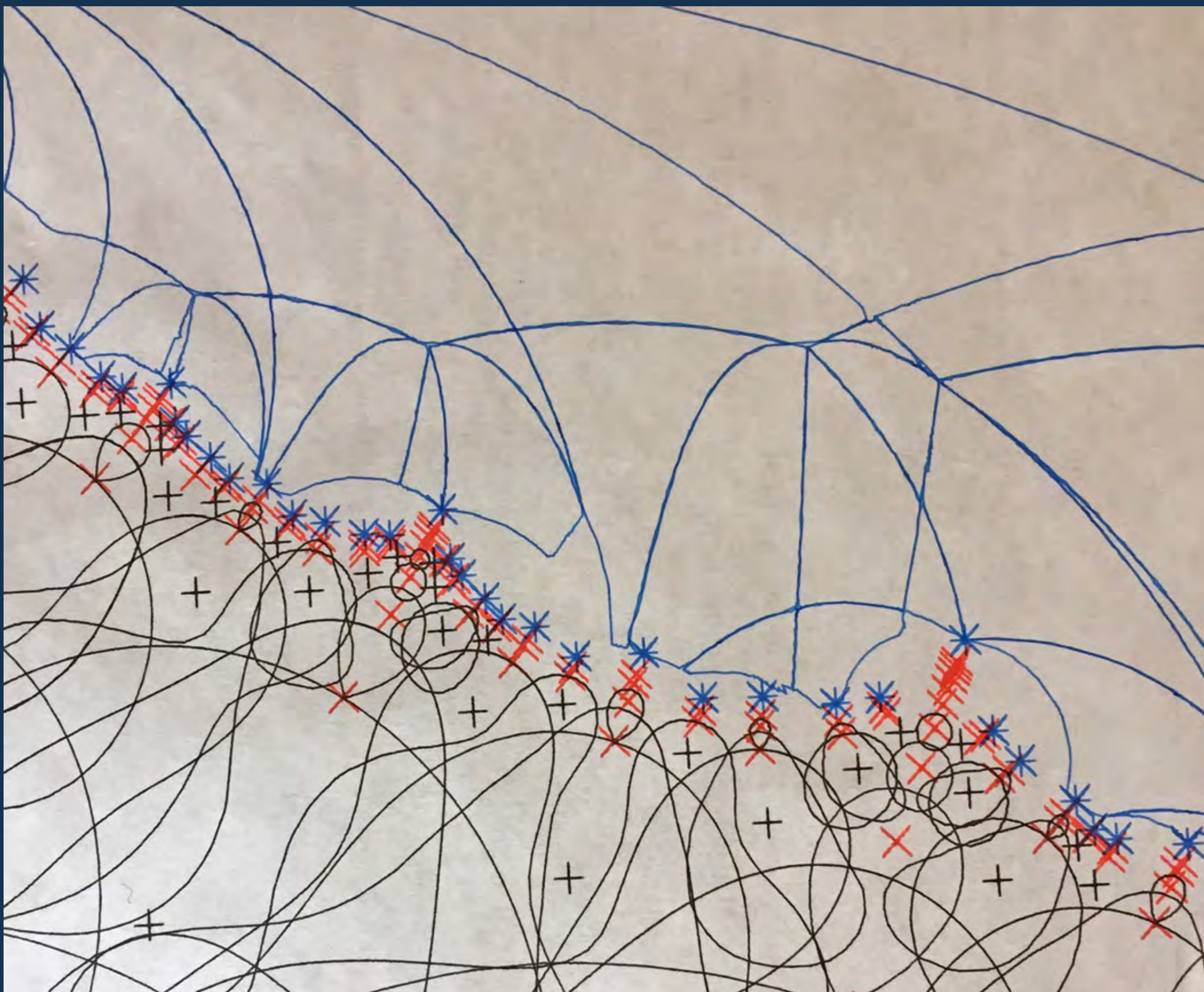
LONDON MATHEMATICAL SOCIETY
EST. 1865

NEWSLETTER

Issue: 489 - July 2020



INTERVIEW WITH THE PRESIDENT

COMPLEX ECOLOGICAL META-NETWORKS

BOB RILEY AND HIS MATHEMATICS

EDITORIAL OFFICE

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Charity registration number: 252660

COVER IMAGE

Riley slice detail from the article ‘Robert (Bob) Riley and his Mathematics’ (page 25)

ISSN: 2516-3841 (Print)
ISSN: 2516-385X (Online)
DOI: 10.1112/NLMS

NEWSLETTER WEBSITE

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FROM THE EDITOR-IN-CHIEF

This issue's cover image is a detailed view of the Riley slice which was the subject of Caroline Series' Presidential Address at the Annual General Meeting last November. It is a computer-drawn image by Bob Riley from the 1970s, where the black plus signs represent groups that correspond to two-bridge knot or link groups, and the red crosses correspond to what Bob called Heckoid groups – as described by David Singerman in his article on *Robert (Bob) Riley and his Mathematics* (page 25).

This leads to an idea that we have had for the *Newsletter*: Do you have an image that might be of mathematical interest to Members? Could you write a short item describing this mathematics, or its historical significance? Our new initiative is to publish cover images submitted by Members, along with a short description of the mathematical or historical significance of the image. To be acceptable, submissions need to be visually, as well as mathematically, interesting and of sufficiently high resolution (at least 300dpi). The image also needs to be suitable for the set-up of our front cover: that is, it will be cropped to (width x height) 178mm x 148mm, so the main part of the photo needs to fit these dimensions. Permission to publish the image must be secured (or granted) by the Member, and submissions should be accompanied by a 200–500 word description. Photo credit will be given on our inside front cover. Images and descriptions are welcome at any time, and should be sent to images@lms.ac.uk.

Eleanor Lingham
Editor-in-Chief

LMS NEWS

LMS Response to Covid-19

At its April meeting, Council established the Society's Covid Response Working Party. This working party has been meeting remotely on a weekly basis. It has been discussing suggestions which members have made on how the Society might best mitigate the effects of Covid-19 on the UK mathematics community, following an email from the Treasurer and the Chair of the Early Careers Research Committee.

A pressing concern of the working party is the effect the pandemic will have on young researchers and hence on the people pipeline into our profession. Other issues include the very considerable efforts staff in mathematics departments have had to and will have to spend setting up their courses online, and the quite different effects that working from home can create for individuals, particularly those with caring responsibilities. It is clear that the societal effects of Covid-19 jeopardize the ability of many mathematicians to carry out research, particularly for long uninterrupted periods.

As a result, the Society has exceptionally created an Emergency Covid Reserve Fund. This fund will be used to alleviate some of the most damaging effects of the virus and to ensure that something positive should emerge from this crisis.

The Society has reopened its Early Career Fellowships this year, so that strong mathematicians who find themselves in limbo between completing their PhD and moving to a postdoc position can be supported during these uncertain times. It is anticipated that up to 20 additional Fellowships will be awarded, trebling the number this year. These will be available to begin by the start of the coming academic year and will be apportioned by the usual process to ensure quality.

The Society is taking the opportunity to support and extend the activities of its existing Scheme 3 Research Groups. Groups in similar areas of mathematics will be encouraged to work together to produce short online courses as introductions to their own speciality. Presentations should be delivered by an early career researcher, but it is hoped that the whole Group will be involved in its production. A sum of up to £1,000 will be made available to support each course, rising to up to £2,000 if two or three Groups collaborate.

A third area in which the working party believes our community needs support is to help people learn how to exploit modern technology in order to deliver online courses and tutorials. Together with the IMA and the RSS, the Society is supporting the TALMO initiative (Teaching And Learning Mathematics Online), talmo.uk. The Society also hopes that its discussion forum, tinyurl.com/y734ztbo, will become a channel for disseminating good practice, flagging problems and sharing experiences, not only about teaching and learning issues, but also about working from home, Equality, Diversity and Inclusion matters, and other matters the current crisis brings to the fore.

Finally, the Society continues to be as flexible as possible with regard to grants already awarded. If an event or a visit has to be postponed then the funds will carry over, and it is recognised that the form in which the activity takes place may need to be modified significantly.

Professor Iain Gordon
LMS Vice President

Professor Robert Curtis
LMS Treasurer

A National Academy for the Mathematical Sciences?

Should the UK have a National Academy for the Mathematical Sciences? This was the topic of an online open meeting hosted by the International Centre for Mathematical Sciences (ICMS) on June 11. The purpose was to bring together interested members of the mathematical community to discuss the role and mission of such an Academy and to map out possible ways forward. This was the first time that there has been an open meeting on this important topic, first mooted in the Bond Review of 2018. With over 250 registered participants the event was something of a challenge to organise, but Zoom, as managed by ICMS and the INI, worked brilliantly. Despite the obvious limitations of online meetings, the upside was that considerably more people were able to attend than might have come to the physical events which were being planned before the Covid-19 lockdown.

Readers will recall that, following the Bond Review, the Council for Mathematical Sciences (CMS) set up two committees to consider its recommendations and discuss how to take them forward. These were the Strategic Committee (SC), chaired by Clair Craig (Provost of The Queen's College, Oxford) and the Implementation Committee (IC), chaired by Sir Bernard Silverman. The SC was to discuss ways of attracting possible outside support and funding, while the IC was to look into the various recommendations of the review in much more detail. Along the way, the task of developing the many suggestions of the Bond Review was renamed the Big Maths Initiative (BMI).

One of the most important proposals in the Bond Review was the setting up of a National Academy for Mathematical Sciences. Although the UK has a number of specialist professional and learned societies, it lacks an overarching body with the ability effectively to bring together its diverse parts, from pure mathematics through industrial and applied mathematics to statistics and operational research, in an effective broad-based forum. The experience of the Royal Academy of Engineering shows that a single Academy is a hugely powerful way to build on the existing specialist bodies, enabling them far more effectively to contribute their individual strengths to the overall discipline. The Academy would act as an enabler, not a competitor, enhancing the work of the existing learned societies and other groups.

The slides presented at the open meeting as a starting point for discussion were the outcome of much hard work by members of the IC and SC. The meeting began with a short introduction from Sir Ian Diamond, Chair of CMS (and of course also the National Statistician), and then moved on to a panel chaired by John Pullinger, (SC Member and past RSS President and National Statistician 2014–19).

After an overview by Claire Craig summarising the work to date, David Leslie (Edinburgh, IC Member) presented slides about the BMI's report on what the purpose and functions of an Academy might be. This was followed by Bernard Silverman with discussion of possible governance models. Finally Caroline Series (SC Member and past LMS President) introduced thoughts about the possible next steps, and everything was rounded up with some inspirational words from Nira Chamberlain (SC Member and current IMA President).

Participants then split into 12 breakout groups (sadly the technology could not accommodate more) for in-depth discussion on the presentations. During a coffee break, six 'synthesisers' were tasked with summarising the feedback from the breakout groups with the aid of some fancy technology called MURAL designed to mimic post-it notes. The summaries were presented during a final general session, with the opportunity for the panellists to comment, while use of 'chat' allowed further discussion and questions among participants.

Among the points raised were:

(i) The need to ensure maximum diversity in all senses of the word, especially taking account of the hugely wide variety of practitioners of mathematics of all kinds.

(ii) Whether the new body should be confined to elected fellows or be broadened into a 'membership body'.

(iii) The exciting possibilities around setting up a 'virtual academy' without the need for costly premises.

(iv) It was regretted that there had been very little consultation with bodies involved with mathematics education.

The documentation, which was sent to the participants in advance of the meeting, also included a one-page document briefly outlining the case for a National Academy. Prepared on the advice of the SC, this is designed to show to busy policymakers and potential funders. The general feeling of the day was that, with some provisos, the ideas outlined in the presentations were on the right track and, as neatly summarised by one of the synthesizers Chris Budd, the Academy initiative should go forward *carpe diem*, that is, 'seizing the day'. There was also a consensus that the community should take up the generous offer of ICMS and INI jointly to provide resources to facilitate setting up an interest group to take the project forward, taking account of the input and feedback from the meeting.

The panel discussion was recorded and further material was sent out following the event, together with opportunity for those who wished to express willingness to join an 'interest group' or otherwise offer their services. Material can be accessed via the BMI website tinyurl.com/bmimtg. It is possible that a second online event will be organised sometime later this year.

Professor Caroline Series
SC Member and past LMS President

First Atiyah Fellows Announced

Ahmad Sabra (left) and Mark Wildon

The LMS UK–Lebanon Atiyah Fellowship scheme was set up in memory of Sir Michael Atiyah OM (1929–2019). The LMS is delighted to announce that the first two, for the academic year 2020–21, have been awarded to Professor Mark Wildon, Royal Holloway, University of London and Professor Ahmad Sabra, American University of Beirut (AUB), Lebanon.

Professor Wildon's main research area is the representation theory of the symmetric and general linear groups. He will be visiting the Centre for Advanced Mathematical Sciences at AUB for four months in 2021. He plans to work on the geometric structure of a family of representations of the general linear group GL(2,C) and on the analogous representations defined over number fields and fields of prime characteristic. The characters of these modules are obtained using the plethysm product on symmetric functions: this brings in a rich circle of ideas from algebraic combinatorics and connects two exciting areas of mathematics. Professor Wildon also hopes to visit Beirut Arab University during his stay in Lebanon and to attend and give seminars at both universities.

Professor Sabra is interested in inverse problems involving surfaces which achieve prescribed optical tasks. He has studied the existence and uniqueness of such surfaces as well as their regularity properties and the stability of such optical systems. Sabra was awarded his PhD from Temple University in 2015 and following a postdoc in Warsaw has been an Assistant Professor at AUB since 2017. He will be visiting Dr Omar Lakkis at the University of Sussex for two months in 2020-21. The aim of their project is to apply Galerkin methods to construct numerical approximations of solutions to the equation that appears in far field refractor problems. During his stay he also plans to visit other researchers in Bath, Edinburgh, Oxford and elsewhere.

Applications for Fellowships to be held in 2021–22 will open in early September. More information is available at https://tinyurl.com/tvweckc.

Philippa Fawcett Collection: Appeal for Book Donations

The Society is seeking donations for one of its special collections, the Philippa Fawcett Collection, which is housed in the Verblunsky Members' Room at De Morgan House.

Members are welcome to access the Collection during weekdays from 9.00 am – 5.00 pm once De Morgan House reopens after the current closure due to Covid-19. The Collection is a wide-ranging library of some 200 books written by and about women who studied or worked in mathematical subjects in the nineteenth and first part of the twentieth century, or earlier. Some are academic texts, others are discourses on science, some are school textbooks, and there is a selection of biographies and reference works. A copy of the current catalogue can be found on the Society's website at tinyurl.com/y9hgxn9m.

The Collection was donated to the LMS by one of its members, A.E.L. Davis, in the hope that it will be a useful resource to scholars of the history of women in mathematics, as well as an inspiration to female mathematicians of the future. Dr Davis named the Collection in honour of Philippa Fawcett, the first woman to come top in the finals examination, in 1890, of the Mathematical Tripos at the University of Cambridge. As mentioned in our feature article on Philippa Fawcett (May 2020 *Newsletter*), in those days, women could not be ranked in the same list as men, so instead, Fawcett was described as 'above the Senior Wrangler'.

The Society is seeking donations from its members of books either written by female mathematicians who came to adulthood prior to the 1940s (classified as primary sources) or about female mathematicians (secondary sources) where the mathematician was working or studying mathematics in the 19th and early 20th centuries. Dr Davis' research discovered that there were 2,500 women who graduated with an honours degree in mathematics from 1878–1940 in Britain and

Ireland alone. The aim is to have a complete collection of the works by women in mathematics up to 1950 to highlight their contribution to the advancement of mathematics. A target list of women has been drawn up by A.E.L. Davis and a copy can be found on the Society's website at tinyurl.com/ycfv2l3c.

Members who wish to make a donation of books to the collection may contact Elizabeth Fisher: librarian@lms.ac.uk; 020 7291 9973. As always, the Society is grateful to its members for their support and hopes to enhance the Collection for the benefit of all.

OTHER NEWS

Postponement of Atiyah Conference

Owing to the difficulties and uncertainties caused by Covid-19, the decision has been taken to postpone the conference on the Unity of Mathematics in honour of Sir Michael Atiyah which was to have been held in the Isaac Newton Institute this September. The conference will now be held in September 2021. It is expected that registration will open in early spring 2021. For further information and updates, please see the conference website tinyurl.com/ybyp3qvv.

Brin Prize 2020

Corinna Ulcigrai

The ninth Brin Prize has been awarded to Corinna Ulcigrai, an LMS member, for her fundamental work on the ergodic theory of locally Hamiltonian flows on surfaces, of translation flows on periodic surfaces and wind-tree models, and her seminal work on higher genus generalisations of Markov and Lagrange spectra.

The Brin Prize in dynamical systems is awarded for an outstanding impact in the theory of dynamical systems or in related fields. The Prize recognises mathematicians who have made substantial impact in the field at an early stage of their careers. For further information about the prize visit tinyurl.com/ycwbzsbd. Ulcigrai is the first female to receive the Brin Prize which was launched in 2008.

