K$ which fixes infinity in the sense that, for some $C > 0$,

$$f(z) = Cz + O(1), \text{ as } |z| \to \infty.$$

Given a sequence of particles $P_1, P_2, \ldots$ and their associated conformal mappings $f_1, f_2, \ldots$, define a sequence of conformal bijections $\Phi_n : D_0 \to \mathbb{C} \setminus K_n$ by setting $\Phi_0(z) = z$ and recursively defining

$$\Phi_n(z) = \Phi_{n-1} \circ f_n(z) = f_1 \circ \cdots \circ f_n(z).$$

Note that $K_n = K_{n-1} \cup \Phi_{n-1}(P_n)$, so the sequence $K_0 \subset K_1 \subset K_2 \subset \cdots$ represents a growing cluster where the unit disk K_0 is the seed particle and at time n the particle $\Phi_{n-1}(P_n)$ is added to the cluster (see Figure 4).

When the particles which are being attached to the cluster are small, relative to the unit disk, the leading order behaviour of each particle map depends only on the attachment angle and the size of each

Harmonic functions

A continuous function $u : \bar{D} \to \mathbb{R}$, where D is some open subset of $\mathbb{R}^d$ and $\bar{D}$ is its closure, is a harmonic function if it satisfies Laplace's equation

$$\Delta u = 0 \text{ in } D \tag{1}$$

where

$$\Delta u = \frac{\partial^2 u}{\partial x_1^2} + \cdots + \frac{\partial^2 u}{\partial x_d^2}.$$

Using the Cauchy-Riemann equations, it can be shown that the real and imaginary parts of any holomorphic function are harmonic functions. Harmonic functions are closely connected with Brownian motion. Let D be a transient domain (i.e. a domain which Brownian motion will exit in finite time almost surely). Then if u is a harmonic function on D

$$u(x) = \mathbb{E}_x u(W_\tau) \tag{2}$$

where W is a d-dimensional Brownian motion started at x and $\tau = \tau_D$ is the first exit time from D. Set

$$\omega(x; dy) = \mathbb{P}_x(W_\tau \in dy).$$

Then we can write (2) as

$$u(x) = \int_{\partial D} u(y)\omega(x; dy).$$

We call $\omega(x; dy)$ the harmonic measure on ∂D as seen from x. Using the symmetry of Brownian motion, it follows that harmonic functions satisfy the mean value property

$$u(x) = \int_{\partial B_r(x)} u(y) dA(y)$$

for all balls $B_r(x) \subseteq D$, where dA is the normalised uniform surface area measure on $\partial B_r(x)$.
A similar notion, with analogous properties, exists on a graph $G = (V, E)$. Given any finite connected subgraph $\emptyset \subset D \subset V$, a function $u : D \cup \partial D \to \mathbb{R}$ is discrete harmonic on D if it satisfies (1), where Δ is now the discrete Laplacian

$$\Delta u(x) = \sum_{y:(x,y)\in E} (u(y) - u(x)).$$

This implies the mean-value property

$$u(x) = \frac{1}{|\partial\{x\}|} \sum_{y:(x,y)\in E} u(y).$$

For any $A \subset \partial D$, the harmonic measure of A with respect to D, $\omega(x, A, D)$, is the unique discrete harmonic function on D with boundary values $\omega(x, A, D) = 1$ when $x \in A$ and $\omega(x, A, D) = 0$ when $x \in \partial D \setminus A$. Harmonic measure is closely connected with simple random walks via the relationship

$$\omega(x, \{y\}, D) = \mathbb{P}_x(X_\tau = y)$$

where X_n is a simple random walk starting from x and $\tau = \tau_D$ is the first exit time from D. If u is any harmonic function on D then

$$u(x) = \sum_{y\in\partial D} u(y)\omega(x, \{y\}, D) = \mathbb{E}_x u(X_\tau).$$

particle. For each particle P_n, let θ_n denote the attachment angle and d_n the diameter. By choosing the sequences $\theta_1, \theta_2, \ldots$ and $d_1, d_2, \ldots$ in different ways, it is possible to describe a wide class of growth models. In the remainder of this section, we will explore how to choose these sequences in order to construct models that correspond to the physical processes described in the first section.

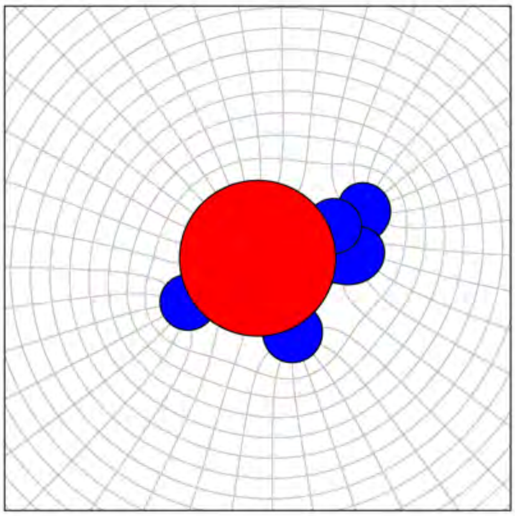


Figure 4. An example of a cluster K_n, grown by successive compositions of conformal mappings, together with the image of the exterior unit disk under the map Φ_n. The seed particle is shown in red and the first five particles to arrive are shown in blue.

In the model for biological growth, the probability of attaching a particle along a section of the cluster boundary should be proportional to the arc-length along that section of the boundary. Specifically, given an open arc in ∂K_{n-1}, there exist $a < b \in \mathbb{R}$ such that $\theta \mapsto \Phi_{n-1}(e^{i\theta})$ maps the interval (a, b) onto this arc. The length of this arc is therefore given by

$$\int_a^b |\Phi'_{n-1}(e^{i\theta})| d\theta.$$

Attaching the next particle onto this arc is equivalent to picking $\theta_n \in (a, b)$ and so we need

$$\mathbb{P}(\theta_n \in (a, b)) \propto \int_a^b |\Phi'_{n-1}(e^{i\theta})| d\theta,$$

or equivalently θ_n must have density function proportional to $|\Phi'_{n-1}(e^{i\theta})|$. Next consider how the diameter d_n should be chosen. For simplicity suppose that the particle being attached is a small slit of length d_n i.e. $P_n = e^{i\theta_n}(1, 1 + d_n]$. In this case, an explicit formula can be written down for the function f_n (see [2]). Furthermore, there exists some β_n (which can be expressed as a function of d_n) such that $\theta \mapsto f_n(e^{i\theta})$ maps the interval $(\theta_n, \theta_n + \beta_n]$ to P_n. Using the chain rule, the particle $\Phi_{n-1}(P_n)$ which is added to the cluster therefore has length

$$\int_0^{\beta_n} |\Phi'_{n-1}(f_n(e^{i(\theta_n+\theta)}))||f'_n(e^{i(\theta_n+\theta)})| d\theta.$$

Since

$$d_n = \int_0^{\beta_n} |f'_n(e^{i(\theta_n+\theta)})| d\theta,$$

the length of $\Phi_{n-1}(P_n)$ is approximately equal to $d_n|\Phi'_{n-1}(r_n e^{i\theta_n})|$ for some $1 \le r_n \le 1 + d_n$. In the physical models, the particles being attached are all the same size, say d. We therefore need to choose d_n in such a way as to ensure that this expression is approximately equal to d for all n. When d is small, a possible choice is to take

$$d_n = d|\Phi'_{n-1}(e^{i\theta_n})|^{-1}. \quad (3)$$

The above choices of θ_n and d_n provide a model for an off-lattice version of the Eden model.

Now suppose we wish to construct an off-lattice version of DLA. Exactly the same argument as above shows that we should again choose d_n satisfying (3). However, this time we need to pick θ_n so that $\Phi_{n-1}(e^{i\theta_n})$ has the same distribution as W_τ where W_t is a Brownian motion started from infinity (viewed on the Riemann sphere) and τ is the first time that the Brownian motion hits K_{n-1}. Equivalently, $e^{i\theta_n}$ should be the hitting distribution of $\Phi^{-1}_{n-1}(W_t)$ on the unit circle. However, Φ^{-1}_{n-1} is a conformal mapping and therefore its real and imaginary parts are harmonic functions (see the aside). Using Itô's formula, $\Phi^{-1}_{n-1}(W_t)$ is a continuous martingale and hence is a time-change of Brownian motion. By symmetry, the hitting distribution of a time-change of Brownian motion on the unit circle is uniform, so θ_n should be chosen with the uniform distribution on $[0, 2\pi)$.

A similar argument can be used to show that one can obtain an off-lattice version of DBM(η) by taking θ_n with density function proportional to

$$|\Phi'_{n-1}(e^{i\theta})|^{1-\eta}$$

and d_n satisfying (3). Note that, as in the discrete case, $\eta = 0$ corresponds to the Eden model and $\eta = 1$ corresponds to DLA. There are several variations on this. In the original Hastings–Levitov model

HL(α) [2], θ_n is picked uniformly on the unit circle and

$$d_n = d|\Phi'_{n-1}(e^{i\theta_n})|^{-\alpha/2}.$$

The HL(α) model corresponds to DBM(η) via the relation $\alpha = \eta + 1$ so HL(2) gives DLA (see Figure 5).

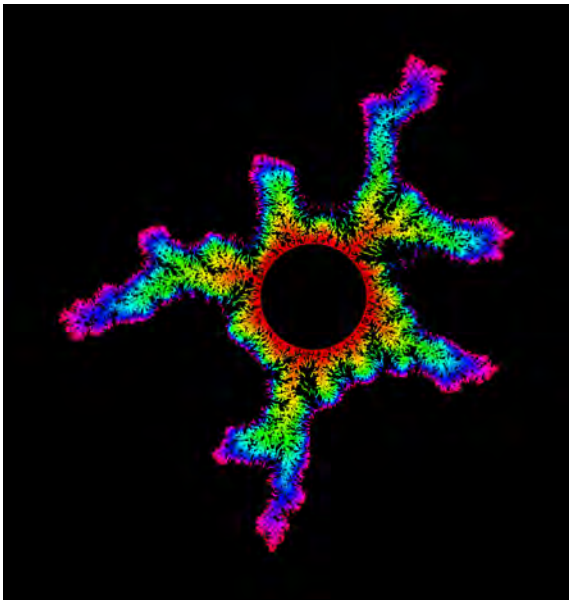


Figure 5. A version of DLA grown by composing conformal mappings using the Hastings–Levitov construction.

Mathematical study of random growth

Random growth clusters in the physical world usually consist of a large number of particles, each of which is small relative to the size of the cluster. Although the randomness typically occurs at a microscopic level through the attachment rule for each successive particle, we observe the clusters at a macroscopic level where we cannot see the individual particles. Random growth processes are completely unpredictable at the level of particles, however large clusters often exhibit predictable or 'universal' behaviour. The aim of studying mathematical models is to extract the principal mechanisms underlying this universal behaviour.

One of the intriguing features of random growth models is that, even though the models are isotropic by construction, simulations suggest that large clusters become anisotropic (see Figure 5). As yet there is no satisfactory mathematical explanation for how such complicated structures arise from the dynamics of the model. Representing random growth clusters using conformal mappings enables one to combine analytic and probabilistic techniques. Research at the interface of these two fields has already given us mathematical objects such as stochastic partial differential equations (SPDEs) and stochastic Loewner evolution (SLE). The recent developments in these areas have suggested approaches and techniques that can be applied in a random growth setting and we are beginning to be able to identify the characteristics of random growth in specific cases [4, 5].

It has been over fifty years since mathematicians and physicists first started seriously thinking about random growth. Although progress has been made in this time, we are still some way from being able to answer the really important questions. Mathematical tools and techniques that may shed light on these questions have recently started to emerge. There is a good chance that we will not need to wait another fifty years for the major breakthrough, so right now this is a very exciting area to be involved in.

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From Gambling to Random Modelling

MARTA SANZ-SOLÉ

Among mathematical fields, probability stands out because of its short history and peculiar origins. Starting with the analysis of games of chance, probability developed as the mathematical theory of uncertainty, and nowadays probabilistic methods impregnate models of random phenomena. Going through the scientific career of the Japanese mathematician Kyoshi Itô (1915–2008), we analyze his theory of stochastic analysis from the perspective of a symbiotic relationship between purely curiosity driven and applied research interests.

Introduction

This aim of this article is twofold, to introduce non specialists to some aspects of stochastic analysis — a main achievement in probability theory of the 20th century — and to give tribute to its founder, Kyoshi Itô (1915–2008). First we set Itô's work in a historical context, then we describe some of his groundbreaking ideas and early contributions in the 40's and 50's; finally, we present some recent developments and highlight the importance of his research in pure mathematics and in stochastic modelling.

In comparison with other mathematical fields, like geometry or arithmetic, probability theory has a very short history. Its origins are in the late Renaissance period, when gambling was a common leisure activity in circles of nobility and learned people, especially in Italy and France. Speculative discussions and conjectures on games of chance attracted the interest of scientists like Gerolamo Cardano (1501–1576), Galileo Galilei (1564–1642), Pierre de Fermat (1601–1655) and Blaise Pascal (1623–1662), who tried to give mathematical answers to challenging questions posed by astute and experienced gamblers.

Until the beginning of the 19th century, probability developed very slowly, possibly because of the absence of suitable mathematical tools (in particular, of combinatorial algebra), moral or religious barriers in the society to the development of the idea of randomness and chance, and the weight given to the connections of probability with philosophy. The publication in the year 1812 of the treaty *Théorie analytique des probabilités* by Pierre-Simon Laplace was a turning point in the early consolidation of the field. For the first time, there was a successful attempt to build a theory of randomness and to give some unity to the subject. However, the independence and blooming of probability only happened at the beginning of the 20th century, thanks to giant figures like Andrei A. Markov, Andrei N. Kolmogorov and Paul Lévy, with stochastic processes in their centre of interest. Around these dates, when he was a student at the University of Tokyo (1935–1938), Itô met stochastic processes. In his own words [4]:

> [...] I was fascinated by the rigorous arguments and the beautiful structures seen in pure mathematics, but also I was concerned with the fact that many mathematical concepts had their origins in mechanics. Fiddling around with mathematics and mechanics, I came close to stochastic processes through statistical mechanics.

This was the beginning of a long and fruitful journey. With his deep mathematical contributions and insight, Itô laid the foundations of stochastic models used now in many fields, like statistical physics, population genetics and mathematical finance. His influence extends to areas far beyond his imagination, as he declared when he was awarded the Gauss Prize in 2006.

Stochastic processes and Markov processes

A stochastic process is a measurable mapping $X : \Omega \times I \longrightarrow \mathbb{R}^d$, where Ω is the set consisting of the random arguments, called the sample space, and I is the set of indices, usually a subset of $\mathbb{R}^k$ or $\mathbb{Z}^k$. By fixing $\omega \in \Omega$, the deterministic mapping $X(\omega) : I \longrightarrow \mathbb{R}^d$ defined by $X(\omega)(t) = X(\omega, t)$, corresponds to an observation of the random evolution described by the process X. The deterministic functions $X(\omega)$ are called the sample paths or trajectories of the stochastic process.

Markov processes are prominent examples of stochastic processes. They are characterized by the lack of memory. If $I = [0, \infty)$, this means that, for any $t \in [0, \infty)$, the future of the process, $X_s, s > t$, is conditionally independent of the past, $X_s, s \le t$, knowing X_t. In other words, the past information gathered by the process is summarized at the present time. Introduced and studied by A.A. Markov in 1905, memoryless random dependence already appears in the work by F. Galton and H.W. Watson (1874) on evolution of populations.

Diffusions and Kolmogorov's equation

Diffusion processes are a fundamental class of Markov processes. They provide models for particles moving randomly in a fluid, like the Brownian motion. Kolmogorov (1931) described diffusions by specifying the behavior of the conditional average of increments of the process over an interval of length h, and of covariances. More specifically, a d-dimensional diffusion $X = \{(X_t^1, \ldots, X_t^d), t \ge 0\}$ satisfies,

$$\begin{aligned} &\mathbb{E}\,(X_{t+h} - X_t | X_s, 0 \le s \le t) = b(X_t)h + o(h), \\ &\mathbb{E}\,[(X_{t+h} - X_t - b(X_t)h)\,(X_{t+h} - X_t - b(X_t)h)^\top] \\ &\quad = a(X_t)h + o(h). \end{aligned} \tag{1}$$

The functions b and a are termed the drift and the diffusion coefficients, respectively.

For fixed $t \ge 0$ and $x \in \mathbb{R}^d$, define $P(t, x, A) := \mathbb{P}\,(X_{t+r} \in A | X_r = x)$, that is, the probability that, if at time r, the process is at point x, after a further t units of time, it visits the set $A \subset \mathbb{R}^d$. Assume that this probability has a density, meaning the existence of a nonnegative function, $p_t(x, \cdot)$, called the transition probability density, such that $P(t, x, A) = \int_A p_t(x, y)\ dy$. Kolmogorov proved that transition probability densities of diffusions are solutions to partial differential equations defined by the partial differential operator $\mathscr{L} = \frac{1}{2} a^{i,j}(x)\partial^2_{i,j} + b^i(x)\partial_i$, and by its dual $\mathscr{L}^*$. These are the backward and forward Kolmogorov equations,

$$\begin{aligned} &\frac{\partial}{\partial t} p_t(x, y) = \mathscr{L} p_t(x, y),\ \forall y \in \mathbb{R}^d, \\ &\frac{\partial}{\partial t} p_t(x, y) = \mathscr{L}^* p_t(x, y),\ \forall x \in \mathbb{R}^d, \end{aligned} \tag{2}$$

respectively.

Existence of Markov processes

In the 1930's, the existence of Markov processes was a central question of study. Using tools of measure theory, Kolmogorov (1931) proved the existence of Markov processes with continuous sample paths. Later on, Feller (1936) proved the existence of Markov processes with jumps.

Itô, who went deeply through the methods of these works, envisioned a differential approach to the problem focussing on the sample paths of the processes, rather than on their probability laws. For this, he viewed the Kolmogorov transition probabilities $P(t, x, \cdot)$ (defined above) as a flow of probability measures; then he introduced a suitable notion of tangent and obtained $P(t, x, \cdot)$ by integration of the tangent. Finally, using this representation, he found a realization of the integral flow $\{P(t, x, \cdot), (t, x) \in [0, \infty) \times \mathbb{R}^d\}$ on the path space of the Markov process, (called nowadays the Itô map). The tangent to the flow should be the best linear approximation. In the probabilistic context, and because of results of Paul Lévy, the role of lines is played by processes of independent increments. In this way Itô (1942) deduced that the sample paths of a diffusion (as defined in (1)) leave the initial state x in the same way as the paths of a Brownian motion B having instantaneous mean and variance $b(x)$ and $a(x)$, respectively. Hence, the probabilistic dynamics of the sample paths of a diffusion is given by the stochastic differential equation

$$dX_t = \sigma(X_t)dB_t + b(X_t)dt,\ \ t > 0,\ \ X_0 = x, \tag{3}$$

where $\sigma(x) = [a(x)]^{1/2}$.

In Itô's words [4]: "It took some years to carry out this idea".

The birth of stochastic calculus

Having achieved a pathwise description of the dynamics of Markov processes, Itô faced two tasks, namely to give a rigorous meaning to (3), and to recover the transition probabilities of the Markov process solution to (3), thereby establishing the connection with Kolmogorov's theory. Itô's stochastic calculus with respect to the Brownian motion was created to solve both questions. With the stochastic integral (1944), equation (3) is rigorously formulated, and using the change of variables formula, called Itô's formula (1951), it is proved that the transition probabilities of $X(t)$ satisfy Kolmogorov's partial differential equations (2).

Let f be a $\mathscr{C}^2$ function and B a Brownian motion. The simplest version of Itô's formula says

$$f(B_t) = f(0)+\int_0^t f'(B_s)dB_s+\frac{1}{2}\int_0^t f''(B_s)ds. \quad (4)$$

From this equality, we see that stochastic calculus does not follow the same computation rules as ordinary calculus. Moreover, the Itô map is not continuous with respect to the Brownian motion and therefore equation (3) lacks a suitable stability property. These facts produced perplexity, especially in applied circles of scientists, motivating further investigations (Rubin and Fisk, 1955; Stratonovich, 1966). Such questions were at the origin of *rough path analysis*, a theory initiated by Terry Lyons in the late 1990's.

Itô's equation and modelling

The connections of equation

$$dX_t = \sigma(X_t)dB_t + b(X_t)dt, \; t > 0, \; X_0 = x, \quad (5)$$

with modelling go back to the notion of Brownian motion, since for $\sigma = 1$, $b = 0$ and $x = 0$, equation (3) defines the Brownian motion. Let us mention some examples. In 1900, Louis Bachelier proposed a model of Brownian motion while deriving the dynamic behaviour of the Paris stock market. Fluctuation of prices in the stock market is due to the selling and buying activity of many agents, producing a similar effect as the movement of pollen particles in the experimental description of Brownian motion. In 1908, Paul Langevin described the velocity v of a Brownian particle of mass m by the equation

$$m \; dv(t) = -\lambda v(t)dt + B(dt), \; t > 0,$$

where B is a Brownian motion independent of the particle. Building on Bachelier's work, McKean and Samuelson (1965) showed that

$$dX_t = \mu X_t dt + \sigma X_t dB_t \; t > 0.$$

is a good model for stock price variations. The process (X_t) is called a *geometric Brownian motion*.

Itô's influence and parallel developments

Generalizations of the Itô stochastic integral (1944) to more general processes than Brownian motion came already in the early 50's. Motivated mainly by the study of the striking connections between stochastic calculus and potential theory, the extensions concern processes with conditionally orthogonal increments (J.L. Doob, 1953), submartingales (P.-A. Meyer, 1962), local martingales (K. Itô and S. Watanabe, 1965), and semimartingales (C. Doléans-Dade and P.-A. Meyer, 1975). In parallel, with very little interaction with other countries, the Russian school made impressive advances in stochastic analysis. Dynkin, Gikhman, Skorohod, Wentzell, Freidlin, Krylov, are in the group of main contributors. Gradually, with the break down of isolation, the groundbreaking work of colleagues in the former Soviet Union became visible and influential.

From finite to infinite dimensions: stochastic partial differential equations

It took several decades to lift Itô calculus to infinite dimensions, in particular, to develop a theory of *stochastic partial differential equations* (SPDEs). At present, SPDEs form a large and blooming field of stochastic analysis, where Itô also made some contributions in relation to measure-valued processes. In this article, we will focus on a class of SPDEs closely related to Itô's equation.

The equation (5) can be understood as an ordinary differential equation with a random forcing nonlinear term $\sigma(X_t)dB(t)$. Some classes of SPDEs obey a similar principle. Consider the heat equation on $\mathbb{R}$,

$$\frac{\partial}{\partial t}v(t,x) - \frac{\partial^2}{\partial x^2}v(t,x) = f(t,x), \; t > 0, \; x \in \mathbb{R}, \quad (6)$$

with a given initial condition, describing the evolution of the temperature along a metal bar under some external influence f. If f contains a stochastic source then (6) gives rise to a stochastic heat equation. For example, f may be a space-time white noise, $\dot{W}(t,x)$ — an infinite dimensional version of a Brownian motion. The linear stochastic heat equation driven by a space-time white noise is one of the most basic examples of an SPDE.

There are good motivations to study SPDEs, as the following two examples illustrate.

The *parabolic Anderson model* is a Cauchy problem for the heat equation with random potential. It has connections with motions in random potentials, trapping of random paths and spectra of random operators, among others. An interesting example of potentials are Gaussian processes. If the choice is a space-time white noise, as before, we have the *stochastic parabolic Anderson model* described by the SPDE

$$\frac{\partial}{\partial t}u(t,x)-\frac{\partial^2}{\partial x^2}u(t,x)=u(t,x)\dot{W}(t,x),\ t>0,\ x\in\mathbb{R}. \tag{7}$$

This equation is related with the Kardar–Parisi–Zhang equation (KPZ, in short) that Martin Hairer successfully solved in 2013,

$$\frac{\partial}{\partial t}h(t,x)-\frac{\partial^2}{\partial x^2}h(t,x)-\left[\frac{\partial}{\partial x}h(t,x)\right]^2=\dot{W}(t,x). \tag{8}$$

Indeed, the stochastic process obtained via the Cole–Hopf transformation

$$u(t,x)=\exp\left(h(t,x)\right),$$

solves (formally) equation (7).

The KPZ equation is a new universality class (similar as the Gaussian law of the Brownian motion in much simpler situations), to describe phenomena like one-dimensional interface growth processes, interacting particle systems and polymers in random environments, which display characteristic scalings and new statistics or limiting behaviors.

Very fundamental questions on SPDEs, like giving a rigorous meaning to the equations, and proving existence and uniqueness of solution, can be very challenging and generate a great deal of wonderful mathematics. A paradigmatic example is the theory of regularity structures of Martin Hairer (2014), motivated by the well-posedness of the KPZ equation.

A question concerning sample paths of SPDEs

Numerical simulations of SPDEs show that their sample paths are complex and intriguing mathematical objects. We next describe a specific problem where the nature of the sample paths plays a very important role.

Let $v=\{v(x), x\in\mathbb{R}^m\}$ be a $\mathbb{R}^d$-valued stochastic process, solution to a system of SPDEs. How likely is it that the sample paths of v visit a deterministic set A? This fundamental question in probabilistic potential theory is clearly related to the regularity of the sample paths and geometric-measure properties of A.

Having upper and lower bounds for the *hitting probabilities*

$$\mathbb{P}\{\omega : v(\omega)(I)\cap A\neq\emptyset\} \tag{9}$$

in terms of notions of geometric measure theory, like the *capacity* or the *Hausdorff measure* of the set A, provides an insight into the problem.

A first result in this direction, proved by S. Kakutani in 1944, states that, up to positive constants, for a d-dimensional Brownian motion, the hitting probability (9) is bounded from above and from below by $\mathrm{Cap}_{d-2}(A)$, the capacity of dimension $d-2$ of the set A. In particular, this implies that a d-dimensional Brownian motion hits points if and only if $d=1$.

Hitting probabilities for the sample paths of solutions to SPDEs have been in the focus of research in the last fifteen years. Since in most cases, the random field solutions to SPDEs fail to have a "suitable" Markov property, Kakutani's method, and further generalizations to different types of Markov processes, cannot be applied. The new successful approach for SPDEs relies on the study of densities of the random field solutions at fixed points, using as mathematical background Malliavin calculus.

Closing the circle: Kolmogorov, Itô, Hörmander, Malliavin

We finish this article with some touches on Malliavin calculus, not only because of its role in the study of hitting probabilities for SPDEs but especially, to close a beautiful circle of ideas that started with Kolmogorov, continued with Itô and Hörmander, and ended with Malliavin.

Recall the forward Kolmogorov equation for densities (in the distribution sense) of diffusions starting from x,

$$\left(\frac{\partial}{\partial t}-\mathscr{L}^*\right)p_t(x,\cdot)=0. \tag{10}$$

In the theory of partial differential operators, there is the notion of *hypoellipticity*. It tell us that, if $\frac{\partial}{\partial t}-\mathscr{L}^*$ is *hypoelliptic* then the solution to the PDE

$$\left(\frac{\partial}{\partial t}-\mathscr{L}^*\right)\alpha=0,$$

is a smooth function. That is, the solution to (10) in the distribution sense is a smooth function $p_t(x,y)$.

In a seminal paper, L. Hörmander (1967) gave sufficient conditions of geometric type for a partial differential operator in quadratic form to be hypoelliptic (see also Kohn (1973), Oleinik and Radkevič (1973)). Hörmander's theorem applies to the operator $\mathscr{L}$, therefore to Kolmogorov's equation.

Malliavin envisioned giving a probabilistic proof of Hörmander's theorem, thereby having an exclusively probabilistic understanding of (10). Starting with Itô's stochastic differential equation (5) he proved that, by expressing Hörmander's assumptions in terms of geometric properties of the coefficients σ and b, the law of the random vector X_t, for any $t > 0$, possesses a smooth density. Existence of densities are obtained by integration by parts formulas on the Wiener space, that rely on a stochastic calculus of variations given the name of *Malliavin calculus*.

Malliavin theory, introduced in [3], was further investigated and expanded by Bismut, Stroock, Ikeda, Watanabe, Bouleau, Hirsch, Meyer, Kusuoka, Shigekawa, Nualart, Bell, Mohammed, Ocone, Zakai, among others. Its scope goes far beyond the initial project of giving a probabilistic proof of Hörmander's theorem. In particular, integration by parts formulas provide explicit expressions for densities of random vectors defined on abstract Wiener spaces. Explicit formulas for densities, along with the Malliavin calculus toolbox, are the basic ingredients to address the problem of hitting probabilities for systems of SPDEs with multiplicative noises (see work of R. Dalang, D. Khoshnevisan, C. Mueller, E. Nualart, M. Sanz-Solé, R. Tribe, C. Tudor, F. Viens, Y. Xiao, L. Zambotti).

Final remarks

By changing the approach to Markov processes of Kolmogorov and Feller, and by giving the sample paths a priority, Itô paved the way to stochastic modelling. His mathematical work consolidated the theory of stochastic processes, that was still in its infancy when he was a student at the University of Tokyo, expanded the theory, by the creation of stochastic calculus, boosted connections with other mathematical fields, initiated the use of stochastic models in sciences, and is still a source of new and challenging problems in probability.

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Colouring Without Colours: Graphs and Matroids

PETER NELSON

The problem of colouring the vertices of the graph so that adjacent vertices get different colours is one of the oldest in combinatorics. Less known are the surprising parallels it has to a problem in the geometry of vector spaces over finite fields. We discuss these connections.

Graph Colouring

This article is about some classical gems in the theory of graph colouring, and their surprising analogues in the world of matroids. However, no discussion of the history of graph colouring is complete without mention of the famous four-colour problem, so that's where I'll start.

This deep question can be phrased in a deceptively simple way: given a political map in which each region is a connected subset of the plane, with some regions bordering others, how many colours are required to fill in the regions so that regions with the same colour do not have a common border? (To make this question interesting, countries whose common border is nonempty but has length zero are not considered to share a border.) The answer appeared to be 'at most four', but in the century following the problem's origins in the mid 1800's, a correct proof of this fact eluded mathematicians, even as graph theory burgeoned from a collection of curiosities into a full-fledged area of mathematics. Indeed, the four-colour problem motivated much of the formative work in the subject, especially in graph polynomials, topological graph theory, and the theory of minors. Only in 1976 did Appel and Haken [1] verify with a computer-assisted *tour de force* that four colours always suffice. Even with this question settled, colouring problems are ubiquitous in the graph theory literature today, and problems close to the four-colour theorem are still key open questions.

When setting up the formalism needed to describe and generalize statements like the four-colour theorem, it is more general and usually more convenient to think about colouring vertices of a graph than colouring regions. To this end, let $G = (V, E)$ be a graph. (By this we mean that V is a finite set of 'vertices' and E a set of 'edges' such that each edge is an unordered pair of vertices; vertices joined by an edge are *adjacent*.) Given a set C of colours, a function $c\colon V \to C$ is a *(proper) colouring* of G if $c(u) \neq c(v)$ for every pair u, v of adjacent vertices of G. If such a colouring exists with $|C| = k$, then we say that G is k*-colourable*. The *chromatic number* of G, written $\chi(G)$, is the minimum k such that G is k-colourable. In this language, the four-colour theorem is equivalent to the statement that, for every graph G that can be drawn in the plane with no two edges crossing, we have $\chi(G) \leq 4$. (The equivalence is via the 'dual graph' of the map to be coloured, which has a vertex for each region of the map, and an edge for each pair of bordering regions.)

Figure 1. A four-colouring of the countries of Europe

The chromatic number is among the most-studied invariants in graph theory, appearing as naturally on the algebraic side of the subject as it does on the combinatorial side. To gain some intuition, it helps to understand its value for a few classes of graphs. For example, a *complete graph* on t vertices, denoted K_t, is a graph for which every pair of vertices is joined by an edge. These graphs need a different colour for each vertex, so $\chi(K_t) = t$. Indeed, $\chi(G) \leq |V|$ is a trivial upper bound for every graph. On almost the opposite extreme of edge-density, a t*-circuit* is a graph C_t with vertex set $\{v_1, \ldots, v_t\}$ and edge set $v_1v_2, v_2v_3, \ldots, v_tv_1$. The chromatic number of C_t depends on the parity of t; it is easy to check that $\chi(C_t) = 2$ if t is even and $\chi(C_t) = 3$ if t is odd. Finally, a connected graph (that is, a graph that is not the 'disjoint union' of two smaller graphs) containing no circuit at all as a subgraph is a *tree*. A simple argument shows that a tree can be coloured with just two colours; simply pick a vertex, colour it red, colour its neighbours blue, their neighbours red, and so on. The absence of circuits implies that no vertex is ever assigned a colour that a neighbour already has, so trees are 2-colourable.

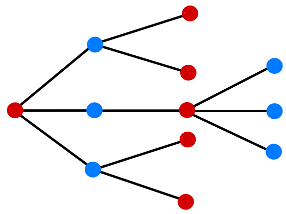


Figure 2. Two-colouring a tree according to distance

In fact, to make this approach to 2-colouring work, we don't have to forbid all circuits as subgraphs, just those of odd size. This holds since any pair of adjacent vertices assigned the same colour by the approach above would give rise to an odd circuit. So we can 2-colour not just the trees, but all the graphs with no odd circuits. Combined with the fact that odd circuits themselves are not 2-colourable, this proves a nice characterisation of 2-colourable graphs:

Theorem 1 (Kőnig, 1936). *$\chi(G) \leq 2$ if and only if G has no odd circuit as a subgraph.*

Two-colourable graphs are also called *bipartite*, since a 2-colouring gives rise to a partition (A, B) of the vertex set for which each edge has one end in A and the other in B. The proof we sketched even comes with an algorithm; a connected graph containing no odd circuit can be efficiently coloured with two colours using the procedure outlined above.

One might hope for a similarly nice characterization of, say, the graphs that are 3-colourable, but such hopes can be quickly dashed, both theoretically and computationally. On the first front, there are graphs with arbitrarily large chromatic number that are, in a sense, 'locally tree-like', and on the second, it is NP-hard to even decide whether a given graph is 3-colourable. On that note, we'll move to the main topic of this article.

Matroids

Matroids are objects defined to capture the abstract essence of the intuitive notion of linear independence, just as a groups do for the notion of symmetry. From optimisation to algebraic geometry, they appear in many guises across mathematics, and have a number of equivalent definitions. The definition we give today really captures only a small subset of matroids, and is intended to emphasise the analogies we will make with graph theory. Broadly, we wish to think of vectors in a binary vector space in the same way as we think of the edges of a graph. To arrive at the definition we will use, we briefly revisit the formalism of graphs and their isomorphisms.

The edge set of a graph with vertex set V is just a subset of the set $\binom{V}{2}$ of all pairs of elements of V. Two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ are isomorphic precisely when there is a bijection $\varphi : V_1 \to V_2$ whose induced map on pairs maps E_1 to E_2. Given a vertex set V, the set $E_0 = \binom{V}{2}$ is an 'ambient space', and the nonisomorphic graphs on V are exactly the subsets of E_0 up to a natural group of permutations of $\binom{V}{2}$ (those arising from permutations of V).

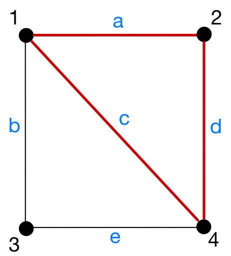
We define matroids along these lines, but with a quite different 'ambient space': instead of $\binom{V}{2}$, it is the vector space $W = \mathbb{F}_2^n$, where $\mathbb{F}_2$ is the two-element field. Here the natural group of permutations of W is the group of its linear automorphisms, and this gives us our definition. A *matroid* is a pair $M = (E, W)$, where $W = \mathbb{F}_2^n$ and E is a subset of W. The integer n is the *dimension* of the matroid. Two matroids (E_1, W_1) and (E_2, W_2) are *isomorphic* if there is a linear bijection $\varphi\colon W_1 \to W_2$ for which $\varphi(E_1) = E_2$. As mentioned, this definition of matroid is very nonstandard. As we have defined them, matroids for which $0 \notin E$ while W is the linear span of E are essentially the same thing as *simple binary matroids* in usual terminology.

One nice class of matroids comes from the graphs themselves. Given a graph $G = (V, E)$, let $B(G)$ denote the *vertex-edge incidence matrix* of G: this is the matrix in $\mathbb{F}_2^{V \times E}$ in which the column corresponding to an edge e is a support-two vector having entries equal to 1 in precisely the two rows corresponding to the ends of e. The set of columns of $B(G)$ is a subset of $\mathbb{F}_2^V$; that is, a matroid M. We call this the *graphic matroid* of G, writing $M = M(G)$.

Of course $M(G)$ is just a set of support-two vectors that trivially encodes G, but a matroid isomorphic to $M(G)$ will not in general be such a simple representation of G. The sensible question to ask is whether G is determined by just the *isomorphism type* of $M(G)$: in other words, given a collection E of vectors in $\mathbb{F}_2^n$ that is known to be the image of a collection F of support-two vectors under an invertible linear map, can we recover F? The answer is essentially yes; a graph G is determined, up to a simple equivalence relation, by just the isomorphism type of $M(G)$.

Circuits and critical number

One particular feature of a graph G that is readily visible from just its graphic matroid $M(G)$ is which sets of edges are circuits. If C is the set of edges in some circuit contained in G, then the vectors corresponding to C in $M(G)$ sum to zero, forming a minimal linear dependency. Strikingly, the converse is also true. If C is any minimal linearly dependent set in $M(G)$, then C corresponds to the set of edges in a circuit of G. In fact, matroid terminology borrows from that of graphs; for any matroid $M = (E, W)$, a minimal linearly dependent subset of E is called a *circuit* of M.



	a	b	c	d	e
1	1	1	1	0	0
2	1	0	0	1	0
3	0	1	0	0	1
4	0	0	1	1	1

Figure 3. A graph with its incidence matrix — circuits such as $\{a, c, d\}$ give minimal column dependencies.

Our goal is to define a good analogue of the chromatic number in the world of matroids. This presents some difficulties, since it is unclear what, if anything, should be 'coloured'. The right notion turns out to have little to do with actual colouring at all. Instead, our starting point is the notion of a circuit. By Theorem 1, we see that 2-colourability of a graph is equivalent to the absence of odd circuits. Now that circuits are matroidal, we can ask what happens when we forbid odd circuits from a matroid. The answer, whose simple proof we omit, is nice.

Proposition 1. *A matroid $M = (E, W)$ contains no odd circuit if and only if there is a hyperplane of W that is disjoint from E.*

In light of Theorem 1, the hyperplane H given by Proposition 1 is a matroidal proxy for a '2-colouring'. In fact, we get our 'matroidal chromatic number' by generalising this idea.

Definition 1. The *critical number* $\chi(M)$ of a matroid $M = (E, W)$ is the smallest integer $\chi \geq 0$ for which W has a χ-codimensional subspace disjoint from E.

This definition requires comment. First, if $0 \in E$ then $\chi(M)$ is not well-defined — we just ignore these cases. Second, an easy argument gives that the matroids (E, W) and $(E, \operatorname{span} E)$ have the same critical number, so when computing χ we may assume that $W = \operatorname{span} E$.

Combining the above definition with Proposition 1, we see that $\chi(M) = 1$ if and only if M has no odd circuits. There are a few ways to think about matroids with larger χ — one comes from the observation that each k-codimensional subspace is the intersection of k hyperplanes, which gives the following.

Proposition 2. *A nonempty matroid $M = (E, W)$ has $\chi(M) \leq k$ if and only if E is the union of k sets $E_1, \dots, E_k$ such that each matroid $M_i = (E_i, W)$ has $\chi(M_i) = 1$.*

This gives us a way to understand χ for graphic matroids. If G a graph with graphic matroid $M(G) = (E, W)$ and $\chi(M(G)) \leq k$, then E is the union of k sets $E_1, \dots, E_k$ as in Proposition 2; it follows that the edge set F of G is the union of k sets $F_1, \dots, F_k$ such that each graph (V, F_i) has no odd circuit. Thus each (V, F_i) is 2-colourable, so there is a partition (A_i, B_i) of V such that neither A_i nor B_i contains an edge in F_i. The common refinement of all the (A_i, B_i) gives a partition of V into 2^k parts, such that no part contains an edge of G: that is, a 2^k-colouring of V. All of this works just as well in the converse, and tidying it up gives a fundamental connection between the graphic and matroidal versions of χ:

Proposition 3. *If G is a graph with graphic matroid $M(G)$, then $\chi(M(G)) = \lceil \log_2 \chi(G) \rceil$.*

When specialised to graphic matroids, the critical number is thus an rough analogue of chromatic number, able to approximate it up to a factor of 2. (In fact, there is even a sense in which critical number is a q-analogue of chromatic number, but we won't discuss this here.) Incidentally, it is an NP-hard problem to even approximate the chromatic number of a graph to a factor of two, which dispels hopes for computing the critical number efficiently, even for a graphic matroid.

The above theory and definitions are all due to Crapo and Rota [3]. In their seminal 1967 paper they take a much more algebraic approach to defining critical number, calling it the *critical exponent*. In fact, their work was partly motivated by the four-colour problem, which is equivalent to the statement that the graphic matroids of planar graphs have $\chi \leq 2$.

These ideas suggest that graph colouring problems should have 'cousins' in the geometry of $\mathbb{F}_2^n$. Over the years, this has been supported by a number of new results. In what follows we discuss two recent but accessible matroidal versions of classical theorems, whose existence exemplifies the surprising relationship between these two worlds. The first is Brooks' theorem, an upper bound for the chromatic number, and the second is the Erdős–Stone theorem, a seminal result in which the chromatic number unexpectedly arises in extremal graph theory.

Brooks' Theorem

The approach we saw to 2-colouring that worked for trees and odd-circuit-free graphs, can be generalised to work for arbitrary graphs. Of course, for this we need more colours. Suppose we wish to colour a graph G, thinking of our available colours as the set of all positive integers. One approach is to simply order the vertices $v_1, \dots, v_n$ and colour them in order, assigning to each v_i the smallest colour that has not already been assigned to one of its neighbouring vertices.

It is easy to see that the resulting colouring is legal, and that the number of colours we use in total is no more than $\Delta + 1$, where $\Delta = \Delta(G)$ is the maximum number of neighbours of a vertex in G. This gives the upper bound $\chi(G) \le \Delta(G)+1$. This bound is far from the truth in many cases, but is tight in two cases we have already seen: complete graphs (which have $\Delta(G) = |V| - 1$ and $\chi(G) = |V|$) and circuits with an odd length (where $\Delta(G) = 2$ and $\chi(G) = 3$). With an argument far less simple than this one, Brooks [2] showed that these are the only tight cases.

Theorem 2. *$\chi(G) \le \Delta(G)$ for every connected graph G that is not a complete graph or an odd circuit.*

There are a few obstacles to making a thing matroidal, the largest of which is that the implicit colouring algorithm just described is difficult to mimic geometrically. The way to proceed is to view the algorithm as an inductive argument; if we want to colour G, we remove a vertex v, inductively colour the resulting graph G' using a colouring c', and then since v has at most $\Delta(G)$ neighbours, one of the $\Delta(G) + 1$ ways to extend c' to a colouring of G must be valid.

The matroidal analogue of a 'neighbourhood' is called a *cocircuit*; a cocircuit of $M = (E, W)$ is a minimal set $D \subseteq E$ for which $\mathrm{span}(E \setminus D) \ne \mathrm{span}(E)$. This condition implies that $\mathrm{span}(E \setminus D)$ has dimension exactly one less than $\mathrm{span}(E)$. Except in degenerate cases, the set of edges incident to a vertex will be a cocircuit in the corresponding graphic matroid.

Let M be a matroid and let $\Delta(M)$ denote the size of a largest cocircuit D of $M = (E, W)$. So the elements of the matroid $M' = (E \setminus D, W)$ span a space of smaller dimension than W. We now wish to show that $\chi(M) \le k$, assuming inductively that $\chi(M') \le k$. The inductive hypothesis implies that $\mathrm{span}(E \setminus D)$ has a k-codimensional subspace U' disjoint from $E \setminus D$. Now we wish to extend U' to a k-codimensional subspace U of $\mathrm{span}(E)$ that is disjoint from E for which $U' = U \cap \mathrm{span}(E \setminus D)$; there are precisely 2^k possible extensions U, and their pairwise intersection is the set U'. If $|D| < 2^k$, then one such extension U must not contain any element of D, implying that $U \cap E = \varnothing$ and so $\chi(M) \le k$ as required.

Using both the fact that $|D| = \Delta(M)$ and the argument above, what the induction gives is that $\chi(M) \le \lceil \log_2(1 + \Delta(M)) \rceil$. This is a bound along the lines of the $\chi \le \Delta + 1$ we got for graphs. But the analogy goes further: if we ask where equality holds, there are again two obvious examples. The first is when $E = W \setminus \{0\}$: that is, where M is a 'complete' matroid, in which case an easy computation yields $\Delta(M) = 2^{n-1}$ and $\chi(M) = n = \lceil \log_2(1 + \Delta(M)) \rceil$. The other is where E is simply a circuit of M of odd size, where $\chi(M) = \Delta(M) = 2$. In beautiful analogy with Theorem 2, Oxley [6] showed the following with a far more intricate argument.

Theorem 3. *If (E, W) is a connected matroid with $0 \notin E$, then $\chi(M) \le \lceil \log_2 \Delta(M) \rceil$, unless E is either an odd circuit, or equal to $W \setminus \{0\}$.*

Here, 'connected' just means that E is not contained in the union of two nontrivial subspaces of W with trivial intersection. So, as with graphs, the natural upper bound one gets for χ in terms of Δ can be improved, except in two quite different exceptional cases, one dense and one sparse. The improvement seems modest, but in fact not even the ceiling function in the bound can be removed in general; Oxley constructed rich classes that demonstrate this.

The Erdős–Stone Theorem

Given a graph H, how many edges can a graph G on n vertices have if G does not contain H as a subgraph? This question is among the most natural in extremal graph theory, and on the surface, has nothing to do with colouring. A classical theorem of Turán gives the answer when H is a complete graph on t vertices; the G with the most edges while having no H-subgraph are the 'balanced complete $(t-1)$-partite graphs', where the vertex set of G is partitioned into $t-1$ sets of as equal size as possible, and the edges of G are precisely those with the two ends in different parts. Such a graph on n vertices has roughly $(1-\frac{1}{t-1})\binom{n}{2}$ edges, with a negligible error term where t is fixed and n is large. (Since a graph on n vertices has at most $\binom{n}{2}$ edges, the $(1-\frac{1}{t-1})$ term here is a 'density'.)

In fact, such a $(t-1)$-partite graph G does not just omit a t-vertex complete graph as subgraphs, but also omits *any* graph H with $\chi(H) \geq t$, since G is by construction $(t-1)$-colourable, as are all its subgraphs. So for an arbitrary graph H there is, for each n, an H-subgraph-free graph G with around $\left(1-\frac{1}{\chi(H)-1}\right)\binom{n}{2}$ edges. The Erdős–Stone theorem [4], strikingly, states that this simple construction is asymptotically optimal.

Theorem 4. *If H is a graph with at least one edge, then the maximum number of edges in an n-vertex graph with no H-subgraph is $\left(1-\frac{1}{\chi(H)-1}+o(1)\right)\binom{n}{2}$.*

Even if this formula is demystified by the preceding discussion, it is on the surface very surprising that the problem of optimally colouring H relates at all to the edge-density of graphs with no H-subgraph. The above theorem was proved in 1960 and remains a cornerstone of modern extremal graph theory.

The question this result answered for graphs has an equally natural analogue for matroids. It is clear what it should mean for a matroid to contain another; we say that $M_0 = (E_0, W_0)$ is a *submatroid* of $M_1 = (E_1, W_1)$ if there is a linear injection $\varphi\colon W_0 \to W_1$ for which $\varphi(E_0) \subseteq E_1$. Otherwise, M is *N-free*.

Say that n is large, and we wish to construct an n-dimensional matroid $M = (E, W)$ that fails to contain some fixed r-dimensional matroid $N = (F, V)$ as a submatroid. If $\chi(N) = t$ and M contains N, then it is easy to verify that $\chi(M) \geq t$; so ensuring that $\chi(M) < t$ guarantees that M is N-free. We can do this while keeping M still rather dense. If U is any $n-(t-1)$-dimensional subspace of W, then the matroid $M = (W \setminus U, W)$ has $\chi(M) = t-1$, as is certified by U itself. And M really is dense; it has exactly $(1-2^{1-t})\,2^n$ elements.

In other words, for each n there is an n-dimensional N-free matroid M with $|N| = (1-2^{1-\chi(N)})\,2^n$. This should seem very familiar to what happened for graphs, with the value of $\chi(N)$ giving a nice construction for dense N-free matroids. As one might hope, this construction is also asymptotically optimal for matroids, as shown by Geelen and the author [5].

Theorem 5. *If N is a matroid, then the maximum number of elements in an n-dimensional matroid M with no N-submatroid is $(1-2^{1-\chi(N)}+o(1))\,2^n$.*

This concludes our tour of colouring without colours, in which we've given a flavour of the geometric world that runs parallel to classical graph theory. Much of what we have discussed runs deeper. For example, the original motivation for studying the critical number came from its links with polynomials associated with matroids and deletion-contraction recurrences, which in turn have beautiful connections to algebraic geometry. In a more analytic direction, the extremal questions brought to light by the matroidal Erdős–Stone theorem are naturally linked to additive combinatorics and the study of pseudorandomness in subsets of groups.

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Peter Nelson

Peter is a New-Zealand-born mathematician who is currently an assistant professor at the University of Waterloo. His research interests are in matroid theory, graph theory and related areas. He is also a fan of his family, jazz piano, cryptic crosswords and craft beer.

Reciprocal societies: Nigerian Mathematical Society

The Nigerian Mathematical Society (NMS) was established in February 1982 at University of Ibadan when sixty-two Mathematicians from almost all the existing Universities and Colleges in Nigeria came together to hold a conference. At that conference, Professor Adegoke Olubummo was elected as the NMS's first President, Professor Christopher. O. Imoru its first Secretary and Professor Haroon Oladipo Tejumola was elected as the first Editor-in-Chief of *Journal of the Nigerian Mathematical Society* (JNMS).

The aims of the NMS are to promote research and applications in mathematical sciences through:

(1) Holding of conferences, symposia, seminars, workshops, etc.,

(2) Publishing of JNMS and other publications;

(3) Awarding of prizes for outstanding mathematical research especially to young mathematicians and;

(4) Cooperating/affiliating with other bodies with similar aims as those of NMS.

In fulfilment of its aims, the NMS has been organising annual conferences in rotational basis in Nigerian Universities since 1982. The 38th annual conference is scheduled to be held at the University of Nigeria, Nsukka in June 2019. Attendance at the annual conferences has increased to over 500 participants in recent years.

The JNMS has been published yearly since inception with at least one volume annually. In 2015, the JNMS operated as the first registered mathematical journal under the Nigeria–Elsevier–Partnership (NEP) which further increased the visibility and readership of the journal. In addition, JNMS increased from one journal issue per year to three issues per year commencing with volume 34 (2015) which contains three issues. However, due to financial challenges, the NEP agreement was terminated in early 2016. A memorandum of understanding was thereafter signed with the Abdus Salam International Centre for Theoretical Physics, Trieste, Italy for JNMS to go live on the Open Journal System (OJS). This has helped to improve the journal contents, maintain high quality and speed of publications. JNMS can be downloaded at ojs.ictp.it/jnms/index.

In 2015, at the 34th annual conference held at the University of Lagos at the NMS, commenced the award of the Fellow of the Nigerian Mathematical Society (FNMS) to outstanding mathematicians who have contributed immensely to the development of Mathematics in Nigeria and the growth of NMS. NMS also instituted annual award of prizes to outstanding PhD which is presented during the annual conferences. The Society commenced the publication of the *NMS Notices* in 2008.

Investiture of FNMS, 2016

NMS is affiliated with the London Mathematical Society and the American Mathematical Society. NMS has been encouraging and supporting the Nigerian Women in Mathematics (NWM) to organize meetings and conferences. A position is reserved for NWM on the Council of NMS. Furthermore, NMS is in collaborating with the National Mathematical Centre, Abuja and other Mathematical associations in Nigeria.

Professor N.I. Akinwande and Professor G.C.E. Mbah are the current President and Secretary respectively. Professor Samuel S. Okoya is the current Editor-In-Chief of the JNMS. Further information about the NMS can be sourced from the website: www.nigerianmathematicalsociety.org.

Ninuola I. Akinwande
President of the NMS

Editor's note: the LMS and the NMS have a reciprocity agreement meaning members of either society may benefit from discounted membership of the other.

Microtheses and Nanotheses provide space in the Newsletter for current and recent research students to communicate their research findings with the community. We welcome submissions of micro and nanotheses from current and recent research students. See newsletter.lms.ac.uk for preparation and submission guidance.

Microthesis: Timescales of Influence on Unobserved Graphs

MATTHEW GARROD

One of the most popular tools for understanding how, for example, ideas or diseases can spread through society is graph or network theory. In practice the network data required to study these phenomena is often unavailable or full of errors and omissions. My PhD project aims to explore the extent to which we can forecast and control processes occurring on graphs when we do not have full information about the graph structure.

Social influence and homophily

Many real world systems can be represented by graphs or networks. Networks or graphs are used to represent systems consisting of many individual units, known as nodes, and the interactions between them, which are referred to as edges or links (see Figure 1). One of their popular recent applications is in the study of social networks, where the nodes represent people and the links represent friendships between them. One of the primary reasons for studying the structure of social networks is to allow us to forecast the outcomes of processes such as the spread of diseases or rumours.

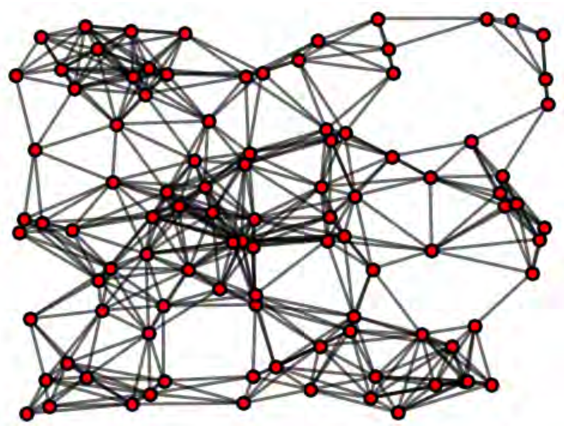

Figure 1. A random geometric graph with $N = 100$, $R = 0.2$. In applications, the nodes might represent people and the links might represent friendships between them.

Given a particular graph, scientists have an armoury of mathematical tools for modelling how opinions and beliefs can spread through a graph [1]. However, in practice we often don't know the structure of the social network itself. This might be because: i) the data we would like is unavailable; ii) of privacy concerns in social networks; iii) the data exists but is full of errors or omissions (in this case we call the data *noisy*). Fortunately, we know a lot about the structure of social networks from decades of past research by social scientists and statisticians. For instance, many social networks are known to be *homophilous* — this means that people who share similar traits are more likely to be friends. For example, two people of a similar age are more likely to be friends than two people with a large age difference.

Random geometric graphs

Inspired by the above we consider a simple mathematical model for homophilous networks known as a *random geometric graph*. To construct a random geometric graph we sample N positions $x_1, x_2, \ldots, x_N$ uniformly at random in $[0,1]^2$. We then connect two points x_i, x_j if:

$$|x_i - x_j| \leq R,$$

where $|\cdot|$ represents the Euclidean distance and R is the connection radius (see Figure 1 for an example).

Positions of nodes may represent the positions of individuals in geographic space or in some "social space" where the coordinate axes might represent attributes such as age, income and education level. Since social networks are homophilous those who are closer together in "social space" are more likely to share a social tie [3].

Algebraic connectivity

An undirected graph can be represented by its symmetric adjacency matrix, A, where $A_{ij} = 1$ if nodes i and j are connected, and $A_{ij} = 0$ otherwise. Another commonly used representation is the Laplacian matrix, L, which has elements:

$$L_{ij} = \delta_{ij}\left(\sum_{j=1}^{N} A_{ij}\right) - A_{ij}$$

The eigenvalues of the Laplacian matrix $\mu_1 \leq \mu_2 \leq \ldots \leq \mu_N$ tell us about the structural and dynamical properties of a graph. In particular, the second smallest eigenvalue, μ_2, is informative of the typical timescale of diffusion processes occurring on graphs. This eigenvalue is often referred to as the *algebraic connectivity*.

One basic question we can ask about a network is: "how long does it take something to spread across it?" We refer to this as the diffusion timescale. The diffusion timescale in a graph is indicative of how well connected the graph is and governs how quickly we might expect a disease, rumour or the adoption of a new behaviour to spread through it. In our recent research we focus on the question:

> *If we do not know the network (but perhaps know some of its properties), how precisely can we know the diffusion timescale?*

We quantify the diffusion timescale using the *algebraic connectivity*, μ_2, of a graph (see "Algebraic connectivity"). We show that different random geometric graphs drawn at random with the same number of nodes and connection radius can have very different μ_2 values. We find that the typical width of the distribution of μ_2 values, $\mathbb{P}(\mu_2)$, across a random ensemble of random geometric graphss can vary a lot given different choices of N, R and spatial dimensions. This demonstrates that, in some cases, we might need precise knowledge of the graph structure in order to estimate quantities such as μ_2.

Our work helps put limitations on how accurately we can forecast the outcome of processes on networks given the available data (which is always imperfect). Future work may involve asking the same questions for real-world datasets. In addition, most of our new results were obtained through computer simulations, meaning that there is also scope for theoreticians in network science and mathematicians to explore the theoretical bases for our results further.

You can read about our research in the paper [2]. This microthesis is adapted from a recent post on Imperial College *Systems and Signals* group blog which is available at: tinyurl.com/ICSandS.

Acknowledgements

My research has been supported by EPSRC grants EP/L016613/1 (Centre for Doctoral Training in the Mathematics of Planet Earth) and EP/N014529/1 (Centre for the Mathematics of Precision Healthcare). I would like to thank Dr Nick Jones and Dr Juvid Aryaman for feedback on the draft.

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Matt Garrod

Matt is a PhD student at the EPSRC Centre for Doctoral Training in the Mathematics of Planet Earth at Imperial College. His main research interests are spatial networks and random geometric graphs, however, he also has an interest in statistical physics and data visualisation. Matthew grew up on a farm in Suffolk, UK. He enjoys rock climbing, heavy metal gigs and playing the bass guitar.

ST JOHN'S COLLEGE
CAMBRIDGE

College Associate Lectureship and Fellowship in Pure Mathematics

Salary: £36,261 - £40,792 p.a. (depending on experience) plus benefits

St John's College is looking to appoint a College Associate Lecturer and Fellow in Pure Mathematics from 1 October 2019. This is an early career development post and is offered for a fixed-term period of five years.

The position offers the opportunity for progression towards an academic career in pure mathematics. The successful candidate will be expected to teach nine hours of small group teaching a week during the twenty teaching weeks of the year (amounting to 180 hours of contact time) and to assume a role as Director of Studies in Pure Mathematics (for which additional remuneration will be paid). He or she will also be expected to pursue scholarly research with a view to building up a high-quality publication record. The position provides the opportunity for collaboration and support from the world leading Cambridge mathematical research community.

Other duties include College examining, participating in the selection of candidates for admission, attendance at Open Days, and general support for the academic development of the Mathematics students in College.

The salary will be in the range of £36,261- £40,792 p.a., depending on experience; the post is pensionable under the Universities Superannuation Scheme (USS). The successful candidate will also be provided with the usual benefits of a Fellowship in the College, and membership of the Faculty of Mathematics in the University of Cambridge.

For further particulars, please visit www.joh.cam.ac.uk/vacancies or telephone 01223 338794.

The closing date for the receipt of completed applications is **12 noon** on **Monday 24 May 2019**.

Heilbronn Institute for Mathematical Research | London Mathematical Society Est. 1865 | STOR-i | Mathematics & Statistics Lancaster University

LMS Prospects in Mathematics Meeting

6–7 September 2019

Lancaster University

All Finalists Maths Undergraduates who are considering applying for a Maths PhD in 2020 are invited to attend the 2019 LMS Prospects in Mathematics Meeting.

The meeting will feature a range of speakers from a wide range of mathematical fields across the UK who will discuss their current research and what opportunities are available to you. 50 places are available, including overnight accommodation and some funding towards travel costs.

To apply, email Dr Nadia Mazza (n.mazza@lancaster.ac.uk) headed Prospects 2019 Application with the statement: "I am on track academically to begin Ph.D. studies in 2020" with evidence of your predicted degree classification. The application deadline is 19 July 2019.

The Eternal Golden Braid: Gödel Escher Bach

Barbican Hall, London, 9 March 2019

Review by Tony Mann

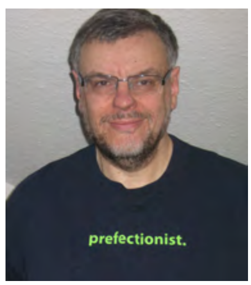
Douglas R. Hofstadter's book *Gödel, Escher, Bach: An Eternal Golden Braid* came out in 1979 and quickly became a cult best-seller. Marcus du Sautoy presented this "performance lecture" celebrating the book's 40th anniversary as part of the Barbican's 2019 *Life Rewired* season, supported by the harpsichordist Mahan Esfahani and the composer Robert Thomas, along with other musicians and a substantial technical team.

The evening started off, as does Hofstadter's book, with an account of J.S. Bach's visit to the court of Frederick the Great, which led to the great composer's Musical Offering. The lecture was interspersed with a video interview with Hofstadter, from which we learned that neither Escher nor Bach were included in his original plan for the book, and with musical examples, including what may have been Marcus's harpsichord debut, assisting Esfahani in Bach's Crab Canon. Marcus illuminated Hofstadter's argument that consciousness comes from what he calls a "strange loop", and Esfahani treated us to a wonderful performance of Bach's *Six Part Ricercar*.

Photo courtesy of Mark Allan/Barbican

The event went on to explore artificial intelligence and creativity (the theme of Marcus's new book *The Creativity Code*). The audience was shown examples of visual art and poetry, and asked which they thought was by a human and which by an AI: impressively, the voting was done by holding up coloured cards which were counted automatically by a camera / computer system. Esfahani then played a collage of music, some by Bach and some by an AI, and the audience indicated, in real time, which they thought was the composer of what was being heard at that moment. We were no more successful in telling the difference than Hannah Fry's Christopher Zeeman Medal audience had been in a similar exercise earlier in the week. Of course, when Esfahani repeated the performance and we were told who had composed each bit, the differences seemed clear! In his illuminating comments afterwards, Esfahani noted that he found the Bach much easier to play than the AI's compositions. We heard further music, some by Robert Thomas and some machine-composed, with the musicians reacting to the audience's preferences as the performance unfolded.

This was a thought-provoking, entertaining, and multi-faceted evening, which kept a full house absorbed and fascinated throughout. Having seen Marcus dance the irrationality of the square root of three, perform in the mathematical play X&Y and now mastermind a remarkable mixture of music, machine creativity and electronic voting, I wonder what he will do next!



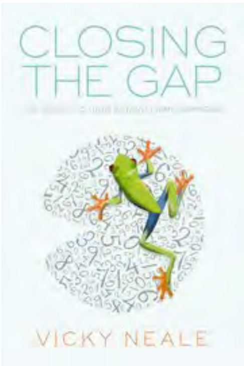
Tony Mann

Tony Mann is Director of Greenwich Maths Centre at the University of Greenwich which works to promote the study of mathematics at all levels. His research interests include topics in the history of mathematics.

Closing the Gap

by Vicky Neale, Oxford University Press, 2017, £19.99, US$ 26.95, ISBN: 978-0198788287

Review by Deborah Chun



Vicky Neale's book tells the stories of two problems in mathematics and along the way illustrates how mathematicians work and how mathematical progress is made. Her main plotline, referenced in the title, is the quest to close the upper bound on the gap between successive primes. This book delves into the progress on the Twin Primes Conjecture, giving the startling result by Yitang Zhang, mentioning the history of the conjecture, and explaining the ensuing online collaboration. Besides this main plot line, Neale leads her audience to a technical result without requiring an understanding of calculus. This compact and well-researched book accomplishes quite a lot in 164 pages!

This book is appropriate for a general audience. Neale uses a warm, conversational tone throughout. She first introduces an analogy. The Inaccessible Pinnacle, a rocky peak on the Isle of Skye in Scotland, seemed at first unclimbable. The draw of the impossible proved enticing enough that eventually a climber reached the summit. This spot has since become a tourist destination for even inexperienced climbers. Similarly, mathematicians aim to make mathematics accessible, and Neale guides anyone interested through the world of mathematical research.

This framework for the two problem arcs shows nicely how new mathematics comes to be. Even numbered chapters cover aspects of the general study of mathematics, while odd numbered chapters lay out the progress of the Twin Primes Conjecture. Neale carefully leads the reader through a set of questions, minimizing technical language and abbreviation. She shows how questions can lead to more questions as easily as they can lead to answers and explains how wrong ideas are just as vital to mathematics as correct ideas are. On page 57, she plainly states, "The process (of doing mathematics) is both messy and creative." She discusses math as an individual pursuit and a collaborative endeavor. She illustrates the process of doing mathematics using the work surrounding Zhang's result.

Hardy and Littlewood show up early on in the Twin Primes Conjecture narrative. Later on, these prominent mathematicians steal the show for a tangential problem. Neale manages to conversationally approach and illuminate an asymptotic formula, which is quite a feat! She discusses Hardy and Littlewood's circle method used to address Waring's problem. In this story arc, we see how every whole number can be written as the sum of four squares (for example, $27 = 0^2 + 1^2 + 1^2 + 5^2$) which generalizes to a whole set of questions about writing integers as the sum of s numbers to the k-th power. This compelling digression can additionally illustrate how problems can draw the mathematician's focus as she is working on another problem.

The main plot line is laid out chronologically by chapter title in Neale's odd chapters, from Chapter 3 "May 2013" to Chapter 15, "April 2014." In these chapters, she describes the progress on the Twin Primes Conjecture. She starts with Zhang's result that there are infinitely many pairs of primes that differ by at most 70 million, which was presented in a Harvard seminar on May 13, 2013. Neale discusses the origin of the conjecture, work done independently by key figures, and details the inception of Polymath, a huge, online collaboration that completed important work on this problem. She shows the successes of Polymath in this problem and clear limitations of the platform. She discusses generally the difficulty of appropriately crediting researchers for their work and explains the slow and painstaking labor of mathematics. On page 29, she writes, "Some improvements would be dramatic, reflecting additional new ideas from the authors, while others would be smaller, highlighting the difficulty of making any progress at all." She brings the reader up the very edge of the known progress on this conjecture.

Overall, this book has a lot to offer. The narration through the text is clear, friendly, and easy to follow. The time you would spend reading this book belies the complex journey you have been guided through. If you are looking for an introduction to the world of Polymath; if you are looking for the story of the Twin Primes Conjecture; if you are looking to show you friends and family what your life as a mathematician is; if you would like a bit of asymptotic mathematics explained to you plainly; if you would like a summary of Waring's problem; or if you just have a couple of hours and are looking for a nice diversion, then you have found it.

Editor's note: Vicky Neale is on the Editorial Board of the Newsletter, but was not involved with this review.

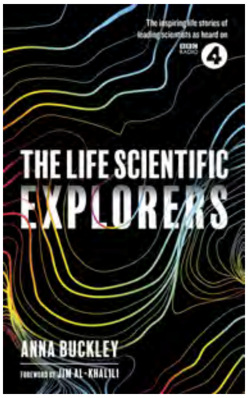
Deborah Chun

Deborah Chun is an associate professor of mathematics and the chair of mathematics at West Virginia University Institute of Technology. Her main research interests are in matroid and graph theory. Deb is an avid player of disc golf and board games and is a reluctant walker of her two large dogs, one of whom adored this book enough to lick the cover.

The Life Scientific Explorers

by Anna Buckley, Weidenfeld & Nicholson, 2018, hardback, pp321. £18.99, ISBN: 978-1-4746-0748-3

Review by Peter Saunders



For the past seven years, the theoretical physicist Jim Al-Khalili has been presenting *The Life Scientific*, a BBC Radio 4 programme in which he interviews scientists about their work and also about their lives as scientists: what made them choose science as a career, how they got into their field of research, how their successes came about and what setbacks they encountered on the way, and so on. A particular strength of the programmes is that we are told enough about the work that we can appreciate what was accomplished without so much detail that we lose the thread of the story.

The range of the series has been deliberately broad. The word "scientific" is, as it should be, taken to include engineering, mathematics, medicine, invention and more. The scientists who are interviewed have all been successful but while some are household names (or as close to being household names as any scientist is likely to get), others are less well-known outside their own field. Some decided on their careers while they were still at school, others came into science much later and by indirect paths.

What they share is that they are all driven by a passion for what they are doing, though the forms this takes and the environments in which they express it can be very different. It's interesting enough for those of us in science to see this, to learn how the other half (or should I say the other 95%?) lives. But it is surely even more interesting for people who are outside the world of science. In particular, anyone in the process of choosing a career should understand that there isn't just one sort of person who becomes a scientist. There isn't just one gender of person either: eight of the twenty subjects are women.

Anna Buckley has been the series producer of *The Life Scientific* since it began, and Al-Khalili credits her with editing "over an hour of chat between two scientists down to a tight 28 minutes of radio gold". She has now converted twenty of the episodes into a book entitled *The Life Scientific Explorers*. It captures the flavour of the interviews very well; whether you prefer the book or the original broadcasts will probably depend on whether you prefer reading or listening. For me an advantage of the book was that bringing the articles together meant I could see both the wide variety of stories and people and at the same time the common threads. Those include, incidentally, the fact that in science as in other creative activities, there's a lot of hard slog beneath the successes.

I enjoyed reading this book and I recommend it. It would also be a good book to give to a young person or put in a school library.

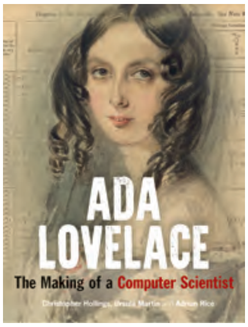
Peter Saunders

Peter is an emeritus professor at King's College London. His chief research interest is mathematical biology and he has also been active in mathematics education.

Ada Lovelace: The Making of a Computer Scientist

by Christopher Hollings, Ursula Martin and Adrian Rice, Bodleian Library, 2018, £20, US$35, ISBN: 978-1-85124-4881

Review by Allan Grady



This fascinating book sets out to be two diverse things: a scholarly work and a volume which has the visual beauty, if not the physical size, of a coffee table book. It succeeds on both counts.

Ada, Countess of Lovelace (1815–1852) was the daughter of Lord and Lady Byron and is frequently referred to as the world's first computer programmer due to her work with Charles Babbage on his Analytical Engine, a device that Alan Turing described as a general-purpose computer. The authors give a readable account of Lovelace's life, making use of previously unpublished archival material including her correspondence with Augustus De Morgan. Samples of this are beautifully reproduced. Even the chapter numbers are copies of Ada's copperplate — another little touch that contributes to making this a visually beautiful book.

The authors assume very little mathematical knowledge on the part of the reader but where any mathematics does appear it is explained clearly and in detail. Ada chose only one of her parents wisely. Her father, Lord Byron, left his wife not long after she was born. Byron's excellence in poetry proved in inverse proportion to his fidelity, and indeed Ada was the only legitimate child he fathered. Her mother, however, raised her well and ensured that she got a good, and more significantly, scientific, education.

For Ada's minor health problems her mother consulted a Dr William King. Just after her 13th birthday her mother extended the range of this consultation and asked Dr King about her daughter's education. He prescribed a course of Euclid and an old-fashioned course of instruction in other areas.

On seeing Pythagoras' Theorem Ada wondered if the squares could be replaced by equilateral triangles, giving a simple example of her inquisitive nature. She also asked probing questions about rainbows, a topic not fully understood at that time.

After her marriage to William King (not Dr King) in 1835 she studied advanced mathematics under Augustus De Morgan. De Morgan (and Babbage) disliked the old-fashioned approach to mathematics and wanted students to understand from first principles. De Morgan effectively prepared a correspondence course for her in university level mathematics and samples of this correspondence are reproduced. Few modern academics would have the time or patience to do this. Charles Babbage started work on his Analytical Engine in the mid-1830s and Ada wanted to contribute to his projects. He wanted to create a calculating machine that could modify its calculation whilst running. Babbage recalls in his autobiography that Ada detected a grave mistake in his paper. Her diagram for the computation by the Engine of Bernoulli numbers is sometimes called the first computer program.

Mathematicians are notorious for destroying scrap paper and making final results look as if they just occurred as a matter of course whereas they usually involve a tortuous route with several blind alleys. The authors include a chapter on some of her scrap paper that has survived. The paper has her thoughts and speculations on some classical problems and they regret that more did not survive.

Although she died at the early age of 36 Lovelace has contributed more to science than many scientists have in a full lifetime. As the authors conclude "her name lives on, in events celebrating women in science, and in books, plays and graphic novels". This book does an excellent job of bringing her life and work to a new generation.

Allan Grady

Allan Grady is a long retired lecturer in mathematics from Dundee. Although no longer doing mathematics he is still active in promoting it.

Membership of the London Mathematical Society

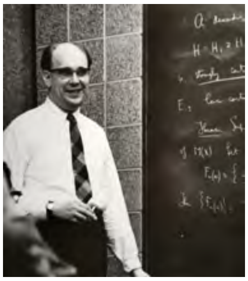
The standing and usefulness of the Society depends on the support of a strong membership, to provide the resources, expertise and participation in the running of the Society to support its many activities in publishing, grant-giving, conferences, public policy, influencing government, and mathematics education in schools. The Society's Council therefore hopes that all mathematicians on the staff of UK universities and other similar institutions will support mathematical research by joining the Society.

If current members know of friends or colleagues who would like to join the Society, please do encourage them to complete the online application form (lms.ac.uk/membership/online-application). Contact membership@lms.ac.uk for advice.

A full list of LMS member benefits can be found at lms.ac.uk/membership/member-benefits.

Obituaries of Members

Norman Blackburn: 1930 – 2018



Norman Blackburn, who was elected a member of the London Mathematical Society on 15 January 1959, died on 24 May 2018, aged 87.

Michael Collins and Charles Leedham-Green write: Norman belonged to that body of mathematicians who are thought of by their colleagues, and perhaps by themselves, as being primarily scholars – their role is to have read, to know, to understand, to appreciate. As such, his fame lies chiefly in his co-authorship, with Bertram Huppert, of the second and third volumes of the magisterial trilogy Endliche Gruppen, or Finite Groups.

He had been involved with reading drafts of Volume I (written by Huppert alone) during summer visits to Kiel in the 1960s, but it is especially Chapter X in Volume III, written at the height of work to classify the finite simple groups, and building too on Chapters VIII and IX in Volume II, where the two authors distilled the then new work of Bender, Glauberman and others. Here they display their sense that understanding the general theory should take precedence over goal driven targets. Indeed, in his review of the book Finite Simple Groups II, which had been based on the 1978 LMS Durham Symposium on that topic and written as the classification reached its climax, Blackburn summarised his feelings as follows: "If the old theory can be compared with the skyline of Dresden, the new one corresponds to that of Lower Manhattan".

Blackburn's original contributions to mathematics were mainly in the area of finite p-groups, as one might expect from a student of Philip Hall. His most influential paper *On a special class of p-groups*, published in 1958, is the founding paper in the study of p-groups of maximal class, which led to the classification (not up to isomorphism) of finite p-groups by coclass, and has overflowed into the study of other nilpotent algebraic objects. It is perhaps worth noting that it was the summary of this work by Huppert in Endliche Gruppen I that brought this work to more general notice. It is still regularly cited, and part of it played an early role in the study of small rank 2-groups that might occur as Sylow subgroups of finite simple groups.

After an initial appointment in Manchester, in 1965 Blackburn joined the new University of Illinois at Chicago Circle (as it was then known) which initially built a mathematical strength in group theory, using the presence of John Thompson and others at the University of Chicago as a magnet. Ten years later he returned to Manchester, where he became the Fielden Professor on the departure of Ian Macdonald. Sadly, in 1984 he was involved in a serious accident in Italy from which he never made a full recovery.

Blackburn was both a Yorkshireman from head to toe and a renaissance scholar. That scholarship extended across music, languages, and art, with a particular interest in opera where he delighted in Wagner, a line of whose music, with libretto, heads the prefaces to each of Volumes II and III of Finite Groups, and he was himself an accomplished pianist. True to form, his first trip abroad after his accident was to Bayreuth. He was a delight to know, and will be long remembered by many friends and colleagues. His wife, Joan, died in 2016, and he is survived by two daughters, Virginia and Caroline.

Lilian Grace Button: 1923–2018

Lilian Button, who was elected a member of the London Mathematical Society on 15 June 1961, passed away peacefully in Kingston Hospital, Kingston on 14 December 2018, aged 95.

Paul Doyle writes: Lilian was born on Snow Hill in the City of London on 24 October 1923 into a large family of six, comprising three girls and three boys; she was the youngest of the three girls. Like many of her siblings she won a scholarship to the then Charterhouse School for Girls. Lilian attended Bedford College for Women (part of the University of London) in October 1942 to study for a BSc Special in Mathematics. Due to the Second World War Bedford College for Women was evacuated to Cambridge and in accordance with the then Ministry of Labour regulations the course was reduced from the normal three years to two years, and in June 1944 Lilian graduated with First Class Honours. Lilian went on to gain her University of London Teacher's Diploma in July 1945 xand in December 1950 was awarded her MSc in Mathematics. Following a spell as Maths Mistress at Okehampton Grammar School in Devon she then embarked on a career spanning over 30 years as a lecturer in mathematics at Exeter University.

Lilian's aptitude and passion for mathematics was matched by her love for life-long learning and she was able to converse across a range of subjects and topics. She could quote at some length weighty passages from Shakespeare and other poetry, discuss past and present politics in some detail and she had an extensive knowledge of history both ancient and modern. She always looked forward to the tennis at Wimbledon each year, was a longstanding member of the Royal Horticultural Society (RHS), the numerous varieties of orchid flower being her favourite.

Always sociable and communicative, Lilian displayed many laudable human qualities and maintained a keen interest in the world around her. Very much a people person, Lilian had a kind and generous nature and a great sense of social justice and empathy and she supported a number of charitable organisations. Strongly opinionated, she never hesitated to challenge conventional or received points of view with insightful knowledge which commanded your attention.

A keen and accomplished pianist and music lover since she was a little girl, Lilian was particularly fond of pieces by Chopin, Rachmaninov and Liszt which she used to play regularly on her piano at home. Despite her failing eyesight and limited mobility in her later years she never gave up striving to continue to play some of their most demanding compositions.

Lilian's resilience, intelligence, compassion, humour and appetite for life will be sorely missed by friends and family alike. Lilian's life, and our memory of her, is perhaps best summed up by one of her former students, colleague and friend at Exeter University, Eddie Evans, who rightly reminds us that: 'We console ourselves by remembering what a full life Lilian had and how she retained her independence to the end.'

Peter Vámos writes: Lilian Grace Button was appointed to the post of Assistant Lecturer at the Mathematics Department of the University College of the South West, later to become the University of Exeter in 1949.

Lilian's area was Geometry and the two courses she taught throughout her long tenure in Exeter were Combinatorics and Graph Theory. She also taught courses in Complex Analysis and Ancillary Mathematics in some years. From 1965 onwards, when Joan Rees joined the Department, Exeter had two female staff which was unusual at the time especially in such a relatively small department. She took an interest in the welfare of women students and this led to her becoming the Warden of Birks Grange in 1952, a Hall of women only. In fact, she was the last mistress of this Hall since it was demolished in 1959. Alas I didn't get to know her personally because she retired at the end of the academic year of 1982-83 and moved away from Exeter.

Desmond John Harris: 1927 – 2019

Desmond John Harris, who was elected a member of the London Mathematical Society on 17 March 1960, died on 6 January 2019, aged 91.

Des Evans writes: Desmond died in La Coruna, Spain, where he had been admitted to hospital after being taken ill on a cruise to the Canary Islands. He was born in Durban, South Africa in 1927. He was a sickly child who would probably be described as a geek in modern terms, having been mainly home schooled due to his persistent illnesses, and with his only sibling being much older, had little other than his studies and books to keep him company. His love of mathematics was awakened when he was presented with a copy of Hardy's Pure Mathematics, but it was to study medicine that he entered the University of Cape Town in 1945. However, when it became clear that he was spending a lot of his time in solving the mathematical problems of his fellow students, he made the decision to change to his first love, mathematics. In 1948 he graduated with a first in mathematics and was encouraged by his teachers to further his studies in Cambridge. His application was successful and he entered St John's College. The next five years in Cambridge were to be very happy and successful ones. He was proud of the fact that he was elected chair of the Archimedians Society, whose secretary was a Ms Lily Brown who later became Lady Atiyah; his successor to the chair was Sir Michael and Desmond always claimed to have played a part in bringing the Atiyahs together! After his BA with a first in 1950, Desmond was taken on by Frank Smithies for a PhD in functional analysis, but a recurrence of the rheumatic fever that had laid him low as a child forced him to cut short his doctoral studies. By now he had met Stella, who worked in the University library, and they were married on 4 July 1953. Desmond and Stella then left Britain for South Africa where Desmond had accepted an offer from the University of Durban. They soon became involved in the politics of South Africa, and were active members of the Liberal Party in its opposition to apartheid. In 1957 they returned to Britain, Desmond having accepted a temporary position in Durham, and in 1958 he was appointed to a lectureship in the Pure Mathematics Department of what was then the University College of South Wales and Monmouthshire, a constituent college of the University of Wales. They were to remain in South Wales for the rest of their lives together, living first in the village of Machen near Caerphilly, and then in Llanishen near Trelech, the birthplace of Bertrand Russell, in the beautiful Monmouthshire countryside.

Desmond had a deep and encyclopaedic knowledge of classical and functional analysis, and an enviable ability to recall obscure facts and details of proofs. His interests fitted in well with mine in spectral analysis and differential equations and it was inevitable that we should collaborate in research after my arrival in Cardiff in 1964. Our first project was to discover how properties like the compactness and measures of non-compactness of Sobolev embeddings depend on geometric properties of the underlying domain. From this knowledge follows, for instance, the dependence of spectral properties of the Neumann Laplace operator on the boundary of the domain. Of particular interest to us were domains with fractal boundaries, and related analytic problems on trees which form a skeleton of the domain. Close ties were established with the Czech analysts in Prague, both in Charles University and in the Mathematical Institute of the Academy of Science, where Desmond's intellect was greatly appreciated. Other off-shoots of the project were investigations on Hardy type integral inequalities for mappings between Banach spaces and the p-Laplace operator, and for our work in these areas we were joined by David Edmunds who had a wide experience and expertise in such problems. A fruitful collaboration between the three of us continued for many years, until well after Desmond's retirement.

In 2009 Desmond's beloved Stella died and in 2013 he decided to leave behind all the happy memories of South Wales and moved to live in Cheltenham. However, the last years of his life were very full and happy ones. He and his friend and companion Peter Wood enjoyed the company of many friends who Peter would entertain on the piano and cello in monthly musical soirées. They regularly visited my wife and me in Cardiff, and also travelled widely, especially in Scotland and Prague. They particularly loved cruises! Desmond was a good friend and a true gentleman. I shall miss him greatly, and so will his many friends.

Slava Kurylev: 1952 – 2019

Slava Kurylev, who was elected a member of the London Mathematical Society on 10 May 1996, died on 19 January 2019, aged 66.

Bill Lionheart writes: Kurylev was known for his work on inverse problems for partial differential equations, especially for the combination of analysis and differential geometry. Yaroslav Vadimovich Kurylev, to give his full name, was born in December 1952 in Kalinin in the USSR (now Tver in Russia). He graduated from Leningrad School 239 (which specialised in Mathematics and Physics) in 1970 and went to read Physics at Leningrad State University. He graduated in 1976 with a masters' degree in mathematical physics. He had initially started reading just physics but he found he had no inclination towards laboratory work and so was naturally drawn to the mathematical side, for which he had a gift. He went on to study for his PhD at the Leningrad Branch of Steklov Mathematical Institute of the Academy of Science, supervised by Vassili Babich. In 1975 he defended his thesis *Asymptotics of the spectral function of a second-order elliptic differential operator.* He continued to work at the Steklov Institute and then held a two year temporary position at Purdue before moving to the UK for a permanent position at Loughborough in 1995.

He made his name from the application of geometric methods to inverse problems for partial differential equations, specifically methods to reconstruct a Riemannian manifold from its boundary spectral data including the "Boundary Control" method he developed with Belishev. Soon after arriving at Loughborough he became interested in applications of inverse problems as well as more theoretical problems. My own first contact with Slava was a long phone call, in which he explained his plan for the widening application of geometric methods of inverse problems, and specifically to gather people together with interests in pure and applied aspects of inverse problems in the UK, from academia and from industry. Slava was rapidly promoted to a chair at Loughborough and had a wide circle of collaborators tackling problems in medical and seismic imaging. In 2007 he moved to University College London. The geometric theory of inverse problems gave rise to counterexamples, in practice things that could not be seen. In 2009 he wrote a paper with Greenleaf, Lassas, and Uhlmann entitled *Cloaking devices, electro-magnetic wormholes, and transformation optics*. In 2018 with Lassas and Uhlmann his paper *Inverse problems for Lorentzian manifolds and non-linear hyperbolic equations* showed how tools developed for general relativity might be used in medical imaging, specifically for elastography. One hopes that that work might help one day in the diagnosis of cancer.

For him he already knew in 2018 that his own cancer was terminal. He spent that time with his close family, watching French comedy films, enjoying tasting wine (although he could no longer drink it properly). He continued working until the very last weeks, with colleagues visiting him at home from Japan, Finland, Spain and the USA. His last work included geometric tomography for connections on manifolds, sometimes reducing his morphine dose so he could concentrate. His collaborators Lauri Oksanen and Matti Lassas told me at his funeral that his last utmost desire was for this work to be finished, leaving them with details of what he hoped could be achieved.

Slava is remembered fondly by all those whose life he touched, and in the mathematical community especially for his encouragement and support of younger colleagues.

Professor Sir Peter Swinnerton-Dyer: 1927 – 2018

Professor Sir Peter Swinnerton-Dyer, who was elected a member of the London Mathematical Society on 20 March 1952, died on 26 December 2018, aged 91.

Jean-Louis Colliot-Thélène, Miles Reid and Gregory Sankaran write: Peter Swinnerton-Dyer is undoubtedly best known for the Birch–Swinnerton-Dyer Conjecture, formulated with Bryan Birch in the early 1960s. This starts from the observation that an elliptic curve E having a large group of rationals points probably has comparatively more points over each finite field; but turning this naive qualitative idea into a precise quantitative statement led to astonishing predictions for global arithmetic: the L-function of the elliptic curve has pole of order the rank of the Mordell–Weil group, and its leading term is also determined by arithmetic properties of E. This problem stands alongside the Riemann hypothesis as a challenge to future generations.

Peter's publications span 63 years. His first paper, which appeared in the Journal of the London Mathematical Society when he was sixteen, was on parametrised families of points on the diagonal quartic surface. Such concrete Diophantine problems remained central to much of his work, and diagonal quartic surfaces are among the subjects of his final papers.

He went to Trinity College, Cambridge, and after completing the Mathematical Tripos began postgraduate work under J.E. Littlewood. This led him to spend a year in Chicago in the mid-1950s: he originally intended to study analysis with Zygmund, but instead came under the influence of André Weil. Many facets of his work as a number theorist and Diophantine geometer can be traced back to this period. He returned to Cambridge, where he submitted his thesis for a Trinity College prize fellowship (rather than for a PhD).

From the late 1950s, Peter was employed in the fledgling Cambridge Computer Laboratory, where his day job included writing the programming language Autocode and the first operating system for the then state-of-the-art new computer Titan. Here "day job" is literal: Bryan Birch writes that they "had relatively low priority: on a good day we would have access to EDSAC 2 [the predecessor to TITAN] from midnight until it broke down some time in the small hours". Nevertheless, their work was the first really serious use of computers to support research in pure mathematics.

Peter was active across a wide range of mathematics research, including analysis, differential equations and dynamical systems in addition to number theory and algebraic geometry. He wrote several papers on differential equations, including one with Mary Cartwright that appeared in Russian translation. His Diophantine work often starts from concrete examples such as diagonal cubic surfaces or varieties fibred in elliptic curves. He initiated several major research directions by finding the right loose ends to pull inside concrete calculations. For example, his calculations for cubic surfaces were a source for the more general Brauer–Manin obstruction to the Hasse principle.

In 1973 he embarked on a second career as an administrator: first in Cambridge, as Master of St Catharine's College and, in 1979-81, Vice-Chancellor (a part-time post in those days). In 1983 he moved to a senior civil service role as head of UGC and its successor UFC, the government body controlling university funding. In this capacity he played a major part in introducing the now familiar system of Research Assessment as an indicator for allocating resources.

While serving as senior civil servant, Peter co-authored important papers, several of which have had major repercussions. His large-scale work with Colliot-Thélène and Sansuc on rational points on the intersection of two quadrics has been extremely influential. It led to the first example of a variety that is stably rational but not rational, even over the complex numbers. This answers Zariski's famous cancellation question, and is a result in geometry that can be stated without reference to arithmetic methods.

Later, he initiated a conditional (and very involved) technique of 2-descent for surfaces with a pencil of curves of genus one, consisting in searching for curves in the pencil having points in every completion of the number field, and with a suitable and small Selmer group. This led in joint work with Colliot-Thélène and Skorobogatov to predictions on the density of rational points on some classes of K3 surfaces.

In 2001 Peter, by then in his seventies, used the method to reduce the proof of the local-to-global principle for rational points on diagonal cubic hypersurfaces over the rationals to the usual conjecture that the Tate-Shafarevich group of an elliptic curve is finite. This opened up areas of research that had been widely viewed as untouchable. In his last 15 years Peter produced 25 research papers, many of which contain ground-breaking results. His 2008 paper on twisting of the Selmer group has been enormously influential.

In 1983, at the time of his move to UGC, he married the distinguished archaeologist Harriet Crawford. In later years he often accompanied her to archaeological meetings: one of his insights into 2-descent occurred to him on a bus journey in eastern Anatolia during one of these trips.

Peter's cultural range was enormous: opera and history were particular enthusiasms. Liberal in every sense and exceptionally open-minded, he often preferred the company of students and younger colleagues: even as Vice-Chancellor, he was frequently to be found with undergraduates, drinking cider in St Catharine's bar with the undergraduates, or playing chess. He played chess very well, but at bridge he was outstanding, playing very successfully at international level in the 1950s.

Peter's varied career brought him into contact with a large number of people, who remember him vividly for his genial character and for his extreme generosity. Through his students, collaborators and other associates his influence spread very wide: the current success in algebraic geometry in Britain, in particular, is difficult to imagine without his encouragement. Some of the many stories on his character and career are told in the preface of *Number Theory and Algebraic Geometry,* 1–30, London Math. Soc. Lecture Note Ser., 303, Cambridge Univ. Press, Cambridge, 2003.

William Benter Prize in Applied Mathematics 2020

Call for NOMINATIONS

The Liu Bie Ju Centre for Mathematical Sciences of City University of Hong Kong is inviting nominations of candidates for the William Benter Prize in Applied Mathematics, an international award.

The Prize

The Prize recognizes outstanding mathematical contributions that have had a direct and fundamental impact on scientific, business, financial, and engineering applications.

It will be awarded to a single person for a single contribution or for a body of related contributions of his/her research or for his/her lifetime achievement.

The Prize is presented every two years and the amount of the award is US$100,000.

Nominations

Nomination is open to everyone. Nominations should not be disclosed to the nominees and self-nominations will not be accepted.

A nomination should include a covering letter with justifications, the CV of the nominee, and two supporting letters. Nominations should be submitted to:

Selection Committee
c/o Liu Bie Ju Centre for Mathematical Sciences
City University of Hong Kong
Tat Chee Avenue, Kowloon, Hong Kong

Or **by email to: lbj@cityu.edu.hk**

Deadline for nominations: 30 September 2019

Presentation of Prize

The recipient of the Prize will be announced at the **International Conference on Applied Mathematics 2020** to be held in summer 2020. The Prize Laureate is expected to attend the award ceremony and to present a lecture at the conference.

The Prize was set up in 2008 in honor of Mr William Benter for his dedication and generous support to the enhancement of the University's strength in mathematics. The inaugural winner in 2010 was George C Papanicolaou (Robert Grimmett Professor of Mathematics at Stanford University), and the 2012 Prize went to James D Murray (Senior Scholar, Princeton University; Professor Emeritus of Mathematical Biology, University of Oxford; and Professor Emeritus of Applied Mathematics, University of Washington), the winner in 2014 was Vladimir Rokhlin (Professor of Mathematics and Arthur K. Watson Professor of Computer Science at Yale University). The winner in 2016 was Stanley Osher, Professor of Mathematics, Computer Science, Electrical Engineering, Chemical and Biomolecular Engineering at University of California (Los Angeles), and the 2018 Prize went to Ingrid Daubechies (James B. Duke Professor of Mathematics and Electrical and Computer Engineering, Professor of Mathematics and Electrical and Computer Engineering at Duke University).

The Liu Bie Ju Centre for Mathematical Sciences was established in 1995 with the aim of supporting world-class research in applied mathematics and in computational mathematics. As a leading research centre in the Asia-Pacific region, its basic objective is to strive for excellence in applied mathematical sciences. For more information about the Prize and the Centre, please visit ***https://www.cityu.edu.hk/lbj/***

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LIU BIE JU CENTRE FOR MATHEMATICAL SCIENCES

Colloquia in Combinatorics 2019

Location: Queen Mary and LSE
Date: 8 and 9 May 2019
Website: tiny.cc/2dayCC

The first day of the 13th Colloquia in Combinatorics will take place at Queen Mary, University of London on 8 May; the second at the London School of Economics and Political Science on 9 May. The talks will be of wide interest to all those working in combinatorics and related fields.

Brackets, Reduction and Integrability

Location: University of Leeds
Date: 17-18 May 2019
Website: tinyurl.com/y3gbmfqk

This is part of a series of meetings in integrable systems and related topics, supported by an LMS Scheme 3 grant. Speakers are: A. Bolsinov (Loughborough), I. Marshall (Moscow), V. Roubtsov (Angers), D. Valeri (Glasgow), R. Vitolo (Lecce), J. P. Wang (Kent). Visit the website for further details.

LMS Education Day 2019

Location: De Morgan House, London
Date: 13 May 2019
Website: tinyurl.com/y65p23kk

This is an opportunity for mathematics lecturers and colleagues to meet and discuss aspects of education directly related to HE. The event will include an update on subject-level TEF, a talk on teaching ethics in the mathematics curriculum, and presentations/small group discussions on sharing practice.

The History of Recreational Mathematics

Location: Birkbeck, University of London
Date: 18 May 2019
Website: tinyurl.com/y5ssc4oc

This one-day conference is organised by the British Society for the History of Mathematics. An exciting programme is planned, with six speakers on a range of topics from ancient riddles to Latin squares and much else besides. Visit the website for more details and to book.

LMS Meeting

LMS Invited Lecture Series 2019

20–24 May 2019: ICMS,The Bayes Centre, 47 Potterrow, Edinburgh EH8 9BT,

Website: https://tinyurl.com/yb7v47yu

Søren Asmussen (Aarhus University) will talk on *Advanced topics in life insurance mathematics*. The accompanying lecturers will be Dr Corina Constantinescu (Liverpool University), who will talk on *Applications of fractional calculus in insurance/risk theory*, and Professor Pauline Barrieu (London School of Economics).

For further details about the LMS Invited Lecture Series and to register for a place, see the website. Travel grants are available; send requests with an estimate of expenses to f.daly@hw.ac.uk.

LMS Meeting

Northern Regional Meeting & Workshop

28 and 29 May 2019, Newcastle University

Website: tinyurl.com/y9d49s5t

The meeting will include lectures by Karin Baur (Universities of Graz and Leeds) and Sibylle Schroll (University of Leicester). All interested are welcome to attend. The Workshop on Higher Homological Algebra will take place on 29 May. There is a registration fee of £25, to be waived for PhD students. Travel grants are available; for further details and to register, email peter.jorgensen@ncl.ac.uk.

Fredholm Theory of Non-elliptic Operators

Location: University of Leeds
Date: 3–7 June 2019
Website: tinyurl.com/y3z9a7jf

Fredholm theory of non-elliptic partial differential operators is a relatively new topic with applications in mathematical physics and other areas of mathematics. Topics of the conference include for example index theory, hyperbolic evolution equations, microlocal analysis.

SIAM–IMA Student Chapter Meeting

Location: University of Reading
Date: 7 June 2019
Website: tinyurl.com/maths-rdg

A student-organised conference in applied mathematics. We have keynote talks from Professor Nick Trefethen (Oxford), Professor Valentina Escott-Price (Cardiff) and Dr Karl-Mikael Perfekt, (Reading). Slots are available for contributed talks and posters from PhD students.

Postgraduate Combinatorial Conference

Location: University of Oxford
Date: 10–12 June 2019
Website: http://tinyurl.com/y2cu9arn

This is the 26th edition of a conference organised by and for postgraduate students in combinatorics under the auspices of the British Combinatorial Committee. All attending students will be encouraged to give a talk. The invited speakers are Imre Leader, Natasha Morrison and Ben Barber. Supported by an LMS Scheme 8 grant.

Mathematics for Social Activism Workshop

Location: University of Leeds
Date: 6–7 June 2019
Website: tinyurl.com/mathsactivism

This workshop will bring together mathematicians and data analysts from academia, NGOs and government, focused on how mathematics can drive social progress. Themes include mathematical modelling of social phenomena and statistical analysis and presentation of data.

Symmetries and Asymptotic Patterns in Nonlinear Partial Differential Equations

Location: Swansea University
Date: 10–12 June 2019
Website: tinyurl.com/y2hbjg97

The aim of the workshop is to bring to Swansea a group of leading international experts on symmetries and asymptotic behaviour in nonlinear PDE for an exchange of ideas and update on most recent advances in these areas.

Research Students' Conference in Probability and Statistics (RSC2019)

Location: University of Exeter
Date: 18–21 June 2019
Website: tinyurl.com/yy3juba7

RSC is an annual conference organised for, and by, postgraduate students. This event is aiming to bring together postgraduates working in probability, statistics and related fields to present their research in the form of a talk and/or a poster. The event is supported by an LMS Scheme 8 grant.

LMS Popular Lectures

Location: London/Birmingham
Date: 26 June/19 September 2019
Website: lms.ac.uk/events/popular-lectures

The LMS Popular Lectures are free annual events which present exciting topics in mathematics and its applications to a wide audience. The speakers for the 2019 Lectures are Professor Peter Higgins (Essex) and Professor Alain Goriely (Oxford).

Defects in Topological and Conformal Field Theory

Location: King's College London
Date: 27–28 June 2019
Website: tinyurl.com/kcldef19

Defects and boundaries are important tools in our understanding of quantum field theories. This short meeting will focus on new mathematical and physical results in this area, with an emphasis on two-dimensional conformal QFTs and topological QFTs. Supported by an LMS Conference grant.

Small Scales and Homogenisation

Location: Cardiff University
Date: 24–26 June 2019
Website: tinyurl.com/y9x54lce

This workshop is intended to connect different communities of mathematicians working in the domain of Asymptotic Analysis of multiscale problems. This is a wide field comprised of several subcategories. Supported by an LMS Conference grant.

(Leaps Through) Loops in Leeds

Location: University of Leeds
Date: 1–4 July 2019
Website: tinyurl.com/loops-leeds

This workshop will revolve around loop braid groups and related braided and knotted structures, throughout different areas of geometric topology, with an eye on applications in physics. Deadline to register is 31 May 2019. Email c.damiani@leeds.ac.uk if you have any questions or comments. Supported by an LMS Conference grant.

LMS Meeting

General Society Meeting & Aitken Lecture

28 June 2019, Mary Ward House

Website: tinyurl.com/ybmyx6on

The meeting will include lectures by Paul Shafer (Leeds) and Bakh Khoussainov (Auckland; 2019 Aitken Lecture), as well as the announcement of the 2019 LMS prize winners. For further details about the event and to register, see the website. A Society Dinner will be held after the meeting at a cost of £35.00, including drinks. To reserve a place at the dinner, email lmsmeetings@lms.ac.uk.

Boundary Integral Methods (UKBIM12)

Location: Oxford Brookes University
Date: 8–9 July 2019
Website: https://ukbim12.com/

Mathematicians, scientists and engineers interested in the theory and application of boundary integral methods are encouraged to attend this twelfth UK Conference on Boundary Integral Methods. Supported by grants from the LMS and IMA.

Computability in Europe 2019

Location: Durham University
Date: 15-19 July 2019
Website: tinyurl.com/y46q9ygj

This annual flagship conference of the Association Computability in Europe promotes the development of all computability-related science. Application areas include computer science, history (of computability), philosophy, physics and biology.

LMS Meeting

LMS Prospects in Mathematics Meeting

Lancaster University, 6–7 September 2019

Website: tinyurl.com/yysds39f

All Finalists Maths Undergraduates who are considering applying for a Maths PhD in 2020 are invited to attend the 2019 LMS Prospects in Mathematics Meeting. There are 50 places are available. See website for details on how to apply. To apply, email Dr Nadia Mazza (n.mazza@lancaster.ac.uk) with the subject heading 'Prospects 2019 Application' and the statement: "I am on track academically to begin Ph.D. studies in 2020", with evidence of your predicted degree classification. Application deadline: 19 July 2019.

Thermodynamic Formalism: Ergodic Theory and Geometry

Location: University of Warwick
Date: 22–26 July 2019
Website: tinyurl.com/y5k8brga

The aim of this workshop, is to progress the theory of thermodynamic formalism as a tool in ergodic theory and its applications. The workshop will also be an opportunity to celebrate the 60th birthday of Mark Pollicott. Deadline for registration: 2 June 2019.

Graph Complexes in Algebraic Geometry and Topology

Location: University of Manchester
Date: 9–13 September 2019
Website: ibykus.sdf.org/graphc/

Graph complexes are simple to define but carry information that manifests throughout mathematics. This conference will bring together experts from vastly different perspectives with the hope of developing a shared understanding of these objects.

Structure Preservation and General Relativity

Location: INI, Cambridge
Date: 30 September – 4 October 2019
Website: tinyurl.com/y4l7449x

Numerical relativity is a powerful approach to understanding complex behaviour of gravitational fields. Einstein's equations are statements about geometry of space times, based on differential geometric structures. Application deadline: 23 June 2019.

The Complex Analysis Toolbox: New Techniques and Perspectives

Location: INI, Cambridge
Date: 9–13 September 2019
Website: tinyurl.com/y5rgpubv

This workshop will showcase new techniques and mathematical ideas within complex analysis. Key themes include the unified transform method of Fokas and advances in the Wiener–Hopf method. Deadline for applications: 9 June 2019.

Clay Research Conference and Workshops

Location: Mathematical Institute, Oxford
Date: 29 September–4 October 2019
Website: claymath.org

Workshops will include *Beyond Spectral Gaps; Modular Representation Theory; Patterns in Cohomology of Moduli Spaces;* and *Conjectures and their Applications*. To register for the conference and to register interest in a workshop, email Naomi Kraker at admin@claymath.org.

Dirac Operators in Differential Geometry and Global Analysis

Location: Bedlewo, Poland
Date: 7–11 October 2019
Website: tinyurl.com/y7pg4ekb

The conference will gather young and experienced scholars working on special geometry, metric contact geometry, index theory and applications to theoretical physics. Limited support available; early registration highly recommended.

UNIVERSITY OF BATH

LONDON MATHEMATICAL SOCIETY
EST. 1865

Random Structures: from the Discrete to the Continuous

LMS Research School, University of Bath
1–5 July 2019

Lecture course topics and speakers

Regularity Structures
Plenary lecture by **Speaker TBC**
Stochastic PDE limits by **Hendrik Weber** (Bath)

Spatial population processes
Plenary lecture by **Alison Etheridge** (Oxford)
Spatial population genetics by **Sarah Penington** (Bath)

Random trees
Plenary lecture by **Christina Goldschmidt** (Oxford)
Scaling limits of random trees by **Nicolas Broutin** (Paris 6)

More information and apply by 29 April 2019 at https://tinyurl.com/LMS-RSDC19.

Financial aid available. Details at https://tinyurl.com/mve2fxb.

SAMBa

Society Meetings and Events

May

20–24 Invited Lecture Series 2019, Professor Soren Asmussen (Aarhus University), ICMS, Edinburgh (482)
28–29 Northern Regional Meeting & Higher Homological Algebra Workshop, Newcastle (482)

June

26 Popular Lectures, Institute of Education, London (482)
28 Graduate Student Meeting, London
28 General Meeting of the Society and Aitken Lecture, London (482)

July

1–5 LMS Research School: *Random Structures: from the Discrete to the Continuous,* Bath (482)
8–12 LMS Research School, *Mathematics of Climate,* Reading

September

6–7 Prospects in Mathematics Meeting, Lancaster (482)
11 Midlands Regional Meeting, Nottingham
19 Popular Lectures, University of Birmingham (482)

November

15 Graduate Student Meeting, London
15 Society Meeting and AGM, London

January 2020

15 South West & South Wales Regional Meeting, Bristol

Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website (www.lms.ac.uk/content/calendar). Please send updates and corrections to calendar@lms.ac.uk.

May

7-10 Interactions between Representation Theory and Homological Mirror Symmetry, University of Leicester (480)
8 Colloquia in Combinatorics 2019, Queen Mary, University of London (482)
9 Colloquia in Combinatorics 2019, London School of Economics (482)
13 LMS Education Day 2019, London
13-17 Optimal Design of Soft Matter, INI, Cambridge (480)
15-17 Modern Applied Probability Workshop, ICMS, Edinburgh (480)
16-17 Idealised Models of Numerical Weather Prediction for Data Assimilation Research, University of Leeds (480)
17-18 Brackets, Reduction and Integrability, University of Leeds (482)
18 The History of Recreational Mathematics, Birkbeck College, London
20-22 Wales Mathematics Colloquium 2019, Gregynog Hall, Tregynon (481)
20-24 LMS Invited Lecture Series 2019, Professor Sören Asmussen (Aarhus University), ICMS, Edinburgh (482)
20-24 Women in Noncommutative Algebra and Representation Theory Workshop, University of Leeds (480)
28-29 LMS Northern Regional Meeting & Higher Homological Algebra Workshop, Newcastle (482)
29 Combinatorics One-Day Meeting, University of Oxford (481)

June

3-7 Fredholm Theory of Non-elliptic Operators, University of Leeds (482)
6-7 Mathematics for Social Activism Workshop, University of Leeds (482)
7 SIAM–IMA Student Chapter Meeting, University of Reading (482)
10 Symmetries and Asymptotic Patterns in Nonlinear Partial Differential Equations, Swansea University (482)
10-12 Postgraduate Combinatorial Conference, University of Oxford (482)
10-14 New Trends and Challenges in the Mathematics of Optimal Design, INI, Cambridge
17-21 MAFELAP 2019, Brunel University London (479)
17-21 Approximation, Sampling, and Compression in High Dimensional Problems, INI, Cambridge
18-21 Research Students' Conference in Probability and Statistics, University of Exeter (482)
24-26 Small Scales and Homogenisation, Cardiff University (482)
24-26 Mathematical Neuroscience International Conference, Copenhagen, Denmark
26 LMS Popular Lectures, Institute of Education, London (482)
27-28 Defects in Topological and Conformal Field Theory, King's College London (482)
28 LMS Graduate Student Meeting, London
28 LMS General Meeting and Aitken Lecture, London (482)

July

1-4 (Leaps through) Loops in Leeds, University of Leeds (482)
1-4 Dense Granular Flows IMA Conference, Cambridge (481)
1-5 LMS Research School: *Random Structures: from the Discrete to the Continuous,* Bath (482)
3-5 Current Status and Key Questions in Landscape Decision Making, INI, Cambridge (481)
8-9 UK Conference on Boundary Integral Methods, Oxford Brookes University (482)
8-12 LMS Research School, *Mathematics of Climate,* Reading
8-12 Measurability, Ergodic Theory and Combinatorics, University of Warwick (479)
8-12 Geometry, Compatibility and Structure Preservation in Computational Differential Equations, INI, Cambridge (481)
15-19 Computability in Europe 2019, Durham University (482)
22-26 Thermodynamic Formalism: Ergodic Theory and Geometry, University of Warwick (482)
22-26 Postgraduate Group Theory Conference 2019, University of Birmingham (481)
28-3 Aug International Mathematics Competition for University Students, Blagoevgrad, Bulgaria (481)
29-2 Aug British Combinatorial Conference 2019, University of Birmingham (479)
31-2 Aug Progress on Novel Mathematics and Statistics for Landscape Decisions, INI, Cambridge (481)

August

12-16 Factorisation of Matrix Functions, INI, Cambridge (481)
26-29 Caucasian Mathematics Conference, Rostov-on-Don, Russia

September

6-7 Prospects in Mathematics Meeting, Lancaster (482)
9-13 Graph Complexes in Algebraic Geometry and Topology, University of Manchester (482)
9-13 The Complex Analysis Toolbox: New Techniques and Perspectives, INI, Cambridge (482)
11 Midlands Regional Meeting, Nottingham
19 LMS Popular Lectures, University of Birmingham (482)
29-4 Oct Clay Research Conference and Workshops, Mathematical Institute, Oxford (482)
30-4 Oct Structure Preservation and General Relativity, INI, Cambridge (482)

October

7-11 Dirac Operators in Differential Geometry and Global Analysis, Bedlewo, Poland (482)
28-1 Nov Complex Analysis in Mathematical Physics and Applications, INI, Cambridge