LONDON MATHEMATICAL SOCIETY
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NEWSLETTER

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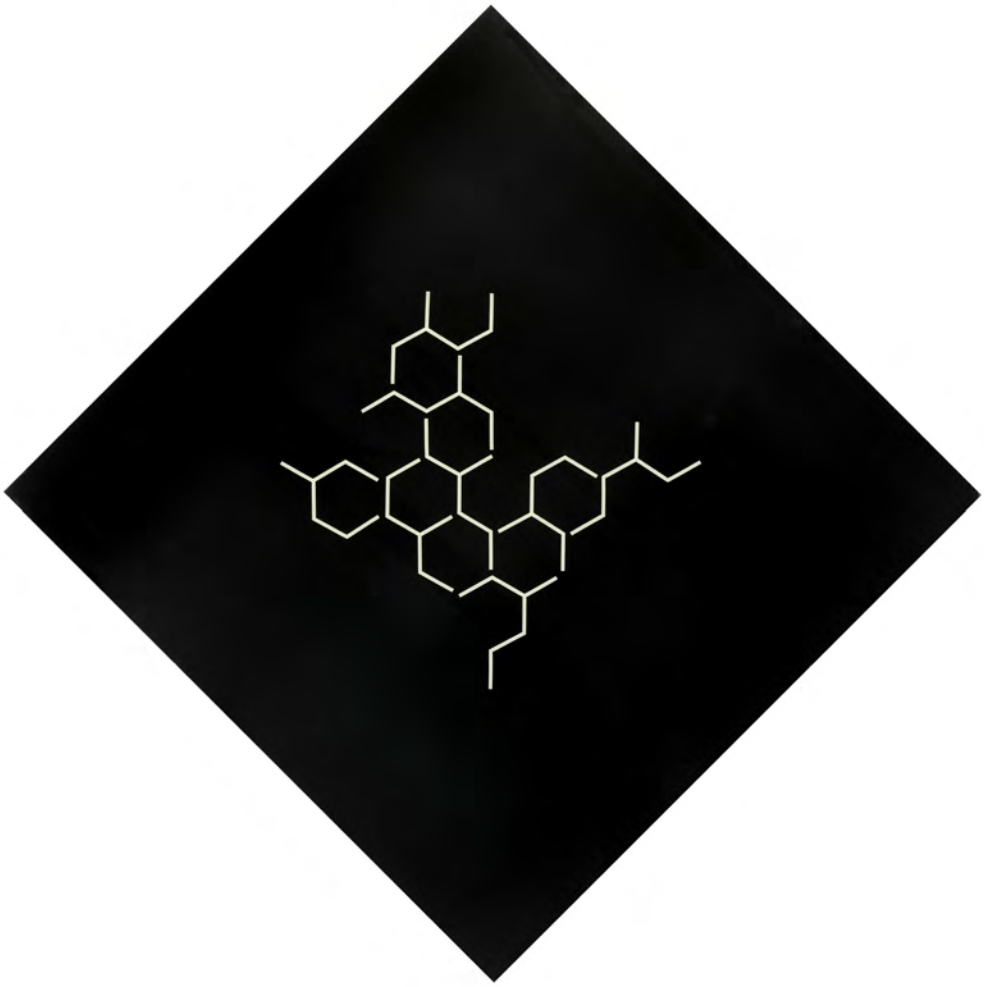

RUBEL'S PROBLEM

MATHEMATICS AND INDEPENDENCE

FOUR DECADES ON

EDITORIAL OFFICE

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Charity registration number: 252660

COVER IMAGE

Anthony Hill: Parity Study No.2 First Version, 1970
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See obituary on page 47.

Do you have an image of mathematical interest that may be included on the front cover of a future issue? Email images@lms.ac.uk for details.

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The Newsletter welcomes submissions of feature content, including mathematical articles, career related articles, and microtheses from members and non-members. Submission guidelines and LaTeX templates can be found at lms.ac.uk/publications/submit-to-the-lms-newsletter.

Feature content should be submitted to the editor-in-chief at newsletter.editor@lms.ac.uk.

News items should be sent to newsletter@lms.ac.uk.

Notices of events should be prepared using the template at lms.ac.uk/publications/lms-newsletter and sent to calendar@lms.ac.uk.

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CONTENTS

LMS Council 2020–21

The results of the 2020 LMS Elections to Council and Nominating Committee were announced at the LMS Annual General Meeting on 20 November 2020. Council membership for 2020–21 is as follows:

President:
Professor Jon Keating FRS (University of Oxford)

Vice-Presidents:
Professor Iain Gordon (University of Edinburgh)
Professor Catherine Hobbs (University of the West of England)

Treasurer:
Professor Simon Salamon (King's College London)

General Secretary:
Professor Robb McDonald (University College London)

Programme Secretary:
Professor Chris Parker (University of Birmingham)

Publications Secretary:
Professor John Hunton (University of Durham)

Education Secretary:
Dr Kevin Houston (University of Leeds)

Member-at-Large (Librarian):
Dr Mark McCartney (Ulster University)

Members-at-Large of Council:
Professor Peter Ashwin (University of Exeter);
Professor Anne-Christine Davis (Cambridge University);
*Professor Elaine Crooks (Swansea University);
*Professor Andrew Dancer (University of Oxford);
*Dr Anthony Gardiner;
Professor Minhyong Kim (University of Warwick);
Professor Niall MacKay (University of York);
*Professor Frank Neumann (University of Leicester);
*Professor Brita Nucinkis (Royal Holloway, University of London);
Professor Anne Taormina (University of Durham) (re-elected to Council);
Dr Amanda Turner (Lancaster University).

*Members elected in 2019 who are continuing with the second year of their two-year term.

The AGM also saw the following members step down as Members-at-Large on Council: Professor Alexandre Borovik, Dr Tara Brendle, Professor David E. Evans, Mr Richard Pinch and Dr Mariya Ptashnyk. The LMS is grateful for their contributions, details of which can be found on page 7.

LMS Nominating Committee

Also at the AGM, Professor Chris Budd (University of Bath) and Professor Gwyneth Stallard (Open University) were elected to the Nominating Committee for three-year terms of office. Continuing members of the Nominating Committee are: Kenneth Falconer (Chair), I. David Abrahams, Beatrice Pelloni, Mary Rees and Elizabeth Winstanley. Council will also appoint a representative to the committee.

Incoming Officers and Members of Council

Treasurer:
Simon Salamon is Professor of Geometry at King's College London, where he was Head of the Mathematics Department 2013–17. He began his career in 1979 as a Visiting Assistant Professor at the University of Maryland, as a CNR fellow at the Scuola Normale in Pisa, and as a member of IAS Princeton. He was a Lecturer and Reader at the University of Oxford 1984–2001, a Reader at Imperial College in 2003/04 and *Professore Ordinario* at the Politecnico di Torino 2000–2011. His research interests lie mainly in differential geometry; specifically, Lie groups, special holonomy, Einstein metrics, nilmanifolds, complex and quaternionic geometry, twistor spaces, and applications to string theory and quantum information. Professor Salamon's LMS service includes being a member of the Editorial Board 1995–98 and Co-Managing Editor of the *Proceedings of the LMS* 1998–2000. He was also Co-Editor-in-Chief of EMS *Surveys in Mathematical Sciences* 2014–17.

General Secretary:
Robb McDonald is a Professor of Mathematics at University College London, which he joined as a Lecturer in 1994 and where he was Head of Department 2011–18. Gaining his PhD from the University of Western Australia in 1991,

Professor McDonald was previously a Royal Society Endeavour Fellow at the University of Oxford 1991–93 and an Australian Research Council Research Fellow at Monash University 1993–94. His research interests are fluid mechanics, vortex dynamics and applied complex analysis, and the application of mathematics to geoscience. Professor McDonald is a Fellow of the Institute of Mathematics and its Applications.

Members-at-Large:
Peter Ashwin is a Professor of Mathematics at the University of Exeter, where he was Head of Department 2010–14. He joined the London Mathematical Society in 1991, the year he received his PhD from the University of Warwick. Previous appointments have been in Postdoctoral Researcher roles at the University of Marburg (Germany) (1991–92), the University of Warwick (1992–95) and INLN Nice (France) (1995–96). He was a Lecturer in Mathematics at the University of Surrey 1996–2000. Professor Ashwin's research interests are in dynamical systems theory and applications, including low dimensional systems, theory of attractors, bifurcation theory, coupled and nonautonomous systems, computational modelling and applications, mostly in life and environmental sciences.

Anne-Christine Davis is Professor of Mathematical Physics 1967 (Emeritus) DAMTP, Centre for Mathematical Sciences, Cambridge University. Her previous appointments have been as a Postdoc at Durham University (1976–78) and Imperial College (1978–80), a Fellow at CERN, Geneva (1980–82), a Member of IAS, Princeton (1982–83) and a Research Council Advanced Fellow (1983–88). From 1988 she moved to Cambridge University as a College Teaching Officer at Kings College (1988–96), Assistant Director of Research (1995–96), a Reader (1996–2002) and Professor of Theoretical Physics, DAMTP (2002–13). Professor Davis is a Mathematical Physicist interested in theoretical cosmology, modified gravity theories, General Relativity, Quantum Theory and in particular particle cosmology. Her service to the LMS has included membership of the Women in Mathematics Committee from 2014 and membership of the Good Practice Scheme Committee from 2014, becoming its Chair in 2018.

Minhyong Kim is Christopher Zeeman Professor of Algebra, Geometry, and Public Understanding of Mathematics, University of Warwick and Distinguished Professor, Korea Institute for Advanced Study. He gained his PhD at Yale University in 1990 and his previous appointments have been at the Massachusetts Institute of Technology, Columbia University, University of Arizona, Purdue University, University College London, Pohang University of Science and Technology and the University of Oxford. Professor Kim's research Interests are Arithmetic Geometry, Topology and Mathematical Physics. His LMS service has been as a member of the Editorial Board of LMS publications (2013–14), Editorial Board of Mathematika (2008–17) and LMS representative on the ICMS Board (2014–19).

Niall MacKay is Professor of Mathematics and Head of Department in the Department of Mathematics at the University of York. He gained his PhD at the University of Durham in 1992; previous appointments have been as a JPS fellow, Kyoto University Research Institute for Mathematical Sceinces (1992–93), PPARC Research Fellow and fellow of Queens' College, Cambridge (1993–95), Stokes Fellow, Pembroke College, Cambridge (1995–98), and Lecturer, University of Sheffield (1998-99). He joined the University of York in 2000, where he has been a Lecturer (2000), Senior Lecturer (2005), Reader (2009), Professor (2014) and Head of Department since 2015. Professor MacKay's research interests are integrable systems and quantum groups; operations research and history. His service to the LMS has been as a member of the Education Committee 2004–09 and 2011–14 and as an Editorial Adviser from 2005–14.

Amanda Turner is a Senior Lecturer in Mathematics and Statistics at Lancaster University. She received her PhD from the University of Cambridge in 2007; previous appointments have been as a College Teaching Fellow at Selwyn College, University of Cambridge (2006–07) and a Visiting Professor at the University of Geneva (2018–20). Dr Turner's research interests are probability, complex analysis and mathematical physics, with a specific interest in random growth models. Her service to the LMS has been as a member of the Research Grants Committee since 2017 and the Publications Committee since 2018 and as Editorial Advisor for the LMS journals since 2016.

Members of Council 2020–2021

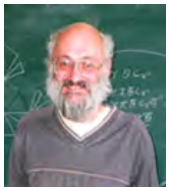
Jon Keating
President

Cathy Hobbs
Vice-President

Iain Gordon
Vice-President

Simon Salamon
Treasurer

Robb McDonald
General Secretary



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Programme Secretary

John Hunton
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Education Secretary

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Andrew Dancer
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Tony Gardiner
Member-at-Large

Minhyong Kim
Member-at-Large

Niall MacKay
Member-at-Large

Frank Neumann
Member-at-Large

Brita Nucinkis
Member-at-Large

Anne Taormina
Member-at-Large

Amanda Turner
Member-at-Large

Retiring Officers and Members of Council

Rob Curtis
Treasurer 2011–2020

After serving as LMS Treasurer since 2011, Professor Rob Curtis stepped down at the Council Meeting on 20 November 2020.

Professor Curtis has supported the LMS over many years as a Council Member-at-Large (2001–07), Librarian (2003–07) and a Regional Coordinator for 5 years; on Prizes Committee (2004–05), Programme Committee (2001–04) and Publications Committee (2018–20). He has also been a member of the Development Committee (2013–20) and he played a major role in the success of the Society's 150th Anniversary celebrations as part of the 2015 Celebration Committee and in particular developing fundraising opportunities.

During his tenure as LMS Librarian, Professor Curtis was Chair of the Editorial Board that oversaw the publication of The Book of Presidents 1865–1965 and the agreement to house the Philippa Fawcett Collection (a wide-ranging library of books written by and about women who studied or worked in mathematical subjects in the 19th and first part of the 20th century, or earlier) at De Morgan House.

As LMS Treasurer, Professor Curtis has seen the importance of the Society's role increase as a well-informed, articulate voice making the case for mathematics and in particular the funding of mathematical research. As financial support for research becomes increasingly difficult to obtain, the Society's role in providing grants became invaluable across the Society's core grant schemes. The amount spent on grants has increased substantially year on year during Professor Curtis's time as Treasurer.

Professor Curtis took measures to ensure the continued financial security of the Society, so that it can support these essential activities now and in the future, by diversifying the Society's investment portfolio into areas such as property, and the Society now owns seven properties in Central London and Birmingham. He also established a high-powered team of external financial advisers and oversaw the movement of the Society's investment portfolio from Morgan Stanley to Schroders, a Charity Multi-Asset investment Fund that maintains the real value of LMS capital while generating a sustainable and reliable return.

Besides finance, Professor Curtis also had responsibility for the Society's membership and made several major contributions in this area. He improved communications between LMS Council and LMS members by re-establishing a network of departmental representatives and there are now 61 LMS Reps at institutions around the UK. He was also instrumental in setting up the Annual Reps Day where representatives meet at De Morgan House to network and exchange ideas.

Another aspect of the work Professor Curtis did in this area was a concerted membership drive to attract a broader cohort of mathematicians to become members of the Society. He oversaw the introduction of Associate Membership for Teacher Training Scholars and Associate (post-doc) Membership categories. In 2019–20 the Society introduced the Associate (Undergraduate) Membership so that students can enjoy membership at a reduced rate. He also visited universities around the UK to give membership presentations. Around 3,100 mathematicians and students of mathematics are currently members of the Society.

Professor Curtis has supported the Society over a number of years in very important roles. His expertise and leadership have helped to shape the Society's membership for the future and he has established a robust model for the Society's finances going forward. The LMS thanks him for his exceptional contributions over the years.

Stephen Huggett
General Secretary 2012–2020

After eight years of service, Professor Stephen Huggett stepped down as LMS General Secretary at the Council Meeting on 20 November 2020.

Professor Huggett has given many years of service to the LMS in a number of different roles dating back to 1992 as a member of LMS Education Committee (1992–2001). He was Programme Secretary (2001–11), Chair, International

Affairs Committee (2004–11), LMS Council member (2001–11 and 2012–20). He has also been a member of the Standing Orders Review Group (2013–19) and Development Committee (2013–19). During the Society's Anniversary celebrations in 2015 Professor Huggett was a member of the 150th Anniversary Committee. He chaired the Website Working Group (2010–19) and the Newsletter Review Group (2015–17) and was a member of the Newsletter Editorial Board (2017–20). He was also a member of the Mentoring African Research in Mathematics (MARM) Board (2008–20).

He has also served on other national and international committees and groups, including as Secretary to the European Mathematical Society (2007–14).

Professor Huggett has been a major supporter of the LMS in a number of influential roles. As Programme Secretary he helped to shape and promote the Society's portfolio of meetings and events, building and maintaining the Society's reputation for excellence in providing meeting programmes presenting high-quality mathematical research in the UK and abroad.

During his time as General Secretary, Professor Huggett was an important member of the Standing Orders Review Group, which oversaw the conclusion of the review and update of the LMS Standing Orders (Charter, Statutes and By Laws) to ensure the Society's governance structure is fit for purpose for the foreseeable future.

In 2015 the Society celebrated its 150th Anniversary with a wide-ranging programme of events. Professor Huggett was an integral member of the 2015 Celebration Committee and was the driving force behind several major 150th Anniversary initiatives including the re-design of the Society's logo and the broader Society brand identity, both for the anniversary year and going forward.

The Society undertook a major redesign of its monthly newsletter in 2016–17. The work was undertaken by the Newsletter Review Group, chaired by Professor Huggett, which sought input from the LMS Membership on the content and appearance of the new Newsletter. Professor Huggett was also a member of the new Newsletter Editorial Board, which oversees the content for the Newsletter, including LMS news and feature articles, and affording the opportunity for current research students to present their research findings to the wider mathematical community.

The Society's website is a vital part of its communications armoury providing a range of online services for its members. Development work over the past 10 years, overseen by the Website Working Group, chaired by Professor Huggett has resulted in progress in both the design of the website and also the services it provides, including online membership applications and grant submissions, registration for events and meetings and an online donation page.

Professor Huggett's expertise on mathematics issues in a wider context was very important in giving the Society an influential voice on the international stage and he has represented the Society at numerous meetings, including as part of the delegations to both the International Congress of Mathematicians (ICM) and the European Congress of Mathematics (ECM).

Professor Huggett's wide experience has benefited not only the Society but the mathematics community as a whole for many years and the LMS is enormously grateful to him for the support he has given to the Society.

Alexandre Borovik
Member-at-Large 2006–12 and 2014–20

Professor Borovik has served as a Member-at-Large on Council 2006–12 and 2014–20. He has served on many committees and groups: Programme Committee (2007–10), Research Meetings Committee (2007–10), Education Committee (2010–12), the Web Working Group (2010–19) and the Standing Orders Review Group from 2013. Professor Borovik was the founder in 2011 and one of the editors of The De Morgan Forum (education.lms.ac.uk), the LMS blog on mathematics education and The De Morgan Gazette, online magazine on mathematics education. The LMS thanks Professor Borovik for his contributions to Council meetings, committee membership and continuing involvement with the work of the Society.

Tara Brendle
Member-at-Large 2014–20

Professor Brendle has been a Member-at-Large on the Society's Council since 2014. She served as the Edinburgh Mathematical Society representative on the London Mathematical Society's Women in Mathematics Committee from 2013 to 2019. Professor Brendle has served as Council Diarist and on various committees including the Strategic Subgroup, Education Committee, Personnel Committee, Prizes Committee and the Zeeman Medal Committee. The LMS is grateful to Professor Brendle for her ongoing commitment to achieving the aims of the Society over several years.

David E. Evans
Member-at-Large 2016–20

Professor Evans has been a Member-at-Large on Council since November 2016, also sitting as a member of the Personnel Committee and as the Council's representative on Nominating Committee for 2019. He also was a member of the David Crighton Medal Committee in 2019. The Society would like to thank Professor Evans for his service and continuing support of various aspects of LMS work.

Richard Pinch
Member-at-Large 2019–20

Dr Pinch joined Council as a Member-at-Large for a one year term in November 2019. From 1998 to 2018 Dr Pinch was a Mathematician at GCHQ; he has also served as Deputy Director of the Heilbronn Institute for Mathematical Research. He is currently Vice President (Professional Affairs and Industry) of the Institute of Mathematics and its Applications. Since joining Council he has served as a member of the Research Policy Committee and the Covid-19 Working Group. Dr Pinch gave the LMS Popular lecture in 1994. The LMS records its thanks to Dr Pinch for his contributions to the work of the Society.

Mariya Ptashnyk
Member-at-Large 2018–20

Dr Ptashnyk has been a Member-at-Large on Council since November 2018. She has served as a member of Computer Science Committee since 2018, Early Career Research Committee from 2019 and Research Grants Committee since 2020. As a Council member who works in Applied Mathematics, Dr Ptashnyk has also represented the Society on the Board of the International Council for Industrial and Applied Mathematics (ICIAM). The LMS would like to thank Dr Ptashnyk for her work on Council and her continuing support for the Society.

Newsletter Editor-in-Chief

The LMS will be seeking a new Editor-in-Chief of the Newsletter shortly. Full details of the role and how to apply will be circulated via the regular membership update and the LMS website.

UCL Library Members Usage Survey

The LMS Library is integrated with the Mathematics section of the University College London (UCL) Science Library. We wish to assess membership use of UCL library facilities by surveying responses based on usage in 2019, as access to library facilities in 2020 was hindered due to the pandemic.

We urge all LMS members, whether they use the library or not, to respond to this brief survey so that the library and the LMS will be better attuned to responding to your needs.

Thank you for participating. The survey can be found at: tinyurl.com/y2jdecza.

Mark McCartney
LMS Member-at-Large (Librarian)

New Editorial Board for *TLMS*

The *Transactions of the LMS (TLMS)* was established in 2014. During 2021 it will set out with a new editorial board, positioning the journal as a high-quality fully open-access publication compliant with the growing number of institutional and funder mandates requiring open access publication including Plan S, which comes into force from January 2021. It continues to welcome papers of a general or specialised nature that represent a significant advance in mathematical knowledge, as well as submissions that are deemed to stimulate new interest and research activity.

A new editorial board of leading experts across mathematics is currently being formed, with Caucher Birkar, Amin Coja-Oghlan, Charles Fefferman, Alessio Figalli, Jelena Grbic, Martin Hairer, Heather Harrington, John King, Jessica Purcell, Carola Schönlieb, Sebastian van Strien, Marcelo Viana, Juncheng Wei and Sarah Zerbes having agreed to serve.

The new editorial board will start accepting submissions from 1 March 2021. The current joint Editorial Board of the LMS journals will continue to receive new submissions for the *Bulletin* and the *Journal*. Papers submitted to the *Transactions* prior to 1 March 2021 will continue to be handled by the existing Board and decisions for acceptance will be held against the same criteria that were in place at the time of submission.

An increasing number of authors now have access to funds to fully cover article processing charges (APCs) through agreements made between their institution or funding agency and Wiley. This includes many researchers based in the UK, Austria, Finland, Germany, Hungary, Norway and Sweden (a list of funders and institutions which offer support for APCs can be found on Wiley's website: http://tiny.cc/OA_agreements). These charges support the cost of managing and publishing the *Transactions*, with surplus income from all of the Society's publications used in support of mathematicians and mathematics research. It is however the Society's intention that no author be turned away owing to funding constraints, those without access to institutional support having their article costs met through alternative means.

To find out more about changes to the *Transactions* visit lms.ac.uk/tlms. John Hunton, LMS Publications Secretary, wrote about Plan S in the January 2019 LMS Newsletter, page 5 (tinyurl.com/y8udwwyd). There is a plan to provide an update on Plan S for the March 2021 LMS Newsletter.

José Rodrigo
Managing Editor, *Transactions of the LMS*

Leaving a Legacy to the Society

Readers receiving a hard copy of the *LMS Newsletter* will notice that this issue includes a leaflet intended to assist anyone who is contemplating leaving a legacy to the Society.

Historically, we have been very fortunate to receive major donations and legacies, including those of Lord Rayleigh, Joseph Larmor, G.H. Hardy, Albrecht Fröhlich and Samuel Verblunsky, and more recently donations from the Liber Foundation and Dr Tony Hill. The Society is extremely grateful to anyone able to make a donation or remember the Society in their Will. The generosity of supporters contributes to an income which enables us to carry out vital work in support of the mathematics community, particularly in response to the impact of the covid-19 pandemic, including research and conference grants, initiatives for students, early career researchers and teachers, and general promotion of mathematics. With income from publications potentially under threat owing to the move to Open Access, the long-term stability of the Society's programmes is increasingly likely to depend on endowments and gifts. At the same time, Council is keen to maintain, and ideally increase, its support for all its activities.

Notes on leaving a legacy to the Society in your Will can be found in the enclosed leaflet. In addition, the LMS website includes more general information on how to make a donation to the Society: see lms.ac.uk/content/donations. There is also the 'De Morgan Friends' scheme for those in a position to make a donation of £1,865 or more; see lms.ac.uk/content/donations/demorgan.

If you would like to discuss any of this further, please contact development@lms.ac.uk.

Professor Jon Keating FRS
LMS President

EPSRC Peer Review

The LMS Research Policy Committee works closely with the Engineering and Physical Sciences Research Council (EPSRC), the UK's main funding body for the Engineering and Physical Sciences, including the Mathematical Sciences. As many LMS members will know, independent peer review is key to EPSRC funding decisions; all such decisions are based primarily on this expert advice, typically received from members of EPSRC's College of Peer Reviewers.

It is important that the College reflects the knowledge, expertise and diversity of the wider research community. EPSRC is keen to maintain a broad and deep coverage of all areas of the Mathematical Sciences. It has an Associate College which offers the opportunity to be promoted to Full College at a later stage. Membership of the Associate College is achieved either through external nomination or through self-nomination. The LMS encourages all of its members, and particularly those at the early-career stage, to apply for membership of the Associate College if they've not already done so, and for Heads of Departments to consider nominating suitable colleagues. When doing so, EPSRC asks for as much detail as possible on nominees' backgrounds and research areas when completing the application: this ensures proposals sent for review are appropriate.

Members may also be aware of the Additional Funding Programme for the Mathematical Sciences, announced by the government at the beginning of this year, which EPSRC is working to implement. Timely peer review will be a highly important factor in the delivery of this programme: the aim is to make funding decisions on at least 80% of proposals within 20 weeks of receipt.

The EPSRC Mathematical Sciences team have commented that a very helpful action would be for people to respond to requests to review as soon as possible. Last year about 20% of requests did not get a response, and this can cause a significant delay when processing a grant application.

Further information on the College of Peer Reviewers and how to join it can be found at tinyurl.com/yxs5saf2 or you can contact a member of the Mathematical Sciences team at EPSRC at tinyurl.com/y6odateo.

Iain Gordon
LMS Vice-President and Chair of Research Policy Committee

Levelling Up Scheme

Thanks to a generous donation from Dr Tony Hill, the LMS has been working on a new venture to support the provision of online tutoring for A-Level Mathematics students who come from backgrounds that are under-represented in the mathematics community.

Ideally working with three or four universities, the broad aims of the pilot are to bring together undergraduate student tutors with A-Level Mathematics students who wish to improve their grades to enable them to read a STEM degree. Universities will identify nearby schools with significant percentages of students from under-represented backgrounds and enlist the help of undergraduate students to tutor and engage with their A-Level tutees.

We have now established a strong team who will be responsible for developing the tutoring programme and the tutorial materials. Collectively their role will be to plan out a suitable programme and to identify and collate tutorial material for the pilot scheme. This will include problem sets and worked solutions, key facts and principles relating to the problem topic, suggested discussion points and extension materials. The aim is to challenge and stretch pupils and also to consolidate A-Level content, drawing on suitable pre-existing material where possible. In addition, the curator/curriculum developer will develop a website to make material available to the universities, tutors and pupils participating in the scheme, using a suitable platform.

OTHER NEWS

Fellow of the AMS

Professor John Greenlees (University of Warwick), past Vice-President of the LMS (2009-19), awarded the LMS Berwick Prize in 1995, has been elected a Fellow of the American Mathematical Society (AMS) for contributions to commutative algebra and algebraic topology, in particular to equivariant stable homotopy theory. For further details of the fellowships see tinyurl.com/y5hnbwgx.

Fellow of the AWM

LMS member Dr Eugenie Hunsicker, Chair of the LMS Committee for Women and Diversity in Mathematics, has been elected a Fellow of the Association for Women in Mathematics (AWM). Dr Hunsicker receives the honour for 'leadership of the United Kingdom community of women in mathematics; tireless advocacy for women in mathematics everywhere through talks, writing, and the film *Faces of Women in Mathematics,* and application of mathematical and statistical expertise to research into equity and diversity issues facing the mathematical community'.

Dr Hunsicker became a member of the LMS Women in Mathematics Committee (now the Committee for Women and Diversity in Mathematics) in 2015 and Chair in 2016. She has been the driving force behind the Society's strategy for women and diversity in mathematics and has achieved notable success in broadening the Society's remit in terms of diversity issues. Other achievements include securing a larger commitment to events, activities and grants, with enhanced support for Women in Mathematics, Girls in Mathematics and Diversity in Mathematics events, grants to help those with caring responsibilities and setting up the Success Stories website. Another major achievement was the award of the Royal Society's inaugural Athena Prize to the LMS Women in Mathematics Committee in 2016. Dr Hunsicker is also Deputy Chair of the Athena Forum.

The 2021 AWM Fellows will be honoured at the AWM Business Meeting and Awards Presentation as part of the Virtual JMM on 8 January 2021. For further details see tinyurl.com/y29t6yub.

Note: A features article about AWM was in the September 2020 Newsletter, pages 42–43: tinyurl.com/y53r8jo2

Leverhulme Trust Exceptional Researchers Awards

The Leverhulme Trust has awarded the Mathematics and Statistics Exceptional Researchers award to LMS members Professor Heather Harrington (University of Oxford) for her work on applied mathematics: algebraic systems biology and topological data analysis and Professor Sasha Sodin (Queen Mary, University of London) for his work on spectral theory of random operators and random matrices. Other awardees are: Dr Ana Caraiani (Imperial College London) for her work on algebraic number theory and arithmetic geometry, Dr Richard Montgomery (University of Birmingham) for his work on combinatorics (random and extremal graph theory) and Dr Nick Sheridan (University of Edinburgh) for his work on homological mirror symmetry and symplectic topology.

For further information, including the other categories, visit the website at tinyurl.com/y2rylws9.

MATHEMATICS POLICY DIGEST

Generating New Ideas

A new initiative launched by the government in October 2020 will 'provide mathematical scientists and researchers with small amounts of funding to support the discovery of ground-breaking and transformative new ideas'. The Engineering and Physical Sciences Research Council (EPSRC), part of UK Research and Innovation (UKRI), will provide the funding programme.

These small grants are aimed at helping mathematical scientists to pursue short term research projects outside their day to day work, which could lead to more substantial programmes of research being developed.

The initiative can also be used to upskill or retrain staff, for example through hiring postdoctoral research assistants to work on specific projects, or for institutions to host secondments that will aid research. More information is available at tinyurl.com/y52m3qwe.

Digest prepared by Dr John Johnston
Society Communications Officer

Note: items included in the Mathematics Policy Digest are not necessarily endorsed by the Editorial Board or the LMS.

EUROPEAN MATHEMATICAL SOCIETY NEWS

From the EMS President

With a partial lockdown throughout Europe, most of the activities of the EMS have turned to the internet. This includes the December meeting of the Executive Committee, the third virtual meeting in succession. It was decided to postpone the EMS's 30th anniversary meeting, previously planned for Edinburgh in October, until it can be held in person. The Treasurer reported the excellent financial situation of the EMS, though this is unfortunately partially due to almost none of the EMS's sponsored meetings, workshops, and schools taking place. Here too we are hoping for an end of the pandemic. The number of individual members of the EMS has for the first time passed 3,000 members which indicates the strong role that the EMS plays in the European mathematical community. A bad incident that occurred was a hacking attack on the EMS website. This is still under repair to protect the personal data and make it safe against further attacks. It was decided that the EMS is joining the European Open Science Cloud (EOSC) initiative to make scientific results and developments available in a F(indabale), A(ccessible), I(nteroperable), R(eusable) way. Good news also came from the Klaus Tschira Foundation which will support a large number of young mathematicians to participate in the ICM 2022. The EMS is now hosting the online Encyclopedia of Mathematics (www.encyclopediaofmath.org/) via EMS Press. I hope that you all stay healthy.

Volker Mehrmann

Erdős–Rényi Prize

Sonia Kófi (CNRS) has been awarded the Erdős–Rényi Prize "for foundational and empirically grounded theoretical research that has advanced network science and its applications in ecology, with a focus on multiple types of interactions among species and the implications for global change, opening the path to new ways to study ecosystems." The Erdős–Rényi Prize is awarded each year by the Network Science Society to a selected young scientist (under 40 years old on the day of the nomination deadline) for their research achievements in the area of network science, broadly construed. While the achievements can be both theoretical and experimental, the prize is aimed at emphasizing outstanding contributions relevant to the interdisciplinary progress of network science.

8ECM

A reminder that calls are still open for Minisymposia and Satellite conferences at the 8th European Congress of Mathematics at Portorož, Slovenia, postponed to 20–26 June 2021. The deadline for applications is 31 January 2021. Participants are reminded that full and active e-participation will be available soon, in addition to the regular format of the conference. Visit 8ecm.si for all details.

EMS News prepared by David Chillingworth
LMS/EMS Correspondent

UPDATE FROM THE EPSRC SAT

A meeting of the EPSRC Mathematical Sciences Strategic Advisory Team (SAT) took place online on Wednesday 4 November 2020.

The meeting welcomed new EPSRC Mathematical Sciences team members Nishtha Agarwal and Jacob Wood. Nishtha takes on responsibilities for these Applied Mathematics areas: Continuum Mechanics, Numerical Analysis, Non-linear systems and Mathematical Biology. She will also be leading on Fellowships. Jacob is responsible for the Statistics and Applied Probability part of the portfolio and will lead on Mathematical Sciences Programme Grants. He will also work on the Mathematical Sciences / Artificial Intelligence (AI) interface; Ruqaiyah Patel continues as the Senior Portfolio Manager with responsibility for AI strategy. Details of the remaining members of the team can be found, as usual, on the UKRI website and the team are always happy to respond to enquiries about grant schemes and hear feedback from the community.

The New Horizons Call, now concluded, was a highly successful exercise in many regards, not just delivering many new awards for Mathematical Sciences research, but also having a significantly higher response rate from peer reviewers than is usual for standard mode proposals. A detailed analysis of all aspects of the Call is now underway and EPSRC hope to be able to share the results of the analysis soon. A survey sent to all those involved in the New Horizons process has gained several hundred survey responses; these will all contribute to the analysis of the Call, in particular to help understand whether the response to the Call was uniform across the research community or not, and how the peer review exercise should be improved.

The EPSRC team noted the successful start to the Lead Agency Opportunity agreed between EPSRC Mathematical Sciences and the National Science Foundation's Division of Mathematical Sciences. This has resulted in five jointly-funded proposals so far.

The Additional Funding Programme for the Mathematical Sciences has a separate Advisory Board and has met monthly since March 2020. It is anticipated that a more complete picture of the status of the Additional Funding Programme will be able to be set out very shortly.

Equality, Diversity and Inclusion (ED&I) is a recurring and embedded aspect of the Additional Funding Programme; Professors Sara Lombardo (Loughborough University) and Gwyneth Stallard (Open University) have been appointed ED&I Champions and attended Additional Funding Programme meetings in that capacity. The Additional Funding Programme broadly seeks to add more resource through existing mechanisms (such as Doctoral Training Partnerships, Fellowships, and Programme Grants) rather than creating new mechanisms, but one new (or at least re-established) scheme is the Small Grants Scheme launched in October 2020 on a long-term basis. Grant applications for up to £80,000 will be reviewed on a quarterly basis in batches, by a Peer Review Panel. This mechanism was selected in order to deliver a fast response to applicants while maintaining a level playing field. The Advisory Board for the Programme has noted that an early review of the effectiveness of this process would be highly desirable.

The SAT meeting went on to discuss Public Engagement which has remained 'embedded within the portfolio' for EPSRC without having specific funds associated with it. In the light of UKRI's vision for public engagement published in September 2019 there is ambition more actively to promote public engagement work. One aspect of this already visible is in the new 'Open Plus' Fellowships announced in July 2020 where a deeper focus on public engagement is one possibility for the 'Plus' aspect of the Fellowship.

Finally the SAT discussed the general need for deeper reflection on the outcomes of funded research and the need to present more coherent and compelling narratives around the use of EPSRC funding for Mathematical Sciences. There is a need to go further than the (true!) statement that mathematical sciences activity underpins and enables all other scientific fields. Conveying the power, excitement and need for fundamental research will be key to the review of the Additional Funding Programme and indeed elsewhere in the coming years.

The EPSRC team thanked Professors David Evans (Cardiff University) and Stephane Launois (University of Kent) who are stepping down from the SAT after this meeting. The appointment of new SAT members will be announced in due course; the next SAT meeting is anticipated to be in March 2021 and an update from that meeting is expected to follow in the May 2021 edition of the *Newsletter*.

Katie Blaney
Head of Mathematical Sciences, UKRI EPSRC

Jonathan Dawes
Chair, EPSRC Mathematical Sciences SAT

OPPORTUNITIES

LMS Grant Schemes

The next closing date for research grant applications (Schemes 1, 2, 4–6 and AMMSI) is 22 January 2021. Applications are invited for the following grants to be considered by the Research Grants Committee at its February 2021 meeting:

Conferences (Scheme 1)
Grants of up to £7,000 are available to provide partial support for conferences held in the UK. This includes a maximum of £4,000 for principal speakers, £2,000 to support the attendance of research students who are studying at universities in the UK, and £1,000 to support the attendance of participants from Scheme 5 eligible countries.

Visits to the UK (Scheme 2)
Grants of up to £1,500 are available to provide partial support for a visitor to the UK, who will give lectures in at least three separate institutions. Awards are made to the host towards the travel, accommodation and subsistence costs of the visitor. Potential applicants should note that it is expected the host institutions will contribute to the costs of the visitor.

Research in Pairs (Scheme 4)
For those mathematicians inviting a collaborator to the UK, grants of up to £1,200 are available to support a visit for collaborative research either by the grant holder to another institution abroad, or by a named mathematician from abroad to the home base of the grant holder. For those mathematicians collaborating with another UK based mathematician, grants of up to £600 are available to support a visit for collaborative research either by the grant holder to another institution within the UK or by a named mathematician from within the UK to the home base of the grant holder.

Collaborations with Developing Countries (Scheme 5)
For those mathematicians inviting a collaborator to the UK, grants of up to £3,000 are available to support a visit for collaborative research, by a named mathematician from a country in which mathematics could be considered to be in a disadvantaged position, to the home base of the grant holder. For those mathematicians going to their collaborator's institution, grants of up to £2,000 are available to support a visit for collaborative research by the grant holder to a country in which mathematics could be considered to be in a disadvantaged position. Applicants will be expected to explain in their application why the proposed country fits the circumstances considered eligible for Scheme 5 funding. Contact the Grants team if you are unsure whether the proposed country is eligible, or check the IMU's Commission for Developing Countries definition of developing countries (tinyurl.com/y9dw364o).

Research Workshop Grants (Scheme 6)
Grants of up to £10,000 are available to provide support for Research Workshops held in the United Kingdom. Research Workshops should be an opportunity for a small group of active researchers to work together for a concentrated period on a specialised topic. Applications for Research Workshop Grants can be made at any time but should normally be submitted at least six months before the proposed workshop.

African Mathematics Millennium Science Initiative (AMMSI)
Grants of up to £2,000 are available to support the attendance of postgraduate students at conferences in Africa organised or supported by AMMSI. Application forms for LMS–AMMSI grants are available at ammsi.africa.

The next closing date for research grant applications (Schemes 8–9 and ECR Travel Grants) is 22 February 2021. Applications are invited for the following grants to be considered by the Early Career Research Committee at its March 2021 meeting:

Postgraduate Research Conferences (Scheme 8)
Grants of up to £4,000 are available to provide partial support for conferences held in the UK, which are organised by and are for postgraduate research students. The grant award will be used to cover the costs of participants.

Grants for Early Career Researchers
Grants of up £500 are available to provide partial travel and/or accommodation support for UK-based Early Career Researchers to attend conferences or undertake research visits either in the UK or overseas.

For full details of these grant schemes, and to find information on how to submit application forms, visit the LMS website: lms.ac.uk/content/research-grants. Queries regarding applications can be addressed to the Grants Administrator Lucy Covington (020 7927 0807, grants@lms.ac.uk), who will be pleased to discuss proposals informally with potential applicants and give advice on the submission of an application.

Early Career Fellowships 2020-21 Awards
The deadline for the Early Career Fellowships 2020–21, with support from the Heilbronn Institute for Mathematical Research, is 14 January 2021. Further details, including how to apply are available at: tinyurl.com/y7npy2q7.

LMS Research Schools and LMS Research Schools on Knowledge Exchange 2022: Call for Proposals
Grants of up to £15,000 are available for LMS Research Schools and LMS Research Schools on Knowledge Exchange which provide training for research students in all contemporary areas of mathematics. Normally the Society supports up to three Research Schools and up to one Research School on Knowledge Exchange. The LMS Research Schools and LMS Research Schools on Knowledge Exchange support participation of research students from both the UK and abroad. The lecturers are expected to be international leaders in their field. The LMS Research Schools and LMS Research Schools on Knowledge Exchange are often partially funded by the Heilbronn Institute for Mathematical Research (heilbronn.ac.uk). Information about the submission of proposals can be found at tinyurl.com/ychr4lwm along with a list of previously supported Research Schools.

Applicants are strongly encouraged to discuss their ideas for Research Schools with the Chair of the Early Career Research Committee, Professor Chris Parker (research.schools@lms.ac.uk) before submitting proposals. A requirement for proposals is that there is a good gender balance amongst speakers.

Proposals should be submitted to Lucy Covington (research.schools@lms.ac.uk) by 22 February 2021.

LMS Undergraduate Research Bursaries in Mathematics 2021
The Undergraduate Research Bursary scheme provides an opportunity for students in their intermediate years to explore the potential of becoming a researcher. The award provides £215 per week to support a student undertaking a 6–8 week research project over Summer 2021, under the direction of a project supervisor. Students must be registered at a UK institution for the majority of their undergraduate degree, and may only take up the award during the summer vacation between the intermediate years of their course. Students in the final year of their degree intending to undertake a taught Masters degree immediately following their undergraduate degree may also apply. Applications must be made by the project supervisor on behalf of the student. For further information and to download the application form, visit tinyurl.com/ya5stelx. Queries may also be addressed to Lucy Covington (urb@lms.ac.uk). The closing date for receipt of applications is 5pm Monday 1 February 2021.

Cecil King Travel Scholarship 2021: Call for Applications
The London Mathematical Society administers two £6,000 travel awards funded by the Cecil King Memorial Foundation for early career mathematicians, to support a period of study or research abroad, typically for a period of three months. One Scholarship will be awarded to a mathematician in any area of mathematics and one to a mathematician whose research is applied in a discipline other than mathematics.

As per the terms of the bequest left to the Cecil King Memorial Foundation, which funds the Travel Scholarship, applicants should be mathematicians in the United Kingdom or the Republic of Ireland who are under the age of 30 at the closing date for applications, and who are registered for a doctoral degree or have completed one within 12 months of the closing date for applications. The LMS encourages applications from women, disabled, Black, Asian and Minority Ethnic candidates, as these groups are under-represented in the United Kingdom and the Republic of Ireland mathematics community.

To apply, complete the application form at tinyurl.com/yarns982 and include a written proposal giving the host institution, describing the intended programme of study or research, and the benefits to be gained from the visit. The application deadline for applications is 31 March 2021. Shortlisted applicants will be invited to an interview during which they will be expected to make a short presentation on their proposal. Interviews will take place in May 2021. Queries may be addressed to Tammy Tran (ecr.grants@lms.ac.uk).

Prospects in Mathematics 2022

UK departments are invited to submit Expressions of Interest to host the LMS Prospects in Mathematics Meeting 2022. Email the Steering Group by 1 March 2021 (ECR.grants@lms.ac.uk). Funding of up to £7,000 is available to facilitate the event to be held in a face-to-face, online or hybrid format.

Expressions of interest should be short (maximum one A4 side in length) and include:

- Confirmation of support from the department.
- Reasons for wanting to host the LMS Prospects in Mathematics Meeting.
- A provisional list of speakers. Speakers should be representative of the UK research landscape both in geographical terms and in scientific terms.
- Speakers from under-represented groups and women speakers; the latter should account for at least 40% of the invited speakers.
- Confirmation that prospective organisers have read and understood the terms and conditions in the Guidelines for Organisers (available from tinyurl.com/y9yn2ryo).
- Willingness to attend the planned LMS Prospects in Mathematics Meeting 2021 (details TBC).

The Early Career Research Committee is interested in innovative approaches to the LMS Prospects in Mathematics Meeting, including online and hybrid formats. For further details visit: tinyurl.com/y9yn2ryo.

HEILBRONN DOCTORAL PARTNERSHIP
PhD in Mathematics

Heilbronn Institute for Mathematical Research

The Heilbronn Doctoral Partnership invites applications for fully-funded PhD studentships in association with the Universities of Bristol, Manchester and Oxford. We are seeking applicants with research interests in Discrete Mathematics, interpreted in its broadest sense, which includes most areas of Pure Mathematics, Computational Statistics, Data Science, Probability and Quantum Information.

In addition to undertaking cutting-edge research in one of the partner universities, students on this programme will have the opportunity to spend nine weeks each summer (in years 1-3) at the Heilbronn Institute for Mathematical Research (HIMR), where they will contribute to the classified research activities of the institute. However, this is not a requirement of the studentship.

Successful candidates who wish to work at the Heilbronn Institute must satisfy vetting in order to engage with the classified research at HIMR and UK resident UK nationals will normally be able to meet this condition.

Students from traditionally under-represented groups are strongly encouraged to apply.

For further information about HIMR and this new initiative, together with details on how to apply, please visit our website: https://heilbronn.ac.uk/postgrad-students/

LMS Council Diary — A Personal View

Council met via video conference on Friday 16th October. The meeting began with the President's business, including note of the Society's congratulations to Sir Martin Hairer and Sir Roger Penrose, winners of the 2020 Breakthrough Prize and 2020 Nobel Prize for Physics respectively, and a brief report on a meeting of the CMS Presidents with EPSRC, where the topics of discussion included the recent government funding uplift for mathematics and the need for increased uptake of invitations to participate in peer review, which is currently a cause for concern. There followed an update on the Michael Atiyah Memorial Projects, where it was noted that the planned conference is now due to take place in 2021 and the two fellowships already awarded will go ahead but have been delayed.

A proposal to offer support for mid-career mathematicians, particularly those with caring responsibilities, was then discussed and it was agreed that the Covid Working Group will reconvene to consider this matter in detail. Council also recorded its decision (taken earlier by email) to broaden the Terms of Reference of the Women in Mathematics Committee, including a change of name to the Women and Diversity in Mathematics Committee, and heard from the Publications Secretary about the ongoing discussions regarding the Society's publications contracts.

The next major item of business was a discussion of the draft Trustees' Report, including the Annual Accounts. A number of items were highlighted by the Treasurer, ranging from the importance of the LMS Representatives to the Society, the fact that two thirds of the Society's income came from Publications, and that overall, the Society remained in a secure financial position. It was noted that the Society had dealt well with its clients following the closure of De Morgan House and its conference facilities, with full deposits having been given to repeat bookers and half refunds to others. The Treasurer also reported on the annual meeting of the investment sub-committee, which had felt that the Society's investment portfolio had performed as well as could be expected in the circumstances, due largely to investments in international markets.

There was an extended discussion on the issue of the diversity of the Society's volunteers and the need to ensure that the Society does not become narrower as a result of the workload implications of being involved in its activities. It was agreed to revisit this matter at a later date after having gathered data on current diversity of volunteers and views of the community on volunteering. We also agreed that the next Council Strategic Retreat should be postponed until an in-person meeting was feasible, but that ways should be explored for how topics that might have been discussed at the Retreat could be addressed in the meantime, and that due to the limitations on in-person meetings due to the pandemic, the Annual General Meeting would be virtual, as the June General Meeting had been.

The meeting closed with the President thanking everyone for their contributions.

Elaine Crooks
Member-At-Large

Long-Standing LMS Members

The London Mathematical Society greatly values the contributions made by all its members and would particularly like to acknowledge the following members, who will be celebrating membership milestones of more than 50 years in 2021.

75 years of membership: Eric L. Huppert.

Over 70 years of membership: David Borwein, Bernard Fishel, Godfrey L. Isaacs.

Over 65 years of membership: John C. Amson, Daniel E. Cohen, Ioan M. James, Hanafi K. Farahat, Francis Rayner, John R. Ringrose, G.E. Wall.

65 years of membership: John F. Bowers, David E. Edmunds, David A. Edwards, Roger Penrose, Roy L. Perry.

Over 60 years of membership: Bryan Birch, Ronald Brown, Aldric L. Brown, David A. Burgess, Roger W. Carter, Gearoid De Barra, A. Howard M. Hoare, Roland F. Hoskins, Glenys Ingram, Donald Keedwell, Lionel W. Longdon, I.G. Macdonald, Joseph F. Manogue, David Monk, Alun O. Morris, Michael F. Newman, Dennis C. Russell, Eira J. Scourfield, Dona Strauss, Ronald F. Turner-Smith, Alan West.

60 years of membership: Patrick D. Barry, Hallard T. Croft, Roy O. Davies, Ian M.S. Dey, Vlastimil Dlab, Frank Rhodes, Stewart A. Robertson, John W. Rutter, Abe Sklar, C. Terence C. Wall.

Over 55 years of membership: J. Clifford Ault, John C.R. Batty, Benjamin Baumslag, Alan F. Beardon, Homer Bechtell, Thomas S. Blyth, William Brown, John H.E. Cohn, Bruce D. Craven, Charles W. Curtis, M.M. Dodson, J. Keith Dugdale, Peter L. Duren, L.C. Eggan, David B.A. Epstein, John A. Erdos, Edward A. Evans, W. Desmond Evans, James O.C. Ezeilo, Matthew P. Gaffney, Cyril F. Gardiner, David J.H. Garling, Peter Giblin, Robin E. Harte, Philip Heywood, Keith E. Hirst, Otto H. Kegel, John F.C. Kingman, J. David Knowles, David G. Larman, Peter G. Lowe, Malcolm T. McGregor, Hugh Morton, Albert A. Mullin, John E. Peters, Frederick C. Piper, John S. Pym, George A. Reid, Joseph B. Roberts, Derek J.S. Robinson, H. Peter Rogosinski, James E. Roseblade, Bruce L.R. Shawyer, Brian F. Steer, Anthony C. Thompson, Graham F. Vincent-Smith, Grant Walker, John F. Watters, Alfred Weinmann, David J. White, Joyce E. Whittington, Sheila O. Williams.

55 years of membership: M.C. Bramwell, M.A.H. Dempster, Barry G. Eke, K. David Elworthy, Terence H. Jackson, E. Christopher Lance, Ian M. Michael, Oliver Pretzel, Colin P. Rourke, Stephan M. Rudolfer, Nelson M. Stephens, Bill Stephenson, W. Brian Stewart, Anthony E. Stratton, Bertram Wehrfritz.

Over 50 years of membership: R.B.J.T. Allenby, Mark A. Armstrong, Irene A. Ault, Roger C. Baker, Anthony D. Barnard, Earl R. Berkson, Nicholas H. Bingham, David A. Brannan, F. Trevor Brawn, Roger M. Bryant, Allan G.R. Calder, Sheila Carter, Munibur R. Chowdhury, Michael J. Collins, Donald J. Collins, H. Garth Dales, P. Laurie Davies, Richard Delanghe, Peter Dixon, John D. Dixon, John Duncan, Martin J. Dunwoody, Roger H. Dye, Roger A. Fenn, Colin R. Fletcher, James W.M. Ford, John B. Fountain, T. Alastair Gillespie, Charles Goldie, Paul R. Goodey, John A. Haight ,Wilfrid A. Hodges, Jonathan Hodgson, A. Geoffrey Howson, Graham J.O. Jameson, David L. Johnson, Michael E. Keating, Monsur A. Kenku, Peter Kopp, Thomas J. Laffey, Earl E. Lazerson, Charles Leedham-Green, David W. Lewis, W.B. Raymond Lickorish, John H. Loxton, Bernard L. Luffman, Bob Margolis, Vassilis Mavron, John McKay, Peter McMullen, William Moran, Kung-Fu Ng, Peter J. Nicholls, Adam J. Ostaszewski, David R. Page, Fredos Papangelou, Jon V. Pepper, Hilary A. Priestley, John F. Rennison, J. Christopher Robson, Harvey Rose, Keith Rowlands, Peter Rowlinson, Philip Samuels, Rodney Sharp, John Silvester, David Singerman, David B. Singmaster, Linda R. Sons, Gabrielle A. Stoy, R.F. Streater, David Tall, Elizabeth Taylor, R. Kenneth Thomas, Brian Thorpe, David Tipple, R.C. Vaughan, Michael R. Vaughan-Lee, Rabe R. von Randow, Martin Antony Walker, Jeffrey R.L. Webb, Dominic J.A. Welsh, Christopher Wensley, Thomas A. Whitelaw, Christopher M. Williams, Geoffrey V. Wood, Douglas R. Woodall, J.D.M. Wright, Abraham Zaks.

50 years of membership: David H. Armitage, Thomas G.K. Berry, W. Meurig Beynon, Roger J. Cook, Alan Cornish, F. Mary Hart, Cherry Kearton, Arthur Knoebel, Paul R. Meyer, Timothy Porter, Andrew Rae, Helen D. Robinson, Klaus Schmidt, Allan M. Sinclair, Patrick F. Smith, Donald Taylor, Peter Walker, Richard Walton, Anthony Wickstead.

Elizabeth Fisher
Membership & Grants Manager

Correction

Attentive readers will have noticed that the purported correction in the November *Newsletter* to an error in an article in the September issue was itself erroneous. There are indeed no continuous functions $f : \mathbf{R} \to \mathbf{R}$ that are both periodic and surjective, although discontinuous examples could easily be constructed using $\tan x$, for example. We apologise for this lapse in editorial vigilance.

REPORTS OF THE LMS

Report: LMS–IMA Joint Meeting

The first fully online joint Zoom meeting between the LMS and the IMA was held on 1–2 October 2020. This meeting celebrated the fact that there are many areas of mathematics of great interest to both 'pure' and 'applied' mathematicians, and that much is to be gained, and celebrated, by exploring such common ground.

The chosen topic was: *Topological Methods in Data Science*. We live in a world increasingly dominated by huge amounts of data, much of which is in the form of images or other non standard media. We are also seeing an explosive growth in methods used to study and compute with this data, such as deep learning methods. However many of these methods are heuristic and without a firm mathematical foundation, and can therefore be unpredictable in their outcome. Sophisticated mathematical ideas are needed to make sense of this, and the topological methods of representing complex data clouds by low dimensional manifolds, which can then be studied rigorously, offer a way of making sense of this otherwise confusing situation. Such topological methods offer a true fusion of data science, and pure mathematics, with a vast range of potential applications. The topic was therefore perfect for this joint meeting.

The speakers at the event were: Gueorgui Mihaylov (King's College London), Vidit Nanda (Oxford), Kathryn Hess (EPFL), Ulrike Tillmann (Oxford) and Ran Levi (Aberdeen). The talks displayed a dazzling range of mathematical ideas taken from such diverse areas as algebraic geometry, algebraic topology, homology and group theory, to gauge theory, and random graphs. The applications of these covered an equally broad range from barcodes and data clustering to deep learning on neural networks and emergent behaviour in complex systems.

The meeting attracted an excellent audience of early career mathematicians, anxious to learn more about this new and exciting field:

Tina Zhou, PhD student, University of Bath: "The talks covered many exciting modelling applications in data science. Speakers presented excellent background introduction for relevant theories, which makes those topics much easier to understand for young students like us.

Speakers (clockwise from top left): Gueorgui Mihaylov, Ran Levi, Kathryn Hess, Vidit Nanda and Ulrike Tillmann

I am amazed by how exciting it is when real-life problems are solved by some of the most abstract maths ideas — especially when the delivery of those ideas maintains a balance of theory and practice. I have been greatly inspired for my research and have learned much more than expected."

Gianluca Audone, PhD student, University of Bath: "Topological data analysis techniques are a topic that has always caught my interest but I never had the chance to really study. The meeting has been a great opportunity to discover the powerful applications TDA has without losing the focus on its mathematical foundations."

Whilst we all miss the social interplay, and the chance for random serendipitous conversations, that we get from a face-to-face conference, the online event went very well. It also had the advantage that many people were able to attend who would not have been able to do so otherwise, and it also ensured a great diversity amongst the attendees. The IMA and LMS would both like to thank the ICMS for providing the excellent support which made this meeting run so smoothly.

Chris Budd OBE
University of Bath

Report: Maths Week Scotland 2020

Olympic athlete and mathematics graduate Eilish McColgan

The fourth annual Maths Week Scotland took place from 28 September to 4 October 2020. Despite the challenges caused by Covid-19, the week was a huge success with a greater emphasis placed on virtual and outdoor activities, as well as an enhanced social media presence — 2020 saw Maths Week Scotland trending at #1 or #2 in all cities across Scotland at the start of the week. Many local activities were funded by the Maths Week Scotland Small Grants Fund, established by the Edinburgh Mathematical Society, the Glasgow Mathematical Journal Trust and the Scottish Government to encourage innovation and participation. Over 160 applications for funding were received, surpassing the number of applications from previous years.

Around 40,000 pupils registered as taking part in Maths Week Scotland via the website from every local authority in Scotland. More than 100 schools and learning institutions received small grants to develop projects and many adapted maths learning within their local environment. The outdoor maths theme was really embraced by teachers this year and maths trails, take-home kits and resource packs meant that outdoor learning, learning at home or socially distanced learning were all accommodated within the programme. For example, a new collaboration with Science Skills Academy saw the creation of Outdoor Maths kits for primary school pupils. A total of 174 kits were delivered, reaching every primary school in the Highland Council area, and these were supported with online training for teachers on how to use the kits during Maths Week Scotland and beyond.

Entrepreneur and *MasterChef* winner Amar Latif

Online sessions during the week for pupils were well attended and featured well-known mathematics communicator and Christopher Zeeman Medalist Dr Hannah Fry, and several individuals using maths in a variety of unexpected settings. For example, comic illustrator Rossie Stone of Dekko Comics shared his story of overcoming dyslexia and struggles with maths by creating comics, showing pupils how to explore mathematical concepts through drawing. Olympic athlete and mathematics graduate Eilish McColgan shared how she uses maths in her training with primary school pupils in Stirling, while Amar Latif shared how maths has impacted his life, from measuring ingredients in *MasterChef* to setting up his own business. Focusing on maths in careers, a social media campaign #ShowYourWorking ran on Twitter across the week encouraging people to share how they use maths in their work, with many organisations taking part.

Various organisations hosted online public events with hundreds of people virtually attending. The LMS Popular Lecture by Dr Diana Davis, *Billiards on Regular Polygons,* hosted by the University of Glasgow, was well attended, as was a headline talk from Marcus du Sautoy, sponsored jointly with the International Centre for Mathematical Sciences and Heriot-Watt University, on whether algorithms can create works of art. Additional talks throughout the week covered topics such as virus spread, coding and origami. Despite restrictions we also had some in-person family day activities at informal learning centres such as Scottish Maritime Museum and the National Mining Museum. The ever-popular Maths Week Scotland Challenges returned, including the photo contest Maths Inside, Maths Week Scotland competitions from Sumdog and Mangahigh, the Deputy First Minister's Challenges and the Maths Wi Nae Borders competition.

Tara Brendle, University of Glasgow
Katie Oldfield, Maths Week Scotland Co-ordinator, National Museums Scotland

Records of Proceedings at LMS–IMA Joint Meeting Ordinary Meeting: 1–2 October 2020

This meeting was held virtually on Zoom, as a joint meeting with the Institute of Mathematics and its Applications (IMA), to showcase some recent developments in pure and applied mathematics that cover topological methods in data science. Over 122 members and visitors were present for the first part and over 88 participants for the second day of the meeting.

The meeting began at 1.30pm on 1 October with the IMA President Dr Nira Chamberlain, CMATH FIMA FORS CSci, in the Chair. Dr Chamberlain welcomed guests, thanked the organising parties, and then introduced the LMS President Professor Jon Keating, FRS who provided the welcome from the LMS. Professor Keating then introduced Liam Holligan of the ICMS, to cover the housekeeping items for the meeting. The meeting was then handed over to Professor Helen Wilson FIMA, who introduced a lecture given by Professor Kathryn Hess (EPFL) on *Trees, Barcodes, and Symmetric Groups*.

Professor Wilson introduced the second lecture given by Professor Vidit Nanda (University of Oxford) on *Geometric Anomaly Detection in Data*.

After the tea break, Professor Wilson introduced the final lecture of Day 1 given by Professor Ran Levi (University of Aberdeen) on *Combinatorial Structures in Neural Networks*.

Dr Chamberlain then thanked the organisers and speakers at the Meeting before he handed over to Professor Keating to close the first day of the meeting.

The meeting continued at 9.30am on 2 October with Professor Keating in the Chair. Professor Keating welcomed guests and then introduced Dr Chamberlain who provided the welcome from the IMA.

Professor Keating introduced a lecture given by Gueorgui Mihaylov (GSK & King's College, London) on *A Gauge Theory of Complex Systems*.

After a brief break, Professor Keating introduced the final lecture of the meeting by Professor Ulrike Tillman, FRS (University of Oxford) on *Homology of Random Geometric Complexes*.

Dr Chamberlain and Professor Keating concluded the meeting by thanking the speakers and all of the organisers and conference from the LMS, IMA and ICMS.

Membership of the London Mathematical Society

The standing and usefulness of the Society depends upon the support of a strong membership, to provide the resources, expertise and participation in the running of the Society to support its many activities in publishing, grant-giving, conferences, public policy, influencing government, and mathematics education in schools. The Society's Council therefore hopes that all mathematicians on the staff of UK universities and other similar institutions will support mathematical research by joining the Society. It also very much encourages applications from mathematicians of comparable standing who are working or have worked in other occupations.

Benefits of LMS membership include access to the Verblunsky Members' Room, free online subscription to the Society's three main journals and complimentary use of the Society's Library at UCL, among other LMS member benefits (lms.ac.uk/membership/member-benefits).

If current members know of friends or colleagues who would like to join the Society, please do encourage them to complete the online application form (lms.ac.uk/membership/online-application).

Contact membership@lms.ac.uk for advice on becoming an LMS member.

Records of Proceedings at BSHM–IMA–LMS Joint Meeting: *Black Heroes of Mathematics* Ordinary Meeting: 26–27 October 2020

This meeting was held virtually on Zoom, as a joint meeting with the British Society for the History of Mathematics (BSHM), the Institute of Mathematics and its Applications (IMA), and hosted by the International Centre for Mathematical Sciences (ICMS) to celebrate Black Heroes of Mathematics. Over 500 members and visitors were present during all or part of the two-day meeting.

The meeting began at 10.00am on 26 October with IMA President Dr Nira Chamberlain, CMATH FIMA FORS CSci, in the Chair. Dr Chamberlain welcomed guests, thanked the organising parties, and then introduced recorded welcome messages from the LMS President, Professor Jon Keating FRS, the BSHM President, Dr Mark McCartney, and the ICMS Director, Professor Paul Glendinning.

He then introduced the audience to the subject of the two-day meeting. Most talks given during the two-day meeting had been previously recorded, and after each talk the speaker answered questions which had been sent in via the chat function of Zoom, in real time.

The order of talks for the first day was as follows:

Dr Angela Tabiri (AIMS Ghana, Ghana): *The Journey of Female African Mathematicians*.

Dr Howard Haughton (King's College London, UK): *On the Use of Probability and Moment Generating Functions for Quantifying Loan Portfolio Credit Losses*.

Professor Tannie Liverpool (University of Bristol, UK): *From Boltzmann to Bird Flocks: Journeys in Non-Equilibrium Statistical Mechanics*.

Following the lunch break, Dr Nira Chamberlain gave a presentation on *The Black Heroes of Mathematics*.

After a short break, Dr Chamberlain chaired a live panel discussion during which the panellists answered questions from the audience.

The first day of the meeting ended at 3.30pm and Dr Chamberlain thanked all the speakers and panellists.

The second day of the meeting began at 1.00pm on 27 October with Dr Nira Chamberlain in the Chair. Most of the talks were pre-recorded, as on the first day, and the speaker answered questions, which had been sent in via the chat function of Zoom, in real time.

The order of talks for the second day was as follows:

Natalya Silcott FIMA (Harrow School & Caribbean Diaspora for Science, Technology & Innovation, UK): *Leaving a Legacy*.

Dr Spencer Becker-Kahn (Cambridge): *Soap Bubbles and Minimal Surfaces*.

After a short break, there were two further talks:

Professor Nkechi Agwu (CUNY, USA): *Mathematical Storytelling: Fostering Creativity, Innovation, Cultural Awareness and Entrepreneurship*.

Professor Edray Goins (Pomona College, USA): *The Black Mathematician Chronicles: Our Quest to Update the MAD Pages*.

After a short break, Dr Chamberlain chaired a panel discussion during which the panellists answered questions from the audience.

The second day of the meeting ended at 7.30pm and Dr Chamberlain thanked all the speakers, panellists, and organisers.

A full report of the event is due to appear in the March edition of the *LMS Newsletter*.

Rubel's Problem: from Hayman's List to the Chabauty Method

EDWARD CRANE AND GENE S. KOPP

Walter Hayman's list *Research Problems in Function Theory* has been updated for its 50th anniversary. Problem 4.27, posed by Lee Rubel, was related to the famous Prouhet-Tarry-Escott problem. Some modern tools of computational number theory can be used to find explicit counterexamples.

Walter Hayman's list

Research Problems in Function Theory [9] is a collection of problems collated by Walter Hayman in 1967 and fondly known as *Hayman's list*. It gave a big impetus to the development of geometric function theory, the part of complex analysis concerned with the geometric properties of analytic functions.

Walter Hayman. Photo credit: MFO

Walter loved to encourage younger mathematicians. He and his wife Margaret founded the British Mathematical Olympiad, and the mathematics genealogy website lists his 20 PhD students and 133 mathematical descendants. His own research was groundbreaking, particularly his work on the Bieberbach conjecture and in the area of Nevanlinna theory, which relates the growth and covering properties of meromorphic functions. He was awarded the LMS De Morgan medal in 1995. Fifty years after the publication of the list, Walter Hayman teamed up with Eleanor Lingham (the editor of this newsletter) to gather the state of the art on each of the problems. The resulting *Fiftieth Anniversary Edition* [10] was published in 2019. We were sorry to learn that Walter Hayman passed away on January 1st 2020.

Rubel's problem

The fourth chapter of Hayman's list is about polynomials. Among the problems there, number 4.27 stands out for the strange reason that it is not a problem in function theory but rather a problem in number theory. It was posed by the American analyst Lee Rubel (1927–1995), whose work spanned differential equations, approximation theory, and the theory of analog computing. Among many other surprising results, Rubel showed that there is a single entire function whose derivatives are dense in the space of entire functions with respect to the topology of locally uniform convergence. Here is problem 4.27, as it appears in [9]:

> *4.27 (Lee Rubel). Let $f(x)$ be a real polynomial of degree n in the real variable x such that $f(x) = 0$ has n distinct (real) rational roots. Does there necessarily exist a (real) non-zero number t such that $f(x) - t = 0$ has n distinct (real) rational roots?*
> *(I can prove this for $n = 1, 2, 3$.)*

You might like to try the case $n = 3$ for yourself.

The phrasing suggests Rubel hoped for an affirmative answer, extrapolating from the low-degree cases.

In a straw poll of our local number theorists at the University of Bristol, all had the opposite intuition to Rubel's. We'll give a calculation-free proof below that the answer is *no*, at least for even degrees $n \geq 8$. What is more interesting is to try to exhibit explicit counterexamples. For each degree $n \geq 4$ we pose the following challenge:

> Problem $\texttt{Rubel}(n)$:
> Exhibit $f \in \mathbb{Q}[x]$ of degree n such that $f - t$ has n distinct rational roots if and only if $t = 0$.

If f is a polynomial of degree n, $t \neq 0$, and both f and $f - t$ have n distinct rational roots, then we can rescale the roots of f and $f - t$ to obtain an ideal solution of the *Prouhet-Tarry-Escott problem* (see

inset). Those are hard to find, but this doesn't mean it is easy to prove that any particular polynomial f is a solution of $\texttt{Rubel}(n)$.

Prouhet–Tarry–Escott problem

Given $k, n \in \mathbb{N}$, find two distinct sets of integers A and B, both of size n, such that

$$\sum_{a\in A} a^i = \sum_{b\in B} b^i \quad \text{for } i = 1,\dots,k.$$

A simple appplication of the pigeonhole principle shows that solutions exist whenever $n > k(k+1)/2$. A solution is called *ideal* when $n = k+1$. In that case we have

$$\prod_{a\in A}(x-a) - \prod_{b\in B}(x-b) = c$$

for some non-zero constant c. The largest k for which an ideal solution is known is 11. This solution was found in 1999 by Nuutti Kuosa, Jean-Charles Meyrignac and Chen Shuwen:

$$A = \{\pm 35, \pm 47, \pm 94, \pm 121, \pm 146, \pm 148\},$$

$$B = \{\pm 22, \pm 61, \pm 86, \pm 127, \pm 140, \pm 151\}.$$

For this example, we have

$$\begin{aligned} c &= 67440294559676054016000 \\ &= 2^{12}.3^9.5^3.7^2.11^2.13^2.17.19.23.29.31. \end{aligned}$$

If you can prove that there is no ideal solution of Prouhet-Tarry-Escott with $k \geq 12$, then you will also have proved that every polynomial of degree $n \geq 13$ with n distinct rational roots is a solution of $\texttt{Rubel}(n)$. See Borwein [5] for more information about the Prouhet-Tarry-Escott problem.

No local obstructions

It is easy to write down an $f \in \mathbb{Q}[x]$ of degree $n \geq 3$ for which there is no $t \in \mathbb{Q}$ such that $f - t$ has n distinct roots. For instance, this is true for $f = x^3 + x$, because it maps $\mathbb{R}$ to $\mathbb{R}$ injectively. For another example, $f = x^3 - 2x$ is an injective mapping from $\mathbb{Q}$ to $\mathbb{Q}$. If $x \neq y$, but $f(x) = f(y)$, then $x^2 + xy + y^2 = 2$, but for $x, y \in \mathbb{Q}$, not both zero, the valuation of $x^2 + xy + y^2$ at 2 is always even. These are both examples of *local obstructions*. In the first case $f - t$ cannot have two distinct real roots, and in the second it cannot have two distinct 2-adic roots.

Ruling out repeated roots stops Rubel's problem from being trivial. For example $f = x(x-1)^2(x+1)^2$ has a local minimum at 1 and a local maximum at -1, so $f - t$ has at most three real roots if $t \neq 0$.

The constraint that f has n distinct rational roots makes the problem a lot more fun. It implies that for $K = \mathbb{R}$ or $K = \mathbb{Q}_p$, every sufficiently small perturbation $\tilde{f}$ of f still factors completely over K. This is because each root of f can be perturbed to a nearby root of $\tilde{f}$ in K. This is shown in the p-adic case by Hensel's lemma. So we cannot hope to solve $\texttt{Rubel}(n)$ by finding a local obstruction.

A non-constructive solution of Rubel's problem

We will answer Rubel's problem by showing that for each $n \geq 4$, there exists a solution to $\texttt{Rubel}(2n)$.

Choose rationals $0 < a_1 < \cdots < a_n$ such that the polynomial $f = \prod_{i=1}^n (x - a_i^2)$ has $n-1$ distinct critical values. This condition holds generically in $\mathbb{A}^n(\mathbb{Q})$.

Case 1: f is a solution of $\texttt{Rubel}(n)$.
Then $F := f(x^2)$ is a solution of $\texttt{Rubel}(2n)$, since it has the $2n$ distinct roots $\pm a_1, \dots, \pm a_n$. However, if $F - t$ has $2n$ distinct rational roots for some nonzero t, then $f - t$ must have n distinct rational roots, contrary to the hypothesis.

Case 2: f is not a solution of $\texttt{Rubel}(n)$.
Define $G(x,w) = (f(x) - f(w))/(x - w)$. Thinking of G as a polynomial in x, its coefficients are polynomials in w, so its discriminant h belongs to $\mathbb{Q}[w]$. The roots of h in $\mathbb{C}$ are the $n-2$ non-critical preimages under f of each of the $n-1$ critical values of f. Each of these is a simple root of h. So h has degree $d = (n-1)(n-2)$ and has no repeated complex root. In particular, $d \geq 6$, and the discriminant of h is non-zero.

If $f - t$ has rational roots $q_1 < \cdots < q_n$, we have $2n$ rational points on the hyperelliptic curve $Y^2 = h(X)$, given by

$$\left(q_k, \pm \prod_{\substack{i<j \\ i,j\neq k}} (q_i - q_j) \right), \quad k = 1, \dots, n.$$

Since $\mathrm{disc}(h) \neq 0$, this is a smooth affine curve over $\mathbb{Q}$ of genus $g = \lceil (d-2)/2 \rceil = (n-1)(n-2)/2 - 1$. Although its closure in the projective plane is not smooth, it can be embedded in a smooth curve in projective 3-space, by a map that takes rational points to rational points. Faltings' theorem says that any smooth projective curve of genus at least 2 has only finitely many rational points (see inset). It follows that the curve $Y^2 = h(X)$ has only finitely many rational points.

We deduce that there are only finitely many rational values of t for which $f - t$ has n distinct rational roots. Let $t_1 < \cdots < t_m$ be the complete sorted list of these rational values. Since f is not a solution of $\texttt{Rubel}(n)$, $m \geq 2$. Consider the degree $2n$ polynomial

$$F := (f - t_1)(f - t_2)\,.$$

By construction F has $2n$ distinct rational roots. We claim F is a solution of $\texttt{Rubel}(2n)$. Suppose $F - t$ has $2n$ distinct rational roots for some $t \in \mathbb{Q} \setminus \{0\}$. Let α_1 and α_2 be the roots of the quadratic $(x - t_1)(x - t_2) - t$, so that $\alpha_1 + \alpha_2 = t_1 + t_2$ and

$$F - t = (f - \alpha_1)(f - \alpha_2)\,.$$

At most n of the roots of $F - t$ can be roots of $f - \alpha_i$, for $i = 1, 2$, so exactly n are roots of each. In particular α_1 and α_2 are rational and they belong to $\{t_3, \ldots, t_m\}$. But because of the sorting this contradicts $\alpha_1 + \alpha_2 = t_1 + t_2$. □

This proof was simple enough, but we used the big hammer of Faltings' theorem to crack the nut! And we haven't yet written down any explicit counterexample to Rubel's problem.

A sextic f such that $f - t$ has six rational roots for infinitely many t

Define four rational functions of u:

$$A = \frac{2u^2 + 6u + 1}{u^2 + u + 1}, \quad B = \frac{3u^2 + 2u - 2}{u^2 + u + 1},$$

$$C = \frac{u^2 - 4u - 3}{u^2 + u + 1}, \text{ and } R = (ABC)^2\,.$$

Then

$$x^6 - 14x^4 + 49x^2 - R = (x^2 - A^2)(x^2 - B^2)(x^2 - C^2)\,.$$

Our argument using Faltings' theorem does not apply to this case because for each choice of u the critical values of this sextic polynomial in x are repeated.

Faltings' theorem

In 1922 Louis Mordell proved the seminal result that the abelian group of rational points on any elliptic curve defined over $\mathbb{Q}$ is finitely generated. At the end of same paper, he conjectured a restricted version of what became known as the *Mordell conjecture*, that any smooth algebraic curve defined over $\mathbb{Q}$ with genus at least 2 has only finitely many rational points.

The Mordell conjecture was proved by Gerd Faltings in 1983, and he won a Fields Medal in 1986 for this work. A few years later, Paul Vojta gave a very different proof using Diophantine approximation and Arakelov intersection theory, and Enrico Bombieri soon gave a more elementary version of this proof.

Faltings' theorem is *ineffective*: it does not tell us how to enumerate all of the rational points. Bombieri's proof in principle gives a computable bound on how many rational points a curve has but no bound on how much ink it takes to write the points down.

An explicit example for $\texttt{Rubel}(7)$

Consider the polynomial

$$f = (x-4)(x-3)(x-1)x(x+1)(x+3)(x+4)\,.$$

Because f is odd, the discriminant of $f - t$ with respect to x is an even polynomial in t. We write $\mathrm{disc}(f - t) = h(t^2)$, where h is the irreducible cubic

$$\begin{aligned} h = &- 823543\,t^3 + 353645809920\,t^2 \\ &- 35639879984676864\,t \\ &+ 95144698561167360000. \end{aligned}$$

The sextic $h(t^2)$ has six real roots, which are the critical values of f. Any $t \in \mathbb{Q}$ for which $f - t$ has seven roots $q_1, \ldots, q_7 \in \mathbb{Q}$ gives us a rational point

$$(X, Y) = \left(t, \prod_{1 \leq i < j \leq 7} (q_i - q_j) \right)$$

on the affine curve $\mathscr{C} : Y^2 = h(X^2)$.

The curve $\mathscr{C}$ is called *bi-elliptic* because it has nonconstant maps to two different elliptic curves. First, $\mathscr{C}$ covers the elliptic curve $E_1 : Y^2 = h(X)$, by the map $(X, Y) \mapsto (X^2, Y)$. This map does not help

us, because the point of $\mathscr{C}$ coming from $t = 0$ has infinite order in E_1, so the group of rational points $E_1(\mathbb{Q})$ is infinite. Second, $\mathscr{C}$ covers the elliptic curve

$$E_2 : Y^2 = X^3 h(1/X) .$$

Any rational point (t, y) on $\mathscr{C}$ such that $t \neq 0$ yields a finite rational point

$$(X, Y) = (1/t^2, y/t^3) \in E_2(\mathbb{Q}) .$$

A calculation in `Magma` shows that $E_2(\mathbb{Q})$ is trivial, consisting only of the point at infinity. Hence $\mathscr{C}$ has no finite point (t, y) with $t \neq 0$, and $f - t$ never factors completely over $\mathbb{Q}$ for $t \in \mathbb{Q} \setminus \{0\}$.

Professor Elmer Rees CBE (1941–2019)

Ed Crane writes: Rubel's problem was the last mathematical question that I discussed with my friend Elmer Rees, whom I got to know when he was the first director of the Heilbronn Institute for Mathematical Research. Although he was an algebraic topologist, Elmer had worked before on problems about the factorization of polynomials over the integers. In Oxford in the 1970s, he wrote a brief note with Walter Feit, *A criterion for a polynomial to factor completely over the integers*. They proved this criterion in order to establish a result about the algebraic topology of complex algebraic varieties. Later, in Edinburgh, Elmer and Chris Smyth proved some necessary divisibility properties of solutions of the Prouhet-Tarry-Escott problem [14]. Even though Elmer was already unwell when we talked about Rubel's problem, it sparked his characteristic enthusiasm.

Another solution of `Rubel`(7) is

$$f = (x-3)(x-2)(x-1)x(x+1)(x+2)(x+3) .$$

To prove this one can use the method of Victor Flynn and Joe Wetherell [8] to enumerate the rational points on the bi-elliptic curve $y^2 = \mathrm{disc}(f - t)$. Their method is elementary, using ideas from the proof of the Mordell-Weil theorem, but we omit the details.

(L-r) Gerd Faltings, Robert F. Coleman. Photo credit: MFO

The Chabauty method

To give an explicit solution to `Rubel`(6), we will use Chabauty's method to enumerate the rational points on a smooth affine curve of the form $y^2 = \mathrm{disc}(f - t)$, where $\deg(f) = 6$. When f has no repeated critical value, this curve has an embedding defined over $\mathbb{Q}$ into a smooth projective curve in $\mathbb{P}^3$.

Chabauty's method originated as a proof of the Mordell conjecture for a restricted class of curves [7], over 40 years before the work of Faltings. Chabauty's method was made effective in the 1980s through Robert F. Coleman's work on p-adic integration.

The Chabauty–Coleman method is a technique for bounding the number of rational points on a smooth curve $\mathscr{C}$ by embedding them in the p-adic points on the Jacobian variety $\mathscr{J} = \mathrm{Jac}(\mathscr{C})$. The *Jacobian* $\mathscr{J} = \mathrm{Pic}^0(\mathscr{C})$ is an abelian variety of dimension g, where g is the genus of $\mathscr{C}$. The *Abel-Jacobi map* is a rational embedding $u_{P_0} : \mathscr{C} \to \mathscr{J}$ defined with respect to a base point P_0. A point P on $\mathscr{C}$ is sent to the divisor class $u_{P_0}(P) = [P - P_0]$.

If p is a prime, base change gives an embedding $\mathscr{J}(\mathbb{Q}) \to \mathscr{J}(\mathbb{Q}_p)$. The p-adic closure of $\mathscr{J}(\mathbb{Q})$ in $\mathscr{J}(\mathbb{Q}_p)$ is a p-adic submanifold. Its dimension $r' = \dim \mathscr{J}(\mathbb{Q}_p)$ is always bounded above by the rank $r = \mathrm{rk}\mathscr{J}(\mathbb{Q})$ of the rational points of the Jacobian as an abelian group.

By composing with the Abel-Jacobi map, we have an embedding $\mathscr{C}(\mathbb{Q}) \to \mathscr{J}(\mathbb{Q}_p)$. When $r' < g$, Chabauty used the properties of this embedding to show there are only finitely many points on the curve.

In fact, an explicit bound may be given, as was shown by Robert F. Coleman. The proof and full statement of Coleman's theorem requires a type of p-adic integration now called *Coleman integration*, that treats degree 0 divisors as paths of integration

and allows one to extend the definition of p-adic integration outside the radius of convergence of the standard definition. We state Coleman's theorem for easy reference (Theorem 5.3 in [13]); see [13] and the references therein for the definition of the Coleman integral and other notation and terms.

Theorem (Coleman). *Let $\mathcal{C}$ be a curve of genus $g \geq 2$ over $\mathbb{Q}$, and let $\mathcal{J} = \mathrm{Jac}(\mathcal{C})$. Let p be a prime of good reduction for $\mathcal{C}$, let $r = \mathrm{rk}\,\mathcal{J}(\mathbb{Q})$, and let $r' \leq r$ be the dimension of the closure of $\mathcal{J}(\mathbb{Q})$ in $\mathcal{J}(\mathbb{Q}_p)$ as a p-adic manifold. Assume that $r' < g$.*

(a) Let ω be a nonzero 1-form in $H^0(\mathcal{C}_{\mathbb{Q}_p}, \Omega^1)$ with the property that, if $Q_i, Q_i' \in \mathcal{C}(\mathbb{Q}_p)$ such that $\left[\sum_i (Q_i' - Q_i)\right] \in \overline{J(\mathbb{Q})}$, then $\sum_i \int_{Q_i}^{Q_i'} \omega = 0$. Such an ω necessarily exists, and we may assume by scaling that it reduces to a nonzero 1-form $\overline{\omega} \in H^0(\mathcal{C}_{\mathbb{F}_p}, \Omega^1)$. Suppose $Q \in \mathcal{C}(\mathbb{F}_p)$, and let $m = \mathrm{ord}_Q\, \overline{\omega}$. If $m < p - 2$, then the number of points in $\mathcal{C}(\mathbb{Q})$ reducing to Q is at most $m + 1$.

(b) If $p > 2g$, then $\#\mathcal{C}(\mathbb{Q}) \leq \#\mathcal{C}(\mathbb{F}_p) + (2g - 2)$.

In the case when $r' < g$ (in particular, when $r < g$), part (b) of Coleman's theorem gives an upper bound on the number of rational points on $\mathcal{C}$. Part (a) may be used to give a narrower upper bound in many cases. However, this bound is not always tight.

In practice, it seems that the Chabauty-Coleman method *can* be used — in combination with a technique called the Mordell-Weil sieve — to enumerate with proof all the rational points on a curve when $r < g$. When the algorithm terminates, it yields a tight upper bound, but it has not been proven that it always terminates.

A solution of `Rubel(6)`

We now show that the following polynomial is a solution to `Rubel(6)`:

$$f = (x-2)(x-1)x(x+1)(x+2)\left(x + \frac{5}{2}\right).$$

Let $\mathcal{C}$ be the curve defined by the equation

$$\begin{aligned} y^2 &= 8^2 \mathrm{disc}(f - t) \\ &= 2985984t^5 + 38231885t^4 - 161118396t^3 \\ &\quad - 811349595t^2 + 1302526656t + 4629441600. \end{aligned}$$

This is a hyperelliptic curve of genus 2, so Chabauty can be applied if the rank of the Jacobian is 0 or 1.

The rank $r = \mathrm{rk}\mathcal{J}(\mathbb{Q})$ of the Jacobian $\mathcal{J} = \mathrm{Jac}(\mathcal{C})$ is found using a 2-descent algorithm, which computes the rank of the 2-part of the Selmer group to give an upper bound, combined with a search for points to give a lower bound. Both are implemented by the `Magma` function `RankBounds`. To reduce the runtime of `RankBounds`, we first compute a minimal Weierstrass model for $\mathcal{C}$, given by the equation

$$\begin{aligned} y^2 &- (x+1)y \\ &= 17915904x^5 - 51347635x^4 - 621566x^3 \\ &\quad + 108253979x^2 - 92802025x + 22173442. \end{aligned}$$

When applied to this model, `RankBounds` returns a lower bound of 0 and an upper bound of 1.

The Jacobian has a nontrivial rational point $u_\infty(P) = [P - \infty]$ coming from the known point $P = (0, 68040) \in \mathcal{C}(\mathbb{Q})$ coming from the factorisation of f. The torsion group of the Jacobian is computed to be trivial with the `Magma` function `TorsionSubgroup`; thus, $u_\infty(P)$ has infinite order. So, $\mathrm{rk}\mathcal{J}(\mathbb{Q}) = 1$.

The `Magma` function `Chabauty` is then used to enumerate the rational points on $\mathcal{C}$. This function implements the Mordell-Weil sieve as described by Bruin and Stoll [4] to rule out conjugacy classes modulo various primes until the Coleman bounds are made tight (under the assumption that $\mathrm{rk}\mathcal{J}(\mathbb{Q}) = 1$). For our curve $\mathcal{C}$, `Magma` does a Mordell-Weil sieve using local information at the primes $\{19, 37, 41\}$ and gives the full set of rational points on $\mathcal{C}$ as

$$\mathcal{C}(\mathbb{Q}) = \{(0, -68040), (0, 68040), \infty\}.$$

Thus, $f - t$ does not factor completely over $\mathbb{Q}$, except when $t = 0$. In fact, this method shows that, for $t \neq 0$, the Galois group of the polynomial $f - t$ is never contained in the alternating group A_6.

A challenge: Can you solve `Rubel(4)` or `Rubel(5)`?

Current developments of Chabauty's method

The past few years have seen the development of new variations on Chabauty's method that allows one to go beyond the $r < g$ regime (or even $r' < g$). The far-reaching theory of non-abelian Chabauty developed by Minhyong Kim allows one

to replace the Jacobian variety with any one of a collection of non-abelian analogues [11]. Balakrishnan and Dogra have refined a special case of Kim's ideas into an effective method called *quadratic Chabauty*. They have used quadratic Chabauty to enumerate the rational points on specific curves, including hyperelliptic curves with $(g,r) = (2,2)$ [1] and $(g,r) = (2,3)$ [2], and, jointly with Müller, Tuitman, and Vonk, to a non-hyperelliptic curve with $(g,r) = (3,3)$ [3].

Meanwhile, classical Chabauty continues to pay off in a wide array of problems. An application to a problem in fluid dynamics may be found in [12]; Lemma 18 therein provides a fully worked example of the Chabauty-Coleman method.

Acknowledgements

The computations in this article were carried out in the Magma Computational Algebra System [6] using algorithms developed by many people. In particular, the `Chabauty` program was primarily developed by Michael Stoll with contributions from others, the `RankBounds` program was written by Brendan Creutz, and the elliptic curve rank algorithms that we used are based on the `mwrank` program of John Cremona.

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Edward Crane

Ed is a senior Heilbronn Research fellow at the University of Bristol. He started out in geometric function theory and now works mainly in probability theory. Ed is trying to eliminate bindweed from his vegetable plot, so he spends time trying to locate roots even when he is not thinking about polynomials.

Gene S. Kopp

Gene is a Heilbronn Research fellow at the University of Bristol. His main interest is number theory. He is especially attracted to problems with an elementary statement and to interplays between algebra and analysis. Gene also enjoys recreational probability theory (playing Dungeons & Dragons) and recreational knot theory (baking soft pretzels).

Photos of the authors by Chrystal Cherniwchan.

Mathematics and Independence: from Euclid to Revolution

IAN STRACHAN

The American Founding Fathers knew what 'self-evident' truths were, often using Euclidean axioms as motivating examples in their thought. But for men such as Adams, Jefferson and Hamilton, whence came this influence? This article explores such influences behind these remarkable people, tracing it back to their early teachers of mathematics and geometry.

Introduction

The opening salvo of the American Declaration of Independence:

> 'We hold these truths to be self-evident'

is then followed by the axioms and postulates that form the intellectual justification of the American Revolution. While the influence of Enlightenment thinking, of Locke and the Scottish Enlightenment thinkers, for example, on Jefferson and the Founding Fathers is well-studied, less so is the influence science had upon their thinking. The very first line of the Declaration, with its reference to *'the Laws of Nature'*, has opened up the study of the Newtonian influence on the political thought of such men as Adams, Franklin, Jefferson and Madison [7].

But what of mathematics, and of geometry? Surely to any mathematician the mention of a *'self-evident'* truth would immediately bring to mind the axioms of Euclid. So can any trace of mathematics and of Euclid be found in the thinking of the Founding Fathers, and if so, from whence came this influence?

John Adams

The mathematical influences on John Adams, he who, more than anyone, argued for independence during the Second Continental Congress in Philadelphia in 1775–76, can be found two decades earlier in his diary entry[1] for 1 June 1756, when he was twenty:

> 'The Reasoning of Mathematicians is founded on certain and infallible Principles. Every Word they Use, conveys a determinate Idea, and by accurate Definitions they excite the same Ideas in the mind of the Reader that were in the mind of the Writer. When they have defined the Terms they intend to make use of, they premise a few Axioms, or Self evident Principles, that every man must assent to as soon as proposed. They then take for granted certain Postulates, that no one can deny them, such as, that a right Line may be drawn from one given Point to another, and that these plain simple Principles, they have raised most astonishing Speculations, and proved the Extent of the human mind to be spacious and capable than any other Science.' [2]

Jefferson, in the original rough draft of the declaration used the phrase *'to be sacred and undeniable'*. It is intriguing to wonder if Adams, who along with Franklin and Jefferson, was a member of the drafting committee, had any influence in the introduction of *'self-evident'* into the final document. The alteration, though, is in Jefferson's hand, so we will never know.

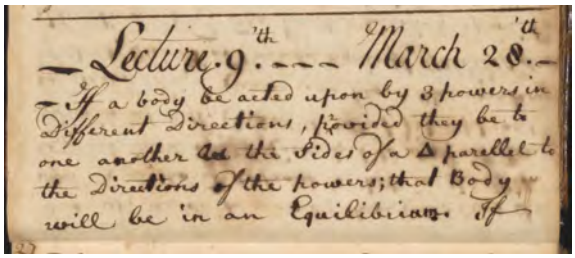
Lecture 9th ---- March 28th
If a body be acted upon by 3 powers in
Different Directions, provided they be to
one another as the Sides of a Δ parallel to
the Directions of the powers; that Body
will be in an Equilibrium. If

Figure 1. Lecture 9 from Winthrop's Lecture Notes on the triangle of forces [15].

So what can be said of the education of John Adams? He was taught at Harvard by John Winthrop, holder of the Hollis Professorship for Mathematics and Natural Philosophy, the second oldest endowed chair in the colonies. Winthrop's observations of the transit of Mercury led to his election as a Fellow of the Royal Society and an honorary degree from the University

of Edinburgh. Amongst the longest entries in Adams' diaries from his time at Harvard are the entries covering Winthrop's lectures, particularly those on Newtonian dynamics [2]. Remarkably, Winthrop's lecture notes still survive from the period [15].

After graduating, and a time spent as a teacher, Adams moved to the study of law, and thence to politics and the science of government, leaving the study of mathematics to future generations. In a famous letter to his beloved wife Abigail (addressed as 'Portia' in these letters) sent from France on 12 May 1780, Adams wrote:

> 'The science of government it is my duty to study, more than all other sciences; the arts of legislation and administration and negotiation ought to take the place of, indeed exclude, in a manner, all other arts. I must study politics and war, that our sons may have liberty to study mathematics and philosophy. Our sons ought to study mathematics and philosophy, geography, natural history and naval architecture, navigation, commerce and agriculture in order to give their children a right to study painting, poetry, music, architecture, statuary, tapestry and porcelain.' [3]

But the influence of his Harvard education ran deep. He continued to use Newtonian metaphors in his arguments in favour of a tripartite system of government, with the balancing of opposing forces to create a political equilibrium, using ideas he learnt from Winthrop's lectures. The repeated use of such arguments led the historian Bernard Cohen to remark:

> 'In reading Adam's statements about forces and powers in his 'Defence of the Constitutions of Government of the United States of America' and other writings, one is sometimes hard put to discern whether he is writing about politics and social issues or about the sciences of statics.' [4]

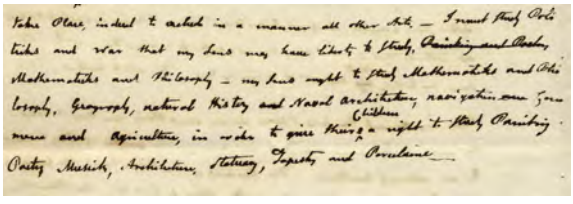

Figure 2. A section from a letter from John Adams to Abigail Adams

In his retirement at Monticello, the home he had designed, Thomas Jefferson wrote to Adams that he had:

> 'given up newspapers in exchange for Tacitus and Thucydides, for Newton and Euclid, and I find myself much the happier,' [5]

to which Adams replied:

> 'Oh that I had devoted to Newton and his Fellows that time which I fear has been wasted on Plato and Aristotle.' [6]

So, at the end of their remarkable lives, these two founding fathers reminisced over their study of Newton and Euclid, two of the founding fathers of our subject.

Figure 3. William Small (1734–1775), painted by Tilly Kettle, c. 1765

Thomas Jefferson

Thomas Jefferson's debt to his mathematics professor is more easy to determine. In his autobiography he wrote:

> 'It was my great good fortune, and what probably fixed the destinies of my life that Dr Wm. Small of Scotland was then professor of Mathematics, a man profound in most of the useful branches of science, with a happy talent of communication correct and gentlemanly manners, & an enlarged & liberal mind. He, most happily for me, became soon attached to me & made me his daily companion when not engaged in the school;

> and from his conversation I got my first views of the expansion of science & of the system of things in which we are placed.' [7]

Elsewhere he described Small as being *'to me as a father. To his enlightenment and affectionate guidance, I am indebted for everything'*. [8]

William Small was born in 1734 in Carmyllie, near Arbroath, and was a student at Dundee Grammar School before becoming an undergraduate at Marischal College, the newer of the two Universities that eventually merged to form the University of Aberdeen. Aberdeen was very much a centre of the new thinking that was emerging at this time. Thomas Reid, for example, one of the founders of the Scottish Enlightenment, was a professor at King's College, the older University in Aberdeen, at this time. Graduating in 1755, Small became a Professor at the College of William and Mary in Virginia in 1758, returning to England in 1764 with a letter of introduction from Franklin. He became a member of the remarkable group of men that would later become the famous Lunar Society, introducing his fellow Scot, James Watt (formerly the *'Maker of Mathematical Instruments to the University of Glasgow'*) to this circle. Thus, Small was a direct influence on two nascent revolutions – the American and the Industrial. The only extant letter from Jefferson to Small – accompanied by three cases of Madeira – was never read by Small, arriving six months after his early death [9].

Jefferson wrote about the importance of mathematics throughout his life, often as a foundation for other areas of study. For example, circa 1773 he wrote:

> '... the faculties of the mind, like the members of the body, are strengthened and improved by exercise. Mathematical reasoning and deductions are, therefore, a fine preparation for investigating the abstruse speculations of the law' [9]

and thirteen years later:

> 'The foundations which you have laid in languages and mathematics are proper for every superstructure. The former exercises our memory while that and no other faculty is yet matured, and prevents our acquiring habits of idleness, the latter gives exercise to our reason, as soon as that has acquired a certain degree of strength, and stores the mind with truths which are useful in other branches of science.' [10]

His carefully catalogued library shows his continued interest in mathematics throughout his lifetime. Figure 4 shows part of the Geometry section of the library catalogue he started in 1783 and which covers the years 1770 to 1812. Other pages cover *'Mathematics pure, arithmetic'* and *'mathematics, mechanics, statistics, dynamics, pneumatics'* and contain entries for many of the standard editions of the time. One prominent entry in the Geometry section is Simson's (misspelt Simpson's) famous edition of Euclid, which was first published in 1756. Simson was Professor of Mathematics at the University of Glasgow (1711–1761), and also served as its first Clerk of Senate [14]. As we will see, he was also the academic grandfather of Alexander Hamilton.

Figure 4: A page, in Jefferson's hand, from his 1783 library catalogue

Jefferson's use of mathematics to develop and design voting systems and cypher wheels is staple material for practically all of his biographers. His design of the mouldboard plough of least resistance tends to be mentioned in less detail; some do not mention this at all. From his letters it is clear that he appreciated that such problems should be solved with calculus, but there appears to be little actual evidence that he solved the problem this way; there is, for example, no mention of calculus at all in his *Proceedings of the American Philosophical Society* article [10]. The calculation of resistance was an advanced exercise in Emerson's (1743) *'Doctrine of Fluxions'*, which was, to quote Jefferson, *'the book I used in College'*. But the calculation of least resistance is an altogether more difficult problem. How – if at all – Jefferson solved this calculus of variation problem I will leave to experts. But Jefferson's long term influence as a Founding Father of mathematics as an intellectual discipline in America can be seen in the prominence it played both at the United States Military Academy

West Point, which he established whilst President, and in his curriculum for the University of Virginia which he founded in 1819.

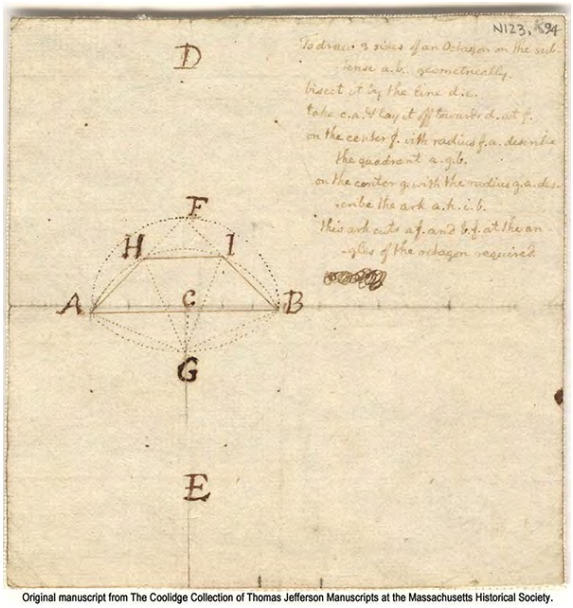

Figure 5: Jefferson's geometric construction of an octagon (1771) [13]. This may have formed part of Jefferson's Building Notebook for the construction of Monticello, which features a 'North Octagonal Room'.

Alexander Hamilton

W.S. Randall, the author of a recent biography [12] of Alexander Hamilton[11], was asked a question – Who was right about America, Jefferson or Hamilton? And so we move from the 18th Century and the Enlightenment towards the 19th Century and international finance, with Hamilton's role in the establishment of the US federal banking system. Hamilton, at the start of his education, was influenced by an Irish born, Scottish trained immigrant, one who was again a product of the Enlightenment education he received while an undergraduate – Robert Harpur.

Harpur was born in County Monaghan in 1731, and studied at the University of Glasgow during the time that Robert Simson was Professor. It is difficult to attribute any direct influence on the young Harpur, but a nineteenth century biographer of Simson notes:

> 'His manner of teaching was uncommonly clear, and engaging to young people; and most of his scholars retained through life an affection and reverence for the Professor.' [12]

He also goes on to say that students *'seldom admitted of that long and nearly exclusive cultivation of one particular science, by which alone, especially in mathematics, eminence usually can be attained'*.

Harpur became Professor at King's College in New York (or Columbia College as it was renamed after the revolution) and so became the teacher of Alexander Hamilton. Hamilton, having rejected admission at Princeton because they would not admit him as an advanced student, started his studies at King's College, formally matriculating in May 1774. But, given his unorthodox, self-taught, education, Hamilton needed extra mathematics lessons. Harpur, upon learning that Hamilton was a scholarship student, declined the customary fee for such private tuition. Hamilton returned to Harpur for help in the following year, on the eve of the American Revolution, to master the mathematics of gunnery.

One can see, a decade later, the influence of Hamilton's mathematical training in *The Federalist Papers* [8]. In 1788, at the very start of Paper 31, *Concerning the General Powers of Taxation*, Hamilton used geometry as an example of a 'primary truth':

> 'IN DISQUISITIONS of every kind, there are certain primary truths, or first principles, upon which all subsequent reasonings must depend. These contain an internal evidence which, antecedent to all reflection or combination, commands the assent of the mind. Of this nature are the maxims in geometry, that the whole is greater than its part; things equal to the same are equal to one another; two straight lines cannot enclose a space; and all right angles are equal to each other.'

And he goes on to say:

> 'the objects of geometrical inquiry are so entirely abstracted from those pursuits which stir up and put in motion the unruly passions of the human heart, that mankind, without difficulty, adopt not only the more simple theorems of the science, but even those abstruse paradoxes which, however they may appear susceptible of demonstration, are at variance with the natural conceptions which the mind, without the aid of philosophy, would be led to entertain upon the subject. The INFINITE DIVISIBILITY of matter, or, in other words, the INFINITE divisibility of a FINITE thing, extending even to the minutest atom, is a point agreed among geometricians, though

> not less incomprehensible to common-sense than any of those mysteries in religion, against which the batteries of infidelity have been so industriously leveled.' [13]

Thus Hamilton is using geometry to argue that certain things are right, even if they go against the natural conceptions of the mind. He goes on to argue that a government has the right to tax its citizens. Surely one of the stranger uses of geometry!

The intellectual debt owed by Hamilton to Harpur is impossible to quantify, but when King's College sacked Harpur, Hamilton gave him five guineas as financial help. Hamilton's untimely and violent death in 1804 means there are no memoirs or correspondence recording his thoughts on his teacher.

Conclusions

Jefferson returned to mathematics in his retirement in Monticello. In a letter – dated 17 August 1811 – to his old friend Dr Benjamin Rush, who had signed the Declaration of Independence along with Adams and Jefferson, he wrote:

> 'Having to conduct my grandson through his course of mathematics, I have resumed that study with great avidity. It was ever my favorite one. We have no theories there, no uncertainties remain on the mind; all is demonstration and satisfaction,' [14]

and again in the same year, to James Madison (the President of the College of William and Mary and cousin of the politician), half a century since he was first taught by Small:

> 'I have been for some time rubbing up my mathematics from the rust contracted by fifty years pursuits of a different kind. And thanks to the good foundations laid at college by my old master and friend Small, I am doing it with delight and success beyond my expectations.' [15]

And in the following year he wrote in a letter to a correspondent:

> 'When I was young, mathematics was the passion of my life.' [16]

So what can be learnt from these musings on the mathematical influences behind some of the Founding Fathers? Perhaps the fact that our biggest influence as teachers of our subject may be on people who never wish – or even aspire – to become professional mathematicians?

But my raison d'être for writing this article is the following: as we move towards the next Research Excellence Framework, with its growing emphasis on the measurable and tangible impact of our work as mathematicians, let us not lose sight of the intangible and immeasurable impact of what we do: its ability to change how people think.

Acknowledgements

I am very much in debt to the late John Fauvel, whose lecture on Jefferson and Mathematics, given in 1999 at the University of Virginia, is still available online (tinyurl.com/y4wgrh8w). Many of the ideas explored in this article originate from this text, which was also where I first found many of Jefferson's quotations on mathematics. I am also indebted to June Barrow-Green for her careful reading of the original draft and her excellent suggestions on how the article could be improved.

I would like to thank the following institutions for their permission to reproduce the figures: Harvard University (Figure 1), the Massachusetts Historical Society (Figures 2,4,5) and The Muscarelle Museum of Art, The University of William and Mary (Figure 3). I would also like to thank Katherine Wright of the LMS for her help in obtaining the permissions to reproduce the various figures.

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Ian Strachan

Ian is Professor of Mathematical Physics at the University of Glasgow. His research interests are in integrable systems and Frobenius manifolds. When not being Head of School, he spends any spare time playing the organ and piano and he has recently acquired a spinet. The mathematics of tuning systems is a black hole he is resisting the temptation of falling into, but it is of interest that Augustus De Morgan wrote a paper on the subject and has a tuning system named after him.

Notes

1: Quotations may be found in the sources contained in the references, but are also available from the easily searchable United States National Archive website: founders.archives.gov.

2: [1], p.32.

3: [3], p.260.

4: [7] p.210.

5: [6], p.291 (21 January 1812).

6: [6], p.294 (3 February 1812).

7: [11], p.2.

8: [5], vol 8, p.200–201 (15 January 1815).

9: [5], vol 7, p.625–631.

10: [4], vol 10, p. 305–309 (27 August 1786).

11: Any reader who has observed that the name Alexander Hamilton has the same number of syllables as William Rowan Hamilton should be directed to tinyurl.com/y42rsqtk.

12: [14], p.5.

13: [8], p.142–143.

14: [5], vol 4, p.87–88 (17 August 1811).

15: [5] vol 4, p.369–370 (29 December 1811).

16: [5] vol 5, p.366–368 (1 October 1812).

Mathematical Society in Hamburg

The Mathematical Society in Hamburg (Mathematische Gesellschaft in Hamburg), founded in 1690, is an academic association concerned with mathematics and located in the German city of Hamburg.

Was gefunden, ich verwahr und noch finde immerdar

This slogan ("What discovered I retain, and still I search again") was one of two competing slogans of the society; the second slogan has since 1699 been the Latin version *Antiqua emendo, substituoque nova* ("The old I improve, and replace by new") which emphasizes the improvement. Both slogans express also the Janus-headedness of the society, which can be seen in the logo. There is the double-headed Janus with a torch, a key and mathematical-astronomical instruments, framed by the slogan of the society.

After the end of the Thirty Years' War, societies of various kinds were founded in Hamburg. Among them in 1690 the Mathematical Society was founded under the name Kunstrechnungs-Liebende Societät by Heinrich Meissner (1644–1716) and by Valentin Heins (1637–1704). Originally the society was a mixture of scientific association and interest group. It was the task of the bookkeepers and arithmetic masters to convey not learned education, but rather practical knowledge. Since there was no uniform teacher training, they had very different levels of mathematical knowledge. One of the objectives of the society was that every member should make every effort to learn and pass on the art of mathematics. To become a member one had to pass a test and submit a piece of work.

In the next two generations it was predominantly the external members who kept the society alive. Initially, northern Germany, Denmark and Schleswig, and also the northern Netherlands formed the main catchment area for membership. Gradually in the 18th century, in addition to higher-qualified teachers and business people, academics were accepted into the society. The board of directors has been sending out a regular annual letter at Johannis (24 July) since 1718.

For the 100th anniversary, the society was reformed. Shortly afterwards, the fate of the city of Hamburg and the society was determined for almost 15 years by the changing events during the Napoleonic Wars. Only with the economic recovery after 1814 was an orderly club life of the society possible again. For the first time officers joined as members from 1815 and military-technical books were placed in the library. Engineers, architects and mechanics were increasingly being accepted as members. Typical examples are the hydraulic engineer Reinhard Woltman (1757–1837) and the astronomer and instrument-maker Johann Georg Repsold (1770–1830).

The teachers were nevertheless still an influential group among the members, while school conditions in Hamburg had a very bad reputation in the 19th century. The schoolmaster at the St. Jakobi Kirchenschule, Johann Hinrich Röding (1732–1800), and the one at St. Michaelis, Christoph Dietrich Westphalen, campaigned intensively for a reform of mathematical teaching. The foreign member Daniel Schürmann (1752–1838) from Remscheid also worked as an important school reformer. On Schürmann's initiative, the government in Düsseldorf issued a teacher examination regulation as early as 1800. Hamburg, on the other hand, did not set up a primary school teacher training college until 1872, having been the last German state to introduce compulsory schooling the year before.

Increasing recognition and professionalization of the engineering profession led in the 19th century to the formation of special technicians', architects' and engineers' associations. They withdrew traditional membership groups from the Mathematical Society, so that at times it had only a little more than 20 local members. New admissions were now recruited more from the group of employees of the observatory and the teaching staff of the higher schools. The society's interest shifted from practice-oriented applied mathematics to pure mathematics. From 1872 the society published its own magazine, the Mitteilungen der Mathematischen Gesellschaft in Hamburg, which appears now every year.

With the establishment of Hamburg University immediately after World War I, the Mathematical Society also received a new impetus. A younger group of mathematicians quickly gathered around the appointed professors Wilhelm Blaschke (1885–1962),

Erich Hecke (1887–1947) and Johann Radon (1887–1956).

Today the Mathematical Society is a registered non-profit association whose five-person board has retained the old names Jahrverwalter, Mitjahrverwalter, Adjunkt, Archivar and Kassenwart. The board of directors is elected at the annual general meeting, where also the topic of the annual *Herbsttagung* (autumn meeting) is determined. Each member can propose to the board a new full member for admission to the society, and the members present will decide on admission at the next event. Any person interested in mathematics is very welcome in the Mathematical Society in Hamburg.

Alexander Kreuzer, Jahrverwalter
Mathematische Gesellschaft in Hamburg
Hamburg Universität

Four Decades On

We are sure that all our members realise that our News Editor Susan Oakes has been with the LMS for a long time — but does it surprise you to know that we have reached the fortieth anniversary of her first day? To mark this occasion we re-print one of her *Newsletter* items, which was written at the end of her first decade with us.

The First Decade

In December 1980 the London Mathematical Society was interviewing for a new Administrative Assistant for the LMS office. They forgot to ask the prospective candidate whether or not they would be prepared to climb Mount Everest as part of their duties. Since January 1981 it is estimated that the number of times I have climbed the elegant staircase of 71 steps to the LMS office in Burlington House equals at least ten trips up Mount Everest.

Many a President too has also since climbed those stairs to visit me: Johnson, Cohn, James, Zeeman, Coates and Kingman. Visiting Officers over the past ten years who have attempted to equal this feat include Robertson, Pym and Brannan (Publication Secretaries), Bailey and Pears (Meetings & Membership Secretaries), Goodey, Mulvey and Sharp (Council & General Secretaries), Schwarzenberger and Wright (Treasurers), Larman, Kestelman, Johnson, Samet and Erdos (Librarians). I leave it to you to guess who is nearest the summit.

The past decade has seen technology change most of our lives, and the office and Officers of the LMS have not escaped untouched. Since 1982 the membership records have been computerised; this enabled direct debiting to be introduced, and the production of a bi-annual List of Members to name but a few initiatives. A fax machine and electronic mail are also now part of everyday life in the LMS office.

Ten years ago the membership was 1,600. One of my main goals has been to see it reach 2,000 and this has now been achieved. However, even with numerous reminder letters the computer and I have sent, there are always some of you who are not keen to pay your subscriptions, so the membership slips slowly back again.

1982 saw the initiation of the Popular Lectures which has provided me and many others not of a mathematical background with an insight into the role of mathematics in everyday life. The office also doubles up as a video shop, stocked with videos of the Popular Lectures.

In 1988 the office moved... to the room next door. Instead of sharing a large office with the British Astronomical Association, the London Mathematical Society moved to the small room next door and could finally claim to have an office of its own.

In recent years I have welcomed more applied mathematicians into membership of the Society. With Christopher Zeeman's enthusiasm the journal *Nonlinearity* was launched in 1989.

New bridges have been built with the American Mathematical Society and in 1990 the LMS went to the AMS's annual summer meeting where Alan Pears and I manned the first ever LMS stand at an AMS meeting. The ground was also prepared for a joint meeting in the UK in 1992.

Ah, but that is into the next decade and who knows how many members we will have, and where the office will be located, and how many more times I will have claimed Mount Everest by 2001.

S.M. Oakes
Administrator

Microtheses and Nanotheses provide space in the Newsletter for current and recent research students to communicate their research findings with the community. We welcome submissions for this section from current and recent research students. See newsletter.lms.ac.uk for preparation and submission guidance.

Microthesis: Quasi-stationary Monte Carlo Methods

ANDI Q. WANG

What connects the distribution of extant species and Big Data? In this microthesis I will describe a surprising connection which has led to an entirely new class of statistical algorithms designed for the Big Data age.

Unfortunately, populations go extinct. This is perhaps keenly felt now more than ever. Suppose we are interested in modelling the size of a population using a stochastic model. That is, we want to construct a Markov chain in continuous time, $X = (X_t)_{t\geq 0}$, taking values in $E = \{0, 1, 2, \dots\}$ which will model some population. The state 0 acts as an absorbing state: setting $\tau_\partial := \inf\{t \geq 0 : X_t = 0\}$, we have that $X_t = 0$ for all $t \geq \tau_\partial$.

Given such a continuous-time Markov chain X which we know is absorbed, or *killed*, at some random time τ_∂, a distribution π on the state space E is called *quasi-stationary* if

$$\mathbb{P}_\pi(X_t \in A \mid \tau_\partial > t) = \pi(A),$$

for each $t \geq 0$ and measurable set $A \subset E$. In other words, when the process is initialised via $X_0 \sim \pi$, it remains in distribution π for any time thereafter, *conditional on the fact that the process is still alive*.

For processes which experience such killing, the quasi-stationary distribution is useful for describing the observed 'equilibrium' behaviour of a stochastic process, and can provide the link between the initial distribution and the (trivial) limiting stationary distribution, which corresponds to extinction.

Bayesian inference

But what about Big Data? As you are no doubt aware, we are currently living in a world awash with data. Online shops, your smart-phone, even perhaps your fridge, are churning out data at an unprecedented rate. How can we make sense of all this?

As a discipline, statistics is the science of learning from data. One broad school of statistical learning is *Bayesian inference*, named after Thomas Bayes (1701–1761). This provides a mathematical formulation for something we all intuitively do: updating beliefs based on evidence. For example, if you are shopping for a new car, you will have some prior beliefs about the suitability of a given car based on the brand and prior experiences. But as you investigate further and read reviews, your beliefs become more nuanced and detailed.

Thomas Bayes: statistician, philosopher, Presbyterian minister

Mathematically, Bayesian statistics provides a rigorous framework for this updating-of-beliefs process. Suppose we want to learn about some parameter $\theta \in \mathbb{R}^d$, and we have some observed data $y_1, \dots y_N$ which depend on θ: this dependence is modelled using a *likelihood function* $L(\theta \mid y_i)$. Given our prior beliefs about θ, encoded in a *prior distribution* $\pi_0(\theta)$, the *posterior distribution* is given by

$$\pi(\theta) \propto \pi_0(\theta) \prod_{i=1}^{N} L(\theta \mid y_i). \tag{1}$$

To perform Bayesian inference, we would like to draw samples $\theta_i \sim \pi$. The traditional way to obtain such draws is to use Markov Chain Monte Carlo (MCMC); see the box on the next page. However, MCMC methods require numerous pointwise evaluations of the posterior density function (1). This is usually not an issue, except when we have *Big Data*: the number of data points N in modern problems can be astronomical! It can be so large that evaluating $\pi(\theta)$, which involves a loop over the entire data set $\{y_i\}_{i=1}^N$, is computationally infeasible.

Monte Carlo methods

Many important scientific quantities can be expressed as an integral, such as

$$I = \int_S f(\mathbf{x})\pi(\mathbf{x})\mathrm{d}\mathbf{x}, \qquad (2)$$

where π is a probability density function. In reality, S could be a complicated subset of $\mathbb{R}^d$ with $d = 100$ or 1000 or even more. Such integrals are impossible to evaluate analytically.

One numerical approach is to use *Monte Carlo*, so-named since one of the pioneers, Stanislaw Ulam, had an uncle who would frequent the casinos in Monte Carlo. The idea is to obtain random draws $X_1, X_2, \ldots, X_n \sim \pi$, and then use an average $n^{-1}\sum_{i=1}^{n} f(X_i)$ as an approximation to I. One very popular and effective class of Monte Carlo methods is *Markov Chain Monte Carlo* (MCMC), where the random draws $X_1, X_2, \ldots$ themselves constitute a Markov chain with stationary distribution π.

Quasi-stationarity and Big Data

To tackle this problem of large data sets, a new paradigm for Bayesian inference was proposed in [1], which I further developed during my doctoral work [2, 3], known as *quasi-stationary Monte Carlo* (QSMC), making crucial use of the above notion of quasi-stationary distributions.

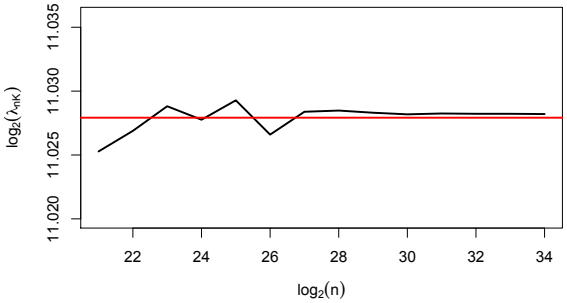
The idea is to build a stochastic process, which experiences killing, such that the quasi-stationary distribution of the killed process coincides with the Bayesian posterior distribution (1). When we choose the killed process to be a diffusion such as a *Brownian motion*, the simulation of the killed process only depends on $\log \pi(\theta)$, and from the expression (1) we know that $\log \pi(\theta)$ will be a large sum.

The QSMC framework enables the use of a computational technique known as *subsampling*, without incurring error. Intuitively, we need access to unbiased estimators of $\log \pi(\theta)$, but this can be done very cheaply: imagine a random variable I which is uniformly distributed on $\{0, 1, \ldots, N\}$, and consider

$$\ell_I(\theta) := (N+1)\log L(y_I \mid \theta),$$

(where $\log L(y_0 \mid \theta) := \pi_0(\theta)$). It is easy to see that $\mathbb{E}[\ell_I(\theta)] = \log \pi(\theta)$. Furthermore this estimate ℓ_I only looks at a single data point y_I, rather than the entire data set of size N.

This is what underlies the ability of QSMC to handle large data sets. There are still several practical and theoretical questions which need to be overcome in order to fully employ QSMC methods, however, as our modern data age presents many new and exciting challenges, it seems that unlikely pairings may be the key to overcoming them.



A figure from [1] showing that the cost of running QSMC is roughly constant as the size of the number of observations n increases. ©Royal Statistical Society/Wiley; reproduced with permission from the authors/Royal Statistical Society

FURTHER READING

[1] M. Pollock, P. Fearnhead, A. M. Johansen, G. O. Roberts, Quasi-stationary Monte Carlo and the ScaLE Algorithm. *J. Royal Statist. Soc.: Ser. B (Methodological)*, 2020, to appear.

[2] A. Q. Wang, Theory of Killing and Regeneration in Continuous-time Monte Carlo Sampling. DPhil thesis, *University of Oxford*, 2020.

[3] A. Q. Wang, M. Kolb, G. O. Roberts, D. Steinsaltz, Theoretical properties of quasi-stationary Monte Carlo methods. *Ann. Appl. Probab.*, 29(1), 434–457, 2019.

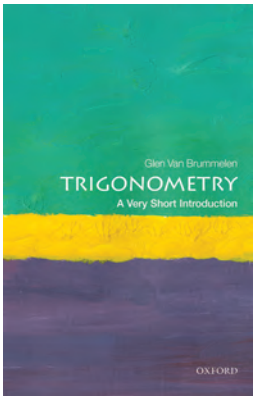
Andi Wang

Andi recently completed his DPhil at the University of Oxford, as part of the EPSRC OxWaSP CDT (oxwasp-cdt.ac.uk/). He is currently a senior research associate at the University of Bristol on the CoSInES project (cosines.org/), researching Monte Carlo methods. Andi is from Cambridge, UK, and also enjoys musing about board games, books and theology.

Trigonometry: A Very Short Introduction

by Glen Van Brummelen, Oxford University Press, 2020, £8.99,
ISBN: 9780198814313

Review by Ruiting Jiang



Going back to ancient times, astronomy and geometry have attracted generations of people to devote their lives to the mysteries and beauty they hide. In particular, trigonometry was developed as a way of relating information on length to that on angles. In the book *Trigonometry: A Very Short Introduction*, the author, Glen Van Brummelen guides the audience to reveal all the treasures in this topic. Van Brummelen accomplishes this by his well-structured discussions which give the reader a thorough understanding of the topic, his humorous style, and the various examples he gives, together with the illustrative graphs that come with them.

The book is part of the *A Very Short Introduction* series by Oxford University Press, which give introductions to a variety of topics. They are all very concise and easy to read, but provide us much more information than the number of pages may suggest. This book is no exception.

The content of the book unfolds in a very natural way, inviting the reader to learn about trigonometry from its beginning and its most simple forms, to the unlimited developments, exceeding our usual perception. In the first chapter of the book, Van Brummelen gives motivation for why people thought of this idea in the first place. From Hipparchus of Rhodes, who wanted to know the distance between the Earth and the centre of the sun's orbit, to Lord Kelvin, who was eager to predict the ocean tides, the author suggests how this mathematical tool may be a solution, and implied how powerful it might be. Intrigued by these questions, the interested audience may naturally ask how the trigonometric functions are defined, and how they may solve these problems. This is precisely what Van Brummelen writes about next. However, although the audience may be convinced that trigonometry is a powerful tool in theory, practical questions arise on how we know the value of $\sin(\theta)$ or $\cos(\theta)$ for an angle θ that we may encounter. In the next chapter, the author disentangles this issue, reassuring his readers. In the last few chapters, Van Brummelen leads his audience to several new areas approachable by trigonometry, namely complex numbers, hyperbolic functions, spherical geometry, and even the more abstract non-Euclidean geometry, where the sum of the angles in a triangle may be more or less than $180°$. Van Brummelen gives historical accounts of each topic of trigonometry he discusses. Therefore, after reading the book, the reader may not only grasp the knowledge, but also why and how these areas became what they are now.

Since this is an introductory book on this subject, Van Brummelen employs a humorous tone. Take as an example, when he talks about using trigonometric functions to predict the elk populations, he writes that "[c]learly, elk considering whether or not to reproduce are not consulting the ratios of sides in a triangle on this romantic evening" (p. 32). This sentence, in a funny way, illustrates the approximation nature of mathematical modelling, and at the same time unveils how potent and magical this mathematical tool is. On the other hand, when Van Brummelen leads the reader to some theorems or identities, despite keeping a light tone as if talking with the audience, he gives the entire, but never too long, deductions with severals graphs helping the reader to follow. After reading the proofs, the reader will understand how and why the statements are true, and may also get a sense on how mathematicians think about the questions. Hence, the book clearly achieves its goal of enabling "anyone with some understanding of mathematics" to appreciate the "magic and mystery of mathematics in general, and trigonometry in particular", as Van Brummelen says in the preface.

There could be no doubt that both someone with some slight interest in maths, and those who want to pursue a mathematical career will benefit from this book. Indeed, the structure and style of the book

make it really accessible for the former. Moreover, for the latter group of people, they could also learn some profound ideas and subtle techniques, which pervade every corner of higher mathematical study. For example, when Van Brummelen talks about the ways to calculate $\sin(1^\circ)$, he introduces the method to approximate this value through an upper bound and a lower bound. Surprisingly, this method gives the value accurately to 5 decimal places. In addition, the author also shows the reader the magic of fixed-point iteration, and discusses the subtle differences in radians and degrees when calculating derivatives. With these topics in mind, the audience will certainly gain some feeling for how higher level mathematics works. On top of that, the 'Further reading' at the end of the book gives precious recommendations to those attracted readers. Therefore, both types of audience will greatly benefit from Van Brummelen's book, and the book may even turn people in the first group to those in the latter.

Thanks to the gradually developed structure of the book, the historical account of the development of trigonometry, and the enjoyable writing style, the book will successfully lead the audience to the broad topics on trigonometry, and grasp many of the core ideas in mathematics. It serves the aim of a great introductory book on a specific topic, but at the same time the book spreads the interest of maths and the logic of maths to all who get a chance to read it.

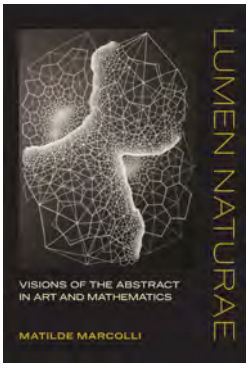
Ruiting Jiang

Ruiting Jiang is a third-year undergraduate student of mathematics at the University of Oxford. His interests are in pure mathematics, but he is also quite interested in analytic philosophy. Ruiting was born in China, and came to study in the UK after graduating from high school. He is learning to play the erhu, a traditional Chinese instrument, to relax during studies, and he is really keen on skiing.

Lumen Naturae: Visions of the Abstract in Art and Mathematics

by Matilde Marcolli, MIT Press, 2020, hardback, 369pp. £36.00,
ISBN: 978-0-262-04390-8

Review by Peter Saunders



Maltide Marcolli describes herself as a mathematical physicist who grew up among art critics and art historians. She sees modern art and modern science as exploring many of the same themes: the nature of space and time, the shape of the cosmos, the structure of matter, and so on. In *Lumen Naturae* she develops this idea, but without attempting to identify direct links between art and science or artists and scientists. On the contrary, she warns us explicitly that we are to expect not close correspondences but "fluid analogies and an interest in common general themes as well as a common preoccupation with certain important abstract concepts".

This makes *Lumen Naturae* quite challenging for the reader, as it must have been to write. If there were a direct connection between the two fields, someone familiar with either could use it as a basis for understanding the other. As things are, however, the reader will need some understanding of each separately, and help with this is provided.

An important feature of the book is the very large number of illustrations. We are not expected to be familiar with the different schools of painting that Marcolli mentions; there are many examples to help us follow her line of thought. One of her stated aims is to introduce contemporary art to those who are inclined to be dismissive of it, and whether we agree with her interpretation of a particular work or not, after we have looked carefully at a number of them to decide what we think, it is difficult to continue insisting there is nothing there.

On the other side, Marcolli provides both formal and informal accounts of concepts of mathematics and physics, having explained in advance that the reader is expected to read as much as they can understand, and skip the bits that are over their head. You do not need to fully understand a mathematical idea to see what it may share with something in art, but you do need more than mere hand-waving.

Marcolli writes that some of the chapters of *Lumen Naturae* began as lectures in a local bookshop, and the favourable reactions they received encouraged her to turn them into this book. Of course the audience in a bookshop a few blocks from Caltech is hardly a random sample of the public at large (randomness is one of the concepts discussed in *Lumen Naturae*, by the way), and Marcolli is exceptionally good at conveying in ordinary language the essence of some quite advanced mathematical ideas, but she is surely right to urge us to be more confident about people's willingness and ability to engage with real mathematics.

An entire chapter in *Lumen Naturae* is devoted to the void. It may surprise you that the void should be considered interesting but this goes back a long time — Lucretius wrote about its importance in 55 BC. Only in the twentieth century, however, did the void come to be seen as more than the mere absence of matter. Marcolli writes that in both art and science the classical notion of the void as passive and undifferentiated has been replaced with "a modern notion in which the vacuum is active, differentiated, and dynamical". In classical physics, empty space is a fixed frame of reference, a stage on which the action takes place. In general relativity, in contrast, it has shape: it can be curved and it can have singularities. In quantum mechanics the vacuum contains virtual particles.

A striking example of the dynamical nature of the void is the Casimir effect. If two conducting plates are placed in a vacuum parallel to each other and very close together, there will be an attraction between them. Marcolli explains that this is because in the gap between the plates only virtual photons with suitable wavelengths contribute to the vacuum energy, whereas there is no such restriction outside. She writes down the relevant equations and supplies references for anyone who prefers a technical description, but the reader who cannot get beyond the plain text account has enough to appreciate the point she is making.

The classical view in art is illustrated by Dürer's famous engraving *The Drawing Frame*, in which an artist is portrayed about to fill an empty grid. Marcolli contrasts this with the work of the Russian artist Kazimir Malevich, best remembered for his representations of the void and in particular his 1913 painting *Black Square*: a black square on a white background.

We are then shown Mark Rothko's *Black on Maroon*, essentially two large maroon rectangles on a black background. There is a small gap between the rectangles, and the brighter figures on either side and the narrowness of the gap make it appear more luminous than the external black regions, even though it is actually the same colour. Marcolli describes this as a sensation of imbalance, of force emanating from the painting. This naturally makes us think back to the Casimir effect, but she pointedly makes no attempt to take the idea further. There is, however, a bibliography at the end of the chapter for readers who want to know more.

I don't expect I or anyone else will agree with absolutely everything in this book, but then as Marcolli reminds us, paintings are not theorems. They do not have unique, unambiguous readings and they leave plenty of room for the sort of debate we're not so accustomed to in mathematics. But I learned a lot and I was left with much to reflect on. I also found *Lumen Naturae* a surprisingly easy read, considering the depth of the subject matter and the seriousness with which the author treats it. And there are lots of nice pictures.

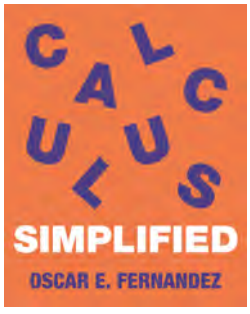
Peter Saunders

Peter Saunders is Emeritus Professor of Mathematics at King's College London. His chief research interest is mathematical biology and he has also been active in mathematics education.

Calculus Simplified

by Oscar E. Fernandez, Princeton University Press, 2019, £16.99, US$19.95, ISBN: 978-0691175393

Review by Ciarán Mac an Bhaird



Calculus Simplified, by Oscar Fernandez of Wellesley College, opens with a preface which reads as a sales pitch for this book, who its target audience is and why someone might use it. The author refers to the book "*as occupying the 'Goldilocks zone' between a calculus textbook and a calculus supplement*" and, that it contains "*just the right amount of challenge and complexity — not too much, not too little*". He continues that "*This book teaches you calculus in at most 110 pages*" but elaborates on the point that though the book was designed to '*streamline*' calculus learning, it should not be confused with watered down learning. So, the reader should be fully prepared about what to expect from the text.

The five chapters cover many of the main topics that you would expect. Chapter 1 briefly introduces calculus as a subject and some of the key concepts including limits and related problems. Chapter 2 deals with aspects of limits

and continuity, and Chapter 3 approaches the basics of differentiation (first derivatives and, very briefly, second derivatives). Chapter 4, the best chapter in my opinion, considers applications of differentiation and Chapter 5 concerns integration, including some applications of integration.

Two of the main features of the book that caught my attention were the large number of examples and exercises (over 500), and the succinct style of writing. The chapters handle all those topics in only 158 pages and, if you leave out the exercises, and focus solely on algebraic functions, the reader has less than 90 pages to consider. This conciseness is achieved primarily through the style of presentation. The chapters contain memory aids such as 'Tips, Tricks, and Takeaways', boxes which focus on key criteria, highlights and frames, and each features 'Chapter Preview' and 'Parting Thoughts' sections. There are motivational materials, examples and applications, definitions and theorems. However, as advertised, there are few obvious signs of standard proofs, and exponential, logarithmic and trigonometric functions play an arbitrary role. They can be found in the appendices and in brief sections called 'Transcendental Tales'. Appendix A (Review of Algebra and Geometry) and Appendix B (Review of Functions) cover pre-calculus material over 52 pages, which is larger than any of the five chapters.

This book delivers exactly what the author sets out at the start, and to that end, it is well written, clear and concise. Motivation is given for much of the material covered, along with practical applications. It could be used effectively by teachers and students alike to supplement their teaching and learning. It is also nice to see some historical context provided via Zeno, Newton and Leibniz, though many more contributed to the development of modern calculus before and since.

When reading books, I consider carefully who I might recommend them to. With this book, I remain unsure. I accept the points made by the author about the size of standard calculus textbooks and their volume of content. Students, especially those who are less disposed towards taking mathematics, can often become overwhelmed by the amount of information such textbooks contain. This is one of the arguments made for streamlined books such as this one. However, as someone with experience of teaching large first year calculus courses and assisting students in a learning centre, I have concerns. When large textbooks are carefully signposted for students from the lecture notes and coursework, they will find much of the same material as contained in this book. The added benefit is that students can be encouraged, in a structured way, to explore the book further and consider relevant proofs. This can act as a launching pad for a broader student understanding and appreciation of the subject. Despite the author's clear warning in the preface, I would be concerned that some readers finishing this text might consider that, in addition to knowing how to carry out the standard introductory calculus operations, they also have a complete understanding of calculus in less than 90 pages.

Ciarán Mac an Bhaird

Ciarán is an Assistant Professor in Mathematics at Maynooth University and Director of the Mathematics Support Centre. His current research interests are mostly in mathematics education, but he also conducts research in the history of mathematics and, occasionally, in algebraic number theory. Ciarán tries to play sport on a regular basis and helps his parents on the small family farm in Co. Monaghan.

Obituaries of Members

Sir Vaughan F.R. Jones : 1952–2020

Sir Vaughan Jones, who was elected Honorary Member of the London Mathematical Society on 25 November 2002, died on 6 September 2020. He was the Hardy Lecturer in 1989.

David Evans writes: In his thesis work at Geneva, under the supervision of André Haefliger and Alain Connes, Vaughan Jones classified finite group actions on the hyperfinite II_1 factor R, extending the work of Connes for cyclic actions. Soon after, during appointments at UCLA (as Hedrick Assistant Professor) and the University of Pennsylvania, Vaughan returned to a problem he had considered as a graduate student, that of the position or embedding of one copy of the hyperfinite factor in another — what became to be known as subfactor theory, a vast generalisation of a group to a quantum symmetry. Whilst dimension in a matrix algebra is discrete, the dimension in the completion, the hyperfinite factor R, becomes continuous. It was therefore surprising that Vaughan found that the index, subsequently called the Jones index, a relative dimension of the subfactor in the factor, took discrete values below 4 at $4\cos^2(\pi/n)$ for integral n.

Vaughan analysed the structure of a subfactor through a construction extending the subfactor to a tower of algebras. The algebraic relations between the projections of the subfactors in the tower led to representations of the braid group, which Vaughan then exploited to get a new invariant of knots and links — the Jones polynomial. This unexpected discovery resolved amongst other things the Tait conjectures left open since the 19th century. These algebraic relations also appeared in work of Temperley and Lieb in statistical mechanics. Through this and the Yang–Baxter equation, an enhancement of the braid relations with spectral parameter, Vaughan's subfactor theory has found applications and connections with statistical mechanics, random matrices, topological, algebraic and conformal quantum field theory in physics — as well as in the topology of DNA strands and protein folding. MSRI ran two simultaneous programmes during 1984–85 on K-Theory, Index Theory, and Operator Algebras and Low Dimensional Topology with Vaughan taking a leading role in their interaction. The modern field of quantum topology is based on his work.

For these breakthroughs Vaughan received the Fields Medal in 1990, and was elected a Fellow of the Royal Society in 1990, Honorary Fellow of the Royal Society of New Zealand Te Apārangi 1991, Member of the US National Academy of Sciences 1999, and Foreign Member of several other national learned societies. He was elected Vice-President of the AMS in 2004 and of the IMU in 2014.

Vaughan was born in Gisborne, New Zealand and educated at Auckland Grammar School and the University of Auckland. He had a long term commitment through time, energy and personal funding to nurture mathematics in New Zealand, in particular through the annual summer schools and workshops where he spent every January since 1994 as Director of the New Zealand Mathematics Research Institute. After the year at MSRI, he was Professor of Mathematics at Berkeley from 1985 until 2011 when he was appointed as the Stevenson Distinguished Professor at Vanderbilt University — supervising over 30 graduate students during his career.

He had longterm links with the UK. He made many visits to Symposia on Foliations, von Neumann algebras and Ergodic Theory, and Operator Algebras, at the MRC Warwick from 1979. He was a member of the first programme at the INI in Cambridge, on Low Dimensional Topology and Quantum Field Theory in 1993 and organiser of a programme on Operator Algebras: Subfactors and their Applications in 2017.

Vaughan's father Jimmy was born in Penbre in the Gwendraeth Valley in Wales, and emigrated with his parents to New Zealand at the age of eight. Vaughan kept in touch with his cousins and rugby in Wales during his frequent visits there – e.g. the IAMP Congress at Swansea in 1988, the centenary celebration of the University of Wales in 1993 when he was awarded an honorary doctorate, LMS Regional Meeting at Gregynog in 2002, and Learned Society of Wales Distinguished Frontiers Lecturer in 2013. He was elected Honorary Fellow of the Learned Society of Wales in 2018.

Vaughan had a distinctive and personal style of research in mathematics. His warmth, generosity, sincerity, humour and humility led him to thrive on social interaction, and for the mathematical community to significantly benefit from his openness in sharing ideas through every stage of development from speculation and conjecture of the way forward to

discussing and explaining results. Vaughan's presence at events and regular interaction with graduate students and colleagues, enriched all who came into contact with him.

Vaughan was an accomplished choral singer, violin player, rugby player, skier, golfer, kite surfer and barista. He is survived by his wife Martha (Wendy), children Bethany, Ian and Alice and grandchildren.

Patrick Dolan: 1939–2020

Patrick Dolan, who was elected a member of the London Mathematical Society on 16 November 1967, died on 29 June 2020, aged 81.

Trevor Stuart writes: Patrick was usually known as Paddy by his friends. He was of Irish nationality, was born on 18 February 1939 and studied Mathematics at undergraduate level at University College Dublin. However, when postgraduate studies loomed he moved to England, settling in Royal Holloway College as part of the University of London. There his advisor was Professor William Hunter McCrea. Patrick graduated PhD of the University of London in 1964, his thesis being titled *Problems in Relativistic and Steady-State Cosmology*. Indeed, he remained faithful to the fields of special and general relativity, including gravitational theory, throughout his professional career.

Doubtless influenced by the presence at Imperial College London of Professor Gerald J. Whitrow, whose interests encompassed Relativity Theory, