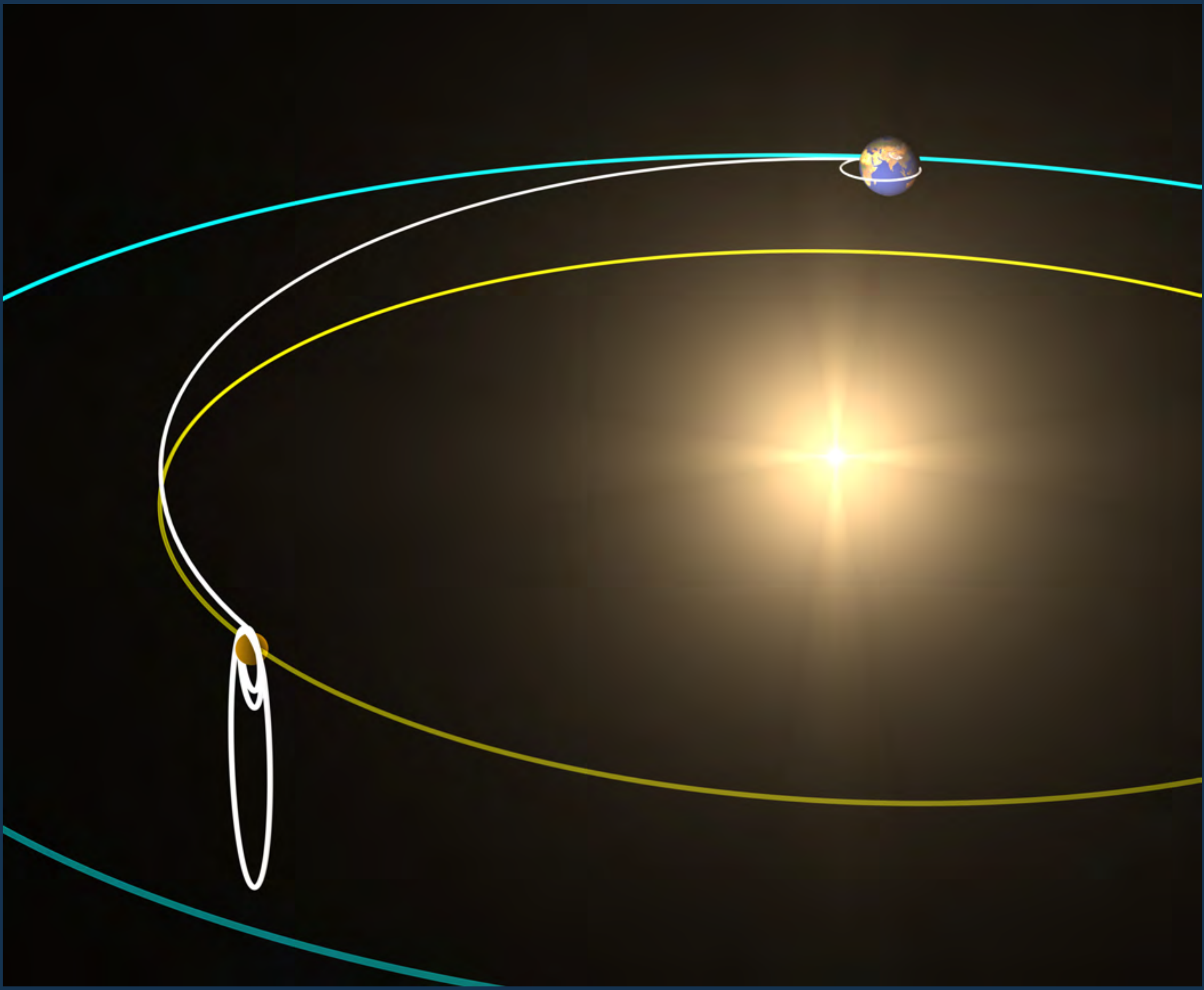
LONDON MATHEMATICAL SOCIETY
EST. 1865

NEWSLETTER

Issue: 487 - March 2020



SIR MICHAEL ATIYAH (1929–2019)

MULTIFRACTAL RANDOM MEASURES

MATHEMATICS OF SPACECRAFT TRAJECTORY

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COVER IMAGE

Venus Express' trajectory to Venus. © ESA/C. Carreau – CC BY-SA IGO 3.0.

ISSN: 2516-3841 (Print)
ISSN: 2516-385X (Online)
DOI: 10.1112/NLMS

NEWSLETTER WEBSITE

The Newsletter is freely available electronically at lms.ac.uk/publications/lms-newsletter.

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The Newsletter welcomes submissions of feature content, including mathematical articles, career related articles, and microtheses, from members and non-members. Submission guidelines and LaTeX templates can be found at lms.ac.uk/publications/submit-to-the-lms-newsletter.

Feature content should be submitted to the editor-in-chief at newsletter.editor@lms.ac.uk.

News items should be sent to newsletter@lms.ac.uk.

Notices of events should be prepared using the template at lms.ac.uk/publications/lms-newsletter and sent to calendar@lms.ac.uk.

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FROM THE EDITOR-IN-CHIEF

It was a sad start to 2020 for me with the death of my co-author Walter Hayman on New Year's Day. We had worked together on a book for the last few years — via Royal Mail — and I very much miss the excitement of having 'actual' letters arrive, and the intrigue/stress involved in deciphering his handwriting and dealing with this slow communication process. On one occasion, I had written a postscript to say that I was looking forward to seeing him at the upcoming ODFTM (the One Day Function Theory Meeting — an annual complex analysis event), and he replied saying that he never did gain a taste for alphabet soup.

Inevitably, perhaps, this has led me to think about life, death, mathematics and why we do what we do. Not the cheeriest of starts to a Newsletter I know, but one which touches on some issues of importance to this publication. In my pre-editor days, I had often read the Obituaries section and wondered at its lack of diversity, particularly in terms of gender. However, the fact of the matter is that it may be a long time before this particular section diversifies. This is because every member deserves an obituary, and while our Society is becoming more diverse, it could be a long time before diversity reaches all levels — as you will see in Eugenie Hunsicker's article on the academic pipeline.

To finish, I think that we should acknowledge the contribution that our obituary writers make to our Newsletter. These are always friends or colleagues (or both) of the deceased and they bear the responsibility of summing up a life — something of huge importance on both a personal, and an historical, level. Through their pieces we learn a little about the mathematician, their work and the way that they lived. While the overall picture may not be one of diversity, the close-up is one of individuality — and each of these individuals should be celebrated.

Eleanor Lingham
Editor-in-Chief

LMS NEWS

Mathematician Honoured in New Year's Awards

Professor Nick Woodhouse was appointed Commander of the Order of the British Empire (CBE) in the 2020 New Year Honours awards for services to mathematics.

Professor Woodhouse has been a member of the LMS since 1977 and was a Member of LMS Council from 1999–2009 (Meetings and Membership Secretary 1998–2001 and Treasurer 2002–09). He served on the Research Meetings Committee/Early Career Research Committee as the Clay Mathematics Institute (CMI) Representative from 2013–18 while the LMS–CMI Research Schools ran from 2014–18. He stepped down as President of CMI in 2018.

Professor Woodhouse is Emeritus Professor of Mathematics, University of Oxford and Emeritus Fellow of Wadham College. He studied for his undergraduate degree at Oxford and gained his PhD from King's College London in 1973. After a postdoctoral position at Princeton he returned to Oxford in 1976. His research interests are at the interface between mathematics and physics, including relativity, and the connections between geometry and physical theory.

Annual Elections to LMS Council

The LMS Nominating Committee is responsible for proposing slates of candidates for vacancies on Council and vacancies on its own membership. The Nominating Committee welcomes suggestions from the membership.

Anyone who wishes to suggest someone for a position as an Officer of the Society or as a Member-at-Large of Council (now or in the future) is invited to send their suggestions to Professor Kenneth Falconer, the current Chair of Nominating Committee (nominations@lms.ac.uk). Please provide the name and institution (if applicable) of the suggested nominee, her/his mathematical specialism(s), and a brief statement to explain what s/he could bring to Council/Nominating Committee.

It is to the benefit of the Society that Council is balanced and represents the full breadth of the mathematics community; to this end, Nominating Committee aims for a balance in gender, subject area and geographical location in its list of prospective nominees.

Nominations should be received by 17 April 2020 in order to be considered by the Nominating Committee.

In addition to the above, members may make direct nominations for election to Council or Nominating Committee. Direct nominations must be sent to the Executive Secretary's office (nominations@lms.ac.uk) before noon on 1 September 2020. For details on making a direct nomination, see lms.ac.uk/about/council/lms-elections.

The slate as proposed by Nominating Committee, together with any direct nominations received up to that time, will be posted on the LMS website in early August.

Visit to Hong Kong and China

Visit to Hong Kong University of Science and Technology

In November 2019, a delegation from the LMS spent a week visiting Hong Kong and China. There were two major aims: to build on existing links between UK and Chinese mathematics and to increase the visibility of the Society's Publications in the region. The President Designate, Professor Jon Keating, and the Publications Secretary, Professor John Hunton, were accompanied by two LMS Members, Professor Jens Marklof (University of Bristol) and Professor Gui-Qiang Chen (University of Oxford).

Despite the ongoing political tensions within Hong Kong at the time of the visit, the planned meeting with the Hong Kong Mathematical Society went ahead over a very pleasant dinner hosted by their President Professor Tong Yang. This provided an opportunity to explore ideas for future collaboration between the two societies.

The delegation also paid a visit to the campus of the Hong Kong University of Science and Technology and met the Dean of Science, Professor Yang Wang, and the Head of the Department of Mathematics, Professor Xiao-Ping Wang, who were also very open to the building of further links with UK mathematics. The LMS delegation then travelled by train to the city of Guangzhou in China to join close to 1,000 mathematicians for the Chinese Mathematical Society's annual meeting and four-yearly Congress at the invitation of their President, Professor Ya-xiang Yuan.

The Chinese Mathematical Society held its elections during their annual meeting, enabling connections to be made between the newly elected President, Professor Gang Tian, and Professor Keating who was also about to begin his term as LMS President. Professor Keating was honoured to be invited to present one of the Congress' three plenary talks, titled *Extreme Values of the Characteristic Polynomials of Random Matrices and the Riemann Zeta-Function*, and to introduce the LMS and its activities. Each of the other delegates gave mathematical lectures in the invited sessions and Professor Hunton also gave an overview of the Society's publishing activities in a session on scholarly publishing which also included representatives from Cambridge University Press and Science China Press.

Meetings with the Chinese Mathematics Society and the Academy of Mathematics and Systems Science (part of the Chinese Academy of Sciences) revealed a mutual desire to establish links on several activities including promotion of publications.

It is hoped that there is scope to take forward many of initiatives and ideas generated during the trip and to strengthen the bonds between UK mathematicians and their counterparts in China.

Forthcoming LMS Events

The following events will take place in the next three months:

Invited Lecture Series 2020: 30 March–3 April, Brunel University (tinyurl.com/yxxxnndz)

Society Meeting at the Joint BMC–BAMC: 8 April, Glasgow (tinyurl.com/yarpowdo)

LMS–IMA David Crighton Lecture, Professor Ken Brown: 23 April, The Royal Society, London (tinyurl.com/t2fzjm5)

A full listing of upcoming LMS events can be found on page 58.

OTHER NEWS

The Unity of Mathematics: Conference in Honour of Sir Michael Atiyah

Sir Michael Atiyah, who died on 11 January 2019, was the dominant figure in UK mathematics for decades and the Society, along with the Isaac Newton Institute, is organising a major meeting in his memory. This will take place from 22–24 September 2020 at the Isaac Newton Institute. The many distinguished posts Sir Michael held in his lifetime included Founding Director of the Isaac Newton Institute and President of the LMS from 1974 to 1976. His Presidential Address, *The Unity of Mathematics*, was delivered on 19 November 1976. The Society, which took the lead in proposing a conference in his honour, suggested this as the title for the conference. As he remarked 43 years ago, "The aspect of mathematics which fascinates me most is the rich interaction between its different branches, the unexpected links, the surprises..."; this fascination continued throughout his life.

The aim of the conference is to show an appreciation of Sir Michael's legacy via a selection of talks which we believe he would have found interesting, and which reflect the interactions that he so much appreciated. The speakers cover a broad range of mathematical generations and topics, ranging from number theory to theoretical physics. They also include representatives from Oxford, Cambridge, Bonn, Princeton and Edinburgh — places which played a fundamental role at various stages of his long life in mathematics.

The speakers will be: Robbert Dijkgraaf (IAS Princeton), Nigel Higson (Penn State), Vaughan Jones (Vanderbilt University), Minhyong Kim (Oxford), Holly Krieger (Cambridge), Rahul Pandharipande (ETH), Oscar Randal-Williams (Cambridge), Peter Sarnak (Princeton), Nick Sheridan (Edinburgh), Catharina Stroppel (Bonn), Maryna Viazovska (EPFL) and Edward Witten (IAS).

Robbert Dijkgraaf will be the Clay Lecturer. The meeting is jointly funded and organised in partnership between the LMS, the INI, the Heilbronn Institute, the Clay Institute and the Oxford Mathematics Department.

For more details see the conference website newton.ac.uk/event/ooew02.

Nigel Hitchin (Scientific Committee)
José Figueroa-O'Farrill (Organising Committee)

International Day of Mathematics: March 14

The International Mathematical Union (IMU) has announced that the 40th General Conference of UNESCO has proclaimed that March 14 (pi day 3.14) will from now on be the International Day of Mathematics (IDM).

As March 14, 2020 is a Saturday, the international official launch will take place on Friday March 13, 2020. There will be two parallel international events: in Paris at the UNESCO Headquarters, and at the Einstein Forum in Nairobi, Kenya.

The theme for IDM 2020 is *Mathematics is Everywhere*. More details can be found at idm314.org, where a map shows events country by country, including at least two in the UK, at the Big Bang Science Fair at the NEC in Birmingham and in the Houses of Parliament. Organisations at all levels are invited to announce their celebrations and there is a page containing material to help organisers.

The IMU is deeply grateful to Christiane Rousseau for her initiative in proposing the IDM, and for leading the process all the way to the successful proclamation by UNESCO.

Ramanujan Prize Winner 2019

Adam Harper (Warwick University), a member of the LMS and the LMS representative for Warwick, was awarded the Ramanujan Prize 2019 on 21 December, the ceremony taking place at SASTRA University in Kumbakonam, India (Ramanujan's home town).

De Morgan Foundation

As many members will know, LMS founder Augustus De Morgan's son William was a prominent member of the Arts and Crafts Movement, working with William Morris and designing furniture and tableware as well as tiles combining romantic design with mathematical appeal. Information about the De Morgan Foundation, which promotes his work, can be found at demorgan.org.uk.

Wolf Prize Winner

Professor Sir Simon Donaldson (Imperial College London) has been awarded the Wolf Prize for Mathematics, jointly with Professor Yakos Eliashberg (Stanford University). Sir Simon receives the award for his contributions to differential geometry and topology.

Sir Simon has been a member of the LMS since 1983. He was awarded the LMS Whitehead Prize in 1984 and the Pólya Prize in 1999. Sir Simon was a member of LMS Council from 2010–13 and he continues to contribute to the work of the LMS as an Editor of the *Journal of Topology*.

MATHEMATICS POLICY DIGEST

Major funding boost for the Mathematical Sciences

The government has announced additional funding for the Mathematical Sciences of up to £300 million, which will more than double the current funding for the Mathematical Sciences delivered by the Engineering and Physical Sciences Research Council (EPSRC), part of UK Research and Innovation (UKRI).

It includes:

- £19 million pa additional funding for PhD studentships, moving to 4-year studentships as standard and offering 5-year funding for research associates to compete with the US and Europe.
- £34 million pa additional funding for career pathways and new research projects, including multi-institutional projects and programmes.
- £7 million pa additional funding for new PhDs/research fellows at the Heilbronn Institute (Bristol), and funding to increase participants/workshops by a third at each of the Isaac Newton Institute (Cambridge) and International Centre for Mathematical Sciences (Edinburgh).

More information is available at tinyurl.com/s8aflcg.

Government announces changes to Tier 1 Visas for researchers

A new, fast-track visa scheme to attract the world's top scientists, researchers and mathematicians opened on 20 February 2020.

- Top scientists and researchers to be given fast tracked entry to the UK with no cap on who can benefit.
- UK Research and Innovation to lead new visa scheme for researchers and specialists.
- Existing visa routes led by National Academies also expanded to cover more UK Research and Innovation fellowships.

More information is available at tinyurl.com/vm2zwkh.

Report on Research Culture

The Wellcome Trust has published a report titled *What Researchers Think About the Culture They Work In*. The report is based on a survey of more than 4,000 researchers in the UK and around the world to ascertain their 'perspectives on and experiences of research culture'. The key findings from the survey are:

- Researchers are passionate about and proud of their work but have concerns about job security.
- Poor research culture is leading to unhealthy competition, bullying and harassment, and mental health issues.
- There is a disconnect between researchers' perception of their management skills and their abilities in practice.
- The system favours quantity over quality and creativity is often stifled.

More information is available at tinyurl.com/wkv8qm3.

Open Access prices and services report

An independent report has been published, which makes a series of recommendations to support greater transparency in the communication of open access publishing prices and services. The report was published by consultancy Information Power and is the outcome of a project funded by UK Research and Innovation (UKRI) and Wellcome on behalf of cOAlition S, an international consortium formed to make full and immediate Open Access to research publications a reality.

The project funders, libraries, publishers and universities worked together to inform the development of a framework intended to provide information about Open Access process in a 'transparent, practical and insightful way'. More information is available at tinyurl.com/sj3orho.

Digest prepared by Dr John Johnston
Society Communications Officer

Note: items included in the Mathematics Policy Digest are not necessarily endorsed by the Editorial Board or the LMS.

From the EMS President

With the EMS turning 30 and the 'golden twenties' just starting, we are looking forward to a decade that will undoubtedly see tremendous changes and trials. I call upon the whole mathematical community to join us in tackling these challenges head-on.

The European Congress of Mathematics (8ECM, 5–11 July 2020, Portorož, Slovenia) is approaching, with its accompanying EMS council meeting. These, and many other upcoming events, will provide opportunities to meet and discuss how to move the European mathematical community into the future.

With artificial intelligence and machine learning now entering everyday life at high velocity, it is essential that Mathematics contributes to the wider understanding of these inherently mathematical techniques, including discouraging their use when a high risk of failure (or disaster) can be anticipated. The EMS and its members must take some responsibility in this regard.

Unfortunately, research funding for Mathematics has declined in many parts of Europe, including at the European Research Council. This is partly our own fault, by not applying for enough grants, and by not stressing to decision makers at every level that research funding for Mathematics is of central importance to the well-being of society as a whole, and to the flourishing of both disciplinary and interdisciplinary progress. I call upon the whole community to become more active here.

Another major focus is the sustainable and open availability of the results of mathematical research. The new EMS Publishing House together with publishing houses of member and sister societies is committed to address this issue by investigating new open access publishing models that follow the basic principles:

- The quality of publications is paramount and beyond compromise.
- Publications shall be accessible and available in perpetuity.
- Pricing models shall be transparent and fair.
- The publishing house serves the mathematics community, and commits surplus funds to community initiatives.
- The publishing house commits to collaborative relationships with other stakeholders within the mathematics community.

A first milestone on this road is the recently announced open access availability of Zentralblatt Math from 2021.

It is heartening to see that the number of EMS members, both individual and corporate, continues to grow. Thus, the EMS's financial position is improving, allowing us to fund more events such as summer schools and conferences. We are striving to ensure that the decision-making process in the distribution of funds is as transparent as possible, and to guarantee equal opportunities, regardless of mathematical field, gender, geography, or age. It is an important goal of the coming decade to continue this trend and to increase, in particular, the numbers of young members and female members, giving them greater representation within the society. At the forthcoming council in Bled we will discuss several moves in this direction, including the formation of a young EMS academy.

As usual, this year we will renew the membership of several committees and elect new members of the executive committee, as well as a new vice president. I call upon EMS members to nominate good candidates to represent all areas of mathematics and the whole continent.

Let me thank all departing and continuing committee members for their hard work in the past year. Let me also thank those who work for the mathematical community more widely — on the boards of mathematical journals and research centres, on meeting and prize committees, and in many other ways. I wish all of you good health and a lot of interesting mathematics in 2020.

Volker Mehrmann
EMS President

zbMATH to become Open Access

Zentralblatt MATH (zbMATH) is the world's most comprehensive and longest-running abstracting and reviewing service in pure and applied mathematics. It is edited by the EMS, the Heidelberg Academy of

Sciences and Humanities and FIZ Karlsruhe–Leibniz Institute for Information Infrastructure. The Federal Republic of Germany has now agreed to support FIZ Karlsruhe–Leibniz Institute in transforming zbMATH into an open access platform.

MSC2020

The revision of the Mathematical Sciences Classification System (MSC2020) has now been completed, by the editors of *Zentralblatt MATH* and American Mathematical Society's *Mathematical Reviews*. The revised classification is available at tinyurl.com/wf6xkcp.

ERC Farewell from J-P. Bourguignon

In 1998 Jean-Pierre Bourguignon stood down as President of the European Mathematical Society. Now, 21 years later, he is stepping down as President of the European Research Council. The EMS offers him profound thanks for these years of service to European Science.

EMS Editorial

The world is in a period of fast transition which also affects societies like the EMS. This concerns not only the impact on our research of digitalization and artificial intelligence, but also on the ways that scientific societies interact and communicate with their members. To react to this transition, during its recent meeting in Yerevan, the EMS Executive Committee decided to change the format of the *EMS Newsletter* from summer 2020 to become the *EMS Magazine*, and to combine the different news platforms of EMS to become purely online, fast, and real time. The articles of the EMS Magazine will appear online first, and will then be joined together in a quarterly issue that will be available in a print-on-demand-and-request format for EMS members (and open access for everybody).

The new EMS publishing house is now publishing under the name EMS Press (ems.press) with a young and energetic team that wants to bring scientific publishing into a new era by serving the mathematical community and giving it immediate and sustainable access to scientific publications of the highest standards. The EMS also wants to involve the young generation more strongly in shaping the future of mathematics in Europe, and will propose to the EMS Council that we start a young EMS academy.

Note: The December 2019 issue of the EMS Newsletter is available at tinyurl.com/yx5js5e6. Feature articles include *A Problem for the 21st/22nd Century* (Non-uniform hyperbolicity and work of Jean-Christophe Yoccoz) by Sylvain Crovisier and Samuel Senti, and *Grothendieck: The Myth of a Break* by Claude Lobry.

EMS News prepared by David Chillingworth
LMS/EMS Correspondent

Note: items included in the European Mathematical Society News are not necessarily endorsed by the Editorial Board or the LMS.

OPPORTUNITIES

LMS Emmy Noether Fellowships 2020

The London Mathematical Society is delighted to announce that, thanks to a generous donation from the Liber Stiftung (Liber Foundation; tinyurl.com/th4xxcg), several Emmy Noether Fellowships with a value between £2,000 and £10,000 will be awarded in 2020, up to a total of £20,000. The amount awarded for each fellowship will reflect the individual requirements of the applicant. The fellowships are designed to enhance the mathematical sciences research, broadly construed, of holders either re-establishing their research programme after returning from a major break associated with caring responsibilities or those requiring support to maintain their research programme while dealing with significant ongoing caring responsibilities.

The grants will be tailored to individual needs allowing applicants the flexibility to submit a fellowship application with the proposed spend based on their own particular case. These expenses may include contributions to extra caring costs, travel costs, research assistance or some other support without which the applicant would have difficulty in re-establishing or continuing their research, on the basis of full justification in the application.

In addition to financial support, each Emmy Noether Fellow will have access to mentoring and networking opportunities supported by the host department and the LMS.

The decision to award is delegated to a subcommittee of the London Mathematical Society Women in Mathematics Committee, and will be made on the basis of intellectual merit, clearly articulated research programme, statement of support from the applicant's institution/host institution, and justification of how the Fellowship will enhance the applicant's career.

In order to be eligible, applicants must have a PhD in Mathematics or closely related subject and be affiliated with a UK University in an academic capacity.

Those interested in an Emmy Noether Fellowship are encouraged to apply online at tinyurl.com/rrtfgj9 by 23.00 GMT on 1 May 2020.

Cecil King Travel Scholarships 2020: Call for Applications

The LMS administers two £6,000 travel awards funded by The Cecil King Memorial Foundation for early career mathematicians, to support a period of study or research abroad, typically for a period of three months. One Scholarship will be awarded to a mathematician in any area of mathematics and one to a mathematician whose research is applied in a discipline other than mathematics.

Applicants should be mathematicians in the United Kingdom or the Republic of Ireland who are under the age of 30 at the closing date for applications, and who are registered for a doctoral degree or have completed one within 12 months of the closing date for applications. The LMS encourages applications from women, disabled, Black, Asian and Minority Ethnic candidates, as these groups are under-represented in the United Kingdom or the Republic of Ireland mathematics communities.

To apply, complete the application form (tinyurl.com/yarns982) and include a written proposal giving the host institution, describing the intended programme of study or research, and the benefits to be gained from the visit. The deadline for applications is 31 March 2020.

Shortlisted applicants will be invited to an interview during which they will be expected to make a short presentation on their proposal. Interviews will take place at the University of Birmingham on 29 May 2020.

Queries may be addressed to Elizabeth Fisher (ecr.grants@lms.ac.uk).

VISITS

Visit of Jan Kolar

Dr Jan Kolar (Czech Academy of Sciences, Prague) will visit University of Birmingham from 30 March to 6 April 2020. Dr Kolar's recent research has concerned extending vector-valued functions with preservation of continuity and differentiability properties, and the chain rule for partial functions on Banach spaces. For further details contact Olga Maleva (O.Maleva@bham.ac.uk). The visit is supported by an LMS Scheme 4 Research in Pairs grant.

Visit of Miquel Oliu-Barton

Dr Miquel Oliu-Barton (Université Paris Dauphine) will visit the University of Glasgow from 4 to 10 April 2020. His main research interest is Game Theory, namely he works on stochastic games, repeated games and games with incomplete information. For further information email Yehuda John Levy (John.Levy@glasgow.ac.uk). The visit is supported by an LMS Scheme 4 Research in Pairs grant.

LMS Council Diary — A Personal View

A little earlier than usual, to be able to attend the Annual General Meeting that afternoon, Council met at Goodenough College, London, on 29 November 2019. Since it had been just over a month since the last Council meeting, the President gave a very short update on her activities. She reported that the DeMorgan@21 anniversary event had been a great success with 70 participants.

This was followed by an update by the Publications Secretary, who reported on a recent visit to China and Hong Kong with the President Designate to investigate the potential for enhancing links between UK and Chinese mathematicians, including the possibilities for developing publication opportunities. There seemed to be a lot of scope and interest expressed from the Chinese mathematicians they met; the visit had been particularly appreciated by mathematicians in Hong Kong.

The Publications Secretary also reported that two excerpts from one of Turing's articles in the *Proceedings of the LMS* will be featured on the new £50 notes, and the article would continue to be made freely available for at least the next two years. Council further approved the recommendation from the Publications Committee that the Society should declare support of the overall principles of the European Mathematical Society Code of Practice which deals with publishing ethics.

Council then went on to discuss committee membership, as several committees had vacancies. We agreed the applications for membership to be proposed to the Annual General Meeting later that day. The General Secretary reported on the preparations for the European Mathematical Society Council 2020, and a delegation was agreed.

Alexandre Borovik then suggested that we engage with the newly formed Cambridge University Ethics in Mathematics Society, and Council agreed that he and the General Secretary liaise with its founders to explore ways the Society could tackle Ethics in Mathematics matters.

And, finally the President thanked all outgoing members for their work for Council, and the President Designate on behalf of Council gave thanks to the President for her outstanding efforts and achievements during her two years as President of the Society.

We then decamped for lunch and the Annual General Meeting.

Brita Nucinkis
Member-at-Large

REPORTS OF THE LMS

Report: LMS Graduate Student Meeting and Annual General Meeting

John Parker

The LMS Graduate Student Meeting and the Annual General Meeting of the Society and Presidential Address were held on Friday 29 November 2019, at Goodenough College in London. The morning graduate session was started off in the key of low-dimensional hyperbolic geometry by Professor Viveka Erlandsson (Bristol). This introduced some of the elementary topics of the area, leading into the topic of Fuchsian and Kleinian groups, and much other material which would prove relevant throughout the later talks. After a brief coffee break, the session was resumed; here the notion of counting curves on hyperbolic surfaces was centre stage, with apotheosis the astounding curve counting theorem of Mirzakhani. An audience with members from diverse mathematical backgrounds was present, with no small amount of graduate students, many of whom are themselves active in low-dimensional topology. They were all no doubt satisfied with the assortment of topics that would be brought up during the day and proved very engaged in the talk: many questions popped up afterwards, including a brief but enlightening discussion as to the etymology of the word *loxodromic*, and why this navigational term is used in the context of Möbius transformations.

A number of graduate student presentations followed this. A wide range of interesting topics was presented, with a slight bias towards the combinatorial and algebraic side. By popular vote, two graduate speakers were each awarded a prize: Leyu Han (Birmingham) for an in-depth talk on Lie superalgebras, and André Macedo (Reading) for an overview of current lines of research on the Hasse principle. After this, rounding up the Graduate Student Meeting, was a talk by Professor David Singerman (Southampton) about Bob Riley, whose eponymous slice would be central in the later Presidential Address. This talk brought up both moving personal reminiscences as well as presenting some of the context in which Riley's work developed, including the work of William Thurston. A fascinating computer-generated picture of the Riley slice, printed in the late 1970s, was shown to the attendees, and this would later be put on display for all to see during the day.

With the Graduate Student Meeting concluded, the Annual General Meeting of the Society could then begin. Among many things, resolutions to amend the Royal Charter, Statutes, and By-Laws of the Society were voted on and passed, finalising a process long in the making. The certificates for the 2019 LMS prize winners — with no shortage of heavyweight names on the list of recipients — were also presented. The first of the talks during the Meeting was given by Professor Marc Lackenby (Oxford) who tied together a number of fields in a thoroughly accessible and entertaining talk, with a common theme the development of methods to recognise whether a given knot represents the unknot, i.e. an identity problem for knots. This problem has been known to be decidable since 1961, but it has frustrated many attempts to pin down an exact complexity. It is known to be in NP since 1999, and in co-NP since 2011 if one assumes the generalised Riemann Hypothesis. One of the speaker's contributions to the area was a 2016 unconditional proof of the problem being in co-NP.

After another well-deserved coffee break, as well as the handover of the Presidential Office to Professor Jon Keating, FRS (Oxford), the time had come for the 2019 Presidential Address by Professor Caroline Series, FRS (Warwick). The talk was on her work on the Riley slice, and outlined a remarkable journey with connections to low-dimensional topology, hyperbolic geometry, and knot theory. It was complemented by stunning computer-generated imagery, including Riley's own aforementioned printout. The various talks that had been given over the course of the day proved invaluable in aiding to make some of the subtleties of the Riley slice all the more striking. Finally, after a wine reception, as well as an address by the new President, the LMS Annual Dinner was held in the stunning main hall of Goodenough College, rounding off the day in a most spectacular way.

Carl-Fredrik Nyberg Brodda
University of East Anglia

LMS South West and South Wales Regional Meeting and Workshop

Tara Brendle

The LMS South West and South Wales regional meeting and workshop took place at the University of Bristol on 15–17 January 2020. The event started with the LMS meeting on the afternoon of 15 January and was opened by Professor Jon Keating FRS, the President of the LMS. This was followed by three colloquium style talks, given by Professor Tara Brendle (Glasgow), Professor Yves Benoist (Paris-Sud) and Professor Mark Pollicott (Warwick). Brendle gave a talk about braids, giving a very nice overview of where the theory of braids appears both in pure mathematics and in applications to the real world. Benoist gave a clear account of classifications of positive harmonic functions for the Heisenberg group and Pollicott's talk on circle packings nicely illustrated the interactions of a several mathematical fields from geometry to number theory and probability. All three gave very beautiful and accessible talks and the event was attended by an audience of over 90 people.

After the meeting there was a wine reception during which we also had a poster session allowing students and early career researchers to display their work. This resulted in many interesting conversations and interactions between young and senior researchers. The reception was followed by a dinner at Kuch Restaurant, attended by 50 people.

The LMS meeting was followed by the two-day workshop *Interactions between Geometry, Dynamics and Group Theory*. The fields of geometry, dynamics, and group theory are closely related and their interplay is a very active research area. A few examples of these connections, all which were covered in the workshop talks: in Teichmüller theory the study of the mapping class group of a surface provides many links between geometry and group theory as well as geometry and dynamics, while graphs and hyperbolic groups naturally combine geometry and group theory, and the study of Lie groups strongly connect the fields of geometry, dynamics, and group theory.

The workshop was attended by over 70 participants coming from all over the UK as well as from abroad. Many of the participants were PhD students and postdoctoral researchers. The talks were excellent — the speakers took care to explain also the background to their research — and the atmosphere was very friendly and interactive. This was particularly appreciated by the many early career researchers in attendance, several of whom afterwards reported they had learned a lot from the talks. The speakers were Benjamin Barrett (Bristol), Rhiannon Dougall (Bristol), Cornelia Drutu (Oxford), Vaibhav Gadre (Glasgow), Selim Ghazouani (Warwick), David Hume (Oxford), Ashot Minasyan (Southampton), Saul Schleimer (Warwick), and Katie Vokes (IHES). This was a diverse group of speakers: it represented all three main mathematical areas of the workshop's focus, included both junior and senior, UK and internationally based mathematicians, and a third of the speakers were women.

Viveka Erlandsson, Thomas Jordan, John Mackay
University of Bristol

Records of Proceedings at LMS Meetings
Annual General Meeting and Society Meeting of The London Mathematical Society, Friday 29 November 2019

The meeting was held in the Great Hall at Goodenough College, London. About 100 members and visitors were present for all or part of the meeting. The meeting began at 2:30pm, with the President, Professor Caroline Series, FRS, in the Chair. Members who had not yet voted were invited to hand their ballot papers to the Scrutineers, Professors Chris Lance and Charles Goldie.

The Minutes of the General Meeting held on 28 June 2019 had been circulated 21 days before the Annual General Meeting and members were invited to ratify the Minutes by a show of hands. The Minutes were ratified.

The Vice-President, Professor Cathy Hobbs, presented a report on the Society's activities and the President invited questions. The Treasurer, Professor Robert Curtis, presented his report on the Society's finances during the 2018-19 financial year and the President invited questions. Copies of the Trustees' Report for 2018-19 were made available and the President invited members to adopt the Trustees' Report for 2018-19 by a show of hands. The Trustees' Report for 2018-19 was adopted.

The President proposed Messrs Kingston Smith be re-appointed as auditors for 2019-20 and invited members to approve the re-appointment by a show of hands. Messrs Kingston Smith were re-appointed as auditors for 2019-20.

The President advised members that, following the membership consultations and discussions about the amendments to the Society's Charter, Statutes and By-Laws, the meeting would proceed directly to the vote. The President invited Members of the London Mathematical Society or any member or non-member acting as a proxy for a Member to vote on the three Resolutions, if they had not already done so online before the Annual General Meeting. Members who had not voted and those amongst the audience who had been appointed as proxies, had received three voting cards at registration and were asked by the President to vote on each resolution by marking the card for the relevant resolution. All cards were collected together following voting on Resolution 3.

46 people were elected to Ordinary Membership: Dr Ahmad Abdi, Dr Gavin Abernethy, Miss Chandi Sravanthi Adharapurapu, Dr Ankush Agarwal, Ms Margaret Apio, Dr Clark Barwick, Dr Mike Blake, Dr Sabine Boegli, Professor Jens Bolte, Dr Helen Christodoulidi, Dr Simon Cotter, Dr Jean-Claude Cuenin, Mr Parthasakha Das, Dr Lorenzo Foscolo, Dr Jeffrey Galkowski, Ms Ashton Green, Dr Raffael Hagger, Dr Yang Han, Dr Adam Harper, Dr Johannes Hofscheier, Dr Xiong Jin, Mr Sachin Kushare, Dr Paul Ledger, Dr Gerry Leversha, Dr Jason Levesley, Dr Yehuda Levy, Dr Fatemeh Mohammadi, Dr Ricardo Monteiro, Mr Ravindra Nikam, Dr Diogo Oliveira E Silva, Professor Dr Mauro Paternostro, Dr Ulrich Pennig, Mr Leonard Pistol, Dr Dhananjaya Reddy, Mr Liu Ren, Dr Gabriela Rino Nesin, Dr Steven Roper, Dr Dmitry Savostyanov, Dr Benjamin Sharp, Mr Arnaud Trebaol, Mr Mahmut Servet Tumkaya, Mr Paul Fatiye Useni, Mr Oliver Vella, Mr Yuzhao Wang, Dr Garth Wilson and Ms Reem Yassawi.

62 people were elected to Associate Membership: Mr Sam Adam-Day, Miss Izar Alonso Lorenzo, Mr Valentin Boboc, Mr David Brown, Mr Marcos Caso Huerta, Mr Alexander Cliffe, Miss Esther Bou Dagher, Miss Purba Das, Mr Karel Devriendt, Dr Marie Farrell, Miss Eliana Fausti, Mr Jonathan Fruchter, Mr Gabriel Garcia Arenas, Mr Mohamed Hammeda, Mr Carl Hines, Ms Adele Jackson, Mr Luka Kovacevic, Miss Reka Agnes Kovacs, Ms Valeriya Kovaleva, Miss Monika Kudlinska, Mr Isaac Kwofie, Mr Saad Labyad, Mr Melvin Yu Xuan Lee, Mr Mario Lezcano Casado, Mr Jared Duker Lichtman, Mr George Liddle, Mr Sam Longden, Dr Somayeh Mamizadeh-Chatghayeh, Mr Paolo Marimon, Mr Lewis Marsh, Mr Avi Mayorcas, Mr Geoffrey Mboya, Mr Ewan Mctaggart, Mr Dimitri Navarro, Dr Kayvan Nejabati Zenouz, Mr Carl-Fredrik Nyberg Brodda, Mr Olusegun Adeyemi Olaiju, Mr Olaniyi Olawale Akeem, Mrs Augusta Oloke, Miss Fulya Ozata, Miss Catalina Pesce, Mr Robin Ramlal, Mr Benjamin Rees, Mrs Felix Ribuot-Hirsch, Mr Matthew Riding, Mr George Robinson, Mr Gustavo Rodrigues Ferreira, Mr Bernard Rybolowicz, Mr Stephen Safee Baah, Ms Valentina Semenova, Mr Muhammad Sinan, Miss Jane Tan, Mr Matija Tapuskovic, Mr Jared Thomas. Mr James Timmins, Mr Ho Lung Tsui, Mr James Van Yperen, Mr Zheneng Xie, Mr Chin Ching Yeung, Mr Hanwen Zhao and Miss Christina Zou.

Three people were elected to Associate (undergraduate) membership: Mr Henry Akers, Mr George Cooper and Mr Aaron Vracar.

10 people were elected to Reciprocity Membership: Dr Saravanan, Dr Abimbola Abolarinwa, Professor Anvarjon Ahmedov, Professor Yomi Aiyesimi, Professor Armen Bagdasaryan, Dr Jennifer Beineke, Mr Sandeep Koranne, Mr Pratham Peshwani, Dr Jitender Singh and Mr Stephen Tierney III.

123 people were elected to Associate Membership for Teacher Training Scholars: Mrs Khadija Abughannam, Mr Shumail Ahmad, Mr Mohammed Ahmed, Miss Katharine Airstone-Thompson, Miss Bamidele Akintayo, Mr Anthony Albuquerque, Miss Devaki Amin, Miss Kathleen Armsby, Ms Susannah Baird, Mr Jonathan Baker, Mr Jack Baldwin, Miss Laura Barber, Dr Adam Barker, Miss Jodie Beard, Mrs Wanyun Vanessa Beaumont, Mr Edward Blissett, Mr James Bloodworth, Miss Hayley Catlow, Miss Lauren Cattanach, Mr Lewis Church, Dr Daniel Clarke, Mr Darren Clarkson, Miss Katherine Coxon, Miss Amy Crowder, Mr Nicholas Dean, Miss Jess Devenney Lees, Miss Chloe Drury, Mr Chris Ely, Miss Anna Flight, Mr Dale Foster, Dr Cesare Gallozzi, Mrs Samantha Garling, Mr Maram Ghani-Zadeh, Mrs Shalini Gnanasubramanian, Mrs Alexandra Gomez, Dr Yilun Gong, Miss Megan Greenwood, Mr Muhammet Guler, Miss Brittany Paige Halpin, Mr Kieron Hamilton, Mr James Harvey, Miss Rhiannon Hayward, Miss Gabriella Herdman, Dr Joseph Hodgskiss, Mr Thomas Holland, Miss Ellen Howell, Miss Sonya Humpage, Miss Joely Hunt, Miss Lauren Hunt, Ms Esther Katharina Indirawati, Mr Mark Jeffreys, Miss Danielle Jenkins, Miss Jemma John, Miss Alison Johnston, Mrs Coral Jolley, Miss Francesca Kemp, Mr Imad Khan, Mr Harry King, Mrs Andres Kolb, Miss Ambigha Krishnakumar, Mr Jeremy Lee, Miss Marina Ley, Mr Connor Lynch, Miss Amy Major, Mr Suhayl Master, Mr Padraig Pearse McCallion, Miss Eleanor McHarg, Mr Reece McIlhatton, Mr William Metcalf, Miss Lily Miles, Mr Ewan Murray, Mr Jim O'callaghan, Miss Siobhan O'kane, Mr Alex O'neill, Miss Dora Olah, Dr Nicholas Owen, Ms Eleanor Owen, Mrs Ekin Ozgur, Mr Micah Pang, Mr Tony Parkes, Mr David Parry, Miss Sarah Pearse, Miss Talya Pedro, Dr Benjamin Pickles, Miss Charlotte Quinn, Miss Tasmeen Rashid, Miss Emily Reeder, Mr Daniel Ridgwell, Mr Isaac Riley, Mr James Roe, Miss Sara Salim, Mr Richard Savory, Miss Adrienne Schneider, Mr David Segura Renau, Miss Kim Sergison, Mrs Jasmine Shea, Mr Samuel Silcock, Mr Sanjay Singh, Miss Naomi Smee, Mr Simon Smith, Miss Laura Smith, Miss Sophie Spurgeon, Mr Jared Stanley, Mrs Daniela Stevenson, Mr Tom Stileman, Miss Lauren Stockton, Mr Daniel Storey, Miss Shria Suchak, Miss Ellen Thompson, Miss Hera Anna Tiborcz, Mr Matthew Treeby, Mr Michael Venables, Mr Jamie Veness, Miss Simona Vicol, Mr Shaun Whiteley, Miss Ellen Whitmore, Mrs Jennifer Widdowson, Mr Josh Williams, Mr Ian Williams, Mr Jonathan Winfield, Miss Zoe Winn, Mrs Dione Witt and Miss Chloe Young.

Four members signed the Members' Book and were admitted to the Society.

The President, on Council's behalf, presented certificates to the 2019 Society Prize-winners:

De Morgan Medal: Professor Sir Andrew Wiles (University of Oxford)

Senior Whitehead Prize: Professor Ben Green (University of Oxford)

Whitehead Prizes: Dr Alexander Buryak (University of Leeds), Professor David Conlon (University of Oxford), Professor William Parnell (University of Manchester). A Whitehead Prize had been awarded to Dr Heather Harrington (University of Oxford) in 2018; her certificate was presented at the AGM in 2019.

Berwick Prize: Dr Clark Barwick (University of Edinburgh)

Anne Bennett Prize: Dr Eva-Maria Graefe (Imperial College London)

The Naylor Prize & Lectureship was awarded to Professor Nicholas Higham (University of Manchester) and a Whitehead Prize was awarded to Dr Nick Sheridan (University of Edinburgh). However, they were unable to collect their certificates and so their certificates have been sent to them.

Professor Marc Lackenby (University of Oxford) gave a lecture on The Complexity of Knots.

After tea, the Chair announced the results of the vote on the Resolutions to amend the Standing Orders and it was declared that the motion to amend the Society's Charter, Statutes and By-Laws had been carried. The President advised the audience that the Society would contact the Privy Council again for their formal approval and that it was hoped to implement the new Standing Orders in the new year, and members would be kept informed.

Then, Professor Lance announced the results of the ballot. The following Officers and Members of the Council were elected.

President: Jon Keating, FRS

Vice-Presidents: Catherine Hobbs, Iain Gordon

Treasurer: Robert Curtis

General Secretary: Stephen Huggett

Publications Secretary: John Hunton

Programme Secretary: Chris Parker

Education Secretary: Kevin Houston

Members-at-Large of Council for two-year terms: Elaine Crooks, Andrew Dancer, Tony Gardiner, Frank Neumann, Brita Nucinkis

Member-at-Large of Council for a one-year term: Richard Pinch

Member-at-Large (Librarian): Mark McCartney

Five Members-at-Large, who were elected for two years in 2018, have a year left to serve: Alexandre Borovik, Tara Brendle, David E. Evans, Mariya Ptashnyk and Anne Taormina.

The following were elected to the Nominating Committee for three-year terms: Beatrice Pelloni and Mary Rees. The continuing members of the Nominating Committee are: Kenneth Falconer (Chair), I. David Abrahams, H. Dugald Macpherson, Martin Mathieu and Elizabeth Winstanley. In addition, Council would appoint a representative to the Committee.

Professor Caroline Series, FRS, handed over the Presidential badge of office to Professor Jon Keating, FRS. The new President thanked members for the honour and privilege of being elected as President and promised to fulfill the Charter, Statutes and By-laws of the Society.

Professor Caroline Series, FRS, University of Warwick, gave the Presidential Address 2019 with the title *All About the Riley Slice*.

Before closing the meeting, Professor Keating thanked the retiring members of Council and the Immediate Past President, Professor Caroline Series, FRS.

Professor Keating also thanked the speakers at the Graduate Student Meeting in the morning; Viveka Erlandsson (University of Bristol) and David Singerman (University of Southampton), congratulated the winners of the Graduate Student Talk Prize, Andre Macedo and Leyu Han.

After the meeting, a reception was held at Goodenough College in the Large Common Room, followed by the Annual Dinner, which was held in the Great Hall at Goodenough College and attended by 88 people. At the start of the Annual Dinner, the Immediate Past President, Caroline Series, FRS, gave a short speech and proposed a toast to the continued health of the Society and Mathematics before the President, Professor Jon Keating, FRS, gave a short speech during which he presented two Certificates for Whitehead Prizes to Dr Toby Cubitt (University College London) and Dr Anders Hansen (University of Cambridge).

Records of Proceedings at LMS–IMA Joint Meeting
Ordinary Meeting: 21 November 2019

The meeting was held at the University of Reading, as a joint meeting with the Institute of Mathematics and its Applications (IMA), to showcase some recent developments in pure and applied mathematics that contribute to a better understanding of the Planet Earth. Over 92 members and visitors were present for all or part of the meeting. The meeting began at 10.30am with Professor Alistair Fitt, CMath, FIMA, CSci, in the Chair. Professor Fitt welcomed guests and then introduced Professor Caroline Series, FRS, who gave the welcome from the London Mathematical Society. She then introduced Phil Newton, Research Dean for Environment at the University of Reading, to give the welcome on behalf of the University of Reading. The meeting was then handed over to Professor Valerio Lucarini, who introduced a lecture given by Professor Michael Ghil (UCLA and École Normale Supérieure).

After tea, Professor Lucarini introduced the second lecture given by Professor Peter Ashwin (University of Exeter) on *Rate-induced and Basin Boundary Effects for the Atlantic Meridional Overturning Circulation*. Professor Lucarini invited Beth Wingate (University of Exeter) to give a talk on *On the Way to the Limit: Oscillations in Fluids and their Role in the Creation of Low Frequency Dynamics*.

After lunch, Professor Lucarini introduced a lecture given by Sandro Vaienti (CPT-Luminy and University of Toulon). Professor Lucarini introduced the fifth lecture given by Professor Tobias Kuna (University of Reading) on *Extreme Value Theory of Generic Observables for Chaotic Dynamical Systems in any Dimension*. After breaking for the second tea, Professor Lucarini invited Professor Jacques Vanneste (University of Edinburgh) to give his lecture on *Gravity-wave Scattering by Turbulence in the Atmosphere and Ocean*.

Professor Lucarini then introduced the final lecture given by Kathrin Padberg-Gehle (University of Luneburg) on *Coherent Behaviour in Geophysical Flows*. Professor Alistair Fitt then thanked the organisers and speakers at the Meeting before he handed over to Professor Caroline Series, FRS, for final words from the LMS. Two members signed the Members' Book and were admitted to the Society. Professor Series then handed back to Professor Lucarini who gave details of the wine reception and dinner.

Records of Proceedings at LMS Meetings
Ordinary Meeting: 15 January 2020

The meeting was held at the University of Bristol, as part of the South West & South Wales Regional Meeting and Workshop on *Interactions Between Geometry, Dynamics and Group Theory*. Over 60 members and guests were present for all or part of the meeting.

The meeting began at 1.30pm with The President, Professor Jon Keating, FRS, in the Chair. There were no members elected to Membership at this Society Meeting. Four members signed the Members' Book and were admitted to the Society.

Dr Thomas Jordan, University of Bristol, introduced the first lecture given by Professor Tara Brendle (University of Glasgow) on *Braids: our Past Informing our Future*. After tea, Dr Thomas Jordan, University of Bristol, introduced the second lecture by Professor Yves Benoist (University of Paris-Saclay) on *Harmonic Functions on the Heisenberg Group*. Dr Thomas Jordan, University of Bristol, introduced the second lecture by Professor Mark Pollicott (University of Warwick) on *Apollonian Circles and their Properties*.

Professor Keating thanked the speakers for their excellent lectures and then expressed the thanks of the Society to the organisers, Dr Viveka Erlandsson, Dr John Mackay, and Dr Thomas Jordan, all of the University of Bristol, for a wonderful meeting and workshop.

Sir Michael Atiyah (1929–2019)

GRAEME SEGAL

This is a personal tribute by one of Sir Michael's Atiyah's former doctoral students that was given at the launch of the Atiyah UK-Lebanon Fellowships. It is an expanded version of his address at a memorial service in Trinity College, Cambridge.

What everyone who met Michael Atiyah remembered was the torrent of energy and enthusiasm he poured forth. It didn't change from his childhood to his ninetieth year. I became his student in Oxford in 1963, a year after he moved there from Cambridge. He was 34, but already had an air of confidence and authority which made him seem ageless rather than young. Beneath the surface of ebullient enthusiasm he presented to the mathematical world, there were many layers he revealed only rarely. Among them were his great pride in both his Arab and his Scottish descent, and his strong political views which, for example, brought him to be President of the Pugwash organisation of scientists campaigning for the reduction of nuclear and other armaments; but now I'm just going to speak of him as a mathematician and teacher.

(Left to right) George Lusztig, Dan Quillen, Graeme Segal and Michael Atiyah at the Institute for Advanced Study in Princeton in 1970.

In 1963 Michael had about six or seven students, as well as a little cloud of de facto students officially belonging to others. His loud voice was always audible in our little mathematics building; we students often hid from its approach, embarrassed to admit we still hadn't thought about his suggestions of the day before. But when he caught us he always fired us up, and made us feel sure we could succeed. He was never discouraging: however muddled and ill-conceived were the ideas we put to him, he never dismissed them, but always managed to twist them round into something promising we could almost believe we had thought of for ourselves. He saw supervising us as like giving artificial respiration to a drowning swimmer: you push them through the motions of research for a time, and then, with luck, autonomic research kicks in. He understood perfectly how lonely and depressing mathematical research can be for a student. He often said: 'If you need to know something, ask me, don't just start reading some long unintelligible book: it will only make you feel hopeless.'

Michael spent all day talking to people, and I always wondered when he did his work. Well, he loved gardening, and claimed he had his best ideas when gardening; but he could work anywhere, at a moment's notice, in the most distracting surroundings. And another of the many paradoxes about him was that, without stopping talking, he was a wonderful listener, interested in everything: after any mathematical gathering you could count on him to go about telling everyone what he had learned from some tongue-tied student no-one else had noticed was there.

Michael's mathematical style and taste were overwhelming. He was always for the big picture, always for simplicity and clarity. If something was true, he was sure it was for a reason whose essence could be explained in a few sentences, and without a blackboard. He had no patience with details, with arguments involving lots of different cases, with anything he thought 'botanical'. He was bored by foundational material. He was famous across the mathematical world for his spellbinding lectures, which linked all kinds of things together in ways whose importance he made instantly convincing. At the time, everyone in the audience thought they had understood everything. 'Giving a lecture,' he told us, 'the thing is to work out what you don't need to say.'

One of Michael's convictions was the unity of mathematics: he didn't like to think he worked in a 'field'. But he was sure he was a geometer. One can ask what he meant by this. There are many ways of being a geometer, and in many of the most natural senses Michael's methods were not very geometrical. I think a historical perspective is helpful: Michael was always interested in the history of mathematics, and thought it should have a greater role in undergraduate teaching. Among his greatest heroes was James Clerk Maxwell, who said in 1870 (in his Presidential Address to Section A of the British Association for the Advancement of Science), that nature reveals 'to the mathematician new forms of quantities which he could never have imagined for himself'. 'New forms of quantities ...'? Maxwell must have had in mind how magnetic fields are described by axial vectors, rotations by quaternions, rigid bodies by inertia tensors, and so on. Structures like these – many would call them algebraic structures – and later ones like the spinors and Clifford algebras Dirac used to describe the spinning electron, were what Michael loved best, and he saw them as geometry.

One shouldn't conclude too hastily from this that his mathematics was motivated by physics: his idea of geometry was more Platonic than that. But a few years before Maxwell's words, in his founding Presidential Address to the London Mathematical Society, De Morgan had presented a conception of mathematics which is in some sense opposite to Maxwell's, and I think also to Michael's. De Morgan, interestingly, conceded that the 'subject-matter' of mathematics is the properties of space and time, but he went on to claim that the real focus of mathematicians is – or should be – the 'laws of thought'. Michael's focus, like Maxwell's, was on the subject-matter and not on the laws of thought.

As a student of Sir William Hodge, Michael began as an algebraic geometer, studying the curves, surfaces, and higher-dimensional structures defined by algebraic equations – the things of which you see beautiful models in older mathematics departments. In the early 20th century Lefschetz had transformed algebraic geometry by recognising that many of its theorems are *topological* in nature: they actually depend only on features of objects in space which can't be changed by bending or stretching or pushing things about: the winding-number proof that an nth-degree polynomial has n roots is the tiny tip of a huge iceberg. Lefschetz created what we call 'algebraic topology'. Hodge was an early follower of Lefschetz, but brought a new perspective by using different tools: those by which Maxwell had described the electromagnetic field – linear partial differential equations.

Michael wrote a memoir of Hodge which gives a fascinating account of this evolution. His own work, in his long collaborations with Hirzebruch, Bott, and Singer, took the story a stage further. The famous 'index theorem' was one of the high points. (Alain Connes once said he'd been to a hundred talks whose title should have been 'Now I too have understood the index theorem'.) The general effect of this development was to shift the focus of geometry from algebraic to less rigid – but still smooth – structures. A crucial step was seeing that, for topological purposes, Dirac's equation for the electron provides a more flexible replacement for the 19th century edifice of complex analysis: the Dirac equation takes the place of the Cauchy-Riemann equations.

Alexander Grothendieck and Michael Atiyah (right) in Bonn in 1958.

Michael's notorious dislike of being called an algebraic topologist fits into this context. Lefschetz's work had been motivated by algebraic geometry, but it created techniques which quickly became focused on much more synthetic – 'artificial' – spaces than those of traditional geometry. It was this artificiality and introversion which Michael disliked about algebraic topology, not the fact that its methods were resolutely algebraic. He was steeped in the algebraic geometer's understanding of commutative algebra, and completely at home thinking of, say, the ring of integers as a geometric object whose points are the primes, however little it might be like most people's idea of space.

A lot of the history of mathematics is about looking at things in new ways. Changing the landscape is what Michael will be remembered for. The topological role of the Dirac equation was part of a larger seismic shift in which, for example, vector bundles evolved from a technical tool in algebraic geometry to a central idea across the mathematical stage. As much as anyone, Michael drove the shift. But all the same, he wasn't a system-builder like, say, Grothendieck. His methods were eclectic, and he liked to do something decisive and move on. He didn't get involved in elaboration, and didn't like to be part of a crowd.

Michael Atiyah (left) and Israel Gelfand in Oxford in 1973.

Michael's commitment to the unity of mathematics, also, took a form that was all his own. He was brilliant at spotting new stars in his geometric firmament, and brought people from all areas of mathematics – established authorities and unknown beginners – to speak in his seminar. Repeatedly he organised programmes of talks – always himself a central speaker – which reached a world-wide audience, and helped start whole new fields of research. He was wonderful at grasping the essence of a new subject and formulating its programme. At the height of his influence he was probably the world's chief arbiter of mathematical excellence. Nevertheless, in areas where he was a leading advocate and inspiration – gauge theory, Floer theory, integrable systems, the geometrising of quantum field theory – he often didn't get deeply involved himself, just stepping in when he had something specific to contribute, like his seminal work on the instanton moduli space. His students worked on diverse problems, and moved on in many directions. He had enormous fatherly pride in them, but no impulse to found a school or empire.

I often wondered how deeply he was interested in physics. Was he like another of his heroes, Gelfand, who felt he could recognise mathematical ideas destined to have a place in physics, and was attracted to ideas of that sort without thinking about physics himself? That would fit most of Michael's work which touched on physics, in particular his beautiful study of the geometry of configurations of particles – which he began quite late in his career, in great doubt whether he could get back into serious research after stepping down from his administrative roles. But I suspect he'd long had a far more ambitious goal, to find the ultimate replacement for quantum theory that Einstein sought so long in vain, and had been keeping his ideas to himself because, like Newton's theology, they were so dangerously heterodox.

I will end with another cleavage in Michael's nature, this one related to elitism. On one side, for him the history of mathematics was the story of giants who saw further than the rest. That side of him was also fascinated by how power is wielded in the world, and was an intrigued observer of other kinds of giants. But he had a very different side: he was always aware that mathematics is a communal activity, and he devoted a lot of energy to the health of the mathematical community in the world, striving to ensure that it transcended national and political boundaries. He always stressed how indispensable are the contributions of people who are *not* giants, and he always stood up for them. He loved to tell the story of Peter Fraser, a lecturer at Bristol University in the 1920s who published nothing at all. Hodge and Dirac were at Bristol for a few years. These two giants – the same age but utterly unalike – went on to spend their careers in the same department and college in Cambridge without, as far as anyone knows, ever speaking to each other; but each of them later recalled Fraser's role in shaping their ideas. Michael's self-confident surface could sometimes seem arrogant; but, beneath the surface, without question, the human qualities he most valued were honesty and humility.

Graeme Segal

Graeme Segal is an Australian who came to Britain as a Commonwealth Scholar. After one year as a student of Hodge he moved to Oxford, where he has remained ever since, except for an interlude 1990–99 in Cambridge. His work is along an axis going from algebraic topology to quantum field theory.

Announcement of the Atiyah UK-Lebanon Fellowships

Saturday 30 November 2019 saw the official launch of the new Atiyah UK-Lebanon Fellowships, with a live video link between De Morgan House and the Center for Advanced Mathematical Sciences (CAMS) at the American University of Beirut (AUB). This scheme is intended to provide a lasting tribute to Sir Michael Atiyah who had frequently visited Beirut and fostered mathematical links there. He chaired the CAMS International Advisory Board; and the Simons Foundation had previously endowed a Michael Atiyah Chair in Mathematical Sciences at the AUB.

Michael had attended schools in the Sudan and Cairo. His father Edward was Lebanese, and his grandfather had studied medicine at the AUB. Edward Atiyah was active in Middle Eastern politics before and after moving to Oxford in 1945, where he met Michael's mother Jean, who was Scottish. The LMS was lucky that its AGM on 29 November fell on the first of a 2-day meeting to celebrate twenty years of CAMS, so the Saturday gathering was a natural corollary. It had perhaps escaped the organisers' attention that Saturday was also St Andrew's Day, acknowledging the Scottish link, and providing a good omen for the future of the Fellowships.

LMS Treasurer Professor Rob Curtis acted as MC in the Hardy Room on Saturday morning. Present were Presidents Caroline Series and Jon Keating, having exchanged the chain of office the day before. Professor Series made the announcement of the Fellowships, and there was a response from the CAMS Director, Professor Jihad Touma, well known to many in the room in London. Present in the audience in Beirut was Nahla Atiyah, Michael's cousin. There followed warm addresses from Dr Fadlo Khuri, AUB President, and from HE Ambassador Rami Mortada, Lebanese Ambassador to the UK and himself an AUB alumnus. Embassy staff had attended the LMS Annual Dinner the night before, and enlightened diners at the Whitehead table about Lebanon and its history.

Professor Graeme Segal gave an address 'Sir Michael Atiyah (1929–2019)' which related personal experiences dating from when he was one of Michael's doctoral students. Graeme's address is reproduced on page 19. It provided insight into the ways in which Michael regarded himself as a Geometer and the relationship of this to Physics, with references to his heroes Dirac, Gelfand and Maxwell. If one has to choose one sentence to inspire future Atiyah Fellows, it could be 'Changing the landscape is what Michael will be remembered for'.

We heard about Atiyah's belief in the unity of mathematics. Those who have been lucky enough to have him as a teacher recall his ability to link apparently disjoint topics and, at a more serious level, this enabled him to formulate some of the deepest theorems of mathematics in collaboration with Hirzebruch, Bott, Singer and others. There will be an international conference entitled 'The Unity of Mathematics' to celebrate his achievements at the Isaac Newton Institute in Cambridge on 22–24 September 2020. The LMS has announced details (see page 6), and it will be funded by the LMS, INI, HIMR, CMI and Oxford University.

The video link with Beirut provided the first extensive test of the new conferencing equipment in De Morgan House, whose split screens provided three different views of the lecture room in Beirut. It was a resounding success, and the ceremony was closed by Rob Curtis and Caroline Series, who had more to say about the scheme.

Details are available tinyurl.com/tvweckc. Applications for this academic year are now closed; those for 2021-22 will open this autumn. The scheme will facilitate a two-way visiting programme for mathematicians between the UK and Lebanon, to be operated jointly by the LMS and CAMS. It will provide for an established UK-based mathematician to visit Lebanon for a period of anywhere between one week and six months, and for a mathematician at any level from Lebanon to visit the UK to further their study for up to a year.

Simon Salamon
King's College London

Multifractal random measures

ELLEN POWELL

Scale invariant characteristics, and the emergence of fractal-like sets, are typical of chaotic systems appearing everywhere in physics and the world around us. Multifractal random measures are a class of models that attempt to account for these observations. They are beautiful mathematical objects giving rise to many intriguing yet apparently universal phenomena, and their rigorous study has provided insight into several seemingly unrelated branches of mathematics.

What are multifractal measures?

Fractal is a general term used to describe objects that display self-similar behaviour, or repeating patterns, on different scales. This phenomenon is widely observed in nature. For example, if one zooms in on some small window on the boundary of a snowflake, or the coastline of a country, then the resulting image (to some extent) looks like a copy of the whole picture (see Figure 1).

Figure 1. Frost crystals forming on cold glass. *Wikipedia*

Associated with a fractal object in mathematics is a single important number. Roughly, this describes its *dimension*. To illustrate this, consider a mathematical model for a coastline. It is reasonable that this should be some sort of curve, filling up no positive area, but on the other hand having infinite length. This gives an example of a fractal with dimension between one and two.

Multifractality is a generalisation of this concept, when a single fractal dimension is not enough to fully describe the behaviour. Returning to the coastline example, a model with identical fractal dimension at every point on the coast is probably not very realistic. Indeed, the dimension should really depend on the point where one stands; for instance, it should be somewhat larger on the western border of Scotland than, say, the east coast of England.

Measures

A measure is a mathematical object that describes the distribution of something, say mass, in a space. More precisely, a measure μ on $\mathbb{R}^d$, is an assignment of values $A \mapsto \mu(A)$ for every (open) subset A of $\mathbb{R}^d$ such that:

- $\mu(\text{the "empty set"}) = 0$,
- $\mu(A) \geq 0$ for all A,
- if $A_1, A_2, \ldots$ are disjoint (open) subsets of $\mathbb{R}^d$ (meaning that A_i and A_j do not intersect if $i \neq j$), and A is the union of all the A_j, then
$$\mu(A) = \sum_{j=1}^{\infty} \mu(A_j).$$

One should think of $\mu(A)$ as describing the amount of mass in the set A.

This article will focus on *multifractal measures*. When dealing with measures displaying self-similar behaviour, fractality is described by how the mass of small balls scale with their size. That is, for points $x \in \mathbb{R}^d$, how the mass

$$\mu(\{z \in \mathbb{R}^d : \|z - x\| < r\}) \tag{1}$$

decays with r. For a multifractal rather than monofractal measure, the *exponent* that characterises this (i.e., the number a such that (1) decays like r^a as $r \to 0$) may vary from point to point. Then the set of points sharing a single exponent will form a fractal set. As a result the multifractal measure determines a whole range of fractal sets, with different fractal dimensions, corresponding to the range of possible exponents a.

Multifractal measures are used to model distributions of mass or energy that display some sort of scale invariance and/or chaotic behaviour. They are also commonly used to describe systems affected by a degree of randomness. This leads to the study of *multifractal random measures*. One of the most important classes of these are known as *multiplicative chaos measures*: these will be discussed in more detail later on.

Random measures in the real world

One early example of random multifractal measures can be found in the 1972 work of Mandelbrot [6], who introduced such objects in an attempt to describe *intermittency* in fully-developed turbulence. Turbulent flow is characterised by the property that the external forces making the fluid move are much higher than its viscosity. This results in highly unpredictable behaviour, and the emergence of many scales on which similar characteristics are displayed.

To illustrate this self-similar behaviour more concretely, let us think about eddies. In a turbulent fluid many eddies will typically form, and these will lose energy to smaller sub-eddies, which in turn transfer energy to even smaller eddies (see Figure 2). Mandelbrot proposed a model to describe where the energy eventually dissipates to. The "scale-invariant" nature of the energy transfer from larger to smaller regions of space, together with the unpredictability, make the resulting distribution a prime candidate for description by a multifractal random measure.

Indeed, this is exactly what Mandelbrot suggested. However, his original model turned out to be rather hard to define mathematically. This is why in 1974 he proposed the simpler model of *multiplicative cascades*: to be discussed more precisely in the next section. In fact, Mandelbrot's original model was only readdressed by Kahane [3] in 1985 in a work representing the initiation of *multiplicative chaos* theory.

Multifractal measures, specifically multiplicative chaos measures, have since been used extensively in far-reaching areas of mathematics and modelling.

For just one more concrete example, let us turn to mathematical finance. If one looks at data describing the *volatility* of a financial asset – roughly, how much it is fluctuating at a given point in time – then this displays many of the properties discussed above; a degree of scale invariance, unpredictability etc. Consequently, the measure encoding this volatility (that is, the measure for which the mass of a time interval is the amount of fluctuation within that interval) should have a multifractal structure. An effective model for this turns out to be a rather simple example of a multiplicative chaos measure.

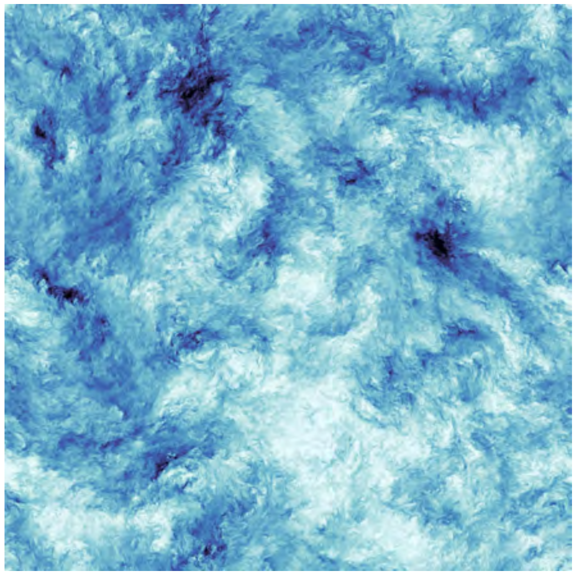

Figure 2. Computer simulation of velocity magnitude in a cross-section of 3D fully developed turbulence.
Cristian Lalescu and Michael Wilczek, Max Planck Institute for Dynamics and Self-Organization

Multiplicative cascades: a simplified model

The model of multiplicative cascades, proposed by Mandelbrot as a simplification of his model for energy dissipation in turbulent systems, is the forefather of multiplicative chaos measures and was one of the first examples of such a measure to be defined in a mathematically rigorous way. The construction of these measures is based on a beautifully simple iteration procedure. For concreteness, we will describe how to build a multiplicative cascade measure on the interval $[0, 1] \subset \mathbb{R}$. Our aim is to define a collection of random “masses” $\mu(A)$ for every subset $A \subset [0,1]$.

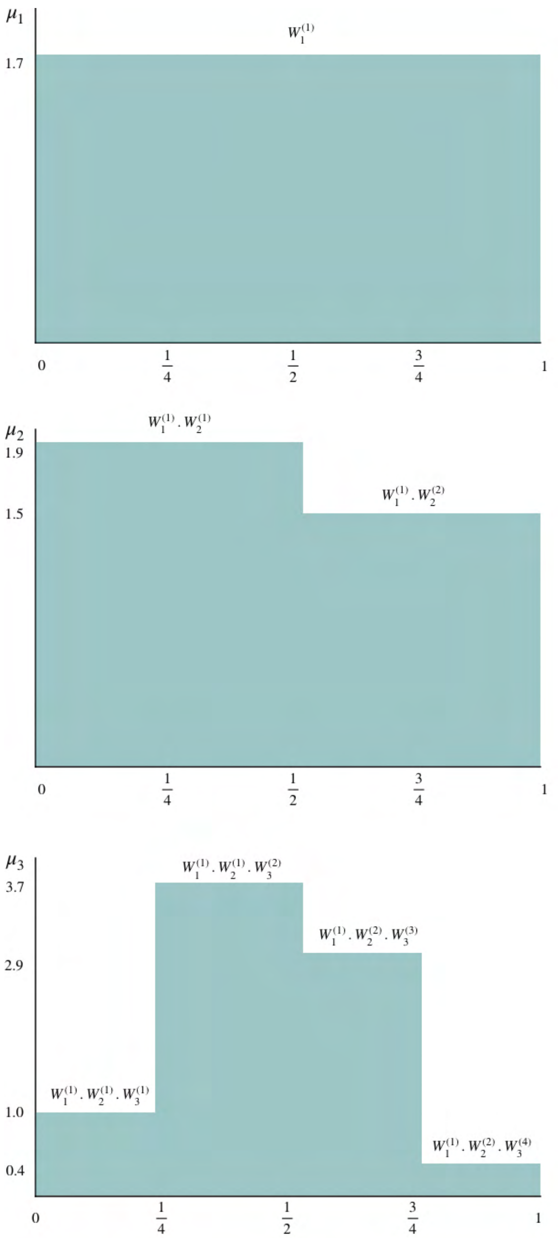

Figure 3. The measures μ_1, μ_2, μ_3 from top to bottom.

The procedure starts by taking a positive random variable with mean one:

$$W_1^{(1)} > 0 \text{ with } \mathbb{E}[W_1^{(1)}] = 1.$$

Then one defines a measure μ_1 by setting

$$\mu_1(\mathrm{d}x) = W_1^{(1)}\mathrm{d}x$$

on $[0,1]$. That is, we simply give $[0,1]$ mass $W_1^{(1)}$, evenly distributed across the interval.

The idea is to iteratively construct a sequence of progressively more complicated measures $\mu_2, \mu_3, \mu_4, \ldots, \mu_n, \ldots$ on $[0,1]$ and then take a limit as $n \to \infty$.

To define μ_2, we sample random variables

$$W_2^{(1)}, W_2^{(2)};$$

each with the same probability distribution as $W_1^{(1)}$, but independently of it and of each other. Then we set

$$\mu_2 = \mu_1 \times \begin{cases} W_2^{(1)} \text{ on } [0,1/2], \\ W_2^{(2)} \text{ on } [1/2,1], \end{cases}$$

to produce μ_2. To define $\mu_3, \mu_4, \ldots$, we repeat (see Figures 3 and 4).

In the example in Figure 3, the $W_i^{(j)}$ were sampled from the uniform distribution on $[0,2]$ and (rounding to one decimal place):

$$\begin{aligned} &W_1^{(1)} = 1.7, \\ &W_2^{(1)} = 1.1,\ W_2^{(2)} = 0.9, \\ &W_3^{(1)} = 0.5,\ W_3^{(2)} = 1.9,\ W_3^{(3)} = 1.9,\ W_3^{(4)} = 0.3. \end{aligned}$$

Now recall, the hope is that as $\mu_n \to \infty$, μ_n will converge to some limiting (multi-fractal) random measure μ. That is, for any fixed subset A of $[0,1]$, the sequence of random numbers $\mu_n(A)$ should satisfy

$$\mu_n(A) \to m_A \text{ as } n \to \infty \tag{2}$$

with probability one, for some random number $m_A \geq 0$. If this holds, a limiting measure μ can be defined by setting

$$\mu(A) = m_A$$

for every such A.

So why does this work? For simplicity, assume that we just have to deal with $A = [0,1]$, and let us look at the measure μ_n defined at the nth iteration. At this point, the interval $[0,1]$ has been divided into 2^{n-1} subintervals, and μ_n has a (different) *density* (the height in Figures 3 and 4) in each one of them.

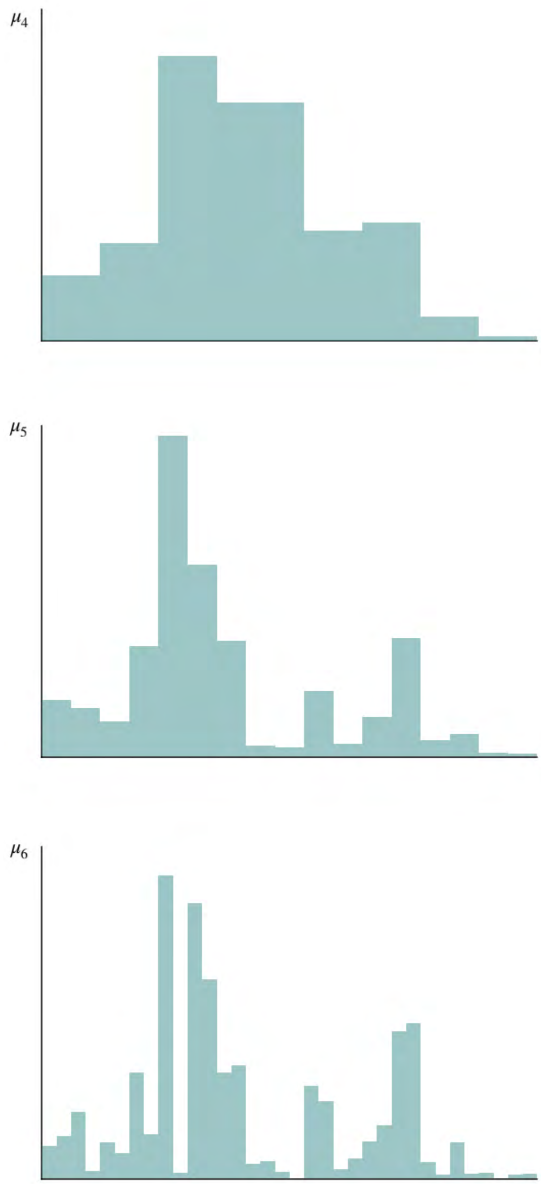

Figure 4. The measures μ_4, μ_5, μ_6, from top to bottom.

The total mass $\mu_n([0,1])$ is simply the average value of these densities, or heights. Very heuristically, the independence structure built into the model – together with the fact that all the W's have mean one, so the densities are not expected to explode or decay – means that things stabilise as $n \to \infty$. The result is that $\mu_n([0,1])$ converges to a limit $m_{[0,1]}$.

More precisely, the reason that (2) holds for every A is because the sequence $\mu_n(A)$ is a *martingale* (think stock prices, or see the summary below). Because it is also *positive* there is a powerful theorem – the *martingale convergence theorem* – that guarantees its convergence.

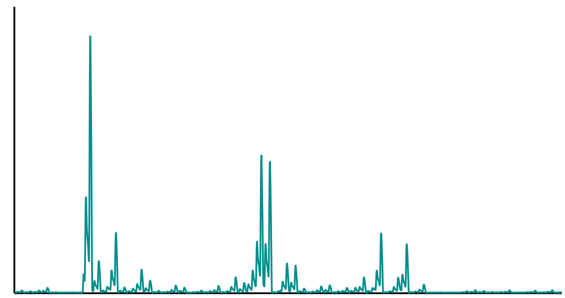

Figure 5. The limiting "multiplicative cascade" measure μ.

Martingales and convergence

The basic definition of a martingale is a process

$$M_0, M_1, M_2, \dots$$

that is *integrable* and satisfies

$$\mathbb{E}[M_n \mid M_0, M_1, \dots, M_{n-1}] = M_{n-1}$$

for every $n \geq 1$.

Example: if $X_1, X_2, \dots$ are a sequence of independent and identically distributed random variables with mean one, and $M_n = X_1 \times \cdots \times X_n$, then the conditional expectation of M_n given $M_0, \dots, M_{n-1}$ is just $M_{n-1} \times \mathbb{E}[X_n] = M_{n-1}$. Hence M_n is a martingale.

The *martingale convergence theorem* says that any sufficiently nice martingale has to have a limit as $n \to \infty$:

$$M_\infty = \lim_{n\to\infty} M_n \text{ exists with probability one.}$$

In particular, this holds for all *positive* martingales.

The fact that a limiting measure μ exists was first observed in the work of Kahane and Peyrière [4], and μ is what is now known as a *multiplicative cascade measure*.

Kahane and Peyrière, followed by many other authors, then began to study its properties, for example, to analyse its multifractal structure. Note that there is (stochastic) scale invariance property built in the model: if you look at an interval of size 2^{-n}, the distribution of mass inside this interval is equal in law (up to a random constant) to the distribution of mass in the whole of $[0, 1]$. This is one of the reasons that Mandelbrot proposed multiplicative cascades as a toy model for systems displaying multifractal behaviour.

As mentioned at the beginning, studying the multifractality of a measure amounts to studying the fractal sets where the measure behaves in a certain way; more precisely, considering the set of points x at which

$$\lim_{r\to 0} \frac{\log \mu[(x-r, x+r)]}{\log r} = a$$

for some a. For a multifractal measure, there will be a range of a for which these sets are non-empty, and form fractal sets having varying dimensions $\delta(a)$. This phenomenon is indeed observed for multiplicative cascade measures, resulting in rather "spiky" distributions of mass (see Figure 5).

In fact, it turns out that if the law of the random variable $W_1^{(1)}$ from which the measure is constructed has enough chance of being large; explicitly, if

$$\mathbb{E}[W_1^{(1)} \log_2 W_1^{(1)}] \geq 1;$$

then the approximating measures μ_n become so localised that $\mu = \lim_n \mu_n$ is just the zero measure. That is, although there will be places where μ_n is very large, as $n \to \infty$ they are too few and far between to actually contribute any mass.

Gaussian multiplicative chaos

Gaussian multiplicative chaos is a theory of random multifractal measures, sharing many features in common with multiplicative cascade measures, that are built from Gaussian fields. This is the sort of measure that Mandelbrot originally proposed for describing fully-developed turbulence [6], but the first rigorous construction only came through the 1985 work of Kahane [3].

Heuristically speaking, Gaussian multiplicative chaos measures are measures of the form

$$\exp(\gamma X(x))\,\mathrm{d}x \tag{3}$$

where $\gamma > 0$ is a parameter, $\mathrm{d}x$ is Lebesgue measure, and X is a *log-correlated Gaussian field* on $\mathbb{R}^d$ for some d.

So first: what is a log-correlated Gaussian field? One would *like* to define such a field to simply be a random function X from $\mathbb{R}^d$ to $\mathbb{R}$, such that

- $X(x)$ is a (centred) Gaussian random variable for every $x \in \mathbb{R}^d$, and
- the covariance between $X(x)$ and $X(y)$ (i.e., $\mathbb{E}[X(x)X(y)]$) blows up like $\log(|x-y|^{-1})$ as x and y get close to one another.

An approximation to such a function is shown in Figure 6. Hopefully this illustrates that log-correlated Gaussian fields are *very rough* objects.

In fact, if the approximation were better, then the field in Figure 6 would be even rougher. In the end this roughness becomes so extreme that trying to define X as a random function just doesn't make any sense. Essentially, the explosion of covariances means that such a function would have to be infinite everywhere!

To make mathematical sense of X, one has to define it as a random *Schwarz distribution*. That is, X does not have a value at any given point, but the integral of X against nice enough functions makes sense. The reader is probably already familiar with other examples of Schwarz distributions; for instance, the Dirac delta "function".

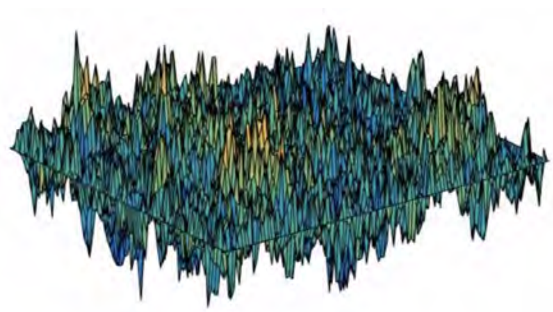

Figure 6. An approximation of the Gaussian free field, defined on a subset of $\mathbb{R}^2$. This is a fundamental example of a planar log-correlated Gaussian field.

The fact that log-correlated Gaussian fields are not actually well-defined as random functions means that it is not obvious how to make sense of measures like (3). Indeed, the basic input $X(x)$ does not make sense!

However, putting this aside for a moment, let us explain why measures of the form (3) might be expected to satisfy some form of scale invariance, and hence be reasonable for modelling systems like turbulent flow. This stems from the basic scaling property of the logarithm: for $a > 0$, $\log(a|x-y|) = \log a + \log(|x-y|)$. An immediate consequence of this is that if one can define a Gaussian field with covariance given exactly by the logarithm function, then (3) will be exactly *stochastically scale-invariant*. This means that if space is rescaled by a constant factor, then the law of the measure will only change by a random constant.

Now returning to the mathematical definition of the measure in (3), the idea – just as for multiplicative cascades – is to construct it via approximation. Of course, this approximation should be done in a way that reasonably interprets the expression (3).

Roughly, given the field X and $\gamma > 0$, a sequence of approximating measures μ_n is defined by:
- *smoothing the field* (say by convolution) to get some nicer fields X_n with $X_n \to X$ as $n \to \infty$;
- replacing X with X_n in (3);
- dividing the resulting expression by a constant, chosen so that the expected mass of $[0, 1]$ is 1.

Due to the initial work of Kahane [3] and much more recent work of many authors, including [1, 9], we now know that such approximate measures μ_n do converge to a limit measure μ as $n \to \infty$. This limit is referred to as a *Gaussian multiplicative chaos* measure.

In fact, in analogy with multiplicative cascade measures, there is a certain regime in which the limit μ will just be the zero measure. Here, this occurs whenever γ is too large; more precisely, when it is greater than or equal to $\sqrt{2d}$.

And again, Gaussian multiplicative chaos measures do exhibit a multifractal structure. Essentially all of the mass is located on a fractal set of points, whose dimension depends on γ. This accounts for the high degree of localisation (small patches of light blue) that can be seen in Figure 7, where colours represent varying intensity. In fact, the images in Figure 7 are made up of small squares having roughly equal GMC mass, and the various shades of blue correspond to different sized squares. The value of γ is larger for the bottom figure, resulting in a "more localised" measure. The sets where the mass concentrates, and more generally the fractal sets where the growth of the measure is governed by a specific exponent, are known as *thick points* of the field. See [1, 2, 10] for a more detailed analysis.

Applications in mathematics

Random metric spaces. One quite fascinating problem, that originates in string theory and turns out to be closely linked with Gaussian multiplicative chaos, concerns the construction of canonical "random metric spaces" in the continuum.

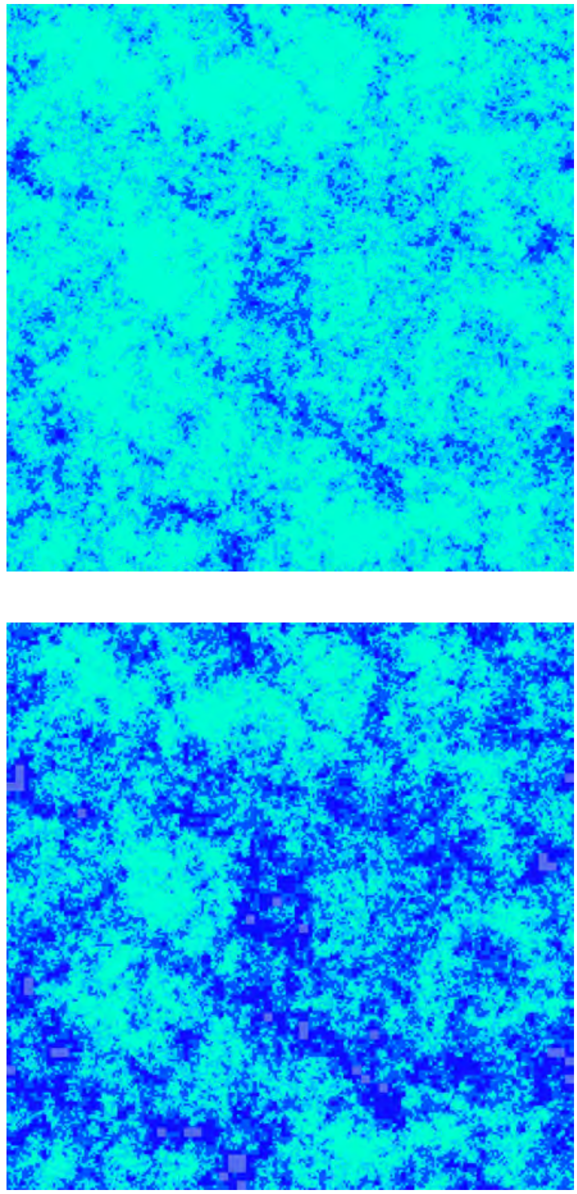

Figure 7. Two approximations of Gaussian multiplicative chaos measures, when the field X is a 2D Gaussian free field and γ takes two different values.
Simulation by Jason Miller

It is not clear a priori what this should mean, but one way to give mathematical sense to such a notion is the following. In very imprecise terms, imagine taking a huge number of triangles and gluing them together according to some stochastic rule; perhaps think "uniformly at random". It is known, at least for some rules [5, 7], that after rescaling things appropriately and taking the number of triangles to infinity, this construction produces a limiting random metric space (see Figure 8).

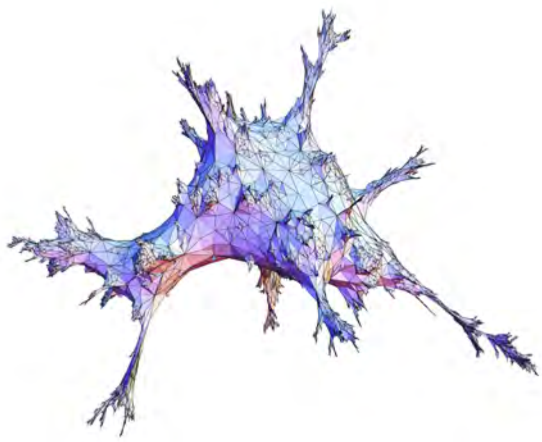

Figure 8. A simulation of a random triangulation with a large number of triangles.
Simulation by Nicolas Curien.

It is conjectured that this limiting metric space actually exists for a wide range of stochastic rules, and comes equipped with a natural measure. The idea is that this measure should be a Gaussian multiplicative chaos measure for an appropriate value of γ. A major breakthrough has been made towards proving such a statement in the case of "uniform gluings", [8]. However, there are still many questions yet to be answered.

The Riemann-zeta function. Somewhat surprisingly, Gaussian multiplicative chaos measures also have a role to play in number theory. They turn out to be relevant for studying the Riemann zeta function; more specifically, when one looks at the function around a *randomly chosen point on the critical line*. Increasingly rigorous results connecting the two models has led to rapid and exciting progress in this area.

Random matrices. As a final example, people have started to notice multiplicative chaos measures appearing in random matrix theory. It seems to be a universal feature of many natural models, that as the size of the matrix goes to infinity, its (rescaled) characteristic polynomial (and powers thereof) can be described by Gaussian multiplicative chaos. This type of result has been rigorously proven for several matrix models, but universality in a stronger sense remains an open problem.

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The Mathematics of Spacecraft Trajectory Optimisation

JÖRG FLIEGE

Spacecraft are big business: the global space economy has an annual turnover of more than $420 billion. However, spacecraft are expensive, with each journey costing hundreds of millions of dollars. We discuss how mathematical optimisation is used to solve a very difficult problem: how to steer spacecraft in the most efficient way.

Introduction

Two years after he became Professor of Geometry at Oxford in 1703, Edmond Halley published his *Astronomiae cometicae synopsis*, in which he used Newton's equations of orbital motion to predict the reappearance of what is now called Halley's comet. The world had to wait for 53 years to find that his predictions were correct; Halley himself did not live to see the comet's return. We do not know what thoughts he entertained, but he probably never discussed the possibility of visiting his or other comets.

In 1865, about a century later, Jules Verne had no such qualms, and published his novel *From the Earth to the Moon*. As far as the underpinning mathematics goes, Wernher von Braun attested 'the science in *From the Earth to the Moon* is nearly as accurate as the knowledge of the time permitted'. (The use of giant cannons to shoot people into space is, however, still not recommended.)

Figure 1. Hermann Oberth (centre, in profile) demonstrates a liquid-fuel rocket engine in Berlin in 1930. Second from the right is Wernher von Braun.

Hermann Oberth, inspired by Verne, finally analysed the basic concepts of spacecraft movement. He submitted his work as a doctoral dissertation in 1922 to the University of Heidelberg, only to have it rejected as 'utopian'. Trajectory analysis did not have it easy in those days. Thankfully, Oberth self-published his work as *Die Rakete zu den Planetenräumen*. Together with works from Tsiolkovsky and Goddard, this laid the groundworks for many further developments.

Optimisation theory provides an appropriate framework for the determination of spacecraft trajectories, that is, flight paths. As on Earth, one often wants to travel from one point to another. (Or from a certain set of points, which we call an orbit, to another set.) As on Earth, considerations of travel time or cost are of high importance. In this article, we will discuss the major concepts in the mathematics of trajectory optimisation.

The rocket equation

Given a rocket, denote by Δv the change in velocity between the start of a journey and the end of a journey. The amount of fuel needed to achieve a certain Δv is given by Tsiolkovsky's classic 'rocket equation',

$$m_{\text{fuel}} = m_{\text{initial}} \left(1 - e^{-\Delta v / I_{\text{sp}}}\right) \tag{1}$$

where m_{fuel} is the mass of fuel we need, m_{initial} is the mass of our rocket at the start of the journey, and I_{sp} is a parameter specific to the engine of the rocket. This exponential relationship clearly shows that large Δv can only be achieved by using rockets which consist almost solely of a fuel tank – not a good option in times where launching one kilogram of mass from Earth costs in the order of $10,000.

Thus, great care is usually taken to find trajectories with as small Δv as possible.

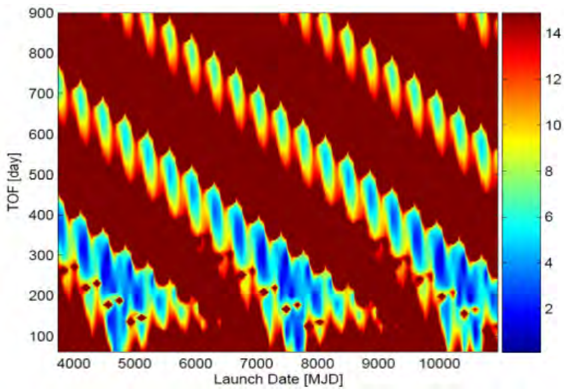

Figure 2. Δv for an example Earth-Apophis mission [1].

As it turns out, there are many different trajectory designs possible with a corresponding small Δv, thus greatly complicating the problem. Figure 2 shows the Δv for a mission from Earth to the asteroid Apophis; all design parameters have been fixed at reasonable values except for the launch date and time of flight (TOF), which vary along the horizontal and the vertical axis, respectively. Several local minima are clearly visible.

The dynamical model

We can re-interpret the rocket equation by saying that we want to consume the smallest amount of fuel possible to reach our destination – a rather natural frame of mind given the costs involved. As we are free to point our rocket in arbitrary directions during the voyage, and fire the engine at different strengths, this gives rise to a dynamic model, that is, a system of ordinary differential equations

$$\dot{x}(t) = f(t, x(t), u(t))$$

where we provide a control function u to evolve the state function x. Typically, $u(t)$ encodes the orientation of the rocket and the thrust we employ at time t, while $x(t)$ corresponds to the location of the rocket and the speed and direction in which it travels. Given a start time of the journey t_0, we have starting conditions $x(t_0) = x_0$. If t_f is the time our journey ends, we might want to enforce additional ending conditions $x(t_f) = x_f$; the time t_f is a free variable as well.

The optimisation problem

Together with the dynamical model described before, we have to consider some performance characteristic, which can simply be Δv, fuel consumption. In this case we just maximise $m(t_f)$, the mass of our spacecraft at the end of our journey. Other performance characteristics to consider are how close we are to a particular orbit at the end of our journey, that is, we want to minimise a term of the form $\|x(t_f) - x_f\|^2$ for some given x_f. Finally, we have to deal with *path constraints*: for example, at each time t, we have constraints on the length of our thrust vector, $\|T(t)\|$, and we do not want to get too close to any planets. We collect all these additional constraints in a function g and demand $g(x(t), u(t)) \leq 0$ at all times t. The general form of our optimisation problem is then

$$\min_{u,t_f} \int_{t_0}^{t_f} L(x(t), u(t), t)dt + M(t_f, x(t_f))$$

subject to:

$$\begin{aligned} &\dot{x}(t) = f(t, x(t), u(t)) \text{ for all } \ t \in (t_0, t_f), \\ &x(t_0) = x_0, \\ &g(x(t), u(t)) \leq 0 \text{ for all } \ t \in (t_0, t_f), \end{aligned}$$

where we optimise over the unknown function u as well as t_f, the time of the end of mission. The function $M(t_f, x(t_f))$ is called the *Mayer term* of the trajectory optimisation problem and denotes the cost of reaching the final state; the integral in the objective is called the *Lagrange term* and refers to the running cost of our mission.

This problem is a control problem: we need to find a particular function u, that is, an element of a particular infinite-dimensional space. A more formal description of our problem would thus involve stating precisely which spaces u and x are from. While this is evidently important, we skip over these technical issues here.

Optimisation techniques

There presently exist two main strategies for tackling trajectory optimisation problems of the form described above. These are often called 'optimise then discretise', and 'discretise then optimise', clearly highlighting their most important features. Both strategies have to approach one of the main difficulties of our control problem: we have to find a point in an infinite-dimensional space, and digital computers with their finite precision cannot represent members of such spaces in a computationally efficient way. This leads us to the concept of *discretisation*: the process of transferring continuous functions, variables, and equations into discrete counterparts. We usually achieve this by discretising a given time interval $[t_0, t_f]$ into discrete points $t_0 < t_1 < t_2 < \cdots < t_n = t_f$. If we have then given function values $u(t_i)$ at this finite number of intermediate points, we can both store these in a computer as well as interpolate values in (t_i, t_{i+1}), if that is so desired.

The 'optimise then discretise' approach starts with a classical question: how can we characterise the minima of a function? For a smooth function $f: \mathbb{R}^n \longrightarrow \mathbb{R}$ to be minimised subject to some constraints, the standard necessary conditions are the Karush-Kuhn-Tucker conditions. These feature an equation of the form

$$\nabla_x \mathscr{L}(x, \lambda) = 0$$

where $\mathscr{L}$ is the Lagrange function of the problem and λ denotes the vector of Lagrange multipliers. Recovering x numerically is still possible, although corresponding algorithms are much more involved. The second generalisation then moves x into some infinite-dimensional Banach space. The Lagrangian is replaced by a Hamiltonian function, and the necessary condition turns out to be a *boundary value problem*, that is, a system of ordinary differential equations $\dot{\phi} = F(t, \phi)$ with some unknown solution ϕ, where the value of the solution is prescribed at t_0 and at t_f to some fixed coordinates. This concludes the 'optimise' step. As we cannot solve the differential equation analytically, we thus resort to some numerical scheme (Runge-Kutta, Adams-Bashforth, etc.) which discretises the time interval given.

The 'discretise then optimise' approach is, in comparison, seemingly crude: start with discretising our time interval $[t_0, t_f]$ into discrete points $t_0 < t_1 < t_2 < \cdots < t_n = t_f$. In our optimal control problem, we will then only consider function values $u(t_i), x(t_i)$ as unknowns. We ask for the finite number of inequalities $g(u(t_i), x(t_i)) \leq 0$ to hold. The integral in the objective is replaced with a Riemannian sum, while the differential equation $\dot{x}(t) = f(t, x(t), u(t))$ is replaced with a discretised version, for example

$$x(t_{i+1}) - x(t_i) = (t_{i+1} - t_i) f(t_i, x(t_i), u(t_i)).$$

The result is a finite-dimensional optimisation problem, ready to be attacked by a variety of optimisation algorithms. Similar to the 'optimise then discretise' approach, we have a lot of leeway in how exactly we discretise the dynamics of the given system – the simple Euler scheme depicted above is only an illustration, and will in practice often not suffice. Good care is necessary in choosing an appropriate discretisation scheme and the number of grid points is often increased during the solution process in an iterative manner – that is, we solve many optimisation problems consecutively. Situations in which we end up dealing with several billion variables are not unheard of.

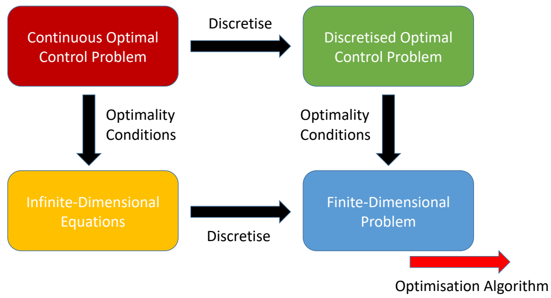

Figure 3. Solving the optimal control problem [2]. Discretise then optimise? Or, optimise then discretise?

Both approaches, 'discretise then optimise' and 'optimise then discretise' can be summarised in a diagram, see Figure 3.

Note that, due to the various discretisation schemes one can employ, several of the arrows in Figure 3 represent whole families of mappings from which one needs to choose particular discretisation schemes carefully. Likewise, the box in the bottom right corner 'Optimization Alg.' contains a multitude of different algorithms. Nevertheless, one might ask if the scheme commutes in any sense, that is, for particular choices of discretisation schemes along some arrows. It turns out that this depends more on the optimisation algorithm chosen than on the discretisation. More precisely, if an optimisation algorithm can be 'lifted' to the infinite-dimensional case and it can be shown that there it solves the optimality conditions for the control problem that we have derived before, then this algorithm is usually a good candidate to ensure that the diagram commutes. In general, Newton-type algorithms show this favourable behaviour.

In practice the 'discretise then optimise' approach usually performs better than the 'optimise then discretise' approach. The latter suffers from a subtle problem of Newton's method: convergence is only guaranteed in a neighbourhood of the optimal solution. For trajectory optimisation problems, this neighbourhood often turns out to be rather small, and one has to engage in a careful manual construction of a first approximation to it. Needless to say, this is often an extremely time-consuming endeavour. In contrast to this, the 'discretise then optimise' approach is correspondingly robust and produces good results even with rather bad starting guesses. This is testament to how much optimisation theory has advanced over the last 70 years.

Outlook

At the beginning of 2018, the last edition of the *Global Exploration Roadmap* reaffirmed the interest of fourteen space agencies to expand human presence on the Moon, in particular via the concept of a *Lunar Orbital Platform-Gateway* (LOP-G), a space station orbiting the Moon [6]. The European Space Agency announced that the LOP-G will be put in a so-called *halo orbit* around the Lagrange point L2, where the station can keep an almost stable orbit with minimal station-keeping involved. This objective is shared by NASA in the Artemis program: by 2024, NASA will send astronauts to the LOP-G, where they will live and work orbiting the Moon. The crew will also take expeditions from this lunar outpost to the surface of the Moon.

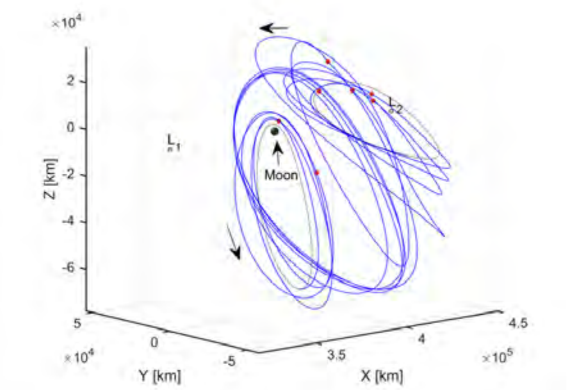

Figure 4. Transfer from a nearly rectilinear orbit around the moon to a halo orbit around the Lagrange point L2 [5].

Travelling to the LOP-G will be a rather involved affair: from Earth orbit to Moon orbit, changing to a Moon orbit almost perpendicular to the Sun-Earth-Moon plane, and then changing to the actual orbit of the LOP-G. Each of these legs will need to be carefully optimised to minimise either travel time (in case of a manned mission) or fuel consumption (in case of an unmanned mission). Figure 4 depicts a starting guess of what is believed to be close to being optimal for the last part of such a journey, crossing from a moon orbit to an L2 orbit.

Another highly interesting area in which much progress has been made in recent years is *low thrust trajectory optimisation*. Traditional chemical rocket engines rely on energy released from the combustion of propellant molecules. In contrast, low thrust engines utilise energy from solar power (via solar power panes), to accelerate and eject charged particles at high energies. The energies involved vary greatly; chemical rockets can produce a thrust of 10^5 Newton, while low thrust engines are content with 10^{-1}–10^0 Newton. However, low thrust engines have a much larger I_{sp} parameter (see (1)), that is, they can obtain much higher Δv. This makes them much more fuel efficient, albeit under a large increase in mission time.

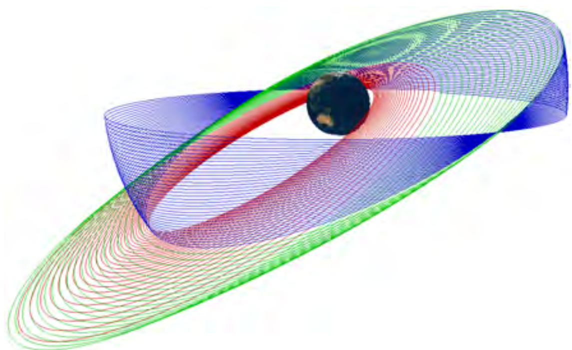

Figure 5. Low-thrust transfer from a starting orbit (innermost red ellipse) to a geostationary orbit (uppermost blue circle) [4].

A typical low-thrust orbital transfer trajectory can be seen in Figure 5. Our rocket starts in a highly inclined elliptical orbit close to Earth, represented by the innermost red circle. This is a typical 'starting orbit' after launch. The fuel-optimal trajectory then first extends the orbits (red ellipses), then reduces the eccentricity (green ellipses) before decreasing the inclination (blue circles). The overall process needs 71 days and uses 611kg of fuel. A classical rocket engine would perform these manoeuvres in less than 15 hours, but use 4011kg of fuel.

Low-thrust trajectories provide some fascinating connections to differential geometry. Consider three bodies (say, Sun, Earth and spacecraft; or Earth, Moon and spacecraft), where the third body has negligible mass and the second body is in a circular orbit around the first. Let $\mu > 0$ denote the mass of the second body and $1 - \mu$ the mass of the first, that is, we normalise total mass to 1. Denote by $r_i(t)$ the distance of the spacecraft from the ith body. If the spacecraft is just coasting, that is, not expending any fuel – a situation of high relevance if we want to save as much fuel as possible – then it can be shown that for its trajectory x we have that

$$\begin{aligned} J(x(t), \dot{x}(t)) \quad := \quad & x_1^2(t) + x_2^2(t) + \mu(1-\mu) \\ & + \frac{2(1-\mu)}{r_1(t)} + \frac{2\mu}{r_2(t)} - \|\dot{x}(t)\|^2 \end{aligned}$$

is a constant in t. This Jacobi constant J is only constant along a particular trajectory; applying a low-thrust force to the spacecraft means moving to a different trajectory and hence to a different Jacobi constant J. This gives rise to the family of five-dimensional manifolds

$$\mathcal{H}(c) := \{(x, \dot{x}) \mid J(x(t), \dot{x}(t)) = c, \forall t\}$$

with c a constant. Analysing these manifolds provides one with realms of possible motion, that is, areas of space that coasting spacecraft can reach. Such knowledge can then be directly employed in the design of fuel-optimal trajectories [3].

Due to longer mission times, any optimisation problem considering low-thrust engines will have many more variables, showing an immediate need for further research in high-performance optimisation algorithms and corresponding implementations. The optimisation problems themselves are also of a distinctively different structure: with solar power panels one needs to avoid the shadow of the Earth and the moon, and long durations spent in parts of space that pose significant danger to the craft, like the Van Allen belt, should be avoided. It is presently unclear how these challenges can be fully addressed by an appropriate mixture of mathematical modelling and optimisation theory, especially when it comes to interplanetary missions using low-thrust engines – exciting times ahead!

Acknowledgements

The author would like to thank Stephen Kemble, Massimo Casasco, Eric Joffre, and Valerio Moro from Airbus Defence and Advanced Systems; and Manuel Sanchez-Gestido from the European Space Agency for many fruitful discussions.

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[2] M. Enriquez, W. Symes, Systemizing the Solution of Simulation-Driven Optimization Problems. Presentation at the National Institute of Standards and Technology (2013).
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Jörg Fliege

Jörg Fliege is Professor for Operational Research at the University of Southampton. His main research interests are in optimisation and applications of optimisation algorithms to real-world problems. He was born in Germany, moved to the UK 15 years ago, and still believes that trajectory optimisation is easier to understand than cricket.

The Current Picture of Gender Diversity in UK Mathematics

EUGENIE HUNSICKER

A report on gender diversity in UK mathematics, commissioned by the LMS Women in Mathematics Committee, was published in summer 2019. It reveals that gender diversity figures have not changed significantly since the last report of this nature in 2012, despite efforts by Departments. Ways forward are suggested.

Background

Research evidence has grown over the past several years as to the benefits of diversity, broadly interpreted, in groups engaged with solving complex problems (see, for example, [1]). There have also long been concerns in the global mathematics community about ensuring that talented mathematicians have equal opportunities to thrive and contribute to the discipline regardless of gender and other diversity characteristics. This motivated the establishment in 1999 of the London Mathematical Society's Women in Mathematics Committee (WIM), and the later development of the Good Practice Scheme (GPS) to support UK mathematics departments in their diversity work. As part of the GPS, the report *Advancing women in mathematics: good practice in UK university departments* was launched at the House of Commons in 2013. The WIM Committee, through the GPS Steering Group, has organised a number of workshops on diversity, principally aimed at supporting Departments in the process of developing applications for Athena SWAN awards. Currently 28 UK Departments hold Athena SWAN Bronze awards and a further 6 hold Athena SWAN Silver awards.

In 2016, a new report, the *National Benchmarking Study 2017* (tinyurl.com/benchmarking2019) was commissioned by the WIM Committee. It was produced by Ortus Economic Research and published in summer 2019. The goal of the study was to learn how the gender profile, as well as gender related diversity initiatives, in UK mathematics departments had changed since the 2013 report. The original study was based on responses of departments to a survey, as well as HESA data about student and staff numbers. The new study used a combination of HESA data and 33 Athena SWAN applications contributed by departments of mathematics, as the committee did not want to burden departments with any additional requests for information on gender initiatives, given the work already associated with Athena SWAN.

Overall, the study shows that the national representation data is not encouraging – it hasn't changed much since the first survey. The study also flagged ongoing problems with participation and inclusion of women in UK mathematics, and challenges with the Athena SWAN process itself.

The 'pipeline' for UK mathematics

A review of HESA data gives us an updated picture of the UK 'pipeline' in mathematics (Figure 1). The latest HESA data available online suggests that the approximate numbers of UK-domiciled women in mathematics for 2017/18 are: Researcher (21%), Lecturer-Reader (21%), Professor (10%).

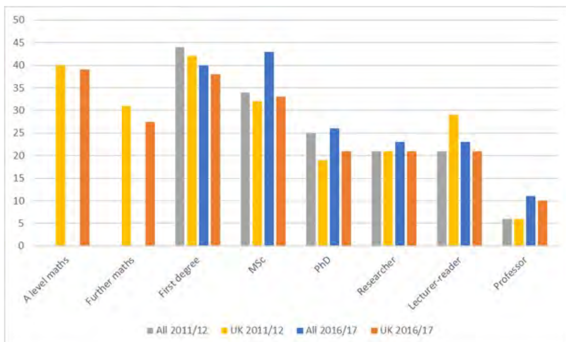


Figure 1. Percentage of women among all and among UK-domiciled mathematics staff in 2011/12 and 2016/17.

Frustratingly, the changes since 2011/12 are not substantial, with the only real upward movement at the rank of Professor. Percentages of women among staff at other levels in the pipeline have remained fairly stagnant. In view of Brexit, it is worrying to see this is particularly true for percentages of UK-domiciled individuals (UK citizens or those with

Table 1: Departmental percentages of women at levels from lecturer/assistant professor through to reader/associate professor.

Quartile	2009/10	2010/11	2011/12	2012/13	2013/14	2014/15	2015/16	2016/17
Minimum	4.3	0.0	3.6	2.8	3.2	4.7	4.9	7.1
1st quartile	9.9	9.9	11.0	10.2	11.0	11.7	13.7	14.0
Median	16.2	18.4	16.0	17.6	19.4	17.9	18.2	18.8
3rd quartile	23.5	24.3	23.2	26.0	24.5	22.8	24.0	24.3
Maximum	39.0	39.5	43.9	45.6	43.7	40.6	47.5	48.4

leave to remain), where over the same period we have in fact seen a substantial drop in the percentage of women at permanent ranks below professor (29% to 21%).

Furthermore, although there is a large relative change in the percentage of women at the professor level (from 6% to 10%), this proportion is still very low given that 50% of the population are women. Finally, there is considerable variation among departments, with the average percentage of women in permanent Teaching and Research roles below professor nationally at 23% but with three-quarters of UK mathematics departments having less than 24.3% women (see Table 1), and half below 19%.

Challenges and practices related to gender

Despite the lack of progress highlighted by the data, undertaking the 2017 Study was nevertheless useful. In particular, considerable detailed information about the practices tried in UK departments, as well as common struggles has been collected, see Table 2. I recommend people read through the full report for more detail on these, which the WIM and GPS committees particularly hope will assist departments in planning their Athena SWAN work and building the case for their applications.

At the department level, even the best changes to improve recruitment or promotion practices can easily be offset by changes in either institutional or national policies. So, statistical noise at the department level makes it difficult to judge by numbers whether the initiatives undertaken are effective. This is why a national picture of effectiveness is important. However, the current Athena SWAN data on such things as promotions, seminar speakers and attitudes of individuals in the department as measured in surveys is not standardised. This makes it impossible at the moment to meaningfully combine data nationally to study what interventions may be most effective and which least.

Proportions of women among staff at different levels of the mathematics pipeline is a measure of representation, but not a measure of participation or inclusion. One conclusion of both the Benchmarking Report and the external Athena SWAN review (also undertaken by Ortus, and available at tinyurl.com/qp676cs) is that there is a lack of clarity among departments about how to evidence the impact of changes other than through increases in staff numbers. Thus, another goal of changes to institutional practices – improving women's inclusion – is not well benchmarked by current Athena SWAN data.

Where next for the LMS and UK mathematics?

Athena SWAN is focused on practices within universities and departments. It seems likely that many of the important changes necessary to improve gender balance and the inclusion of women are at the disciplinary rather than institutional level, relating to policies and practices around grants, peer review, editorial practices, fair citation, workshop organisation and so on.

These all point to three roles for the LMS in ongoing work on gender in mathematics:

(1) to help departments standardise their measures of cultural change so as to permit national collation of evidence of how these may be influenced by various interventions at departmental level.

(2) to coordinate collection of these measures in mathematics departments to inform a national picture, and correlate them to interventions introduced, to study which are most effective.

(3) to coordinate work on changes at the national level in the discipline, and work to influence journals, funders and REF panels to consider how their practices may be improved to bring about increases in representation and inclusion at all levels by women.

Regarding the first of these, the WIM and GPS Committees are awaiting the results of the ongoing review of Athena SWAN, expected to be released soon, before beginning work on national efforts at standardisation in mathematics. This review may suggest changes to the Athena SWAN process along these lines, and we want to ensure that the work of these committees aligns with any changes. We will hold a GPS meeting within the next year to discuss ideas around standardisation and collect input from departments.

In order to obtain a clearer picture of obstructions to the progression of women (and others) in UK mathematics departments, I have recently undertaken the Pipeline Survey for UK Academic Mathematics. This survey collected dynamic information about how individuals move through academic grades together with information about how progression is influenced by international and national mobility, movement among conjugate disciplines, and caring responsibilities and part-time working. The goal of the project is to provide the community with numerical data on barriers to career progression, where individuals are lost to UK mathematics, and the importance of foreign academics in UK mathematics departments. The results will be completed in summer 2020, and shared with the community through an article in the *Newsletter*, as well as with LMS Council and the other organisations in the Council of Mathematical Sciences (CMS), which is the consortium that undertakes work regarding national policy of relevance to UK mathematics.

Finally, two workshops are planned for this spring on the topic of Equity in Peer Review. The first of these is a GPS workshop that will take place in Glasgow on the afternoon of Thursday, April 9, after the conclusion of the BMC-BAMC. It is aims to hear from the UK mathematics community about their experiences, good and bad, with peer review. The views collated from the community will then feed into a second smaller workshop to be held at ICMS in May. This workshop will bring together members of the community involved in various peer review mechanisms, including publications, REF and EPSRC, with researchers in peer review. The goal is, taking account of the community's experiences and concerns, to consider how peer review mechanisms can be improved within the current constraints; what policy changes would be required for further improvements; and what additional studies we need to undertake as a community to decide the best way forward.

How you can help

It is clear from the Benchmarking Study that considerable effort is being put into issues of gender equality and inclusion in UK mathematics departments, and the community should be justly proud of the work we have done. Now is the time to consider how we can make this work more efficient and effective, and the LMS is committed to these efforts. We encourage members to contact us with their ideas.

Table 2: Ortus Economic Research analysis of Athena SWAN applications.

Most successful practices	Most common struggles
Review/improve promotional material	Attracting female students
Improve staffcareer support	Low numbers of female staff
More targeted/proactive recruitment	Data gaps
Promoting postgraduate opportunities	Committee constituency
Review/improve student recruitment opportunities	Leaky pipeline
Review/improve promotions processes	Role models
Review/improve recruitment materials	Awareness/perception of promotion system
Improve staff mentoring	Workload model
Review/improve staff support information	Appointing female staff
Review/improve staff support processes	Awareness of development and support policies

There are three ways we are hoping that members of the mathematics community will help with this work:

(1) if you did not previously complete the Pipeline Survey, you can fill it in at: tinyurl.com/tnfo892. New data will not be involved in any research publications, but will be included in the reports to the community and members of the CMS.

(2) send us comments about your views and experiences with peer review, including what works well and what doesn't work well with current systems. Comments may either be submitted through comment boxes and boards at the BMC-BAMC in Glasgow in April, or at the GPS meeting to be held on Thursday 9 April, or by email at E.Hunsicker@lboro.ac.uk. These will be collated and fed into the discussion at the workshop at ICMS in May, and a report will be written up on the discussions for circulation to the community.

(3) contribute through the GPS workshop that will be announced later this year about standardising data collection for national evaluation of the impact of gender diversity initiatives.

Of course, the Women in Mathematics Committee and I are also very happy to hear your other thoughts regarding diversity in UK mathematics and how the LMS can support the community.

FURTHER READING

[1] Page, S.E. et al, The Diversity Bonus, Princeton University Press, (2017).

Eugenie Hunsicker

Eugenie Hunsicker is a Senior Lecturer at Loughborough University and the Director for Equality and Diversity in the School of Science. Her research is in analysis and topology, as well as applied statistics, especially for high dimensional and non-Euclidean data. She is the Chair of the LMS Women in Mathematics Committee and Deputy Chair of the Athena Forum.

Lebanese Society for the Mathematical Sciences

The Lebanese Society for the Mathematical Sciences (LSMS) was founded in summer 2008 following the initiative of a group of Lebanese mathematicians in Lebanon and the diaspora. An official statement of acknowledgement numbered 1802 was issued by the Lebanese Ministry of the Interior on 3 December 2008. Accordingly, the Society operates under Lebanese law with a general assembly electing a board for a three-year term.

The first board was elected on 16 July 2009, including mathematicians from various universities in Lebanon. Professor Nabil Nassif (American University of Beirut) became the first President of the Society. Since May 2012, three other boards were elected, and Dr Rachel Hobeika Kallas (Lebanese University) has been serving as president for a third consecutive term. Society members are mathematicians from universities in Lebanon and abroad. The door is also open to secondary school teachers and to postgraduate students.

The LSMS's mission is to promote the mathematical sciences in Lebanon through seminars, conferences, regular meetings, publications, and different activities related to research and teaching. To achieve this mission, the successive boards have adopted the following action policies:

- Strengthening the links between mathematicians from Lebanon and abroad through various activities such as scientific meetings, summer schools, and assistance to Lebanese students in order to engage them in education and research programs in Lebanon and abroad.
- Establishing sustainable cooperation with universities and government organizations, particularly the Lebanese National Council for Scientific Research (CNRS-L), and with the economic sector, chiefly banking and industry.
- Establishing sustainable relationships with similar societies and associations in the Middle East and elsewhere, especially the Euro-Mediterranean and American regions, in order to enhance the flow of young mathematicians to universities in Lebanon.
- Enhancing the publication process for LSMS members, particularly those from the younger generation.
- Enabling the participation of secondary school students in international mathematics competitions.

Since January 2010, one key activity of the Society has been the organization of a yearly conference that gathers mathematical scientists from Lebanon and abroad. Universities and research institutions involved in mathematical sciences in Lebanon welcome and support these conferences. Generous support is constantly provided by the CNRS-L in addition to the Center for Advanced Mathematical Sciences of the American University of Beirut (AUB CAMS).

From the 2019 LSMS Annual Conference

Moreover, the Society organizes workshops and a variety of public lectures open to a large public of people interested in mathematical sciences. In addition, it organizes an annual meeting with the heads of local mathematics departments and laboratories to discuss the problems in teaching and research. Officials from the pre-university sector actively participate, and recommendations are issued.

The Society, which celebrated its tenth anniversary in 2019, has been steadily growing. It currently gathers a large community of Lebanese mathematicians from Lebanon and the diaspora, working in the various areas of pure and applied mathematics. It is also an important supporter of many activities related to mathematical sciences. Most notably, the Lebanese American University has been holding a yearly mathematics tournament for second and third-year undergraduate mathematics students in Lebanon since 2017, with the support of the Society.

A key development has recently occurred with the creation of the Atiyah UK-Lebanon Fellowships, by the London Mathematical Society, in memory of British Lebanese mathematician Sir Michael Atiyah. The fellowships were launched in December 2019 and operate in partnership with the AUB CAMS. As part of its mission, the LSMS is committed to promoting these fellowships within the large Lebanese community of mathematical scientists.

Partnership with sister mathematical societies in the Euro-Mediterranean and American regions has been initiated. A letter of support was drafted in 2011 by the President of the Society for Industrial and Applied Mathematics (SIAM). Also, an article has recently appeared in the Société de Mathématiques Appliquées et Industrielles (SMAI) journal. Discussions with other international societies are under way.

More information about the Society can be found at https://www.lsms.net.

Rachel Hobeika Kallas
President, Lebanese Society for the Mathematical Sciences

Careers in teaching – some options

"Dear X, I am a PhD student/postdoc. I'm very interested in teaching. What are my options in terms of teaching in schools or universities, and how can I develop my career in this direction?" — We invite three experts to comment from different perspectives.

Michael McEwan *is a lecturer and Head of Subject in Academic and Digital Development at the University of Glasgow. For the last six years he has taught early career academics about teaching, learning and assessment and supported hundreds of academics to gain professional recognition as a teacher in higher education.*

If you are interested in teaching in a university then you should become aware of the UK Professional Standards Framework for teaching and supporting learning in higher education (https://www.advance-he.ac.uk/guidance/teaching-and-learning/ukpsf). The UKPSF is a framework that helps you unpack the job of teaching into its many building blocks, but it is also a framework for you to work towards professional recognition as a teacher – something that nearly all universities in the UK now require of their teaching staff.

The framework breaks teaching down into 15 dimensions of practice including activities that a teacher does e.g. planning and design (A1), teaching itself (A2) and assessment (A3); and knowledge a teacher should have e.g. knowledge of different teaching methods (K2). The framework also states the professional values expected of a teacher in UK higher education; values around diversity (V1) and equality of opportunity (V2). For a new teacher in HE the framework is a tool to understand and reflect on what you know and have experience of, and, importantly, what you don't.

My next piece of advice is to be proactive. Ask around your subject area for teaching opportunities – university subject areas are always looking for tutors and demonstrators! Find out who coordinates teaching in your subject area (this might be a Head of Teaching, for example) and let them know you are available. Look outside your subject at cognate disciplines also (e.g. discrete maths → computer science; statistics → social sciences). Your first teaching roles will probably be as a tutor in small groups, or as a demonstrator in a practical class or workshop, but you can get opportunities in assessment, planning and developing classes, and lecturing in larger classes (usually as a guest or cover).

Your first classes as a new teacher can be daunting so be aware that this is normal! Teaching is difficult and challenging, but rewarding. An interesting piece by Peter Kugel [1] demonstrates how the typical teacher in higher education develops from their first class to their final one. It might prove a useful read to give you some reassurance that whilst it takes time, and you do need to put in the work (more on that later), you will become a confident teacher.

My final piece of advice is to invest in your role. Becoming a researcher takes time, learning and qualification – so does becoming a teacher. Many universities offer training and support for their new teachers, tutors and demonstrators. Look around your university for an 'academic development' or 'learning and teaching' unit (e.g. within my own university we have LEADS: the learning enhancement and academic development service). These units are there to support new teachers in their professional development and usually offer courses, programmes, standalone CPD sessions and professional recognition frameworks. It is possible to gain Associate Fellowship against the UKPSF (professional recognition) through a role as a tutor or demonstrator, and some roles might afford Fellowship. Gaining professional recognition is demonstration of your reflective and reasoned approach to teaching, learning and assessment but will also give you ideas for your classes to help you stand out as an individual, give you more confidence in your role (helping you make your way through Kugel's stages) and will provide you with evidence of your teaching in a language that will help you answer the teaching questions in a future interview!

FURTHER READING

[1] P. Kugel, How professors develop as teachers, Studies in Higher Education, 18 (1993) 315–328.

Giovanna Scataglini Belghitar *is Key Stage 5 Mathematics Coordinator at Roundwood Park School. She was awarded a PhD in algebraic geometry at Durham University, and then worked as a maths lecturer at St Peter's then Balliol College, Oxford, where she developed an interest in mathematics education.*

If you are interested in teaching in schools, you should first of all get some direct experience. You can contact a couple of schools either where you study, or where you live, and ask to visit their maths department so you can talk to the teachers and see directly what school life is like: mid-autumn and early summer are usually quieter times in school. If you are spurred on by what you see, you could either offer to volunteer on a regular basis (perhaps once a week for a few hours) or you could ask if they can arrange for you to shadow some teachers over a period of a week or so.

Once you've decided that teaching is for you, there are different routes you can take to gain Qualified Teacher Status (https://getintoteaching.education.gov.uk/explore-my-options). The main ones are a postgraduate certificate in education (PGCE) at a higher education institution, or school-based training which could be either 'salaried' or with a bursary; however, there are other avenues such as the 'Researchers in Schools' programme, Teach First or training while working in the independent sector.

Whichever route you eventually select, some direct experience of classroom practice is usually a prerequisite and certainly an advantage. Additional useful experience can be gained at university, running problem classes or tutorials, or in your spare time, for example by helping youth organisations.

One more word of advice: do some research before you start, not so much on 'how to teach' as you will learn all about that while training, but specifically about how students learn mathematics. If you don't know where to start, Mr Barton Maths (http://www.mrbartonmaths.com/index.html) has a series of mathematics and education podcasts, amongst many other useful things, which you may find interesting.

Jonathan Tan *is Head of Allocation and Operations at Teach First. He has a BA in mathematics and computer science from the University of Oxford, and a PGCE in mathematics from the University of Reading.*

It's fantastic to hear that you want to teach. There's a severe shortage of maths schoolteachers in the British education system, and with a postgraduate degree in maths you'll be hugely desirable and much needed in the classroom.

There are many different teacher training routes, and it's worth considering which one best meets your needs. If you want to make a difference, think about where you can have the most impact. Not all pupils have access to the same quality of education, and a large part of this is lack of teachers. Some pupils may not ever be taught by specialist maths teachers, with schools having to use teachers from other departments or supply teachers to fill gaps. As a result, pupil outcomes across the country are vastly unequal. For example, just 26% of disadvantaged pupils make it to university, compared to 85% from independent schools.

You may want to apply to the Teach First training programme, which aims to address this inequality. We're a school-based, salaried teacher training route and we partner with schools across England to place trainees in disadvantaged communities and areas of greatest need. About 40% of our trainee teachers are 'experienced hires', who have additional qualifications or experience beyond an undergraduate degree, so you're well placed to train as a PhD student or postdoc. You'll also be well set to handle the academic rigour of our Postgraduate Diploma in Education and Leadership (PGDE).

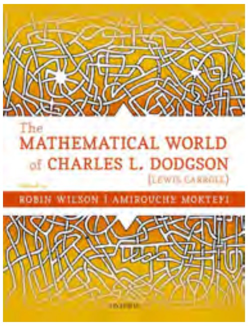
No matter how and where you train, teaching will vastly improve your ability to communicate and to lead. You'll be putting your mathematical knowledge to great use and gaining a whole new skill-set at the same time. It's an enriching, joyous career, and you'll play an integral role in shaping the future of young people who may never have found their own love of mathematics without you.

The Mathematical World of Charles L. Dodgson (Lewis Carroll)

Edited by Robin Wilson and Amirouche Moktefi, Oxford University Press, 2019,

£29.99, US$ 39.95, ISBN: 978-019881700

Review by Mark McCartney



Lewis Carroll is almost universally known for his *Alice* books. What is less well known, even amongst the mathematics community, is that Lewis Carroll was the pen name of Charles Lutwidge Dodgson MA (1832-1898), student (equivalent to fellow) and mathematical lecturer at Christ Church, Oxford.

Born a child of the rectory into a large and happy family, Dodgson was educated at home and then Rugby before matriculating at Oxford in 1850. He graduated with a first in mathematics in 1854 and remained in Oxford teaching mathematics for the rest of his career. The centre of gravity of Victorian mathematics in Britain was most certainly Cambridge rather than Oxford, and to say that Dodgson was a key figure in Victorian mathematics would be generous. However, as this volume shows, Charles Dodgson's mathematical work remains fascinating in its own right, regardless of the fact that Charles Dodgson was also Lewis Carroll.

In *The Mathematical World of Charles L. Dodgson* Robin Wilson and Amirouche Moktefi have picked a subject who is well known, and investigated a major aspect of his life which is not. By doing so they open a door for the reader into some lesser known aspects of Victorian mathematics. After a first chapter by the editors giving a biographical overview of Dodgson, five chapters look at his contributions to geometry, algebra, logic, voting theory and recreational mathematics. The remaining two chapters survey his mathematical legacy and provide a bibliographic essay of Dodgson's mathematical work.

As a teacher at Christ Church Dodgson taught Euclid's *Elements* from freshman to honours level (covering Books I to VI). Seen as an excellent way to inculcate logical reasoning, Euclid was a staple of Victorian education and Dodgson was a firm supporter of it in teaching. However, not all agreed. During Dodgson's career alternative texts to teach geometry began to appear, and in his 1869 Presidential Address to the British Association for the Advancement of Science J.J. Sylvester bluntly stated that 'Euclid made me a hater of geometry' (p. 43). In Chapter 3, Robin Wilson sets Euclid, his detractors and Dodgson's 200-page defence (*Euclid and his Modern Rivals*) in their Victorian context. Dodgson's defence comes in the form of a four act play, complete with two judges from Hades, a German professor and Euclid's ghost. Do not be deceived however, it is not a book of whimsy. Rather it is technical comparison of Euclid against a dozen rival Victorian geometry texts written by a man who was irked by the direction of flow of mathematical education and whose eccentricity could sometimes get in the way of clarity. It was not a battle that Dodgson won, and by the beginning of the twentieth century Euclid's centrality as a teaching text had been lost.

In Chapter 4, Adrian Rice examines Dodgson's contribution to determinants. Dodgson produced an efficient way to evaluate determinants via a method he called 'condensation'. The version of this work published in the *Proceedings of the Royal Society* in 1866 is a concise and clear account of the matter. Dodgson's book-length treatment of determinants which was published the next year was neither. A true devotee of Euclid's style, the *Elementary Treatise on Determinants* was formal, not particularly readable and suffered from the fact that Dodgson insisted on using a notation of his own devising. Amongst other things, even though he was aware of the wide use of the term *matrix*, he insisted on using the term *block* instead. As Rice states 'Dodgson's idiosyncratic style is symptomatic of his unorthodox approach to mathematics' (p. 79).

This idiosyncrasy follows though into both Dodgson's work on logic and his work on recreational mathematics. His work on logic is probably the one mathematical topic where his interest comes as no surprise to the reader of Lewis Carroll. The *Alice* books are replete with linguistic sleight of hand and logical paradox. In Chapter 5, Amirouche Moktefi examines Dodgson's work on logic in the light of work by Boole, Venn and others. Edward Wakeling's chapter on Dodgson's recreational mathematics provides a wealth of examples of his puzzles, from simple arithmetic and probability to the famous 'Monkey Puzzle'. In solving his own puzzles, Dodgson's thorough and logical mind occasionally let him down. Thus, for example his solution to "If an infinite number of rods be broken: find the chance that one at least is broken in the middle". is 1-1/e (pp. 205–6).

In the chapter on voting theory, Iain McLean gives a readable overview of the history of the subject and argues that 'Dodgson is one of the great figures in the axiomatic theory of voting' (p. 121). It was a subject which came to his interest because of voting procedures in Christ Church, but then extended to tennis tournaments and parliamentary elections. McLean argues that the lack of recognition of Dodgson's work on voting has at root a simple explanation: 'He could be pedantic, quarrelsome, and (when not quarrelsome) obscure' (p. 138).

Bertrand Russell, somewhat ungallantly, stated that Dodgson's 'works were just what you would expect: comparatively good at producing puzzles and very ingenious and rather pleasant, but not important...None of his work was important' (p. 114). Well Bertrand, the sad fact of the matter is that virtually no mathematician does work that is really 'important'. We make minor contributions, chip away at the edifice of knowledge, teach our students, and will likely be forgotten with the memories of those who knew us. However, the contributors to this volume argue that Dodgson's contributions to mathematics are not so easily swept aside. This book shows, through the study of one life, that 'unimportant' contributions, placed in historical context by well versed scholars, make for a fascinating read. Wilson and Moktefi's volume will be the standard work on Dodgson's mathematics for many years to come.

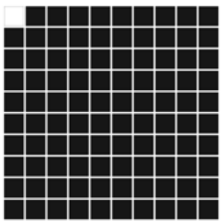
Mark McCartney

Mark McCartney is senior lecturer in mathematics at the University of Ulster. He is of the view that most situations can be improved by drinking tea and reading a book.

99 Variations on a Proof

by Philip Ording, Princeton University Press, 2019, £20.00, US$ 24.95, ISBN: 9780691158839

Review by Chris Sangwin



In *99 Variations on a Proof* Philip Ording provides 99 arguments to find or justify the solution of a single theorem. However, this is really a deep and thoughtful examination of the nature of mathematical arguments, of mathematical style, and of proof itself. The theorem considered is:

> **Theorem.** If $x^3 - 6x^2 + 11x - 6 = 2x - 2$, then $x = 1$ or $x = 4$.

What might appear, at first sight, as a rather trivial result provides an excellent vehicle through which to pursue this discussion. Ording's exposition of mathematical style is not a dry abstract discussion, but pursued by presenting the arguments in a wide range of styles. That is, the book puts into practice each style considered. This is a work of singular clarity of vision, executed with flair and economy. As he says: 'My motivation for this project, from the beginning to end, has been to try to conceptualize mathematics as a literary or aesthetic medium.' Indeed, in this sense, Ording is in very good company. M. Aigner and G. Ziegler [1] collect together *Proofs from THE BOOK*, as a way of discussing the aesthetic of mathematics. There have also been other collections of proofs, notably [6] (curiously not cited in this work), but I am not aware of a similar approach to the discussion of mathematical style.

Any mathematician who reads this book is likely to have favourite proofs. I particularly liked 23 Symmetry and, for somewhat personal reasons, 22 Substitution with 33 Calculus. Conversely there are some arguments which, clearly, do not merit the label 'proof' and some downright outlandish. For example, 49 Outsider reminded me very much of the work of Reddivari Sarva Jagannadha Reddy, see [5]. 99 Prescribed was somewhat predicable and nicely balanced 0 Omitted.

Given the author's clear appreciation of, and interest in, mathematical style it is hardly surprising that 2 Two-Column is accompanied by slightly pejorative comments: '...the gains in logical transparency come at a rhetorical cost. The two-column method absolves the student from having to bother with style, not to mention grammar.' Over the last four hundred years or so there has been a gradual increase in mathematical symbolism in the form of algebraic notation, through to logical symbolism. Compare 7 Found with 4 Elementary. We have made substantial progress, although there has always been some criticism of symbolism. Contemporary mathematicians regularly use position on the page to clarify and communicate meaning. For example, row operations on linear equations clarify by writing less, i.e. omitting the variables in the equations, and using only the coefficients. Furthermore, for novice students learning to write proofs for the first time (the intended audience of two-column proofs) separating out the algebraic steps, logical reasoning and proof structure is very likely to create the kind of scaffolding which helps people learn. The *expert reversal effect* [4] might help to explain why experts eschew this style in their own work.

It seems strange then that many mathematicians are, and continue to be, squeamish about more formal ways of writing proof. 18 Indented is, in my view, a more refined and promising form and has been proposed by [2] and enacted as a complete textbook [3]. This kind of formalism will be necessary if mathematics is ever going to progress from replication of moveable type in LaTeX to electronic communication which might then better interface with automated theorem checkers, or semantic search.

I am also not at all sure that 54 Arborescent does justice to the work of Gerhard Gentzen. The above discussion, in which I give voice to some of my own personal prejudices and conceits, is for me really the point of the book. This catalogue of styles provides opportunities for direct comparison, from which a more meaningful discussion can take place.

Where space permitted, each argument is given a page of its own. Given the generosity of presentation I was slightly disappointed that the corresponding discussion was on the reverse of the page, and not on the facing page. I also found the endnotes irritating[1], particularly since many of them just contain a reference requiring a further lookup in the separate reference section. But these are small niggles about an otherwise excellent book.

This is the kind of book which generates correspondence to the author. Missed proof types perhaps, comments on the text and so on. Indeed, it would be an interesting exercise to identify those styles you accept as legitimate, to choose a favourite elementary result and attempt to write your own proofs in each style. It will be interesting to see if this book starts a modest corpus of similar writing: mathematics might well be richer as a result. Requiring students to use a modest number of proof styles of the more commonly accepted kind would certainly be a useful exercise.

This is a book which will have an unusually wide readership. The theorem itself is elementary, and the discussion is clear and straightforwardly written in the best traditions of Plain English. This book could be read by any undergraduate student on a mathematics degree, or anyone teaching mathematics at school. Anyone teaching proof could profitably read this book, for examples and inspiration. Professional mathematicians will also find much of interest, and plenty of food for thought. There is a lot of history of mathematics, and interesting observations on the sociology and folk traditions of the contemporary mathematical community. I strongly recommend Ording's *99 Variations on a Proof* to any member of the LMS, and a copy should be in any serious library: this book deserves to become a classic.

FURTHER READING

[1] M. Aigner and G. Ziegler. *Proofs from THE BOOK*, Springer (2004).
[2] R. J. Back. Structured derivations: a unified proof style for teaching mathematics, *Formal Aspects of Computing*, 22(5) (2010) 629–661.
[3] R. J. Back. *Teaching Mathematics in the Digital Age with Structured Derivations*, Four Ferries Publishing (2016).
[4] S. Kalyuga, R. Rikers, and F. Paas. Educational implications of expertise reversal effects in learning and performance of complex cognitive and sensorimotor skills, *Educational Psychology Review*, 24(2) (2012) 313–337.
[5] G. Leversha. A proof of the exact value of π, by Reddivari Sarva Jagannadha, *The Mathematical Gazette*, 87(509) (2003) 368–369.
[6] E. Loomis. *The Pythagorean Proposition*, National Council of Teachers of Mathematics, Washington (1968).

Chris Sangwin

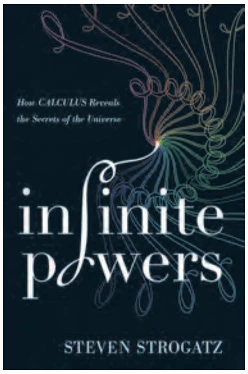
Chris Sangwin is Professor of Technology Enhanced Science Education at Edinburgh. His learning and teaching interests include (i) automatic assessment of mathematics using computer algebra, in particular the development of the STACK system, and (ii) problem solving using student-centred approaches. Chris is keen on hill walking and mountaineering.

[1] As a matter of style I think endnotes and footnotes should be avoided, (as should parenthetical comments). On reflection, it occurs to me ironic that in a book on style Ording has used endnotes in this way.

Infinite Powers: How Calculus Reveals the Secrets of the Universe

by Steven Strogatz, Atlantic Books, 2019, £20, ISBN: 978-1786492944

Review by David Singerman



This wonderful book has a rather strange title. A less glamorous, but more accurate, title would be 'History and Applications of Calculus'. The author states that 'I've written *Infinite Powers* in an attempt to make the greatest ideas and stories of calculus accessible to everyone.' He starts with a nice story: the physicist Richard Feynman met the novelist Hermann Wouk. As they parted, Feynman asked Wouk if he knew calculus. Wouk admitted he didn't. 'You had better learn it,' said Feynman, 'It's the language God talks.' This book is written for the Wouks of this world.

It is explained that calculus is really about understanding infinity. Near the beginning he states his big idea: 'To shed light on any continuous shape, object, motion, process, or phenomenon – no matter how wild and complicated it might appear – imagine it as an infinite series of simpler parts, analyze these, and then add the results together to make sense of the original whole.'

As I wrote, Strogatz takes a historical view of the subject so he starts with the ancient Greeks and then Archimedes. He writes that Archimedes calculated pi (he uses pi rather than π) by considering a circle with a large number of sides. Pi is fundamentally a child of calculus. It is defined as the unattainable limit of a never-ending process. The infinity principle is the key to unlocking the mystery of curves and it arose first in the mystery of pi.

The author states that Archimedes is one of the originators of integral calculus. To find the area of a parabolic segment he imagines it as being infinitely many triangular shards glued together, basically the start of integral calculus. He now tells an amazing story: twenty-two centuries ago Archimedes wrote a letter to Eratosthenes outlining what he called his *Method*. This was thought to have been lost. Then in 1998, a battered medieval prayer book came up for auction at Christie's. Barely visible under its Latin prayers were faint geometrical diagrams and mathematical text. Archimedes' *Method* had been found!

In this historical account, the author then jumps 1800 years to Galileo in Renaissance Italy. In a chapter called 'Discovering the laws of motion' the work of Galileo is discussed. He was a champion of the Copernican theory that the earth moves round the sun, bringing him in conflict with the Catholic Church. Einstein felt that Galileo was the founder of modern physics. Galileo's final book, *Discourses and Mathematical Demonstrations Concerning Two New Sciences*, was written when he was under house arrest after falling foul of the inquisition. On page 66 it is stated that 'Its radical insights helped launch the scientific revolution and brought humanity to the cusp of discovering the secrets of the universe'.

Of course, one of Galileo's most famous discoveries was about the pendulum, when he noticed that it always took the same time to complete its swing whether it transversed a small arc or a big one. This idea gave humanity the first way to measure time accurately. Strogatz then goes on to discuss the Longitude Problem and on to the present day and GPS, which he states is a prime example of the hidden uses of calculus. From Galileo Strogatz proceeds to Kepler, 'What Galileo did for the motion of objects on earth, Kepler did for the motions of planets in the universe.' Kepler's three laws of planetary motion are described and then we move onto Descartes and Fermat. Fermat paved the way for calculus in its modern form with his work on tangent lines.

This is the end of the first part of the book. We are at Chapter 5 'The Crossroads', which is where calculus becomes modern. 'It is where calculus starts to wonder about the rhythms of the universe, its ups and downs, its ineffable patterns of time. In the four centuries since calculus reached this crossroads it has branched out from algebra and geometry to physics and astronomy, biology and medicine, engineering and technology.'

The author now does some mathematics, which he explains simply. We are introduced to power functions, exponential functions and logarithms, and then in the next chapter, 'The vocabulary of change', he introduces the derivative $\frac{dy}{dx}$.

Strogatz explains the three central problems of calculus (1) The forward problem: given a curve, find its slope everywhere; (2) the backward problem: given a curve's slope, find the curve; and (3) the area problem: given a curve, find the area under it.

We now know that these problems are linked by the Fundamental Theorem of Calculus which he attributes to Newton and Leibniz. This made the problem of finding areas under curves, which went back to Archimedes, rather routine. 'The theorem connected areas to slopes and thereby linked integrals to derivatives. Like a twist out of a Dickens novel, two seemingly distinct characters were the closest of kin.' He spends some time carefully explaining the Fundamental Theorem.

Most of the rest of the book is devoted to the applications of calculus. I point out a few which particularly interested me. One was the work of the African-American mathematician Katherine Johnson. While working for NASA she used calculus to compute the angle at which a spacecraft re-enters the atmosphere so it does not burn up. This was to bring back John Glenn, the first American to orbit the earth, and was the subject of the movie *Hidden Figures* which I wrote a review of in the April 2017 *Newsletter*.

One of the more surprising applications of calculus is in fighting HIV. Patients were given drugs called protease inhibitors. The viral load V is a function of the time t since the drug was administered. This obeys a differential equation which after solving showed that V decreased exponentially fast, which helped to lead to development of treatments. Similar equations apply to the Hepatitis C virus. (As your reviewer had this virus for 40 years, I now know that my life was saved by a differential equation!).

Another of the biological applications is to the structure of DNA. In reality DNA is a discrete collection of atoms so there is nothing continuous about it. But a good approximation of it can be treated as though it is a continuous curve. As Strogatz writes on page 276, 'I expect in the future we will see many more examples of calculus being brought to bear on the inherently discrete players of biology. There is simply too much insight to be gained from the continuous approximation not to use it.'

This is a marvellous book about calculus. It does not teach the techniques e.g. how to differentiate of integrate; there are thousands of books that do this. As the author writes 'My goal in this book has been to show calculus as a whole, to give a feeling of its beauty, unity and grandeur.' This is a book that should be recommended to students studying the history of mathematics. It will also provide a wider vision to calculus teachers and their students.

David Singerman

David Singerman is an emeritus professor at the University of Southampton. His main interests have been on Fuchsian groups and Riemann surfaces, in particular the theory of maps (or *dessin d'enfants*) on Riemann surfaces.

Obituaries of Members

Walter K. Hayman: 1926 – 2020

Professor Walter Kurt Hayman FRS, who was elected a member of the London Mathematical Society on 20 March 1947, died on 1 January 2020, aged 93. Professor Hayman was LMS Vice President (1982–84) and was awarded the Society's Junior Berwick Prize (1955), Senior Berwick Prize (1964) and De Morgan Medal (1995). He was elected to the Royal Society in 1956, aged just 30, and was the second longest serving member of both the Royal Society and the LMS.

Phil Rippon writes: Walter Hayman was a world-leading complex analyst who, in addition to making outstanding contributions to his subject, created a strong complex analysis group at Imperial College, London, which was for about thirty years a key international centre for research in the subject. At IC the weekly 'Hayman seminar' was famously the place where complex analysts the world over came to speak and visit, to work with Hayman and learn from him. At times a formidable presence, he became very much the father figure of his subject, with a great sense of responsibility towards colleagues and students, who held him in the greatest esteem and fondest regard. MathSciNet records Hayman as writing over 200 papers with 78 collaborators, and of his twenty PhD students, nine went on to chairs.

Hayman's childhood was spent in Germany. His grandfather was the mathematician Kurt Hensel whose grandmother was the composer and pianist Fanny Mendelssohn, sister of Felix. Though of the protestant religion, the Hayman family ancestry was Jewish and in 1938 he was sent to the UK to study at Gordonstoun School. From there he went to St John's College, Cambridge, and on graduation his research was supervised by Mary (later Dame Mary) Cartwright and also influenced by JE Littlewood. He was awarded the first Smith's Prize in 1948 and shared the 1949 Adams Prize. He then held posts at Newcastle and Exeter before moving to Imperial College in 1956 as its first Professor of Pure Mathematics, where he was head of the pure maths section for many years and also served as Dean of the Royal College of Science (1978-1981). Retiring in 1985, he held a half-time professorship at the University of York for some years, before returning to Imperial College as a Senior Research Fellow in 1994.

Hayman had been encouraged by Littlewood to attack what were seen as the most difficult problems in complex analysis, often related to finding the very best estimates for the rate at which some complex quantity of interest can grow. His work is characterised by the use of novel insights and highly technical arguments deployed in order to arrive at the sharpest possible results, often with ingenious examples to demonstrate sharpness. As one example, in 1955 he gave a technically brilliant proof of the 'asymptotic Bieberbach conjecture'. The Bieberbach conjecture concerns the size of the coefficients of the power series for any univalent (i.e. one-to-one and analytic) function defined in the unit disc and Hayman's result stood as the strongest general result on the conjecture until the full conjecture was proved by Louis de Branges in 1984; moreover, for most coefficients Hayman's result is stronger than Bieberbach conjectured. Other influential results include the so-called 'Hayman alternative' in the value distribution theory of meromorphic functions (his most highly cited paper), his remarkable estimates for the minimum modulus of large entire functions, and the Hayman-Wu theorem on the lengths of level sets of univalent functions.

In addition to his research papers, which solved hard problems across the full range of topics in analytic functions of one complex variable, as well as for harmonic and subharmonic functions in higher dimensions, Hayman published influential monographs: *Multivalent Functions* (1958; 2nd edition 1994), *Meromorphic Functions* (1964) and *Subharmonic Functions* (Vol 1, with P.B. Kennedy, 1976; Vol 2, 1989). Written with great care and clarity, these texts helped shape future research directions and continue to be standard sources for the fundamental results and techniques needed in complex analysis research, and for applications of the subject. Also highly influential are Hayman's lists of open problems in complex analysis. Starting with his *Research problems in function theory* (1967), he was the main author of several additions to this first list, often compiled at conferences, culminating in the publication in September 2019 of the Springer text of the same title, authored jointly with Eleanor Lingham, containing updates (where known) to previous problems and a selection of new problems.

Hayman was an invited speaker at the International Congress of Mathematicians in 1954 and 1970, a British Mathematical Colloquium speaker on four occasions, and served on the Council of the Royal Society (1962-63). He received honorary degrees from the University of Exeter (1981), the University of Birmingham (1985), Uppsala University (1992), Giessen University (1992) and the National University of Ireland (1997), and he was elected to the Finnish Academy of Science and Letters (1978), the Bavarian Academy (1982), and the Accademia dei Lincei of Rome (1985).

Outside mathematics research, Hayman had a profound interest in and knowledge of music, and colleagues at Imperial College and at conferences have fond memories of him accompanying them on the piano. In 1966, Hayman and his wife Margaret founded the British Mathematical Olympiad and ran its International Olympiad team for many years, the UK being one of the first western countries to participate in this. He was also much involved in supporting the human rights of individuals around the world, an instinct he attributed in part to his own background as a refugee in 1938.

Finally, in 2014 Hayman published an autobiography entitled *My Life and Functions*, covering in typically honest fashion his mathematical career, as well as the ups and downs of his life experiences, relationships, and views on topics from music to religion. The book includes many of the anecdotes and witticisms that he was well known for and provides a fascinating account of his unusual and highly influential mathematical life.

Walter Hayman was widowed three times and is survived by his daughters Daphne, Carolyn and Sheila, his six grandchildren, and five great-grandchildren.

Peter W. McOwan: 1962 – 2019

Peter McOwan, who was elected a member of the London Mathematical Society on 19 November 1999, died on 29 June 2019, aged 57.

Colin Bailey writes: Peter made a profound contribution to Queen Mary University of London. He joined Queen Mary in January 2000, became a Professor of Computer Science in 2006, and served as Vice-Principal (Public Engagement and Student Enterprise) between 2012 and 2018.

In some ways, one can think about Peter as having had two successful careers: firstly as an internationally renowned scholar of Computer Science — a scientist operating very close to the top of his profession, who attracted millions of pounds of research funding — and secondly as an innovative and creative communicator, with a particular flair for engaging with children and young people.

Indeed, it is this second area of work that Peter became best known, and where his impact will be felt forever at Queen Mary. His use of magic as an inspiration to explain and provoke thought about mathematics and computer science is widely cited as a textbook example of public engagement — reducing incredibly complex mathematical phenomena into exciting, accessible, and explainable concepts. He wrote several free mathematics magic books and co-founded the magazine *Computer Science for Fun* (CS4FN) which promotes Computer Science in schools. CS4FN went on to have a global impact and still does today.

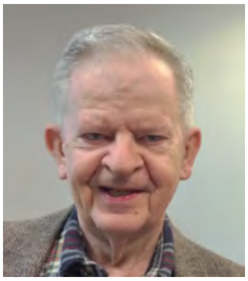
Peter's inspirational leadership led to Queen Mary achieving the distinction of becoming the first UK University to achieve an Engage Watermark Gold Award — the equivalent of a royal charter mark — for its work in public engagement.

Peter was a serious researcher of biologically-inspired computing, though he found fun twists to everything he did. He worked on many topics including understanding how we really 'see' the world and how, in the future, we will live with robots, as well as intriguing applications of artificial intelligence.

A lot of Peter's work was about the mathematical modelling of the human vision system and, together with his PhD students and Professor Alan Johnston at UCL, he created a computational model that could 'see' as a human saw things. The model originally appeared to have a bug in it as it said one test pattern was moving when it was not. After failing to find the bug they realised it had discovered an optical illusion as people thought the pattern moved too. This suggested that it was seeing the world the way a person does. This work was presented at the Royal Society Summer Exhibition and his research was selected four times for presentation in total. The technology was also commercialised into a Spinout company called DragonFly. As with much of Peter's work, his incredible personal drive and energy was not devoted to personal accolades or grand-standing, but putting in place structures and people to ensure that his work endured in a sustainable way.

More than anything, however, Peter will be remembered for his generosity of spirit, his enthusiasm, his humour, and his warm and gentle nature. It is no exaggeration to say that we will never see the like of Peter again. He was such a special individual, and he will be greatly missed by all of us.

Richard Askey: 1933 – 2019



Professor Richard Askey, sometime member of the LMS, died on 9 October 2019.

George Andrews writes: Richard Allen (Dick) Askey was born on 4 June 1933, in St. Louis, Missouri, to Philip and Bessie Askey. He received his BA from Washington University in 1955, an MA from Harvard in 1956 and from there went to Princeton University where he received his PhD in 1961 under the direction of Salomon Bochner. His thesis was titled *Mean Convergence of Orthogonal Series and Conjugate Series*, foreshadowing extensive and important contributions to the study of orthogonal polynomials.

After instructorships at Washington University and the University of Chicago, Dick joined the faculty of the University of Wisconsin, Madison in 1963 where he remained for 40 years until retirement in 2003. At Madison, he was named Gábor Szegő Professor of Mathematics in 1986 and in 1995 was awarded a John Bascom Professorship.

His many honors included Fellowships in the American Academy of Arts and Sciences (1993), the National Academy of Sciences (1999), the Society of Industrial and Applied Mathematics (2009), and the American Mathematical Society (2012). Also, he was appointed an Honorary Fellow of the Indian Academy of Sciences and received an honorary doctorate from SASTA University in India (2012). He was an invited speaker at the ICM in Warszawa (1983).

Dick's impact and influence in the world of special functions and orthogonal polynomials cannot be overstated. His early work with George Gasper yielded an inequality that was central to Louis deBrange's proof of the Bieberbach conjecture. In the mid–1970's, building on long-neglected work of L.J. Rogers, Dick wrote two AMS memoirs, the first with Mourad Ismail and the second with Jim Wilson. The latter memoir not only introduced the now widely used Askey-Wilson polynomials but also included the famous Askey-scheme which provided a hierarchical classification of the classical orthogonal polynomials. The influence of L.J. Rogers is also especially evident in the series of papers Sieved Orthogonal Polynomials, a topic initiated by Askey and extended by Ismail and others. This latter series pointed to a new connection with number theory which is in its infancy.

Dick was not just a powerful researcher. He was a charismatic force in the mathematics community generally. Early in his career, he determined that the three-volume work, *Higher Transcendental Functions*, (a.k.a. the Bateman Project) needed to be extensively updated. His vision and persistence inspired many, and it is not unreasonable to suggest that the Digital Library of Mathematical Functions owes much to the influence of Richard Askey.

Dick was also passionate in his efforts to improve mathematics education at all levels. He was extremely skeptical of many fads that promised instant improvement and delivered little.

All of the above points to the importance of Askey's contributions; however, the charm, the kindness and the mischievous sense of humor are perhaps impossible to convey here. I will close with a short anecdote that at least hints at these latter qualities:

In the early 1990s, Paul Halmos wrote an article for the MAA Focus entitled *The Calculus Turmoil*. I thought it was a wonderful defense of the traditional teaching of calculus and wrote to tell him so. Halmos wrote back an equally warm response. I then called Dick to discuss the article and to tell him of my exchange with Halmos. Dick's first words were, "I hated that article". Adrenaline surged in my system. It seemed to me there were only two possible reasons for Dick's words. Either I had completely misread the article (unlikely), or this was going to be my first real educational clash with Dick. Having seen how effective he was in debate, I was not looking forward to the rest of the conversation. I managed to stammer out: "What makes you say that?" He replied, "I HATED it! I cannot stand to agree with a single word Paul Halmos writes, and I agreed with every word in that article."

Wilson Alexander Sutherland: 1935 – 2019

Wilson Sutherland, who was elected a member of the London Mathematical Society on 18 April 1963, died on 7 October 2019 not long after his 84th birthday.

Brian Steer writes: Wilson was born in Forres on the Moray Firth in 1935 into a modest family. Like his sister Jean before him he attended the local school. There he gained a scholarship to St. Andrews University and was one of a distinguished year. On graduation he was awarded a Carnegie scholarship for graduate work in Oxford and in October 1957 joined the Geometry and Topology group, then directed by Henry Whitehead and Ioan James. The latter became his supervisor. His abilities, and particularly his clarity of exposition, were quickly realized and so it was not a surprise to his fellow students when he was appointed Junior Lecturer in 1960.

In 1963 he travelled to the USA to take up an Instructorship at MIT, returning to a post in Manchester. He did not stay there long, for in 1967 he was elected a Fellow of New College Oxford. At New College he acted as Vice-Warden for a period of time and later, on retirement, was voted unanimously to the one year post of Acting Warden. He served on the Board of the Faculty of Mathematics and was, with Edward Thompson, for many years editor of the Quarterly Journal of Mathematics.

As a College Fellow, Wilson's duties were to teach small classes, to lecture, and to carry out research. I believe that with his natural modesty and his sense of service he gave greater importance to his teaching than to his research. He was an outstanding tutor and cared about his pupils. They in their turn responded with affection. Past pupils have insisted on this and I have heard that at his retirement lunch New College hall was full to overflowing with past and present pupils. Something of his clarity and style can be seen in his book *Introduction to Topological and Metric Spaces* (published by OUP). It is now in a second edition and a standard text for many universities.

He wrote several notable papers. The majority were in classical homotopy theory and concerned Stiefel manifolds, Thom complexes and sections of bundles. (There is a particularly fine one on the Brown–Browder–Dupont invariant.) Later years saw a long and fruitful collaboration with Michael Crabb.

In Oxford Wilson neither lost his Morayshire accent — though maybe it was a little dulled — nor his beguiling smile. His wife Ruth, whom he married in 1964, supported him in all he did as, indeed, he did her. Both had a deep attachment to St. Columba's, the United Reformed Church in central Oxford. Wilson became an Elder and served for many years as secretary, whilst Ruth took part in "Homestart", a scheme for young families in difficulty and also helped another charity, this one for the lonely and homeless.

Wilson had many talents not mentioned above: he was a good cricketer and played the violin — or fiddle as he would say — very well. He was deeply attached to his Scottish mountains and to the Lakeland fells. Above all he was a modest and a warm caring man with a strong sense of service and a gentle sense of humour, to me a wonderful colleague and friend and to his family a beloved husband, father and grandfather.

LMS Meeting

LMS Invited Lecture Series 2020

30 March – 3 April 2020, Brunel University London

Website: boguslavsky.net/lms2020

The invited Lecturer will be Professor Yulia Mishura (Taras Shevchenko National University of Kyiv), who will talk on *Fractional Calculus and Fractional Stochastic Calculus, Including Rough-Paths, with Applications*. Accompanying Lecturers will be Elena Boguslavskaya (Brunel University London), Vassili Kolokoltsov (Warwick University), Nikolai Leonenko (Cardiff University), Joseph Lorinczi (Loughborough University), Hao Li (UCL) and Enrico Scalas (Sussex University). Funds are available for partial support to attend; email the organiser, Dr Elena Boguslavskaya (Elena.Boguslavskaya@brunel.ac.uk) with an estimate of expenses. Visit the website for further details and to register.

Topics in Category Theory: A Spring School

Location: ICMS, Edinburgh
Date: 11–13 March 2020
Website: tinyurl.com/rnhslec

This meeting will unite PhD students and junior researchers who use category-theoretic ideas or techniques in their work. The emphasis is on interactions of category theory with geometry, topology, algebra and logic. Mini-courses will be presented by Olivia Caramello, Constanze Roitzheim and Greg Stevenson. There will be short talks contributed by PhD students and postdocs.Supported by an LMS Scheme 8 grant.

International Network for Didactic Research on University Mathematics

Location: Bizerte, Tunisia
Date: 27–29 March 2020
Website: tinyurl.com/vd9qt6s

The 3rd conference of the International Network for Didactic Research in University Mathematics (INDRUM) will bring together mathematics education researchers and mathematicians from around the world with an interest in the teaching and learning of mathematics at university level. The conference themes will focus on teacher and learner practices at university level as well as the teaching and learning of specific mathematical topics at undergraduate and post-graduate level across disciplines.

Recent Developments in Copula Research

Location: Durham University
Date: 25 March 2020
Website: tinyurl.com/rj32blr

The meeting aims to discuss recent advances in copula research. Copulas have become popular tools for modelling dependence between random quantities in many application areas. Copulas are attractive due to their ability to model dependence between random quantities separately from the marginal distributions. Talks will be given by Aristidis Nikoloulopoulos, Tim Bedford and Tahani Coolen-Maturi. Those interested in attending should email tahani.maturi@durham.ac.uk by 4 March.

Young Functional Analysts' Workshop

Location: Lancaster University
Date: 31 March–2 April 2020
Website: tinyurl.com/yce6j3gy

YFAW is an event aimed at early-stage researchers (PhD students and postdocs) in functional analysis and related areas. It is an opportunity to bring together people who will present their work in front of peers from related areas of study, and so participants are encouraged to present their work in front of a sympathetic audience, but those who prefer not to are always welcome. Supported by MAGIC, Lancaster University, Elsevier/Mendeley and an LMS Scheme 8 grant.

Random Graphs and Random Processes

Location: King's College London
Date: 3 April 2020
Website: tinyurl.com/vg6kwne

The workshop looks at recent work in the area of random structures and algorithms and random processes on networks. Examples are properties of discrete random structures, random walks and interacting particle systems, threshold behaviour, design of randomized algorithms, and applications.

ICFT 2020 Conference

Location: ICMS, Edinburgh
Date: 3–4 April 2020
Website: tinyurl.com/sb6jl5k

This is the annual UK meeting on Integrable Models, Conformal Field Theory and Related Topics (ICFT). The meeting will both showcase the novel work of internationally-leading speakers and give young researchers the opportunity to present. The meeting is supported by an LMS conference grant.

LMS Meeting

LMS Meeting at Joint BMC–BAMC

8 April 2020; 11:30am, University of Glasgow

Website: tinyurl.com/yarpowdo

The Opening of the Meeting & Society Business is at 11:30, followed by the LMS Plenary Lecture given by Catharina Stroppel (University of Bonn). The lecture is aimed at a general mathematical audience. All interested, whether LMS members or not, are welcome to attend this event. This meeting takes place during the Joint British Mathematical Colloquium–British Applied Mathematical Colloquium from 6–9 April 2020.

Mirror Symmetry, Degenerations, and Fibrations Workshop

Location: Loughborough University
Date: 16 April 2020
Website: tinyurl.com/sv8nfu7

This afternoon workshop will focus on recent developments in the geometric theories of degenerations and fibrations, and how mirror symmetry relates them. Talks will be given by Charles Doran, Sara Filippini, and Alan Thompson. Some financial support is available for PhD students. Visit the website for more information and to register. Supported by an LMS Scheme 9 grant and the Loughborough Department of Mathematical Sciences.

LMS–IMA David Crighton Award: Lecture and Medal Presentation

Location: Royal Society, London
Date: 23 April 2020, 6:30pm
Website: tinyurl.com/t2fzjm5

The David Crighton Lecture, *Mathematical Science PhDs — Past, Present and Future*, will be given by Professor Ken Brown CBE FRSE (University of Glasgow), winner of the 2019 Award. Admission to the lecture and reception is by ticket only; email Alison Penry (alison.penry@ima.org.uk) by 6 April 2020. Please confirm whether you would like to attend the lecture and reception, or the lecture only. Admission is free of charge.

Postgraduate Combinatorial Conference

Location: University of Glasgow
Date: 27–29 April 2020
Website: tinyurl.com/tw9r777

The PCC is an established conference organised for, and by, current research students in all areas of combinatorial and discrete mathematics, under the auspices of the British Combinatorial Committee. It will co-locate with the Scottish Combinatorics Meeting. The PCC is mainly aimed at UK-based students but is also open to those from abroad.

Microlocal Analysis and PDEs

Location: University College London
Date: 29 April–1 May 2020
Website: tinyurl.com/yftnf92o

This workshop is devoted to recent trends and developments in microlocal analysis and partial differential equations. It aims to bring together different communities working in microlocal analysis, to share ideas and establish new collaborations. Limited funds are available to support the participation of junior researchers. Supported by an LMS Conference grant.

Swinnerton-Dyer Memorial Workshop

Location: Newton Institute, Cambridge
Date: 5–7 May 2020
Website: tinyurl.com/vpflew4

The meeting will celebrate the wide-reaching contributions to mathematics of Peter Swinnerton-Dyer, one of the most influential number theorists of his generation. He is best known for the conjecture of Birch and Swinnerton-Dyer, but he was also one of the founding figures in the arithmetic of surfaces and higher-dimensional varieties.

Wales Mathematics Colloquium 2020

Location: Gregynog Hall, Tregynon
Date: 18–20 May 2020
Website: gregynogwmc.github.io/

The Colloquium is a forum for the promotion and discussion of current research in Mathematics in Wales. The invited speakers are Chris Breward, Brita Nucinkis and Stefan Weigert. Attendance and contributed talks in any area of Mathematics are very welcome. Supported by an LMS Conference grant.

Scottish Combinatorics Meeting

Location: University of Glasgow
Date: 30 April–1 May 2020
Website: tinyurl.com/scotcomb

This event will feature eight invited talks, a session on applications of Combinatorics, and opportunities for research students to present their work. Attendance is free but participants are asked to register via the website for catering purposes. Limited travel support is available for research students. Supported by an LMS Conference Grant.

Partial Differential Equations and Fluid Mechanics

Location: University of Bath
Date: 4–7 May 2020
Website: tinyurl.com/tjmhfy7

A conference on rigorous methods for nonlinear problems in PDEs and fluid mechanics, marking the 71st birthday of Professor John Toland FRS, his contributions to PDEs and fluid mechanics. Featuring mini-courses aimed at a general mathematical audience on topics from water waves to the Navier–Stokes equations.

Burnside Rings for Profinite Groups

Location: Lancaster University
Date: 7 May 2020
Website: tinyurl.com/y4mnkdjx

This meeting will tie in with our previous group meeting on Mackey functors for profinite groups and will focus on the generalisation of Burnside rings from finite to profinite groups and their applications in representation theory in particular. Funding is available for PhD students. Supported by an LMS Research grant.

Integral Equations and Operator Theory Workshop

Location: University of Reading
Date: 18–22 May 2020
Website: tinyurl.com/yfmv8e2h

This workshop brings together experts in the areas of integral equations and operator theory who use methods of complex analysis, linear algebra, functional analysis and numerical analysis to address problems in mathematical physics, random matrix theory and engineering. Registration deadline: 30 April 2020. Supported by an LMS Conference grant.

Young Researchers in Mathematics

Location: University of Bristol
Date: 8–10 June 2020
Website: tinyurl.com/rg7rr3b

This conference is for all UK PhD students, and an opportunity to meet researchers from all areas in a welcoming environment. There will be opportunities to present your own work during the conference, and attend the plenary talks.

Tropical Geometry, Berkovich Spaces, Arithmetic D-modules and p-adic Local Systems

Location: Imperial College, London
Date: 15 – 19 June 2020
Website: tinyurl.com/stxlp7w

This conference will bring together experts to explore the role of tropical geometry in the application of Berkovich theory to areas such as arithmetic D-modules and non-archimedean representation theories.

Hanna Neumann Day

Location: University of Hull
Date: 12 June 2020
Website: tinyurl.com/rnm2bdc

Hanna Neumann (1914-71) was an outstanding mathematician whose research has significantly influenced the development of modern group theory. This conference includes lectures and a number of public events.

7th IMA Conference on Numerical Linear Algebra and Optimization

Location: Birmingham
Date: 24–26 June 2020
Website: tinyurl.com/yxfgxy4y

The success of modern codes for large-scale optimization is heavily dependent on the use of effective tools of numerical linear algebra. The purpose of the conference is to bring together researchers from both communities and to find and communicate points and topics of common interest.

Point Configurations: Deformations and Rigidity

LMS Research School

University College London
20-24 July 2020

Organisers: Codina Cotar (UCL) and Mircea Petrache (PUC)

The aim of the research summer school is to present several different modern perspectives on the rigidity and deformability of optimum point configurations. Three main points of view are presented in the lecture courses: the point of view of material sciences and elasticity; the point of view of approximation theory; and the point of view of the methods of Viazovska, based on linear programming bounds.

The three main lecture course topics are:

- *Crystallization in classical particle systems* (Gero Friesecke, Technical University of Munich)
- *Discrete Energy on Rectifiable Sets* (Douglas Hardin and Edward Saff, Vanderbilt University)
- *Modular forms, universal optimality and Fourier interpolation* (Danylo Radchenko, ETH Zurich)

These lecture courses will be supplemented by tutorial sessions.

Additionally, there will be three plenary talks by: Keith Ball (Warwick University), Henry Cohn (Microsoft Research New England) and Sylvia Serfaty (New York University).

For further information on the Research School please visit: tinyurl.com/rsucl2020.

Apply for a place on the Research School

Research students, post-docs and those working in industry are invited to apply at https://www.surveymonkey.co.uk/r/RS51ApplnForm. Applications will be reviewed starting from March 31st 2020 and on a rolling basis until all places are filled, with a processing time of at most two weeks; information about individual applications will not be available before April 14th 2020.

Registration Fees:

- Research students*: £150 (no charge for subsistence costs)
- Early career researchers**: £250 (no charge for subsistence costs)
- Other participants: £250 (plus subsistence costs)

* defined as MSc students and PhD students.
** defined as within five years of completing their PhD (excluding career breaks).

Fees are not payable until a place at the Research School is offered but will be due by June 20th, 2020.

Financial Aid (towards travel costs and/or registration fees): Research students who have successfully secured a place on the Research School following the instructions above, will then be invited to apply for financial aid using the Financial Aid Application form, which will be sent to them by the organisers. Details on the information that will be required appear at tinyurl.com/mve2fxb.

__LMS Research Schools__ provide training for research students in all contemporary areas of mathematics. Students and post-docs can meet a number of leading experts in the topic as well as other young researchers working in related areas. The LMS is the UK's learned society for mathematics. Registered charity no. 252660 (www.lms.ac.uk)

Society Meetings and Events

March 2020

3-13 Apr Invited Lecture Series 2020, Brunel University

April 2020

8 Society Meeting at the Joint BMC–BAMC, Glasgow
19-25 LMS Research School, Graph Packing, Eastbourne
23 LMS–IMA David Crighton Lecture and Presentation, London

May 2020

11-15 LMS Research School, Methods for Random Matrix Theory & Applications, University of Reading

July 2020

20-24 LMS Research School, Point Configurations: Deformations and Rigidity, University College London

Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website (www.lms.ac.uk/content/calendar). Please send updates and corrections to calendar@lms.ac.uk.

March 2020

11-13 Topics in Category Theory, ICMS, Edinburgh (487)
16-20 Interactions between Group Theory, Number Theory, Combinatorics and Geometry, INI Cambridge (485)
23-27 Algebraic K-theory, Motivic Cohomology and Motivic Homotopy Theory, INI Cambridge (485)
25 Recent Developments in Copula Research, Durham University (487)
26 Mathematics Adapting to a Changing World, Imperial College, London (486)
27-29 International Network for Didactic Research on University Mathematics, Bizerte, Tunisia (487)
30-1 Apr New Perspectives on SYZ Mirror Symmetry, Imperial College London (486)
30-2 Apr Young Functional Analysts' Workshop, Lancaster University (487)
30-3 Apr UK Easter Probability Meeting , University of Manchester (486)
30-3 Apr LMS Invited Lecture 2020, Brunel University London (487)
30-3 Apr Arithmetic Geometry, Cycles, Hodge Theory, Regulators, Periods and Heights, INI Cambridge (485)

April 2020

1-3 UKACM 2020 Conference, Loughborough University (486)
3 Random Graphs and Random Processes, King's College London (487)
3-4 ICFT 2020 Conference, ICMS, Edinburgh (487)
6-9 Joint BMC/BAMC Meeting, University of Glasgow (485)
7-8 Group Theory at BMC 2020, Glasgow (486)
8 LMS Meeting at the Joint BMC-BAMC, Glasgow (487)
15-16 Algebraic Groups and Geometric Invariant Theory, University of Essex

16 Mirror Symmetry, Degenerations, and Fibrations Workshop, Loughborough University (487)
19-25 Graph Packing, LMS Research School, Eastbourne (486)
23 LMS–IMA David Crighton Lecture and Presentation, London (487)
27-29 Postgraduate Combinatorial Conference, University of Glasgow (487)
29-1 May Microlocal Analysis and PDEs, University College London (487)
30-1 May Scottish Combinatorics Meeting, University of Glasgow (487)

May 2020

1-3 UKACM 2020 Conference, Loughborough University (486)
4-7 Partial Differential Equations and Fluid Mechanics, University of Bath (487)
5-7 Swinnerton-Dyer Memorial Workshop, Newton Institute, Cambridge (487)
7 Burnside Rings for Profinite Groups, Lancaster University (487)
6-9 Joint BMC/BAMC Meeting, University of Glasgow (485)
11-13 Heilbronn Distinguished Lecture Series 2020, University of Bristol (487)
11-15 Methods for Random Matrix Theory & Applications, LMS Research School, University of Reading (486)
18-20 Wales Mathematics Colloquium 2020, Gregynog Hall, Tregynon (487)
18-22 Counting Conjectures and Beyond, INI, Cambridge (486)
18-22 Integral Equations and Operator Theory Workshop, University of Reading (487)

June 2020

1-5 Mathematical Physics: Algebraic Cycles, Strings and Amplitudes, INI, Cambridge (486)
8-10 Young Researchers in Mathematics 2020, University of Bristol (487)
11 Mary Cartwright Lecture, Professor Dorothy Buck, De Morgan House, London (487)
15-19 Tropical Geometry, Berkovich Spaces, Arithmetic D-modules and p-adic Local Systems, Imperial College, London (487)
12 Hanna Neumann Day, University of Hull (487)
24-26 7th IMA Conference on Numerical Linear Algebra and Optimization, Birmingham (487)

July 2020

5-11 8th European Congress of Mathematics, Portorož, Slovenia (486)
12-19 14th International Congress on Mathematical Education Shanghai, China
14-16 IMA Modelling in Industrial Maintenance and Reliability Conference, Nottingham (486)
20-24 Point Configurations: Deformations and Rigidity, LMS Research School, University College London (487)
24-26 7th IMA Conference on Numerical Linear Algebra and Optimization, Birmingham (487)
27-7 Aug Integrable Probability Summer School, University of Oxford

July 2020

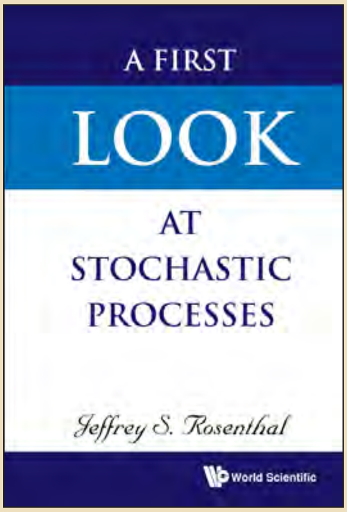
24-30 27th International Mathematics Competition for University Students, Blagoevgrad, Bulgaria (487)
27-7 Aug Integrable Probability Summer School, University of Oxford (487)

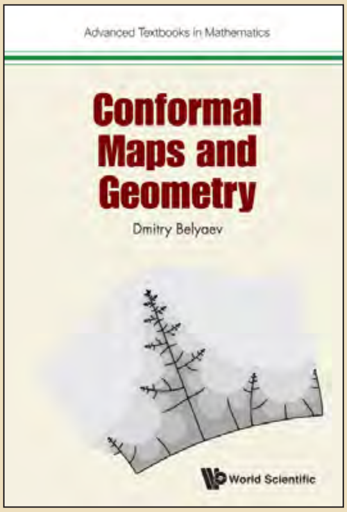
August 2020

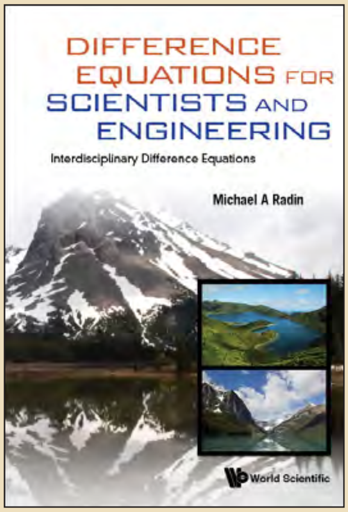
17-21 IWOTA 2020, Lancaster University (481)

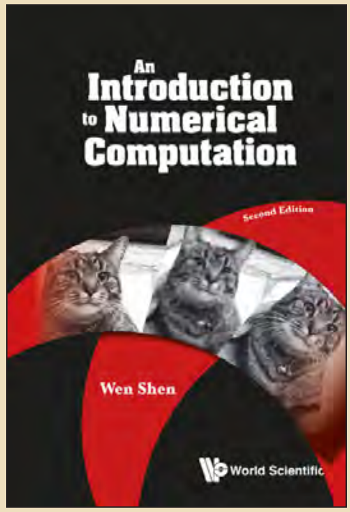
September 2020

22-24 Conference in Honour of Sir Michael Atiyah, Isaac Newton Institute, Cambridge (487)









A First Look at Stochastic Processes

Written by COPSS Presidents' Award Winner

by **Jeffrey S Rosenthal** (*University of Toronto, Canada*)

This textbook, for senior undergraduate and graduate students introduces the theory of stochastic processes. Using concrete examples like repeated gambling and jumping frogs, it presents fundamental mathematical results through simple, clear, logical theorems and examples. The focus is always on making the theory as well-motivated and accessible as possible, to allow students and readers to learn this fascinating subject as easily and painlessly as possible.

212pp | Oct 2019 | 9789811207921(ebook) | £25

Conformal Maps and Geometry

by **Dmitry Beliaev** (*University of Oxford, UK*)

This textbook provides an accessible foundation of the theory of conformal maps and their connections with geometry. It offers a unique view of the field, as it is one of the first to discuss general theory of univalent maps at a graduate level, while introducing more complex theories of conformal invariants and extremal lengths.

240pp | Dec 2019 | 9781786346155(ebook) | £30

Difference Equations for Scientists and Engineering
Interdisciplinary Difference Equations

by **Michael A Radin** (*Rochester Institute of Technology, USA*)

We introduce interdisciplinary research and get students and the audience familiarized with the difference equations; solving them explicitly, determining the long-term behavior of solutions. We help to develop intuition in analyzing convergence of solutions in terms of subsequences and analyzing patterns of periodic cycles. Our book helps you learn applications in biology, economics and business, computer science and engineering.

332pp | Oct 2019 | 9789811202988(ebook) | £40

An Introduction to Numerical Computation
(2nd Edition)

by **Wen Shen** *(Penn State University, USA)*

"For a compact volume, the homework is ample and well-conceived. Homework problems concluding each chapter include applications, programming tasks, and a list of items to turn in. A complete set of solutions is available for instructor upon request... This is an excellent resource as a semester-long text, textbook adjunct, introduction for self-instruction, or a handy reference for practical implementations."

MAA Reviews

340pp | Sep 2019 | 9789811204432(ebook) | £35

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