Mathematicians Among New Royal Society Fellows

The Royal Society announced the appointment of new Fellows for 2020, among whom is LMS member Professor Jack Thorne (University of Cambridge).

Other mathematicians to receive the honour are Professor Ehud Hrushovski (University of Oxford) and Professor Andrew Stuart (California Institute of Technology). Others elected include computer scientist Professor David Harel (Weizmann Institute of Science) and Professor Hugh Osborn (University of Cambridge). Professor Wendelin Werner (ETH Zurich) is elected a Foreign Member. Professors Stuart and Thorne have both been recipients of the LMS Whitehead Prize, in 2000 and 2017, respectively.

The full list of new Royal Society Fellows is available at tinyurl.com/y7x6elyf. The Fellows Directory, which has extended biographies of all Fellows, is available at royalsociety.org/fellows/fellows-directory/.

IMU Committee for Women in Mathematics Newsletter

The International Mathematical Union (IMU) has published its latest Committee for Women in Mathematics (CWM) Newsletter: tinyurl.com/yad3ob3w. It features an interview with LMS Honorary Member, Professor Cheryl Praeger, an article about the launch of the UNESCO International Day of Mathematics first held on 14 March 2020, and a number of personal testimonies of CWM ambassadors from around the world on what the current Covid-19 pandemic has meant for their lives as women in mathematics.

Working from home during a pandemic has been noted as potentially impacting women scientists more than their male counterparts, and the CWM is collecting further testimonials on this. Finally, the now annual 'May 12, Celebrating Women in Mathematics' event – this year conducted online – is highlighted. This event is a joint initiative of European Women in Mathematics, the Association for Women in Mathematics, African Women in Mathematics Association, Indian Women and Mathematics, Colectivo de Mujeres Matemáticas de Chile and the Women's Committee of the Iranian Mathematical Society with the date chosen as the birthday of Fields' Medallist Maryam Mirzakhani.

First Woman Appointed Gresham Professor of Geometry

Professor Sarah Hart, who is Professor of Mathematics at Birkbeck, University of London has been appointed Gresham Professor of Geometry. The position is thought to be the oldest mathematical chair in Britain and Professor Hart is the first woman to be appointed in its 423-year history. She is Vice President of the British Society for the History of Mathematics and a keen maths communicator.

IDM 2021

International Day of Mathematics (IDM) Governing Board has decided the theme for IDM 2021 on 14 March will be 'Mathematics for a Better World'. This choice is motivated in part by the present pandemic of Covid-19 and the role that mathematical sciences can play in understanding the dynamics of epidemics and proposing strategies to control them. Subthemes and explanations of the theme will be posted on the website idm314.org.

To be kept informed of new developments, including more information on the IDM 2021 theme, register on the website for the IDM Newsletter.

George Temple and Albert Green

Albert Green (left) and George Temple

As part of a project with Chris Hollings (Oxford) on the Sedleian Professors of Natural Philosophy at the University of Oxford I am currently researching the lives and work of two of the holders of the chair: George Temple FRS (1901–92) and Albert Green FRS (1912–99). I would be extremely grateful to hear from any readers of the LMS Newsletter who may have met, worked with, or been taught by either of these men, and are willing to share their memories with me. I would also be interested to hear from readers who carried out any form of correspondence with either man and would be willing to allow access to it.

Dr Mark McCartney
Ulster University
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MATHEMATICS POLICY DIGEST

Royal Society President Elect

Sir Adrian Smith, former chair of the Council for the Mathematical Sciences, has been confirmed as President Elect of the Royal Society. He will take up the post of President on 30 November 2020. Sir Adrian is a distinguished statistician and was elected a Fellow of the Royal Society in 2001. He has an outstanding academic record, as well as a wealth of experience of working with government and leading word-class research institutes, such as Queen Mary University of London, where he was Principal (1998–2008); the University of London, where he was Vice-Chancellor (2012–18); and The Alan Turing Institute (2018 to present). More information is available at tinyurl.com/y98wkxtu.

New UKRI Chief Executive

Professor Dame Ottoline Leyser DBE FRS has been appointed as the new Chief Executive of UK Research and Innovation (UKRI). Professor Leyser started her new role in June 2020, succeeding Sir Mark Walport. More information is available at tinyurl.com/yb4p9uhx.

Wellcome Trust: Science Education Tracker 2019

More than 6,400 young people in Years 7–13 (aged 11–18) in schools and colleges across England have taken part in a survey into attitudes towards and experiences of science education and careers. Some key findings include:

- Many young people don't see science as relevant to their everyday lives or their future plans.
- Gender gaps are a major issue – both in the types of science subjects that young people do and don't choose to study, and how they perceive their ability. Male students in Years 11–13 are more likely to choose maths, physics and computer science subjects, while female students are more likely to choose biology, arts and social science subjects. Chemistry is more balanced by gender. Female students in Years 10–13 are less likely than male students to rate themselves as good at maths (63% males, 51% females), physics (46% males, 28% females) and chemistry (42% males, 34% females).
- Students from disadvantaged backgrounds are interested in science, but have fewer opportunities to engage with it — both inside and outside of school.

More information is available at tinyurl.com/ybsrp66p.

New UK Research Funding Agency

In its March 2020 Budget the government stated that it would 'invest at least £800 million' in a 'blue skies' funding agency (first announced in the December 2019 Queen's Speech), which would fund 'high risk, high reward science'.

The House of Commons Science and Technology Committee opened a formal inquiry into the nature and purpose of this new UK research funding agency. The deadline for written submissions was 30 June 2020. More information on the progress of the inquiry is available at tinyurl.com/y6vf4sok.

Digest prepared by Dr John Johnston
Society Communications Officer

Note: items included in the Mathematics Policy Digest are not necessarily endorsed by the Editorial Board or the LMS.

JMI Special Issue

The *Journal of Mathematics in Industry*, published by the European Consortium for Mathematics in Industry, is preparing a special issue on Covid-19. Manuscripts can be submitted now, will be processed as they arrive, and published online as and when they are ready. Visit tinyurl.com/y7y9khph for further details.

ERC Resignation

The President of the European Research Council, Mauro Ferrari, resigned on 7 April at the unanimous request of the 19 active members of the ERC Scientific Council. The statement from the Scientific Council with the background to the resignation can be seen at tinyurl.com/vhnvmdl. The EMS expresses its public support to the ERC Scientific Council in its decision and declaration, and would like to thank EU Commissioner Mariya Gabriel and Director General Jean Eric Pacquet for standing strongly by ERC and for the leading role they have taken to facilitate new initiatives within EU Research & Innovation in response to the Covid-19 crisis.

8ECM Rescheduled

The 8th European Congress of Mathematics 2020 has been rescheduled because of to the Covid-19 pandemic. The new date for the Congress is 20–26 June 2021 in Portorož, Slovenia.

EMS Prizes

Ten EMS prizes are awarded annually to young researchers not older than 35 years, of European nationality or working in Europe, in recognition of excellent contributions in mathematics. The EMS prizewinners for 2020 are:

- Karim Adiprasito (Hebrew University of Jerusalem/University of Copenhagen)
- Ana Caraiani (Imperial College London)
- Alexander Efimov (Steklov, Moscow)
- Simion Filip (Chicago)
- Aleksandr Logunov (Princeton)
- Kaisa Matomäki (Turku)
- Phan Thành Nam (LMU Munich)
- Joaquim Serra (ETH Zurich)
- Jack Thorne (Cambridge)
- Maryna Viazovska (EPFL, Lausanne)

The Prize Committee, consisting of European mathematicians drawn from across the continent and representing the diversity of mathematics, held their decisive meeting at De Morgan House on 23 January 2020. The Chair of the Prize Committee, Martin Bridson, expressed the EMS's sincere gratitude to the LMS for the use of both the building and the excellent video-conferencing facilities.

The Felix Klein Prize is awarded "to a scientist, or a group of at most three scientists, under the age of 38 for using sophisticated methods to give an outstanding solution, which meets with the complete satisfaction of industry, to a concrete and difficult industrial problem." The 2020 Felix Klein Prize winner is Arnulf Jentzen (University of Münster). The core research topics of his research group at the University of Münster are machine learning approximation algorithms, computational stochastics, numerical analysis for high-dimensional partial differential equations, stochastic analysis, and computational finance.

The Otto Neugebauer Prize is awarded "for highly original and influential work in the field of history of mathematics that enhances our understanding of either the development of mathematics or a particular mathematical subject in any period and in any geographical region." The 2020 Otto Neugebauer Prize winner is Karine Chemla (Université de Paris and CNRS). She has a particular interest in the mathematics of ancient and medieval China.

David Chillingworth
LMS/EMS Correspondent

Note: items included in the European Mathematical Society News represent news from the EMS and are not necessarily endorsed by the Editorial Board or the LMS.

OPPORTUNITIES

Prospects in Mathematics 2021: Call for Expressions of Interest

UK departments are invited to submit Expressions of Interest to host the LMS Prospects in Mathematics 2021 meeting to the Prospects in Mathematics Steering Group.

Up to £7,000 is available to support the annual two-day events (usually taking place in September) for final-year mathematics undergraduates who are considering applying for a PhD after they have completed their current studies. This includes funding to cover fares and accommodation for up to 50 students, travel and accommodation for speakers and subsistence for participants including a social event.

LMS Prospects in Mathematics Meetings should feature speakers from a wide range of mathematical fields across the UK who discuss their current research and opportunities available to prospective PhD students.

Prospective organisers should send an expression of interest (maximum one A4 side in length) to the Prospects in Mathematics Steering Group (ECR.grants@lms.ac.uk) by 15 September 2020 with the following details:

- Department's confirmation of support to host the LMS Prospects in Mathematics Meeting.
- Reasons to host the LMS Prospects in Mathematics Meeting.
- A provisional list of speakers who are representative of the UK research landscape both geographically and scientifically.
- Speakers from under-represented groups should be included and women speakers should account for at least 40% of the invited speakers.
- Confirmation that prospective organisers have read and understood the terms and conditions in the Guidelines for Organisers (available from tinyurl.com/y9yn2ryo).

For further details about LMS Prospects in Mathematics visit: tinyurl.com/y9yn2ryo.

Ferran Sunyer i Balaguer Prize

Ferran Sunyer i Balaguer (1912–1967) was a self-taught Catalan mathematician who, in spite of a serious physical disability, was very active in research in classical mathematical analysis, an area in which he acquired international recognition. Each year, the Ferran Sunyer i Balaguer Foundation awards an international mathematical research prize in his honour, open to all mathematicians. It was awarded the first time in April 1993.

The 2021 prize will be awarded for a mathematical monograph of an expository nature presenting the latest developments in an active area of research in mathematics, in which the applicant has made important contributions. The monograph must be original, unpublished and not subject to any previous publication commitment. The prize consists of €15,000 and the winning monograph will be published in Birkhäuser series *Progress in Mathematics*. The deadline for submission is 27 November 2020. For further information visit the website ffsb.iec.cat.

Early Career Fellowships 2019–20

Second Round of Applications: Deadline 10 July

Applications are now open for a second round of the LMS Early Career Fellowships 2019-20. Recognising that one impact of the Covid-19 pandemic on Early Career Researchers is the unexpected turbulence in the job market and to support early career mathematicians in the transition between positions, the LMS offers a number of Early Career Fellowships of between 3 and 6 months to mathematicians who have recently or will shortly receive their PhD. The award will be calculated at £1,000 a month and offers no travel allowance.

For further details and the online application form, visit tinyurl.com/ycfrpz4s. The application deadline is: 10 July 2020. If you have any queries, please email fellowships@lms.ac.uk.

LMS Council Diary – A Personal View

On Friday 24 April 2020, Council met via video-conference. The meeting began with the President extending a warm welcome to the new Executive Secretary, Caroline Wallace, who gave members of Council a brief description of her background together with an update on activities during the first weeks in her new role, and thanked members of the society and colleagues for the warm welcome she had received.

After noting that the Publications Secretary and Publications Committee are leading the Society's response to the UKRI Consultation on Open Access, which has the potential to impact significantly on both the Society's publishing activities and the publishing opportunities for UK mathematicians, the President moved to his own business and introduced an item on the impact to date of Covid-19 on LMS activities. Council heard that De Morgan House had closed in the second week of March, and LMS staff, and almost all tenants of De Morgan House, were now working remotely. Inevitably, the closure of the building had led to a loss of conference facilities income, and many events that the Society had wished to either run or fund had had to be cancelled, postponed, or moved online. In particular, the Chair of the Society Lectures and Meetings Committee reported that the 2020 Hardy Lecture, due to have been given by Peter Sarnak, would have to be either postponed or moved online. The Treasurer made the point that the unprecedented situation would justify a modest increase in expenditure over the agreed annual budget to enable the Society to resource activities in response to the Covid-19 situation. Following discussion, it was agreed to form a working group to consider the Society's response to the pandemic and to make recommendations on how it can best support the mathematical community.

Initial plans for the next strategic retreat of Council, due to take place in February 2021, were presented. The value of the socialising that such retreats affords was noted and led to the President proposing that Council should hold monthly virtual social meetings to facilitate the exchange of ideas in an attempt to replicate the conversations over coffee and lunch that are necessarily absent when Council meetings are held via video-conference.

Following presentation by the Treasurer of the Half Year Financial Review and Indicative Operational Plans 2020–21, the General Secretary reported that it would unfortunately not be possible to hold a physical general meeting in June, but that options to hold a virtual meeting instead were being explored. We also discussed the list of ideas for Honorary Members and heard updates from several committees, including a report from Frank Neumann on the Mentoring African Research in Mathematics (MARM) Board, which had agreed several mentorships. These are now to be postponed until at least November, but it was noted that technology could be useful in providing support to colleagues in Africa and, further, that access to online activities run by the LMS, which may increase in the current climate, would be welcome.

The meeting concluded with the President thanking everyone for the constructive discussions during the meeting, which he felt had gone very well despite the new virtual environment, and in particular the Society Governance Officer for having arranged the technical aspects of the meeting.

Professor Elaine Crooks

Elaine Crooks is an LMS Council Member-at-Large and the Council diarist. She is a Professor of Mathematics at Swansea University.

LMS Grant Schemes

For full details of these grant schemes, and for information on how to submit an application form, visit lms.ac.uk/grants.

Research Grants

The deadline is 15 September 2020 for applications for the following grants, to be considered by the Research Grants Committee at its October meeting.

Conferences Grants (Scheme 1): Grants of up to £7,000 are available to provide partial support for conferences held in the United Kingdom. Awards are made to support the travel, accommodation, subsistence and caring costs for principal speakers, UK-based research students and participants from Scheme 5 eligible countries.

Visiting Speakers to the UK (Scheme 2): Grants of up to £1,500 are available to provide partial support for a visitor to the UK, who will give lectures in at least three separate institutions. Awards are made to the host towards the travel, accommodation and subsistence costs of the visitor. It is expected the host institutions will contribute to the costs of the visitor.

Research in Pairs (Scheme 4): For those mathematicians inviting a collaborator to the UK, grants of up to £1,200 are available to support a visit for collaborative research either by the grant holder to another institution abroad, or by a named mathematician from abroad to the home base of the grant holder. For those mathematicians collaborating with another UK-based mathematician, grants of up to £600 are available to support a visit for collaborative research.

Collaborations with Developing Countries (Scheme 5): For those mathematicians inviting a collaborator to the UK, grants of up to £3,000 are available to support a visit for collaborative research, by a named mathematician from a country in which mathematics could be considered to be in a disadvantaged position, to the home base of the grant holder. For those mathematicians going to their collaborator's institution, grants of up to £2,000 are available to support a visit for collaborative research by the grant holder to a country in which mathematics could be considered to be in a disadvantaged position.

Research Workshop Grants (Scheme 6): Grants of between £3,000–£5,000 are available to provide support for Research Workshops held in the United Kingdom, the Isle of Man and the Channel Islands.

African Mathematics Millennium Science Initiative (AMMSI): Grants of up to £2,000 are available to support the attendance of postgraduate students at conferences in Africa organised or supported by AMMSI. Application forms for LMS-AMMSI grants are available from the AMMSI Administrator, School of Mathematics, University of Nairobi, P.O. Box 30197, GPO 00100, Nairobi, Kenya (email: ammsi.africa@gmail.com or ammsi@uonbi.ac.ke; tel: +254 786 234 678).

The deadline is 15 December 2020 for applications under the Joint Research Groups in the UK scheme (Scheme 3), to be considered by the Research Grants Committee at its January meeting. Grants of up to £4,000 are available to support joint research meetings held by mathematicians who have a common research interest and who wish to engage in collaborative activities, working in at least three different locations (of which at least two must be in the UK). Potential applicants should note that the grant award covers two years, and it is expected that a maximum of four meetings (or an equivalent level of activity) will be held per academic year.

Maths/Computer Science Research Grants

The deadline is 15 October 2020 for applications for Scheme 7 grants, to support visits for collaborative research at the interface of Mathematics and Computer Science either by the grant holder to another institution within the UK or abroad, or by a named mathematician from within the UK or abroad to the home base of the grant holder. Grants of up to £1,000 are available.

Grants for Early Career Researchers

The deadline is 15 October 2020 for applications for the following grants, to be considered by the Early Career Research Committee in November.

Postgraduate Research Conferences (Scheme 8): Grants of up to £4,000 are available to provide partial support for conferences held in the United Kingdom, which are organised by and are for postgraduate research students. The grant award is to be used to cover the costs of participants.

Celebrating new appointments (Scheme 9): Grants of up to £600 are available to provide partial support for meetings held in the United Kingdom to celebrate the new appointment of a lecturer in mathematical sciences at a UK university.

Travel Grants for Early Career Researchers: Grants of up to £500 are available to provide partial travel and/or accommodation support for UK-based Early Career Researchers to attend conferences or undertake research visits either in the UK or overseas.

Interview with the President

Eleanor Lingham interviews LMS President Jon Keating FRS on his life, work and thoughts about the Society.

When you meet someone new, how do you describe your work?

I say that I'm a mathematician, and if pressed further a mathematical physicist. I am interested in many areas of mathematics and I like to think of myself as a generalist. I suppose what excites me most is understanding connections between different, seemingly unrelated areas of mathematics. Much of my work involves searching for, or exploiting such connections. For example, I am interested in using connections between random matrix theory and number theory to understand better the statistical properties of the Riemann zeta-function and fluctuations in the distribution of the primes.

I am also interested in applications of mathematics, and especially applications of modern mathematical ideas. Depending on who asks me, I might also talk about teaching, which I enjoy, and perhaps even the work I do in supporting other people to do mathematics; for example, at the Heilbronn Institute for Mathematical Research and, of course, with colleagues at the London Mathematical Society.

What is your area of mathematics?

I have interests in several areas. My first degree and my PhD were in physics – my PhD advisor was Michael Berry. He encouraged me to think that theoretical physics and mathematics are so closely intertwined as to be inseparable. It is much more interesting to understand what discoveries are enabled by new ideas, or what novel connections they suggest, than to worry about whether those ideas should be labeled as belonging to mathematics or to physics. However, I suppose it is fair to say that over the years my interests have moved closer to the centre ground of mathematics.

I started out working in the area of Quantum Chaos, where the focus is on quantum properties of classically chaotic systems in the semiclassical limit, that is when the quantum wavelength is asymptotically small. I am still interested in that area, but in recent years I have moved more towards random matrix theory. This is a beautiful area of mathematics that has an extremely broad range of applications, including to complex quantum systems, data science, high-energy physics, machine learning, mathematical finance, numerical linear algebra, population dynamics, quantum information theory, and telecommunications.

Jon Keating. Photo credit: Chrystal Cherniwchan

Random matrix theory also has deep connections with other areas of mathematics, including combinatorics, integrable systems theory, number theory, representation theory, statistical mechanics, and stochastic analysis. I find these connections fascinating. I have a particular interest in links between random matrix theory and number theory. It was conjectured by Montgomery in 1973 that the statistical distribution of the zeros of the Riemann zeta-function coincides, asymptotically, with the statistical distribution of the eigenvalues of random unitary matrices.

This highly surprising conjecture is supported by extensive numerical and theoretical evidence and, assuming that it is true, it provides an extremely powerful model for various statistical problems in number theory. However, we still are far from really understanding why it might be true. I have spent a good deal of my research life working on this, and on other problems relating to random matrices.

Are you working on anything particularly exciting at the moment?

I hope so! I am certainly enthusiastic about work I am involved with currently. I am thinking about problems relating to the extreme values taken by the characteristic polynomials of random matrices, and what these might tell us about the extreme values taken by the Riemann zeta-function and other L-functions. In the case of the zeta-function and other L-functions, the size and frequency of the extreme values is a deep and longstanding mystery. We would like to model this using random matrix theory, but the corresponding problem there is also extremely subtle and challenging. Analysing it seems to require ideas from many other areas of mathematics, and I am enjoying trying to put the pieces of the jigsaw together.

For several years I have also been interested in number-theory-inspired questions relating to polynomials defined over finite fields. Here the connection with random matrix theory can be proved, in a certain limit, and so the analogues of formulae that are conjectures in number fields can, in this setting, be proved, allowing one to go much further.

Finally, I have also recently been working on the asymptotics of the moments of the Riemann zeta-function. It is known that these are related to correlations in the values taken by the generalised divisor function, but we have been stuck for over twenty years in sorting out how to utilise this connection in general, because we have been missing at least one key idea. The approach that works for the first few moments fails spectacularly for the higher ones, and understanding why this is the case has been a major puzzle. I have been working with Brian Conrey on this and we now believe we have understood the problem and how to fix it. In fact, it turns out that ideas developed in the theoretical physics literature in the 1990's play a critical role, as do a different set of ideas, due to Manin, relating to Diophantine geometry. I am very excited by this. In collaboration with Henryk Iwaniec, Kannan Soundararajan and Trevor Wooley, we are trying to establish a general picture of what is going on. We have a long way to go, but so far the journey is proving to be an interesting one.

What do you enjoy most about university life?

I love ideas, and so being in an environment where I can learn new ideas and see them being created is highly stimulating for me. Going to a first-rate mathematics lecture is one of life's joys; for many mathematicians this is every bit as important as music, film, theatre or literature.

I also like working with students. I have been fortunate to have had some excellent students and have found working with them to be highly stimulating too. Universities are changing, and many are becoming more commercial and corporate in their outlook. This doesn't sit well with me, but I find that academics themselves are still very much motivated by a sense of scholarship, service and community, which certainly aligns with my own philosophy.

Where has your interest in the LMS stemmed from?

I have been a Member for many years, but my first active involvement began in the 1990s, when the Society introduced its regional structure and I became the Regional Coordinator for the South-West of England and South Wales. I suppose the thing that attracted me in the first instance is that I feel the Society's values and ethos, its focus on scholarship, teaching and community, resonate with my own.

Earlier in my career I benefitted enormously from the time and energy others put into the mathematical community, and I like the fact that that spirit is still alive and well. I admire people who contribute to the community, often behind the scenes and as volunteers, and I hope I can make a similar contribution. I also find the history of mathematics fascinating, and so am interested in societies like the LMS that have played such a central role in the past, as well as currently. The LMS is quirky, which is an aspect I like, and it reflects well the individualistic nature of many of its members. I am, like many mathematicians, naturally drawn to eccentrics and obsessives, and the LMS is an excellent place to find them!

What do you see as the main challenges that the LMS is facing?

If you had asked me this five months ago, then I would have said that the main challenges were:

- Diversity: while we have made significant progress in this area, there is more that we could be doing. We are fortunate to have received earlier this year a most generous donation to support our work in this area, one that we believe will make a major difference to what we can achieve. I am personally extremely interested in issues relating to widening participation. Previous generations in my family did not go to university, and the (state comprehensive) school to which I went was not one that would be considered academically strong. I know the difference higher education can make to one's opportunities and would like to see how we can continue to widen access.

- Income: the current Open Access Consultation could potentially have an outcome which would lead to a large reduction in our income, and so in the charitable support that we are able to give to the UK mathematics community. It is critically important that we continue to build sustainability into all of our activities, and that we look for ways to diversify our income streams.
- International relations: with Brexit, it is more important than ever to ensure that the LMS is outward-looking and engaged with our international colleagues and institutions.
- Working with Government: we currently have a government that is unusually supportive of science, and in particular of mathematics. We need to think how we can make the most of this opportunity. The recent uplift in funding is a good example of what can be achieved by working with them.
- Bond Review: the mathematical community has been challenged regarding communication and knowledge exchange. We need to ask ourselves some hard questions: Is it possible for our somewhat inhomogeneous community to work together, despite there being differing agendas? How best can we cooperate with our sister societies? What is the most useful way for us to contribute to the debate around the possible establishment of an Academy for Mathematical Sciences?

With the current Covid-19 pandemic, we now face new and unexpected challenges. Inevitably the UK higher education landscape will change. Teaching, learning and assessment is moving online. Student numbers will be affected; and so too will conferences and research collaborations. Universities will change, and some may falter. We need to work out how to protect the interests of the mathematical community, and how best to support our members.

What do you see as the main advantages of the LMS as a society?

Our openness – the LMS is a genuinely democratic organisation that seems to me to represent rather well the mathematics community in the UK, from the grassroots upwards. We are non-corporate and diverse. We have a long-standing history and reputation. We are relatively financially secure, and we are supported by wonderful staff at De Morgan House.

If you could have met any Member of the LMS – who would you choose?

G.H. Hardy. I have been strongly influenced by his and Littlewood's mathematics. Moreover, Hardy led an interesting life – one which wasn't always easy – and I would like to know more about him as a person. His book *Ramanujan: Twelve Lectures on Subjects Suggested by His Life and Work* is a favourite of mine. It is mathematically beautiful, but also underscored by the moving story about their relationship. I would like to ask him more about that collaboration. Finally, I would really like to thank him for his bequest to the Society, which has had a significant impact and helped a large number of mathematicians over the last seventy years.

G.H. Hardy c. 1927

Can you tell me a little about your life outside of mathematics?

My family would say that I don't have one! But I do like to cook, and to swim; and I have a healthy obsession with cricket.

What about work/life balance?

Again my answer may cause raised eyes in my family, but this is extremely important to me. It is difficult to combine travel, research, and teaching commitments with family life, but I have always tried to give this the highest priority. My wife, who is also an academic scientist, would be right to feel that she was more successful in this than I have been, but I hope I would get reasonably good marks. One thing I am learning from the current Covid-19 crisis is that travel is less essential than I suspect I once thought. I am strongly supportive of colleagues getting the balance right, and am aware that this is increasingly difficult as workloads grow. We need to support the community in this debate. Here the impact of the current public health crisis may not be helpful, as we are forced to move more of our teaching online.

Do you have any message for our Members?

I would encourage all Members to engage with the Society – either to become involved in the work of the LMS, or to communicate with us about the work that we are doing. Share with us how you think we can best help you, and tell us what we should be doing more of. I'm sure that we all want to see our subject and our community thrive, and contributing to the Society's work is as good a way as I can think of to achieve that.

Stability in Complex Ecological Meta-Networks

GAVIN M. ABERNETHY

What does it mean for a spatial meta-network to be stable? A model of coupled food webs assembled by ecological and evolutionary processes is described, and used to examine what properties allow an ecosystem to withstand perturbations representing catastrophic habitat destruction.

Introduction

How would you answer the question: is the Amazon rainforest a stable ecosystem? We can model an ecosystem by constructing a food web – a directed graph representing the predatory relationships, where the nodes are species (or groupings thereof) and edges indicate a predator's food sources. Twentieth century ecologists had assumed that food webs hold together *because* of their sheer complexity and interconnectedness. However, in 1972 the late Robert May's results on linear stability of random graphs [1] seemed to indicate that stability was favoured by low diversity (the number of species in the ecosystem), low connectance (the fraction of possible relationships that are realised), and weaker feeding relationships. In other words, according to May's criterion, more complex food webs ought to lack stability. As an inspection of the natural world shows that highly-complex ecosystems have continued to exist for some time, linear stability of equilibria therefore must not be the most appropriate criterion for an ecologist. Consequently, as models increase in scale from low-dimensional population dynamics models to complex adaptive networks of many species, which may not have populations at an equilibrium, the notion of stability needs to be rephrased for a more practical analysis of model food webs. Some contenders for this measure are given in the sidebar (for more see [2]). In the ecological context, adaptive foraging, allometry (using body-size scaling to affect energy transfer and the feeding relationships), multiple species sharing the same predators and prey, and employing realistic non-linear functional responses have all been suggested to enhance web stability in some regard.

Stability has inherent links to the non-random structure of the network, and in the case of food webs, this structure cannot be untangled from the evolutionary processes that have shaped it. To address this, researchers have developed eco-evolutionary models that assemble the networks through both the evolutionary dynamics of speciation and extinction, and the ecological processes of predation, competition, reproduction and mortality present in the classical population dynamics approach employed in mathematical biology courses.

Measures of stability

Linear stability analysis: the question of whether small disturbances from a dynamic equilibrium are dissipated or amplified.

Node deletion stability: the fraction of species in the web which, when they alone are deleted, do not result in any further extinctions.

Community robustness: the fraction of species which must be artificially deleted in sequence to induce a total loss (including resulting secondary extinctions) of 50% of all species. A food web lacking robustness would be highly sensitive, with the loss of one species precipitating a significant collapse.

Persistence: whether (or how many) species can endure the duration of the perturbation.

These models can construct complex food webs with realistic distributions of species in different trophic roles, and we use these as the basis of our stability experiments. Furthermore, by coupling multiple food webs and allowing populations to migrate between those adjacent, we expand our eco-evolutionary food web model to a spatially-explicit meta-community model. Again, the question of stability can be appropriately "scaled-up", so that we ask what the properties are of patches (nodes in a graph of the spatial meta-network) whose loss or perturbation causes the greatest damage to the whole system? By answering these questions, models of complex networks can be deployed in service of conservation and preserving geographic biodiversity.

Model equations

Feeding scores:

$$S_{i,j} = \max\left\{0, \sum_{m=1}^{10}\sum_{n=1}^{10}\beta_{i_m,j_n} \times \frac{1}{1.5\sqrt{2\pi}} \exp\left(\frac{-(r_i - r_j - 3)^2}{2(1.5)^2}\right)\right\} \tag{1}$$

Competition scores:

$$\alpha_{i,k} = c + q_{i,k} \times \frac{1-c}{0.6\sqrt{2\pi}} \exp\left(\frac{-(r_i - r_j)^2}{2(0.6)^2}\right) \tag{2}$$

Foraging efforts:

$$f_{i,j} = \frac{g_{i,j}}{\sum_{k\in K_i} g_{i,k}} \tag{3}$$

Functional response:

$$g_{i,j} = \frac{S_{i,j} f_{i,j} N_j}{bN_j + \sum_{k\in P_j} \alpha_{i,k} S_{k,j} f_{k,j} N_k} \tag{4}$$

Population dynamics:

$$\frac{\mathrm{d}N_i}{\mathrm{d}t} = -2\,\mathrm{e}^{-0.25 r_i} N_i + \lambda \frac{N_i}{s_i} \sum_{j=0}^{n} g_{i,j} s_j - \sum_{k=1}^{n} N_k g_{k,i} \tag{5}$$

Model

The model we shall be using is as follows [3]. We define a species i by which ten discrete binary traits $\{i_m | m = 1, \ldots, 10\}$ it possesses, and its body-size s_i which is a continuous variable converted to a log scale as $r_i = \log(s_i)$. Each possible trait p is randomly assigned a non-negative score $\beta_{p,q}$ against every other trait q. Then the potential feeding score $S_{i,j}$ of species i on species j is determined by the sum of the scores of each pair of their traits, scaled by the probability density function of a normal distribution of relative body-size differences centered at three. This means that the most effective predatory relationship is upon a prey with body-size $\exp(-3)$ that of the predator (1).

Any pair of species i, k who utilise the same prey experience a base level of competition $c = 0.6$, which is then increased by an amount that is modified by the similarity of the species: linearly with the fraction of shared traits $q_{i,k}$, and according to a normal distribution probability density function of relative body-size differences centered at zero (2). Thus, if two very different species use the same resource they experience weaker competition and can feed with greater efficacy. In real ecosystems, members of the same or related species are likely to consume the same parts of their prey, while for example a flying-fox that feeds on eucalyptus nectar will experience less competition with the koala that primarily eats the leaves of the same plant.

At a given timestep, each local population of each species must decide what available prey it is going to feed upon, and how to allocate the proportion of its hunting effort among them. To do this, a pair of equations for the foraging efforts $f_{i,j}$ of species i on j (3), and the corresponding functional responses $g_{i,j}$ (4) that govern the actual transfer of energy from prey to predator, are updated between each iteration of the population dynamics, so that species are gradually able to adjust their strategies in response to changing conditions and the success of their previous efforts. Here $b = 0.005$ controls the effectiveness of predation in the ratio-dependent functional response, K_i is the set of current prey of species i, and P_j is the set of current predators of species j.

All local populations are then updated according to ODE (5), with terms accounting for loss due to natural mortality, gains due to feeding where the ecological efficiency $\lambda = 0.3$ controls the flow of biomass to the next trophic level, and losses to predators. If the model allows multiple patches, populations may then move to adjacent patches on the grid according to the processes described in the second box. Two mechanisms of movement, henceforth referred to as diffusive and adaptive migration respectively, are proposed. Both are designed to generate meta-communities with distinct local food webs, as if we allow all species to move by diffusion between any neighbouring patches (even at low rates) typically the food webs become highly synchronised and overall biodiversity is constrained [4]. This rate is also allometrically scaled, so that creatures with a larger body-size are able to move faster.

The above mechanisms describe the ecological dynamics of the model. The evolutionary dynamics then consist of occasionally, after many iterations of the population dynamics, introducing a new species into the global ecosystem by mutation of an existing species. A parent species is selected from the ensemble, and a child introduced with minimum population size. This new species has one of its ten traits randomly exchanged with another choice, and its body-size is uniformly selected from within 20% of the parent's body-size. A model simulation begins with a single species, and in each patch a unique resource that can never move, mutate or be fully depleted, then over time these rules assemble a complex meta-community of hundreds of species with different ecosystems in each patch.

Model communities

We will consider results from three sets of model communities. First, a single food web (with no spatial implementation) can be constructed over 110,000 speciation events and we can examine patterns of robustness in these individual ecological communities. Then we shall consider two meta-communities, assembled on 6×6 spatial grids, using the two different movement mechanisms (7-8). As these simulations are far more computationally expensive, we shall only use 10,000 speciation events for these scenarios. The four patches in the top-right corner of the meta-network constructed with diffusive movement are illustrated in Figure 1.

Movement in the meta-network

Let $N^t_{i,x,y}$ denote the population of species i in patch (x,y) during the t^{th} ecological timestep. Then movement between patches is implemented by

$$N^t_{i,x,y} \mapsto N^t_{i,x,y} + \sum_{j=1}^{x_{max}} \sum_{k=1}^{y_{max}} \delta_{j,k,x,y} \mu_{i,j,k,x,y} N^t_{i,j,k} - \sum_{j=1}^{x_{max}} \sum_{k=1}^{y_{max}} \delta_{x,y,j,k} \mu_{i,x,y,j,k} N^t_{i,x,y} \tag{6}$$

where $\delta_{j,k,x,y} = 1$ if the patches (j,k) and (x,y) are connected and distinct, and zero otherwise. The parameter $\mu_{i,j,k,x,y}$ denotes the fraction of the local population of species i in patch (j,k) that migrates to patch (x,y).

We contrast two choices for this migration:

(1) *Trait-gated diffusive migration:* we associate all links between adjacent patches with 100 randomly-selected traits

$$\mu_{i,j,k,x,y} = \max\left\{1,\ 10^{-4}\frac{s_i}{s_0} \times M_{j,k,x,y}(i)\right\} \times D^{-1}_{j,k}. \tag{7}$$

Here, $s_0 = 1$ denotes the body-size of the resources, and $D_{j,k}$ the degree of patch (j,k). $M_{j,k,x,y}(i)$ returns the number of the traits of species i associated with the link between patches (x,y) and (j,k) and so scales the total migration by how well adapted the species is to traverse this link.

(2) *Adaptive migration:* we allow any species to traverse any link, but only when they have experienced a decline in their local population

$$\mu_{i,j,k,x,y} = \begin{cases} \max\left\{1,\ 0.03\frac{s_i}{s_0} \times \frac{N^{t-1}_{i,j,k} - N^t_{i,j,k}}{N^{t-1}_{i,j,k}}\right\} \times D^{-1}_{j,k}, & \text{if } N^{t-1}_{i,j,k} > N^t_{i,j,k} \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

Meta-community: Trait-gated diffusive migration on a 6×6 lattice

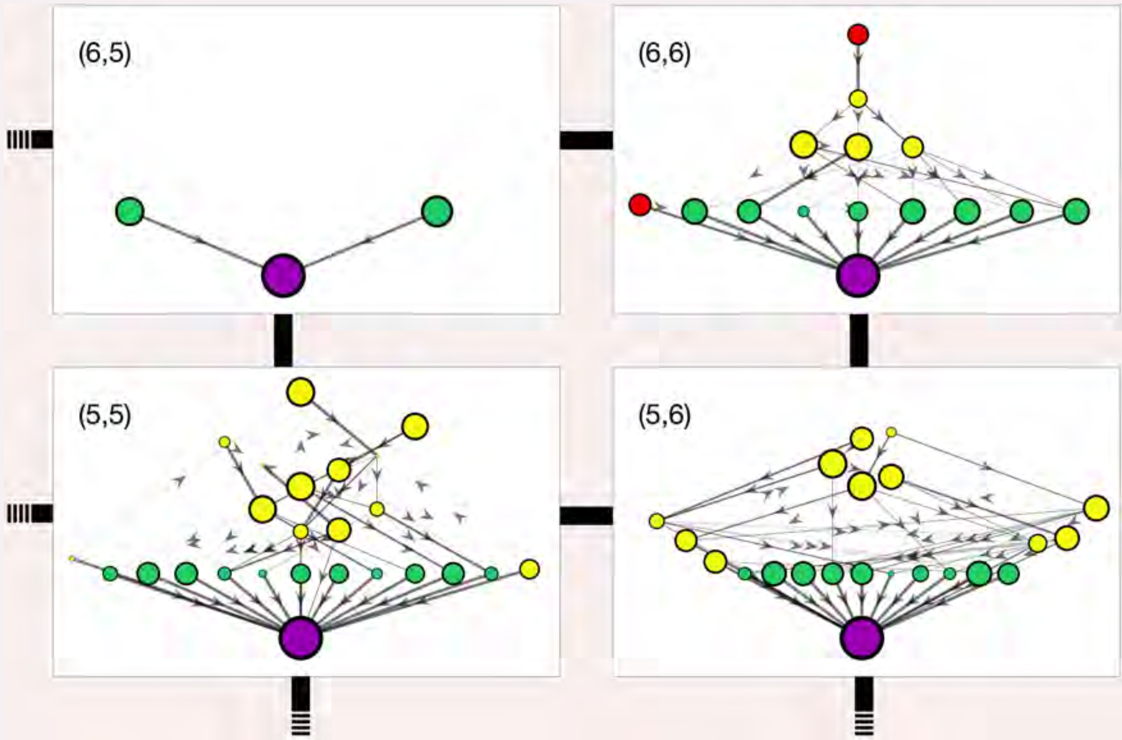

Figure 1. Four local food webs in a corner of the 6×6 meta-network with 458 total biodiversity (and 36 unique resources), assembled over 10,000 speciation events from a single initial species. Purple denotes the resource, green a basal species (feeds only upon the resource), red a top predator, and yellow otherwise. Line thickness is proportional to feeding effort, with arrows from predator to prey, and vertical height corresponds to trophic level. Node radius is proportional to population size on a logarithmic scale.

Stability of an individual food web

First, a result already known to theoretical ecologists [5] is confirmed: when we consider food webs existing on a single isolated patch, network stability in the sense of community robustness (see box 1) is positively correlated with connectance of the food web (Figure 2), that is, when proportionally more of the potential feeding relationships in the network are actively realised [3, 6]. (Connectance of the food web is measured by $L/S(S-1)$, where S is the number of species and L the number of feeding links.) In such cases, species tend to be more flexible and less dependent on a few essential non-resource prey for their survival. This can be achieved in a relative sense by reducing the size of the community, or by allometric scaling influencing the structure of the web to a more pyramidal shape (Figure 3), so that more species feed on the resource which is excluded from being deleted.

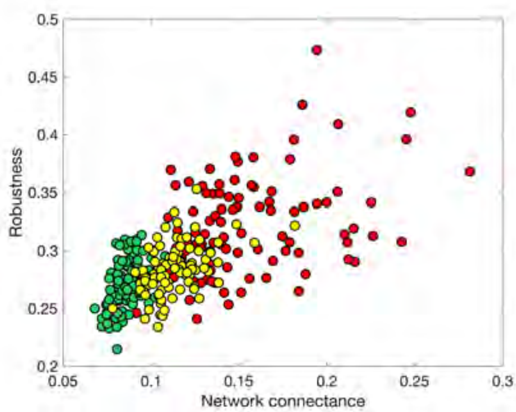

Figure 2. Robustness of individual food webs against their connectance. Red indicates our model, with green and yellow indicating two choices of parameters in a version that lacks the influence of body-size effects.

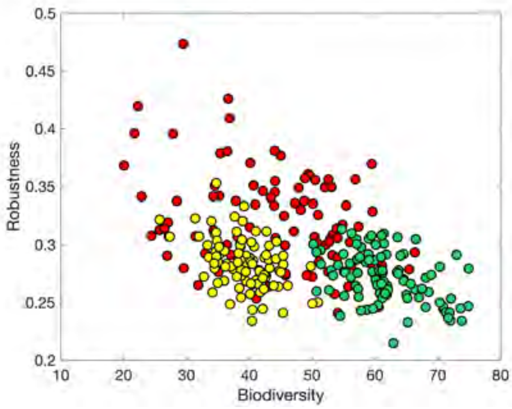

Figure 3. Robustness of individual food webs against their biodiversity. Red indicates our model, with green and yellow indicating two choices of parameters in a version of the model that lacks the influence of body-size effects.

Stability of a meta-network

To investigate the stability of our two evolved meta-communities of many species occupying many patches, let us model the effect of habitat destruction by subjecting them to either individual, or sequences of, perturbations by removing patches from the meta-network, and observing the resulting impact on the biodiversity of the global ecosystem.

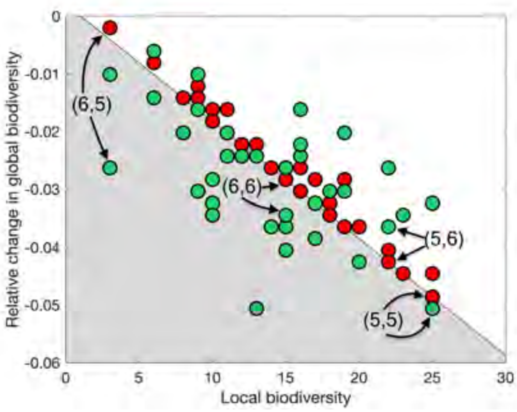

Figure 4. Effect of disrupting a single patch in the meta-community with diffusive migration. Red indicates eliminating the local population, while green denotes the result of displacing them to neighbouring patches. The shaded area indicates a loss of more species than there were in the disturbed patch.

We contrast two forms of disturbance: either the local populations of the afflicted patch are instantly eliminated, or they are uniformly dispersed amongst all neighbouring patches.

First, we confirm the obvious hypothesis that global biodiversity loss due to eliminating the occupants of a given patch is strongly correlated to the prior diversity of that perturbed patch (Figure 4, red), although even this depends on the rules governing dynamics and especially species movement. Both of the movement mechanisms we have employed give rise to fairly isolated local food webs in each patch, with most species occupying only one or two patches by the time of the experiment. If a more liberal diffusion mechanism was permitted, the food webs would co-evolve as highly synchronised, and there would consequently be little effect of eliminating the local population of any patch [4]. However, the current model shows a more interesting response to *displacement* (Figure 4, green). In Figure 1, patch $(6,5)$ has only two resident species compared to the 24 in its neighbour $(5,5)$. Displacing these 24 yields an overall loss of 25 species, but if the two species of $(6,5)$ are forced to emigrate, the end result is a quite disproportionate 13 extinctions! As one of the pair have another population elsewhere, if there was a need to temporarily clear that patch of wildlife it seems that no conservation effort would have been better than one that carelessly introduces non-native species into the complex ecosystem of patch $(5,5)$. Of course, a better (though costly) approach would attempt to transfer the two species to a suitable distant habitat. This captures the highly destructive potential of real invasive species such as the introduction of *rattus rattus* to island communities, although the overall role of invasions in continental extinctions is more contentious [7].

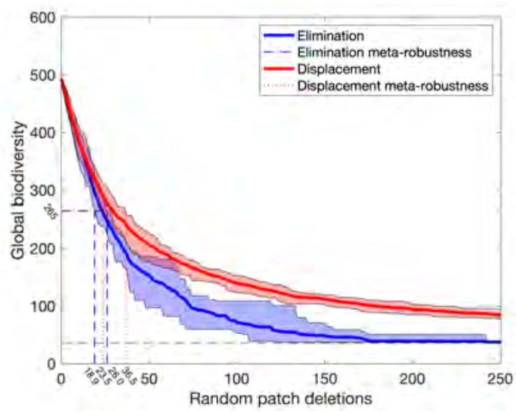

Figure 5. Sequentially perturbing random patches in the meta-community with diffusive migration.

When we consider sequences of disrupting the local populations of randomly-selected patches, displacing the affected individuals is about as damaging to the global ecosystem as eliminating them for around the first 20 events (Figure 5). Subsequently, although it continues to be limiting, the severity is reduced compared to killing the affected populations and a reasonable number of non-resource species can survive indefinitely. If we extend our concept of robustness of a network to define the "meta-robustness" of the system as the number of patches we must disrupt to induce a 50% or greater loss of biodiversity, the model with the diffusive migration mechanism (7) yields 18.9-26.0 for elimination and 23.5-36.5 for displacement from ten trial sequences. This overlap is still present when using adaptive migration (8), but with values increased to 21.8-34.0 and 31.2-45.3 respectively (Figure 6), so we can say that this system is "more robust" to patch deletion of both kinds.

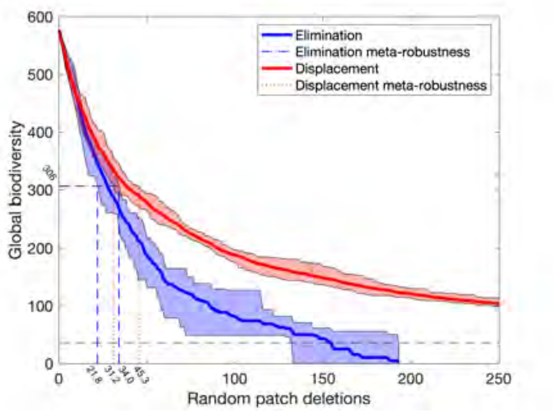

Figure 6. Sequentially perturbing random patches in the meta-community with adaptive migration.

The stability of the ecosystem in the sense of persistence exhibits a stronger dependence on *and* a reversal regarding the migration rules. When adaptive movement is employed, an initial meta-network with greater diversity and complexity is assembled, but it is unable to endure sequences of random elimination of patches indefinitely (Figure 6, blue), while a model using constant low-level diffusion may do so but with very few survivor species (Figure 5, blue). This can be understood in terms of "tall" vs. "wide" play in how species make use of the spatial network – adaptive movement causes initial wide play as species spread out to find better pastures at the beginning of the simulation, but subsequently settle on tall strategies to make the most of the best patch they find themselves in. This allows them to flourish for a time and the patches become relatively isolated, thus when a neighbouring patch experiences catastrophe, the local populations cannot take advantage of it and simply await the destruction of their own habitat. On the other hand, a diffusive mechanism essentially causes all species to always have a "slightly-wide" strategy, which is not so beneficial initially but does allow them to re-colonise destroyed patches and so ensure that some species persist.

As we are witnessing currently, mathematical and computational models do indeed have potentially significant influence on public policy when it comes to human health, wellbeing and the economy. More sophisticated ecological and evolutionary modelling, such as spatial models which explore the optimal size and placement of nature reserves, will be needed to inform responsible human stewardship of the environment and mitigate the impact on biodiversity of advancing urbanisation, deforestation and exploitation of natural resources.

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Gavin Abernethy

Gavin is a Lecturer in Engineering Mathematics at Sheffield Hallam University. His current interests are in ecological modelling and nonlinear dynamics. He enjoys growing chilli peppers, frying prawns, and playing *Animal Crossing*.

Robert (Bob) Riley and his Mathematics

DAVID SINGERMAN

Robert Riley was a US mathematician who spent many years at the University of Southampton. He combined his mathematical and computing skills to discover the Riley slice, which was the subject of Caroline Series' LMS Presidential Address. Here his PhD supervisor, friend and colleague describes his mathematical work.

Introduction

Robert (Bob) Riley was a US mathematician who spent most of the years 1968–1980 at the University of Southampton. During these years he made some remarkable discoveries which we will explore later.

Bob Riley

First, some background: Bob was born in New York City in 1935. In 1957 he studied at Cornell University and after graduating, he proceeded to MIT for graduate studies in number theory. However, he did not get on too well with some of the algebraic geometry, and gave up his course, and went to work for a computing company. This was in the early days of computing, and the result was that he became expert in the usage of computers, which would become valuable for later mathematical pursuits.

In 1966, we find Bob in Amsterdam. Why did he go there? I first heard that this was to escape being drafted to go and fight in the Vietnam War. But more recently, an old university friend of Bob told me that he had fallen madly in love with a Dutch girl, and went to Holland to marry her. A more romantic story which is more likely to be true, but I never again heard about this Dutch girl.

While in Amsterdam, Bob developed an interest in knot theory [5]: "On settling in Amsterdam in October 1966 I wrote off to virtually everyone publishing in knot theory for their reprints and preprints. I recall with gratitude that R.H. Fox and H. Seifert were especially generous. An unassuming little paper by Fox written in Utrecht some 20 miles away, took my fancy." Fox was looking at representations of a knot group (the fundamental group of the complement of the knot in the three-sphere) into A_5. Now A_5 is isomorphic to PSL$(2,5)$ and this led Bob to study *parabolic representations* of knot groups into PSL$(2,F)$, where F is a field. These are representations such that meridians in the knot group map to parabolic matrices, that is matrices with trace ± 2.

The Riley Slice

H.B. (Brian) Griffiths, a professor from Southampton, met Bob in Amsterdam and invited him to take a temporary position at Southampton, which he did in 1968. I started working at Southampton in 1970, after getting my Ph.D. at Birmingham under the supervision of Murray Macbeath. I worked on topics related to Fuchsian groups, discrete subgroups of PSL$(2,\mathbb{R})$, with particular interest in their quotient Riemann surfaces, so I did have an interest in discrete matrix groups and their quotients. On my first day at Southampton I was taken to meet with Bob, and the first thing he said to me was:

"Consider the group generated by the matrices $\left(\begin{smallmatrix}1&1\\0&1\end{smallmatrix}\right)$ and $\left(\begin{smallmatrix}1&0\\c&1\end{smallmatrix}\right)$, where c is a complex number. When is this group discrete and free?"

At the time I did not realise the importance of this question. It led to the *Riley slice* which many mathematicians have been investigating in recent years.

Here are some mathematical preliminaries: a Kleinian group is a discrete subgroup of PSL$(2,\mathbb{C})$, the group of complex Möbius transformations. As Poincaré observed, these groups have an action on hyperbolic 3-space $\mathbb{H}^3$. To see this, we think of this space as the set of points (z,t), where $z \in \mathbb{C}$, $t > 0$. The hyperbolic metric is defined by:

$$ds^2 = \frac{|z|^2 + t^2}{t^2}.$$

Elements of PSL$(2,\mathbb{C})$ map circles to circles in the complex plane. Now each Möbius transformation is a product of an even number of inversions in circles. We extend these by looking at the inversions in the upper-half of the spheres. This gives an action of PSL$(2, \mathbb{C})$ on upper-half 3-space, which is our model of hyperbolic 3-space. More importantly, it gives PSL$(2, \mathbb{C})$ as the orientation-preserving isometry group of $\mathbb{H}^3$. The quotient space $\mathbb{H}^3/\Gamma$ is then a hyperbolic 3-manifold. Indeed, if M is a complete hyperbolic 3-manifold, then there exists a Kleinian group Γ such that $M = \mathbb{H}^3/\Gamma$. A Kleinian group Γ may act discontinuously on some region of the extended complex plane $\hat{\mathbb{C}}$. The maximal open set in the extended complex plane where Γ acts discontinuously is called the ordinary set of Γ and is denoted by Ω. We can then extend M to $(\mathbb{H}^3 \cup \Omega)/\Gamma$ which is M with its conformal boundary.

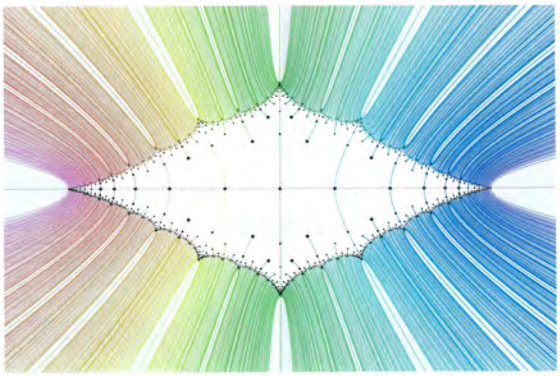

Figure 1. Riley slice. Photo credit: Y. Yamashita

In Figure 1, there is a white region with a fractal boundary which we call $\mathbb{D}$. This is the *Riley slice*. The points c which lie in the exterior of $\mathbb{D}$ correspond to free discrete groups, but there are points inside $\mathbb{D}$ marked in the diagram where the groups are still discrete but not free.

Let us look at part of $\mathbb{D}$ in more detail: in Figure 2, the black plus signs represent groups that correspond to two-bridge knot or link groups. (All two-bridge knot or link groups appear [1]). The red crosses correspond to what Bob called Heckoid groups. These are Kleinian groups with non-empty ordinary set generated by two non-commuting parabolic elements and which contain elliptic elements (elements of finite order). All these points correspond to non-free discrete groups. Also included are the Hecke groups at the points $\pm 4\cos^2\frac{\pi}{n}$. These are groups isomorphic to a free product $C_2 * C_n$. The whole Riley slice goes from -4 to $+4$ on the real axis and from $-2i$ to $+2i$ on the imaginary axis.

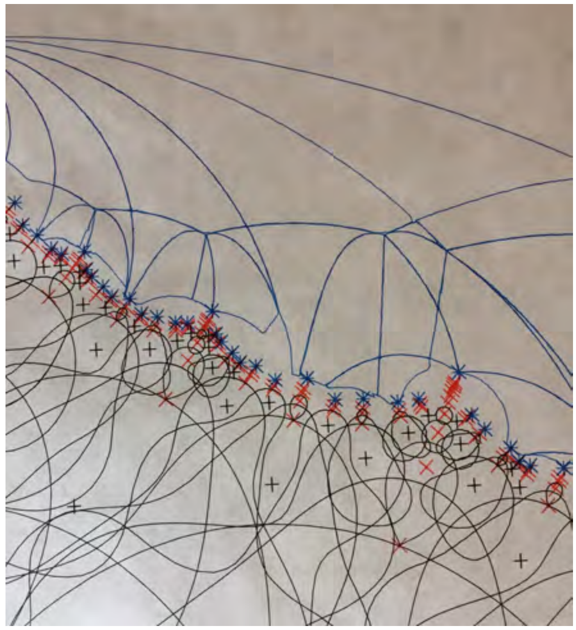

Figure 2. Riley slice detail

The Riley slice picture (Figure 3) was constructed by Bob with the aid of a computer in Southampton. Riley's early employment as a computer programmer certainly paid off! Figure 3 shows the part of the Riley slice in the first quadrant of the complex plane. It is dated 26 March 1979. When Bob left Southampton, he gave a copy to David Chillingworth who then passed it onto Caroline Series at Warwick. She then passed it onto John Parker, who was a postdoc at Warwick at the time. In his lecture on the complex Riley slice, John Parker [4] said "I have a computer printout in my office in Durham which is dated 1979. It was on a huge piece of paper produced by one of those printers with an arm and a pen."

Figure 3. Riley slice by Bob Riley

Figure 1 is a rather beautiful representation of the whole slice by Yasushi Yamashita. Luckily, John kept his diagram of the Riley slice, which as far as I know was the only version available in the UK. This diagram has proved useful for Caroline Series who has written on this topic, and it was the subject of her LMS Presidential Address [8] in November 2019. The only other versions were held by Bob. The Japanese mathematician Masaaki Wada picked up a copy from Bob when he visited Binghamton much later. This appeared on page VIII of [2]. After the Riley slice diagram they write: "Riley's pioneering exploration of groups generated by two parabolic transformations. The computer-drawn picture has been circulated among the experts and has inspired many researchers in the field of Kleinian groups and knot theory. This specific copy of the picture was obtained directly from Professor Riley when M. Wada visited SUNY, Binghamton in February 1991."

Hyperbolic structure on the figure eight knot complement

In 1973, Bob made what was possibly his most important discovery: the hyperbolic structure on the figure eight knot complement. Before we describe this let us return to some personal history: in October 1972 his fourth temporary position at Southampton came to an end, and Bob found himself with a large pile of computer output and no prospect of further employment. He managed to get a six-month appointment at Strasbourg, and after the summer vacation of 1973, he returned to Southampton where the mathematics professors granted him the use of an office and all university facilities *except* for the use of the computer, which was heavily overloaded.

The result was that Bob had to read about Kleinian groups, and in particular Poincaré's theorem on fundamental polyhedra. This made more progress possible and he looked again at the parabolic representation θ of the group of the figure eight knot. This is a two generator group and its image under the parabolic representation is the group generated by $\left(\begin{smallmatrix}1&1\\0&1\end{smallmatrix}\right)$ and $\left(\begin{smallmatrix}1&0\\\omega&1\end{smallmatrix}\right)$, where $\omega = -\frac{1}{2}(1+\sqrt{-3})$ is a cube root of unity. Note that this point corresponds to one of the black plus signs in the Riley slice!

As Bob wrote [5]: "This group is obviously discrete and only its presentation was in doubt." He could find the presentation because he knew Poincaré's theorem: "I remember my surprise at finding this p-rep is faithful. The first version of my account in [6] was received by the editors on 30 November 1973 and it didn't mention the orbit space $\mathbb{H}^3/\pi K\theta$ because I had not even thought of it."

By constructing a fundamental domain for $\pi K\theta$, Bob proved that the orbit space is the figure eight knot complement and so he had now obtained a hyperbolic structure on this knot complement! Even in his revised version of [6] he did not make much of a fuss about the hyperbolic structure. He just had a corollary stating that the identified fundamental domain is homeomorphic to the knot complement. I remember Bob telling me of his result one day when we went walking on Southampton Common. As I had experience of quotients of Fuchsian groups giving hyperbolic Riemann surfaces, I knew what he had done, although I did not fully understand the three-dimensional topology.

Thus by 1975, Bob had a major result but no permanent academic job. The reason was that he had left MIT without getting his Ph.D. and so in 1975, I formally took him on as a research student. Of course, I did not need to give him a research problem; he had many of his own. I also proposed that we apply to the Science Research Council (SRC – a forerunner of ESPRC) to support a hyperbolic project at Southampton. As Bob wrote "the plan was to time the submission of the proposal so that the referee would be at the summer 1975 conference on Kleinian groups at Cambridge where I would publicise hyperbolic structure. Whether or not the plan worked, the Kleinian groupies liked my examples, especially because these examples pointed up the importance of their own work. The SRC did fund the project generously, ultimately for four years, from 1976 to 1979." In 1980, Bob was awarded a Ph.D. for his thesis "Projective representations of knot groups" [7] which was examined by David Epstein from the University of Warwick.

William Thurston

An interesting part of the Riley story is his connection with Bill Thurston. In 1982, Thurston produced his seminal work *Three dimensional manifolds, Kleinian groups and Hyperbolic Geometry* [9]. This was the main work which led to Thurston being awarded the Fields Medal in 1982. Thurston acknowledges his debt to Riley. His Theorem 2.3 tells us which three-manifolds have a hyperbolic structure. As a corollary he states that if $K \subset S^3$ is a knot, then $S^3 \setminus K$ has a geometric structure, if and only if K is not a satellite knot. It has a hyperbolic structure, if and only if it is not a torus knot. Thurston wrote: "This corollary was conjectured by R. Riley based on his construction of a number of beautiful examples with the aid of a computer. His work gave me a big impetus to prove Theorem 2.3."

Bob finally met Thurston in Warwick in 1975. From Bob's account [5]: "On hearing my name, a tall man sprawled over three chairs sprang up. He said his name was Bill Thurston, that he wanted to meet me, and that for about a year he had been working on a general conjecture which included everything I was doing. The shock was immense."

Binghamton

Now that Bob had finally got a Ph.D. he could seriously apply for a permanent position. In 1980, he received a grant to work with Thurston in Boulder, and then in 1981 obtained a permanent position at Binghamton, New York. He would often come back to England to visit. One reason was that he could then go for cycling tours of Scotland. He would keep his bicycle in my garage, and cycled from Southampton to Scotland (a distance of at least 430 miles), and then cycle up Scottish hills. It was because he had trouble climbing these hills that he sought medical attention for his heart. Tragically, following successful heart surgery, he died from complications in Binghamton on 4 March 2000. He was 64 years old.

At Binghamton he had become close to Matt Brin and Ross Geoghegan. Matt Brin was an expert on the Richard Thompson group which many in Southampton were also interested in, and so we had an extended visit from Matt. We found that some of us were close to Bob at different times in his career. We also knew that Bob had written a personal account of his discovery of hyperbolic structures so we prepared this document for publication [3, 5].

We end with some remarks of Ross Geoghegan from Riley's obituary: "Riley holds a a unique position in American mathematics. In the circle of ideas in which he was expert he really was an innovator. His were very original ideas that have had a large impact on mathematics in the last couple of decades. Personally Riley was a character. He was one of a kind, slightly eccentric, delightful company. There was just one Riley. He was interested in Science, particularly Physics, and was an Anglophile who listened to the BBC World Service on his short-wave radio twice a day. He wasn't English but he loved all things English."

Bob regarded himself as a 19th century mathematician with the added advantage that he was proficient in the use of computers. One of his prized possessions was a letter of rejection from a very reputable British journal saying that they "no longer publish 19th century mathematics"!

Acknowledgements

I would like to thank John Parker, Caroline Series and David Chillingworth for their helpful comments.

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David Singerman

David is an Emeritus Professor of Mathematics at the University of Southampton. His original research was in Fuchsian groups and Riemann surfaces, but he then moved to the study of dessins d'enfants (maps on Riemann surfaces). His other interests are musical, with a particular interest in the music of Anton Bruckner. For exercise he likes to run (slowly).

The International Mathematical Union

On 20 September 1920, during the International Congress of Mathematicians (ICM) in Strasbourg, France, representatives from Belgium, Czechoslovakia, France, Greece, Italy, Japan, Poland, Portugal, Serbia, the United Kingdom, and the United States signed the statutes for the International Mathematical Union (IMU), electing C.J. de la Vallée Poussin (Belgium) as President and W.H. Young (UK) as Vice President. Thus the IMU came into being. Perhaps surprising to the modern mind, the initial statutes were to be valid for a period of just twelve years.

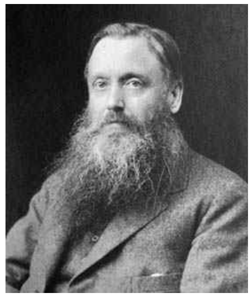
This was in the aftermath of the First World War and it was the time for the creation of scientific unions (the International Union of Biological Sciences (IUBS) and the International Union of Pure and Applied Chemistry (IUPAC) were founded in 1919; the International Union of Pure and Applied Physics (IUPAP) in 1922). Mathematicians tried to heal the wounds from the war and to promote international collaboration in mathematics. The main vehicle to accomplish this was to provide the scientific framework for future ICMs being organised under its auspices[1].

The first decade of the IMU proved to be a challenging one, and with political tensions unresolved, the IMU was dissolved during the ICM in Zürich in 1932. Paradoxically, it was during a period without an existing IMU, namely at the ICM in Oslo in 1936, that the most coveted prize in mathematics, the Fields Medal, was introduced.

C.J. de la Vallée Poussin, the first IMU President

After the Second World War attempts were made to re-establish the IMU and in 1952 the modern IMU was created as an international non-governmental and non-profit scientific organisation. IMU's objectives are: to promote international cooperation in mathematics; to support and assist the International Congress of Mathematicians and other international scientific meetings or conferences; and to encourage and support other international mathematical activities considered likely to contribute to the development of mathematical science in any of its aspects, pure, applied, or educational.

These objectives drive the current IMU and its activities. Membership has increased considerably and today the IMU has around 90 member countries, which fares well compared to other international unions, but remains noticeably behind the 193 members[2] of the United Nations.



W.H. Young, the first IMU Vice President

The quadrennial ICMs remain a focal point of the IMU. The practical demands of organising an ICM have become enormous and the financial commitments are huge – and this responsibility remains with the local organisers. On the other hand, the IMU is responsible for the scientific content: the Structure Committee decides the disciplinary sessions (relative size and content), the Program Committee selects all speakers, and the various prize committees determine the recipients of all IMU awards. With increased and justified attention regarding diversity (geography, gender, mathematical discipline, etc.), and the problems associated with conflicts of interest and unconscious bias, it is no small task to compose all these committees.

In addition to the ICMs, the activity of the IMU is focused on the five commissions and committees:

The Commission for Developing Countries (CDC) runs a plethora of programs for mathematicians in developing countries. In addition to funding directly from membership dues, the CDC receives generous donations from the winners of the Breakthrough Prizes, the Simons Foundation, the Abel Board,

[1] The first ICM had been hosted in Zürich in 1897. To this day, the hosting of the Congresses remains one of the main focuses of the IMU.
[2] Albeit with a different definition of 'member'.

and individual donations from several mathematical societies, including the LMS (thanks!).

The International Commission on Mathematical Instruction (ICMI) focuses on activities related to mathematics education. Being older than the IMU (established at the ICM in Rome in 1908), it runs its own quadrennial congresses, the ICMEs, as well as its own General Assembly.

The International Commission on the History of Mathematics (ICHM) is a joint union between the IMU and the Division of History of Science and Technology of the International Union of History and Philosophy of Science and Technology, bringing together mathematicians interested in history with historians interested in mathematics.

The Committee on Electronic Information and Communication (CEIC) serves as an advisory committee to the IMU regarding a field that has been, and still is, in an amazingly rapid transition, namely that of how we communicate and publish.

The Committee for Women in Mathematics (CWM) is the most recent committee. Having secured a substantial grant from the International Science Council (ISC) on the Gender Gap, the committee has very quickly become an active and effective addition to the IMU's activities.

A major turning point for the IMU was the decision of the General Assembly in 2012 to accept the generous offer of Germany to support a permanent IMU Secretariat in Berlin, located in the heart of the city and hosted by the Weierstrass Institute for Applied Analysis and Stochastics (WIAS). This has transformed the IMU completely, giving the IMU stability and robustness. The annual support from Germany exceeds the sum of all membership dues. In addition to providing a safe IMU Archive, the fact that the IMU has a stable office makes it possible to disperse and receive funds worldwide, in a setting where the measures against money-laundering are becoming ever more complicated.

Mathematics, like all science, has developed phenomenally over the course of the last century. Looking back at the first ICMs, completely dominated by males from Europe and North America, and comparing them to the audience at the amazing opening ceremony of the 2018 ICM in Rio de Janeiro, attended by people of all creeds and colours from across the world, one realises how international mathematics has become. Indeed, for the first time, the Executive Committee of the IMU has members from all continents. This is a promising development that bodes well for the future as we embark on our second century.

Helge Holden
Secretary General of the IMU

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The references above give a multifaceted view of the IMU. On the occasion of the centenary, the IMU has solicited Norbert Schappacher to write a book to appear next year on the IMU's history, generously supported by the Klaus Tschira Foundation. Furthermore, the book *Mathematical Communities in the Reconstruction after the Great War (1918–1928): Trajectories and Institutions* (L. Mazliak, R. Tazzioli, eds.), containing a chapter by G.P. Curbera on W.H. Young, will appear this autumn.

Mathematics News Flash

Aditi Kar reports on new path breaking developments in mathematics from the past few months.

The world remains in the grip of Covid-19; nevertheless there is much to celebrate in the world of mathematics. We report on the resolution of several famous conjectures: the Schinzel-Zassenhaus from number theory, the non-sliceness of Conway's knot, the analytic description of the diffusion equation representing lattice random walks in finite space having discrete parameters and finally, a non-linear dynamical model that promises to save lives at sea.

A Proof of the Schinzel-Zassenhaus Conjecture on Polynomials

AUTHORS: Vesselin Dimitrov
ACCESS: https://arxiv.org/pdf/1912.12545

In the 1960s, Schinzel and Zassenhaus conjectured the existence of an absolute constant $c > 0$ such that for any algebraic integer $\alpha \neq 0$ which is not a root of unity, we have

$$\max\{\ |\alpha_i|\ |\ 1 \le i \le n\ \} > 1 + \frac{c}{n}.$$

Here, the α_i's are the conjugates of α. A disproof of the above conjecture implies an affirmative solution to the Lehmer conjecture (1933) in Number Theory.

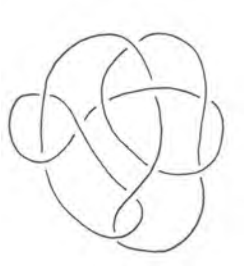
Vesselin Dimitrov has proposed a proof of the Schinzel Zassenhaus conjecture. He shows that for any integer polynomial $P(x) \in \mathbb{Z}[x]$ with constant term 1 and degree n, there is always the following dichotomy: either $P(x)$ is a product of cyclotomic polynomials or at least one of the complex roots lies outside the disc $|z| \le 2^{\frac{1}{4n}}$. Consequently, the conjectured absolute constant can be taken to be $\frac{\log 2}{4}$.

The Conway Knot Is Not Slice

AUTHORS: Lisa Piccirillo
ACCESS: https://arxiv.org/abs/1808.02923

Lisa Piccirillo's much talked-about proof appeared in the *Annals of Mathematics* earlier this year. Newsflash takes this opportunity to celebrate her achievement as a graduate student and also to salute the late John Conway, after whom the mathematical protagonist of her work is named. The world sadly lost Conway to Covid-19 in April.

In mathematics, a knot is an embedding of a topological circle S^1 into the 3-sphere, considered up to continuous deformations. A knot is *trivial* if it bounds a disc in S^3. Fox extended this notion to 4D through concordance: a knot is trivial in concordance if it bounds a smoothly embedded disc in the 4-ball B^4. A knot is *slice* if it bounds a smoothly embedded disc in B^4 but *topologically slice* if it bounds a locally flat disc in B^4.



The Conway Knot. Photo credit: Victoria Dixon

John Conway discovered a pair of knots, each with 11 crossings, one of which came to be known as the Conway knot. The Conway knot remained a mystery for many years – Freedman showed that both of Conway's knots were topologically slice. However, all modern techniques failed to verify whether the Conway knot was slice or not. Piccirillo's result established that the Conway knot is not slice. This completed the classification of slice knots with under 13 crossings and gave the first example of a non-slice knot which is both topologically slice and the positive mutant of a slice knot.

Exact Spatiotemporal Dynamics of Confined Lattice Random Walks in Arbitrary Dimensions

AUTHORS: Luca Giuggioli
ACCESS: tinyurl.com/ya5qd8oa

A lattice random walk is a stochastic process comprising a random path traversed on a lattice. It is commonly referred to as Pólya's walk when the steps occur to any of the nearest-neighbour sites. The random movement is modelled via a diffusion equation and in real life applications, the equation is set in finite space. Finding the analytic description of the space-and-time dynamics of such a confined random walk with discrete parameters has remained an open problem for more than a century.

A definitive solution to this has now been found by Bristol mathematician Luca Giuggioli. Using a set of analytic combinatorics identities with Chebyshev polynomials, Giuggioli developed a hierarchical dimensionality reduction of the diffusion equation to find the exact space and time dependence of the occupation probability for confined Pólya's walks in arbitrary dimensions. The findings are directly relevant to a vast number of applications such as molecules moving inside a cell, animals foraging for resources in their home ranges, robots searching in a disaster area, and humans passing information or a disease.

Non-linear dynamics to trace castaways at sea

AUTHORS: Serra, M., Sathe, P., Rypina, I. et al.
ACCESS: https://doi.org/10.1038/s41467-020-16281-x
Every year, hundreds of people lose their lives in sea accidents. The chance of finding a survivor falls drastically after the first six hours as rescue teams battle ocean currents, coastal tides and unfavourable weather conditions. An international research team led by George Haller, Professor of Nonlinear Dynamics at ETH Zurich has used tools from dynamical systems theory and ocean data to develop a new algorithm that predicts where objects and people floating in water will drift. Results of the study, which promises to save lives at sea, have recently been published in Nature Communications (Search and rescue at sea aided by hidden flow structures. Nat Commun 11, 2525; 2020.)

Rescue teams already use dynamical models to predict the trajectory of floating objects but inaccuracies arise from missing data or uncertainties in parameters of tidal behaviour, weather forecasting, et cetera. The mathematical methods developed by Haller's research team can trace special curves using instantaneous ocean data. These curves, which they call TRAPs (Transient Attracting Profiles) enable more precise planning of search routes than is currently possible. The new system has been tested in two experiments located off the north-eastern coast of the US. Buoys and mannequins thrown into the coastal waters near Martha's Vineyard were found to gather faithfully along the identified TRAPs.

Dr Aditi Kar
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Microtheses provide space for current and recent research students to communicate their findings with the community. We welcome/invite submissions – see newsletter.lms.ac.uk for guidance, and award authors with LMS associate membership for one year.

Microthesis: The Erdős Primitive Set Conjecture

JARED DUKER LICHTMAN

A subset A of $\mathbb{Z}_{>1}$, the set of integers greater than 1, is primitive if no number in the set divides another. Erdős proved in 1935 that the sum of $1/(n\log n)$ for n running over a primitive set A is universally bounded over all choices of A. In 1988 he asked if this universal bound is attained for the set of prime numbers. In this microthesis, I describe some recent progress towards this conjecture.

On a basic level, number theory is the study of the set of integers $\mathbb{Z}$. Maturing over the years, the field has moved beyond individual numbers to study sets of integers, viewed as unified objects with special properties. A set of integers $A \subset \mathbb{Z}_{>1}$ is primitive if no number in A divides another. For example, the integers in a dyadic interval $(x, 2x]$ form a primitive set. Similarly the set of primes is primitive, along with the set $\mathbb{N}_k$ of numbers with exactly k prime factors (with multiplicity), for each $k \geq 1$. Another example is the set of perfect numbers $\{6, 28, 496, ..\}$, that is, those equal to the sum of their proper divisors, which has fascinated mathematicians since antiquity.

We define

$$f(A) := \sum_{n\in A} \frac{1}{n\log n},$$

and let p be prime. After Euler's famous proof of the infinitude of primes, we know $\sum_p 1/p$ diverges, albeit "just barely", with

$$\sum_{p\leq x} \frac{1}{p} \sim \log\log x.$$

On the other hand, we know $\sum_p \frac{1}{p\log p}$ converges (again "just barely") and we may compute $f(\mathbb{N}_1) = \sum_p \frac{1}{p\log p} \approx 1.6366$. In 1935 Erdős generalised this result considerably, proving $f(A) < \infty$ uniformly for all primitive sets A. In 1988 he conjectured that the maximum is attained by the primes $\mathbb{N}_1$:

Conjecture 1. $f(A) \leq f(\mathbb{N}_1)$ for any primitive A.

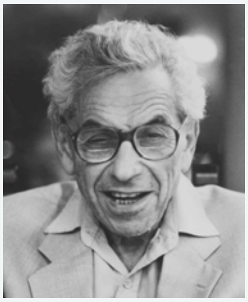
Since 1993 the best bound has been $f(A) < 1.84$, due to Erdős and Zhang [3]. Recently, Pomerance and I [6] improved the bound to the following:

Theorem 1. *$f(A) < e^\gamma \approx 1.78$ for any primitive A, where γ is the Euler-Mascheroni constant. Further $f(A) < f(\mathbb{N}_1) + 0.000003$ if $2 \in A$.*

Primitive from perfection

In modern notation, a number n is perfect if $\sigma(n) = 2n$ where $\sigma(n) = \sum_{d|n} d$ is the full sum-of-divisors function. Similarly n is called abundant if $\sigma(n)/n > 2$, and deficient if $\sigma(n)/n < 2$.

Since $\sigma(n)/n$ is multiplicative, one sees that perfect numbers form a primitive set, along with the subset of non-deficient numbers n whose divisors $d \mid n$ are all deficient.



Paul Erdős (1913–1996)

It is a classical theorem that non-deficient numbers have a well-defined, positive asymptotic density. This was originally proven with heavy analytic machinery, but Erdős found an elementary proof by using primitive non-deficient numbers (this density is now known $\approx 24.76\%$ [4]). His proof led him to introduce the notion of primitive sets and study them for their own sake.

This typified Erdős' penchant for proving major theorems by elementary methods.

One fruitful approach towards Conjecture 1 is to split up A according to the smallest prime factor, that is, for each prime q we define

$$A_q := \{n \in A : n \text{ has smallest prime factor } q\}.$$

We say q is Erdős strong if $f(A_q) \le f(q)$ for all primitive A. Conjecture 1 would follow if every prime is Erdős strong, since then $f(A) = \sum_q f(A_q) \le f(\mathbb{N}_1)$.

Unfortunately, we don't know whether $q = 2$ is Erdős strong, but we do know now that the first 10^8 odd primes are all Erdős strong, [6]. And remarkably, assuming the Riemann Hypothesis, over 99.999973% of primes are Erdős strong, [7]!

A conjecture of Banks & Martin

In 1993, Zhang proved $f(\mathbb{N}_k) < f(\mathbb{N}_1)$ for each $k > 1$, which inspired the following by Banks and Martin [1]:

Conjecture 2. $f(\mathbb{N}_k) < f(\mathbb{N}_{k-1})$ for each $k > 1$.

They further conjectured that, for a set of primes $\mathbb{Q}$,

$$f(\mathbb{N}_k(\mathbb{Q})) < f(\mathbb{N}_{k-1}(\mathbb{Q})), \qquad \text{for each } k > 1,$$

where $A(\mathbb{Q})$ denotes the numbers in A composed of primes in $\mathbb{Q}$. Banks and Martin managed to prove this conjecture in the special case of sufficiently "sparse" subsets $\mathbb{Q}$ of primes.

This result, along with Conjectures 1 & 2, illustrates the general view that $f(A)$ reflects the prime factorisations of $n \in A$ in a quite rigid way. Beautiful though this vision of f may be, it appears reality is more complicated. Recently in [5], I precisely computed the sums $f(\mathbb{N}_k)$ (see Fig. 1), and obtained a surprising disproof of Conjecture 2!

Theorem 2. $f(\mathbb{N}_k) > f(\mathbb{N}_6)$ *for each* $k \neq 6$.

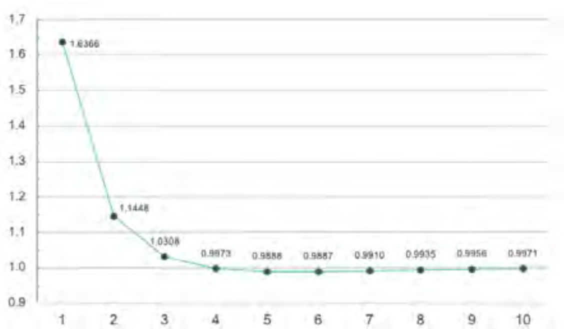


Figure 1. Plot of $f(\mathbb{N}_k)$ for $1 \le k \le 10$, [5].

I also proved $\lim_{k\to\infty} f(\mathbb{N}_k) = 1$, however much about this data remains conjectural. For instance, the sequence $\{f(\mathbb{N}_k)\}_{k\ge 6}$ appears to increase monotonically (to 1), and the rate of convergence appears to be exponential $O(2^{-k})$, while only $O(k^{\varepsilon-1/2})$ is known.

I hope this note illustrates Erdős' conjecture spawning new lines of inquiry. For example, researchers are now studying variants of the problem in function fields $\mathbb{F}_q[x]$. Also, in forthcoming work [2] we manage to prove Conjecture 1 for 2-primitive A, that is, a set where no number in A divides the product of 2 others.

The full Erdős primitive set conjecture has remained elusive, but working towards it has led to interesting developments. In the words of Piet Hein:

> "Problems worthy of attack
> prove their worth by fighting back."

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Jared Duker Lichtman

Jared is a DPhil student at the University of Oxford under the supervision of James Maynard. His research interests lie in number theory, especially problems of analytic and combinatorial flavour. Jared is originally from the United States, and enjoys sports of all kinds. He was converted to maths from his native math, but maintains that football ≠ soccer.

The Universe Speaks in Numbers: How Modern Maths Reveals Nature's Deepest Secrets

by Graham Farmelo, Faber & Faber, 2019,
£20, US$30, ISBN: 978-0571321803

Review by Noel-Ann Bradshaw

Graham Farmelo has opened my eyes to the world of theoretical physics and, more importantly, its links with pure mathematics. As a teenager I loved maths, music and art. I was interested in chemistry and history, but physics left me cold; partly because I had endured the same teacher for five years and she had failed to inspire me. Since embarking on my academic career, I have become somewhat embarrassed by my aversion to all things physics-related, and consequently I saw the opportunity to review this book as a way of addressing this.

In true academic style Farmelo begins each chapter with a summary of what will be covered, but then each chapter unfolds into a beautiful historical account of the development of ideas and the relationships between those who created them. It reminds me of tracing the genealogy of intertwined family trees, showing where they overlap, come together and then separate for a time before coming back to create powerful new dynasties.

This book clearly depicts the development of mathematical physics, starting with a brief overview of the very early history of the subject. As an enthusiastic mathematical historian I enjoyed reading about Aristotle's rejection of Plato's view, that mathematics was fundamental to understanding science and the world. And also then about the impact of mathematical giants such as Euclid, Kepler, Galileo and Newton, who paved the way for the likes of Laplace and Maxwell (said by one of his teachers to practice maths with "exceeding uncouthness"). This journey and progression of ideas is important for mathematicians and physicists to understand and, in my opinion, should be taught in schools where the subjects are still presented very separately.

Farmelo's style of writing particularly brings alive the work of Maxwell, demonstrating how he linked electromagnetism and optics with his fascination for topology and knots. It is clear that he believed in the importance of mathematics for understanding the universe, whereas his friend and collaborator, Thomson, is described as seeing mathematics as a servant of physics rather than a guide.

Next on the scene is Einstein, who is reported as realising that advanced mathematics was a physicist's most valuable tool. There follows a delightful account of the relationship between Hilbert and Einstein as they race to complete the theory of gravity, coming at it with very different backgrounds. For me the book really starts to come alive with the entry of Dirac and his vision for the future of these distinct, but linked, disciplines.

Dirac proposed a new theory: the beauty of mathematics. As someone who has recently been told by a teacher not to mention the b-word in talks to school children because apparently this turns them off the subject, I was delighted to read that he mentioned the beauty of mathematics seventeen times in his talk on the relationship between mathematics and physics at the Royal Society of Edinburgh in 1939 – a talk which is crucial to Farmelo's book. Here he urged theoretical physicists to learn a lot of advanced mathematics, concluding that "big domains of pure mathematics will have to be brought in to deal with the advance in fundamental physics."

The history and theories discussed by Farmelo develop swiftly with much of the book understandably focussed on more recent developments, some of which are less easy for a non-physicist to understand. However, understanding the detailed theory is not essential as it is the accounts of the relationships and collaborations between the likes of Dyson and Feynman, Penrose and Hawking, Weyl and Wigner that are fascinating and thought-provoking. I defy any pure mathematician not to be moved as Farmelo's account becomes personal. He describes his conversations with Atiyah, Dyson, Langlands, Uhlenbeck and other modern day greats, demonstrating not only the link between mathematics and physics but also the desperate necessity for the two disciplines to collaborate and work together. As I am personally aware that the mathematical modeller needs ideas and problems from industry in order to perfect his/her craft, I appreciated seeing this echoed by Uhlenbeck, who is quoted as saying how "Research mathematicians need physicists' ideas."

I believe this is an important book that should be read by both mathematicians and physicists. It challenges, but yet is sympathetic, to the different histories, backgrounds and indeed prejudices of the two disciplines. Farmelo sometimes presents his own opinion but much more frequently uses the words, actions and works of others to put his point across. What makes his call for intimate collaboration more powerful is the acknowledgement that he was not of this opinion when he started out in his career. Over time, his experience has shown him the importance of working together to further developments in areas such as String Theory, Supersymmetry and the discovery of the Higgs particle.

In my opinion this book should be on the reading list for every mathematics and physics A-level student and every new undergraduate of both these disciplines. Wherever their interests in these subjects currently lie, they should be made aware of the overlap of mathematics and physics, the power of cooperation and where the sharing of ideas can lead.

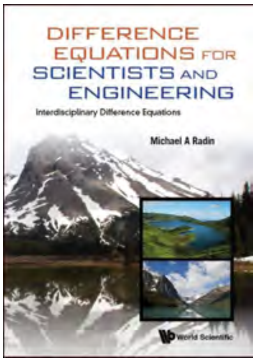
Noel-Ann Bradshaw

Noel-Ann Bradshaw is Head of Computing and Digital Media at London Metropolitan University. She recently survived a brief foray into the world of Data Science at Sainsbury's Argos in 2018. In her spare time she enjoys the company of her two cats, skiing, making cocktails, sunbathing, learning Italian and taking holidays that combine as many of these as possible.

Difference Equations for Scientists and Engineering

by Michael A. Radin, World Scientific, 2019, £50, US$58, ISBN: 978-9811203855

Review by Mark McCartney



Difference equations have always been very much the poor relations of differential equations. They are less likely to be included in the undergraduate curriculum at any level of depth, and while there is a constant flow of texts on differential equations, there is merely a drip of texts to be found on their finite difference cousins. (And yes, dear reader, that was a somewhat weak attempt at a mathematical joke.) This lack of coverage is a pity, because difference equations are not only interesting in their own right, but are also pedagogically helpful in, amongst other things, building basic skills in pattern spotting; developing algebraic confidence; and highlighting the fact that mathematical modelling in discrete time and/or space can be very powerful techniques.

In *Difference Equations for Scientists and Engineering* Michael A. Radin provides a student-centred introduction to the subject. He states in the introduction that the aim of the book is to provide 'repetitive type examples to enhance the understanding of the fundamentals of difference equations and their applications' (p. 3). This repetition of examples is most clearly found, as might be expected, in the exercise sections at the end of each chapter. In total within the confines of just six chapters there are over 400 questions (with solutions for all the odd numbered ones at the end of the book). The questions test students' engagement with the material and provide good consolidation of the key ideas in each chapter.

But the chapters themselves also have a repetition of examples, with the author gradually leading the student from the concrete to the abstract. Thus, for example in Chapter 2, on first order linear difference equations, we start with the concrete $x_{n+1} = 4x_n$, gradually building by example all the way up to $x_{n+1} = a_n x_n + b_n$. For the coefficients a_n and b_n solutions arising from period 2 and 3 sequences are studied before moving on to general odd and even periods $2k + 1$ and $2k$. At first glance this may seem much too slow, after all the general solution of $x_{n+1} = a_n x_n + b_n$ can be easily enough found, but the author's methodical technique means that as the student reads through the chapter she gains a clear grasp of the possible behaviours of this general solution. It is a style which is repeated throughout the book as it moves on to chapters on first order nonlinear difference equations, second order linear and nonlinear difference equations, and finally a chapter introducing higher order and coupled systems.

As might be expected from the title, peppered through the book are examples of difference equations which appear in applied mathematics (e.g. the Beverton-Holt model, the logistic and Ricker maps and the Rulkov model) but the emphasis of the book is firmly on the methods rather than the models. Although chaos gets a mention, it is only a brief one. Indeed it is *too* brief to be helpful to the reader. For example, the classic Feigenbaum diagram for the logistic map is presented but not explained, apart from stating that it 'evokes chaos as period doubling' (p. 118).

I have to admit that the book has some quirks of style which jarred with me, but the only one I feel was substantive enough to allow me to mention it in a review is that axes on graphs were habitually not labelled. This is an omission, which as any student taught by me knows, I consider to be punishable by death. (Unfortunately my employer feels that, aside from the inevitable human rights issues involved, such a response is an overreaction.)

However, my personal bugbears aside, I think Radlin has done a nice job in producing a textbook which provides a learner friendly introduction to difference equations. It would suit as a core text for a first year course in the topic, aimed, as the title suggests, at physical science or engineering undergraduates. The student who is prepared to work through the book will get a good grounding in basic techniques and gain a feel for the possible behaviours of standard equations. He will also be given some indication of the usefulness and potential complexity of discrete systems in modern science and engineering. It is thus a pity that the hefty £50 price tag is likely to put him off the idea of purchasing a copy.

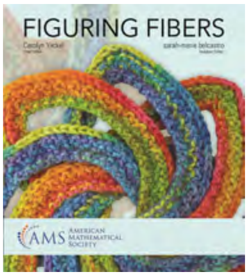
Mark McCartney

Mark McCartney lectures in mathematics at Ulster University. His wife and children suspect that during the COVID-19 lockdown he was responsible for the regular disappearance of substantial quantities of chocolate biscuits from the kitchen cupboard. He categorically denies this.

Figuring Fibers

edited by Carolyn Yackel and Sarah-Marie Belcastro,
American Mathematical Society, 2018, US$40.00, ISBN: 978-1-4704-2931-7

Review by Julia Collins



It is always wonderful to see another book published that celebrates and explores the connections between mathematics and fibre arts. I am of the belief that it is impossible to do craft (of any kind) without an implicit understanding of mathematics, and that mathematics can be understood more deeply when explored through craft. *Figuring Fibers* presents eight chapters that uncover more of these connections. Some start with a craft project and investigate the mathematics arising from it; others start with a mathematical idea and use this to generate a craft project. In the best chapters, craft and mathematics develop together.

Figuring Fibers is the third such book edited by Sarah-Marie Belcastro and Carolyn Yackel, following in the successful footsteps of *Making Mathematics with Needlework* and *Crafting by Concepts*. Although it is beautifully presented, with colourful pictures and photographs, this book is not for the casual reader. Each of the chapters arose from presentations given at the American Mathematical Society's Joint Mathematics Meetings: they are therefore aimed at an audience with a serious mathematical background. At the same time, the crafting aspect of each chapter assumes that the reader has a working familiarity with fibre arts: knitting, crochet and quilting. Beginners in either fibre arts or mathematics will likely find this book to be too technical for comfort.

The book begins with two different overviews of the chapters: one aimed at mathematicians and one aimed at crafters. This is an excellent idea and helps the reader get into both mindsets needed for the book. It also allows the audience to quickly find the projects that will interest them most, whether it is picking an area of maths they appreciate or a type of craft they enjoy. The mathematical ideas range over: fractals and space-filling curves, graph theory, topology, linear congruences (solving systems of equations in modular arithmetic), knot theory, polyhedra and tessellations, and hyperbolic geometry. Craft-wise, three of the chapters use needlework and quilting techniques, three use knitting and two use crochet. (British readers should be wary that the crochet chapters use US terminology, so 'single crochet' (US) means 'double crochet' (UK).)

My favourite project is Chapter 2 by Kyle Calderhead, 'Gosper-like fractals and intermeshed crochet'. This is the chapter in which the mathematics and the fibre-art develop most naturally side by side, without one or the other dominating. Inspired by the Hilbert curve, a space-filling curve generated iteratively in a fractal way, the author develops a new fractal space-filling curve to fit within a hexagonal grid instead of a square one. He proves that there is only one such feasible tiling before going on to design a new hexagonal-intermeshing crochet technique to implement the pattern. Both the mathematics and the crochet are carefully and effectively explained, without using overly technical language yet without leaving out any details. As the author explains, the project is a great example of how "discoveries in both mathematics and fibre arts are made that might never have happened otherwise".

Chapter 4, by Mary D. Shepherd, is inspired by a quilting pattern called *Snake in a Hollow Maze* and is another great example of craft and mathematics mutually inspiring one another. A Truchet tile is a square tile whose motif consists of two quarter-circles joining midpoints of adjacent sides. When these tiles are arranged into a grid, the quarter-circles join up to create a winding 'snake trail' around the grid. A *Snake in a Hollow Maze* pattern results when the snake trail is a single connected path starting and ending on the edges of the quilt. Historically the instructions on how to create this were a carefully guarded secret, passed down from quiltmaker to quiltmaker. But no longer! This chapter provides an algorithm for generating such a pattern from any random initial configuration of the tiles. The final result is some novel mathematics that is easy to follow, and which inspires more questions that are likely to keep both mathematicians and quilters thinking for a while. Readers who are not quilters (like me) can also explore the ideas using printed tiles.

In Chapter 6, Sarah-Marie Belcastro wishes to knit torus knots (those that can be drawn on the surface of a torus) using a particular configuration of circular knitting needles. The constraints generated by this technique necessitate a mathematical investigation to find out which knots can be knitted in this way. While the mathematics is technical, it is not very deep, and therefore even a relative newcomer to knots and braids should be able to follow it. Once this problem is resolved, the chapter concludes with several methods for knitting torus knots, creating both the beautiful image shown on the front cover as well as some more practical bracelets and cowls.

Chapter 7, by S. Louise Gould, will appeal to those readers intrigued by tessellations and polyhedra. The mathematics in this chapter is not new, but it is well-explained and provides an excellent introduction to the idea of 'triply-periodic polyhedra'. First, the reader is introduced to regular and semiregular tessellations, Platonic and Archimedean solids. Just as these tessellations and solids are each composed of gluing together regular polygons with the same configuration at each vertex, so it is possible to create infinite polyhedra with the same constraints but with negative curvature at each vertex. That is, the sum of the angles at each vertex is greater than 360°. The final geometric object being crafted in this chapter is a model made of regular pentagons, arranged five around each vertex, which has translation symmetries in three independent directions. Detailed instructions are provided for how to construct such a model from cloth, but those readers not adept with a sewing machine will appreciate the additional instructions for a model made from card.

The remaining chapters, while having interesting starting points in terms of either the mathematics or the craft, did not, to me, suitably balance the two aspects. Some chapters were too mathematically technical with the craft appearing as an afterthought, while in others the mathematics was too trivial to be of interest.

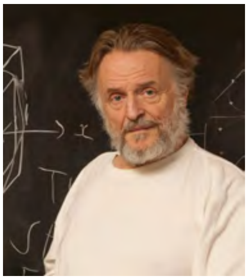
Overall this is a beautiful, intriguing and interesting book with plenty of ideas to explore and create, so long as the reader approaches it with patience and curiosity. I hope it will inspire more mathematicians to express their ideas in craft, and inspire crafters to investigate the mathematics in their projects.

Julia Collins

Julia Collins works as a Lecturer at Edith Cowan University in Perth (Australia) with scholarly interests in mathematics education, outreach and knot theory. Her latest popular maths book, *Numbers in Minutes*, has just been published. When not teaching or writing, Julia will be found knitting mathematical objects or hiking in the Australian bush, accompanied by her trusty sheep Haggis.

Obituaries of Members

John Horton Conway: 1937 – 2020



John Horton Conway, who was elected an Honorary Member of the London Mathematical Society in 2015, died on 11 April 2020, aged 82. He was awarded the LMS Berwick Prize in 1971 and the first LMS Pólya Prize in 1987.

Rob Curtis writes: John was one of the most celebrated British mathematicians of the latter half of the 20th century. It would be difficult to think of anyone who has made such substantial contributions to so many branches of Mathematics as he did.

Conway matriculated at Gonville and Caius College, Cambridge in 1956. As an undergraduate he was already undertaking mathematical research, often in collaboration with Mike Guy, for instance finding all solutions to Piet Hein's SOMA cube. They drew a graph whose vertices were the 240 distinct solutions, joined if one could be obtained from the other by withdrawing two pieces, twisting and replacing them. The resulting SOMAP had a single connected component and an isolated vertex. He proceeded to a PhD under the number theorist Harold Davenport and set about solving the Waring problem expressing every integer as a sum of fifth powers. Conway proved the result using a combination of analytic techniques for large integers together with the ad hoc methods for which he would become famous for the small integers. But by this time his interest had moved on to logic and infinite numbers, leading to a thesis on transfinite numbers.

In 1964 Conway became a University Lecturer and Fellow at Sidney Sussex College, Cambridge. I well remember him striding across the College lawns in the snow, long hair and gown flowing out behind him, followed by his wife and a retinue of young daughters. It was during his time at Sidney that John McKay introduced Conway to John Leech's recently discovered 24-dimensional lattice, and suggested that it might have an interesting group of symmetries. Indeed it did, and in 1968 in an epic piece of work Conway constructed this new simple group and found that it contained two further new simple groups as subgroups. The paper announcing this discovery was a highlight of Volume 1 of the Bulletin of the LMS. This dramatic event motivated Conway to produce a reference book devoted to the finite simple groups and some years later, with co-authors Robert Curtis, Simon Norton, Richard Parker and Robert Wilson, the *Atlas of Finite* Groups appeared. It has since become a standard reference book for any mathematician whose work involves symmetries of finite configurations.

The Leech lattice had been discovered in connection with sphere-packing problems. For instance, what proportion of *n*-dimensional space can be occupied by non-intersecting *n*-dimensional spheres of the same size? This took Conway into the world of coding theory and sphere-packing and eventually to a long collaboration with Neil Sloane which resulted in the monumental *Sphere-Packing, Lattices and Groups*, an essential companion for a sojourn on a desert island.

By this time Conway had acquired international fame. He had moved back to Caius College in 1970 having resigned from Sidney when he considered that behaviour in connection with the election of the new master was underhand, and in 1971–2 he took up a sabbatical at the California Institute of Technology. During his time there he collaborated with David Wales in winning the race to construct the Rudvalis group. He also gave talks around the States on *The Game of Life* which had been invented in the common room at Cambridge where Conway was a fixture, and which had acquired a cult following, occupying millions of hours of computer time around the world. He used to worry that he would be best remembered for *Life* but that apparently frivolous pastime has now gained scientific respectability as an example of a universal cellular automaton. When it appeared in Martin Gardner's column on recreational mathematics in *Scientific American*, to which he had contributed on many occasions, Conway offered a prize of $50 to anyone who could produce a configuration which could be shown to grow indefinitely. This was won by Bill Gosper of MIT who produced a 'glider gun' and, happily for Conway as it was a substantial sum of money at that time, *Scientific American* paid the bet!

Conway's other great passion was mathematical games and in 1982 he produced, together with Elwyn Berlekamp and Richard Guy, Mike's father, the wonderful *Winning Ways for your Mathematical Plays*. Some of the games in this book, such as the well-known 'dots and boxes', had been around for many years, but many more, such as 'sprouts' and 'hackenbush' were invented by Conway himself. It is indeed ironic that the three authors of this very special book have all died within a few months of one another. The analysis of these games involves a great deal of serious mathematics and it is no exaggeration to say that for Conway, 'Mathematics is a Game and Games are Mathematics'.

However, the discovery for which Conway would most like to be remembered is that of what Donald Knuth christened surreal numbers, which combined the approaches of Dedekind and Cantor to produce a rich system of finite, infinite and infinitesimal numbers with many remarkable properties. It is a striking fact that in 1968–69, the year Conway referred to as his annus mirabilis, he had discovered the three finite simple groups named after him, *The Game of Life* and the surreal numbers. Conway continued to be immensely productive and original. Again John McKay came on the scene, pointing out that the smallest degree of an irreducible complex representation of the Monster group is just one less than a coefficient in the Fourier expansion of the modular function J. This innocent observation led to the Conway–Norton theory of Monstrous Moonshine and to the Monstrous Moonshine conjecture for the proof of which Conway's student Richard Borcherds was awarded the Fields Medal.

By the late 1970s Conway's personal life had become chaotic and both his marriages to Eileen, with whom he had four daughters (Susie, Rosie, Ellie and Annie) and Larissa Queen, with whom he had two sons (Alexander and Oliver) ended in difficult divorces. It does seem that he was so utterly committed to mathematics that he could not afford to let human relationships interfere with his research. Some of those closest to him undoubtedly paid a price for his brilliant work. He had moved to Princeton with Larissa in 1986 to take up the von Neumann chair and, following this break-up, he married his third wife Diana, with whom he has a son Gareth. But the inventive brilliance continued, not least in the Free Will Theorem, which he proved together with Simon Kochen in 2004.

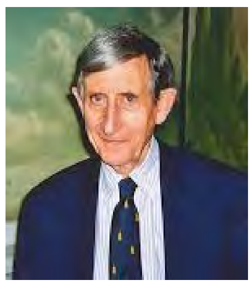
Working closely with Conway was a joy and an inspiration. When dealing with some new structure or concept he would hone and hone the notation until he felt it conveyed all the information required of it as concisely as possible. I often came into work in the morning to find that the language with which I had become familiar had been ditched overnight to be replaced by a more elegant version. Despite his being rather sniffy about combinatorics for its own sake, feeling that every mathematician should possess the required skills, he was a consummate master of the art as is evident in so much of his work. His lectures, both to undergraduates and to research seminar audiences, were refreshing and spontaneous. In the early days he invariably lectured without notes, off the top of his head, spending just a few moments in contemplation before going 'on stage', although I understand that in later years he prepared his lectures meticulously. Indeed, at one time he had a cult following among Cambridge mathematics undergraduates who founded the Conway Appreciation Society.

Such is his international fame that he already has a prize-winning biography written about him: *Genius at Play, the curious mind of John Horton Conway* by Siobhan Roberts.

Conway received many honours during his distinguished career. Apart from his LMS prizes, he was awarded the Nemmers Prize in Mathematics in 1998 and the Leroy P. Steele Prize for Mathematical Exposition in 2000. He was elected a Fellow of the Royal Society in 1981.

John Conway was my mentor, inspiration, co-researcher and friend. I shall miss him enormously.

Freeman J. Dyson: 1923 – 2020



Professor Freeman J. Dyson, who was elected a member of the London Mathematical Society on 17 March 1943, died on 28 February 2020, aged 96. He was elected LMS Honorary Membership in 2000.

Michael Th. Rassias writes: Freeman J. Dyson was one of the world's most famous and vocal scientists. We were all saddened by his passing, as – even at his 96 years – he seemed unstoppable, with his most recent book having been published in 2018. Honored and humbled to be surrounded by such pillars of science as a visiting researcher at the Program in Interdisciplinary Studies of the Institute for Advanced Study, Princeton, over the last years, I had the great privilege of meeting Dyson around 2015. Since then, I had the opportunity to spend some time with him, hoping to absorb some of his wisdom. Inspired by his accomplishments, I was always being carried away by his beautiful narrations of the numerous interesting events of his life.

Dyson, born at Crowthorne in Berkshire, England, was an American theoretical physicist and mathematician whose academic stature had reached that of a historical figure of science, long before his passing.

At the age of 17, in 1941, he arrived at Trinity College, Cambridge, as an undergraduate at a period when Hardy, Littlewood, Besicovitch and other already famous professors were there, with whom he became personal friends: "Especially with Besicovitch, who was the owner of the billiard table", he smilingly pointed out raising his finger. In 1945 he obtained his BA in Mathematics from Cambridge University, for the period 1945–46 he was an Instructor at Imperial College and in 1947 he went to Cornell University as a graduate student, where he worked with Hans Bethe and Richard Feynman. Subsequently, for the period 1948–49 he was a Member at the Institute for Advanced Study, Princeton, and for the period 1949–51 he was a Research Fellow at the University of Birmingham. He then became Professor at Cornell University where he remained until 1953. Surprisingly, he was made Professor at Cornell notwithstanding the fact that he did not have and never actually obtained a PhD. Throughout his career he was a harsh critic of the PhD system, which he strongly believed should be abolished. In a discussion we had, he said that he considered himself "lucky to have been educated in England at a time when the PhD was not required as an entrance ticket to an academic career". He was very much bothered by the fact that the current rigid and lengthy PhD system is one of the main reasons why talented women drop out of academic careers.

In 1953, Dyson became a permanent Professor at the Institute for Advanced Study, Princeton, where he remained throughout the rest of his career.

Dyson has made numerous, profound and versatile contributions in a broad spectrum of subjects of Mathematics and Physics. Among his most important contributions is the unification of the three versions of quantum electrodynamics invented by Richard Feynman, Julian Schwinger and Shin'ichirō Tomonaga, all three of whom were awarded the Nobel Prize in Physics in 1965. His work and lectures on Feynman's theories played a decisive role in making them understandable to physicists of the time and this very much helped Feynman's work being accepted by the academic community. Dyson's work on this subject impressed J. Robert Oppenheimer – who was at the time the Director of the Institute for Advanced Study, Princeton – and had an impact on him being offered a permanent position there. Curiously, despite his stellar accomplishments, Dyson was never awarded the Nobel Prize. He somehow missed his chance. On this subject, he humorously said with his playful character that it is better for people to ask you why you did not get a Nobel prize, rather than why you actually did.

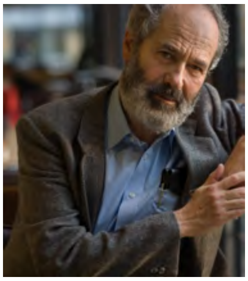
In 1958, at the age of 35, he was a member of the design team under Edward Teller for a small and really safe nuclear reactor called TRIGA used throughout the world in hospitals and universities for the production of medical isotopes. Some of these reactors are still in use, sixty years later.

Teller, Feynman, Hardy, Littlewood, Besicovitch, Gödel, and many other legendary names were just a few of the people entangled with Dyson's spectacular academic life. I must admit that I often caught myself being mesmerized by the surreality of discussing with a living piece of history when I immersed myself in one of the two opposite armchairs in his office at the Institute for Advanced Study.

Throughout his career, Dyson had the characteristic passion to delve into the exploration of problems through which Mathematics can be usefully applied. His span of scientific interests and his everlasting appetite for research quests and the pursuit of the truth, had lead him to investigate problems not only in Mathematics, Physics and their interconnections, but also to other fascinating subjects, such as Astrobiology.

During his career, he had been bestowed with a plethora of awards and distinctions. However, no award bestowment provided him with greater joy than that of unraveling the mystery and beauty of Nature.

Jan Saxl: 1948 – 2020



Jan Saxl, who was elected a member of the London Mathematical Society on 19 March 1976, died on 2 May 2020, aged 71.

Martin Liebeck writes: Jan Saxl was born and grew up in Brno, Czechoslovakia. He began his university studies in 1966 at Masaryk University in Brno, but his life changed dramatically in the summer of 1968, when he was away on holiday in the UK. On the train back home, he learned of the Soviet invasion of his country. He got off the train in Frankfurt and decided not to return, instead making his way to Bristol, where he had relatives. Jan continued his studies at Bristol University, graduating in 1970. He was not able to return to his native country until 1988.

Jan then moved to Oxford to do a DPhil under the supervision of Peter Neumann, finishing in 1973 with a thesis entitled *Multiply Transitive Permutation Groups*. Oxford was an exciting place to do research in algebra at that time, and Jan's student contemporaries included Cheryl Praeger, Peter Cameron, Gareth Jones, Rosemary Bailey, Donald Taylor, Steve Smith, Derek Holt and Jonathan Hall, all of whom went on to distinguished academic careers. After the DPhil, Jan spent a year at the University of Illinois at Chicago Circle, and was then awarded Research Fellowships, first at Hertford College Oxford, and then Downing College Cambridge. He was appointed as a lecturer at Glasgow University in 1978, but returned to Cambridge in 1979, where he spent the remainder of his career, eventually retiring in 2015 as Professor of Algebra and Fellow of Gonville and Caius College. Jan married Ruth Williams, another Cambridge mathematician, in 1979, and their daughter Miriam was born in 1980.

Jan was a leading figure in algebra for over 40 years, publishing around 100 papers and books on a wide variety of topics: permutation groups, finite simple groups, maximal subgroups, representation theory, probabilistic group theory, algebraic groups, algebraic combinatorics, and applications to other areas such as number theory, Galois theory and model theory. He was tremendously collaborative in his research, publishing with 55 different co-authors, and holding visiting appointments at Chicago, Perth, Rutgers, Princeton, Jerusalem and Caltech. Some of these collaborations were very long-lasting, particularly those with Cheryl Praeger, Bob Guralnick and me.

Let me describe briefly just one of the many themes of Jan's research. Starting in the 1980s, many group theorists worked on developing the theory of the subgroup structure of the finite simple groups. Jan was at the forefront of this work, and one of the offshoots was his classification (together with Cheryl Praeger and me) of the maximal factorizations of the simple groups – that is, expressions $G = AB$ of a simple group G as a product of two maximal subgroups A and B. These factorizations have proved to be fundamental to many applications. For example, together with Bob Guralnick and others, Jan used them in an ingenious way to solve problems going back to Dickson, Schur and Carlitz in the theory of exceptional polynomials over finite fields – these are polynomials which induce permutations on infinitely many finite extensions of the field of coefficients; later on, they extended their theory to rational functions over number fields, and found a beautiful connection with elliptic curves.

Jan served as editorial adviser for the LMS journals from 1998–2001; he was also an editor for the *Journal of Algebra* and the *Mathematical Proceedings of the Cambridge Philosophical Society* for many years. He organised two memorable LMS Durham Symposia in 1990 and 2001, and many other high profile conferences. He supervised ten Cambridge PhDs, and was an inspiring and dedicated teacher of hundreds of undergraduates.

Orienteering and skiing were two of Jan's favourite pastimes, and he also had a deep love of classical music, particularly opera. He combined warmth and generosity with an irresistible self-deprecating wit, and was wonderful company. He is deeply missed by his many friends, colleagues and students.

Jan is survived by his wife Ruth, daughter Miriam, and granddaughters Maya and Eva.

Death Notices

We regret to announce the following deaths:

- Professor Peter J. Bushell of University of Sussex, who died on 26 May 2020.
- Professor Mark H.A. Davis of Imperial College London, who died on 18 March 2020.
- Dr Kirill C.H. Mackenzie of University of Sheffield, who died on 2 May 2020.

Covid-19: Owing to the coronavirus pandemic, many events may be cancelled, postponed or moved online. Members are advised to check event details with organisers.

LMS Meeting

LMS Northern Regional Meeting

University of Manchester, 7–11 September 2020

Website: tinyurl.com/yamy8uvq

This conference is held in celebration of the 60th birthday of Bill Crawley-Boevey. The representation theory of quivers has developed through a strong interaction of general theory and the investigation of examples. Crawley-Boevey has had a major influence on the field, his work exemplifying this interaction as well as the entwining of finite- and infinite-dimensional representation theory. Quivers and their representations appear in many parts of mathematics, in physics and in an increasing variety of applications, and this range will be represented by the speakers.

Supported by the LMS, EPSRC, the Clay Mathematics Institute, the Heilbronn Institute and the Alexander von Humboldt Foundation.

LMS Meeting

LMS Prospects in Mathematics (online)

10–11 September 2020, hosted by the University of Bath

Website: tinyurl.com/yaqtyzmm

All finalist mathematics undergraduates who are considering applying for a mathematics PhD in 2021 are invited to attend this meeting, which will feature speakers from a wide range of mathematical fields across the UK who will discuss their current research and what opportunities are available to you. To register, send an email headed ‘Prospects 2020 Application’ to prospects2020@bath.ac.uk with the statement “I am on track academically to begin PhD studies in 2021” and evidence of your predicted degree classification. The registration deadline is 15 August 2020.

RSC Student Conference

Location: University of Nottingham
Date: 21–24 July 2020
Website: tinyurl.com/u8vy5u5

The Research Students’ Conference is returning to the University of Nottingham for a fourth time since it began in 1980. This conference is for PhD students based in Probability and Statistics fields, organised by PhD students every year in the UK. If you’re a student and interested in speaking at the conference, complete the validation questionnaire when booking your place. Supported by an LMS Early Career Research grant.

IMA Induction Course for New Lecturers in the Mathematical Sciences 2020

Location: Isaac Newton Institute, Cambridge
Date: 16–17 September 2020
Website: tinyurl.com/t5ryxgg

This course, designed by members of the mathematics community, is suitable for anyone new to or with limited experience of teaching mathematics/statistics in UK higher education. Session leaders will have significant experience of teaching in the mathematical sciences, and delegates will have the opportunity to discuss their own ideas, challenges and experiences.

Covid-19: Owing to the coronavirus pandemic, many events may be cancelled, postponed or moved online. Members are advised to check event details with organisers.

Society Meetings and Events

September 2020

7-11 Northern Regional Meeting, Conference in Celebration of the 60th Birthday of Bill Crawley-Boevey, University of Manchester

10-11 Prospects in Mathematics Meeting, University of Bath

October 2020

29 EMS/EdMS/LMS Meeting, ICMS, Edinburgh

November 2020

19 Computer Science Colloquium, London

19 Society Meeting and AGM, London

December 2020

5 Midlands Regional Meeting, Lincoln

Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website (www.lms.ac.uk/content/calendar). Please send updates and corrections to calendar@lms.ac.uk.

July 2020

21-24 RSC Student Conference, University of Nottingham (489)

24-30 27th International Mathematics Competition for University Students, Blagoevgrad, Bulgaria (487)

27-31 Integrable Probability Summer School, online event (488)

September 2020

10-11 Heilbronn Annual Conference 2020, online event (488)

16-17 IMA Induction Course for New Lecturers in the Mathematical Sciences, Isaac Newton Institute, Cambridge (489)

July 2021

7-9 22nd Galway Topology Colloquium, University of Portsmouth (488)

12-16 New Challenges in Operator Semigroup, St John's College, Oxford

12-19 14th International Congress on Mathematical Education Shanghai, China

14-16 IMA Modelling in Industrial Maintenance and Reliability Conference, Nottingham (486)

20-26 8th European Congress of Mathematics, Slovenia

August 2021

16-20 IWOTA, Lancaster University (481)

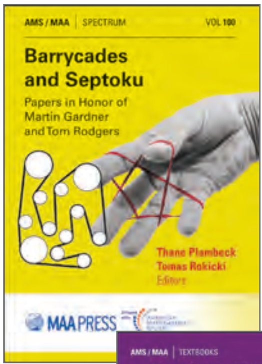
September 2021

19-24 8th Heidelberg Laureate Forum, Heidelberg, Germany

21-23 Conference in Honour of Sir Michael Atiyah, Isaac Newton Institute, Cambridge (487)

July 2022

24-26 7th IMA Conference on Numerical Linear Algebra and Optimization, Birmingham (487)



BARRYCADES AND SEPTOKU

Papers in Honor of Martin Gardner and Tom Rodgers

Edited by Thane Plambeck, Counterwave, Inc., Palo Alto, USA & Tomas Rokicki

Consists of papers originally presented at the Gathering 4 Gardner meetings. Recreational mathematics is prominent with games and puzzles, including new Nim-like games, chess puzzles, coin weighings, coin flippings, and contributions that combine art and puzzles or magic and puzzles. Anyone who finds pleasure in clever and intriguing intellectual puzzles will find much to enjoy in *Barrycades and Septoku*.

Spectrum, Vol. 100

MAA Press

May 2020 234pp 9781470448707 Paperback £64.95

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A textbook for a second course or capstone course in analysis for advanced undergraduate or beginning graduate students. By assuming the existence and properties of the Lebesgue integral, this book makes it possible for students who have previously taken only one course in real analysis to learn Fourier analysis in terms of Hilbert spaces.

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MAA Press

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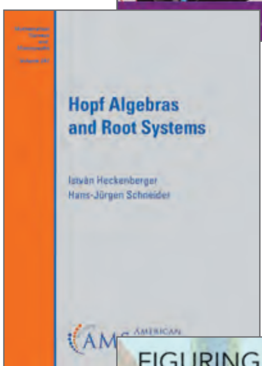


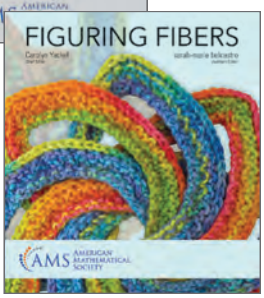
HOPF ALGEBRAS AND ROOT SYSTEMS

Istvan Heckenberger, Philipps Universität Marburg, Germany & Hans-Jurgen Schneider, Ludwig-Maximilians-Universität München, Mathematisches Institut, München, Germany

Provides an introduction to Hopf algebras in braided monoidal categories with applications to Hopf algebras in the usual sense. The main goal is to present from scratch and with complete proofs the theory of Nichols algebras (or quantum symmetric algebras) and the surprising relationship between Nichols algebras and generalized root systems.

Mathematical Surveys and Monographs, Vol. 247

Jun 2020 582pp 9781470452322 Hardback £139.00



BESTSELLER

FIGURING FIBERS

Edited by Carolyn Yackel, Mercer University, Macon, USA & Sarah-Marie Belcastro, MathILy, Mathematical Staircase, Inc., Holyoke, MA and Smith College, Northampton, USA

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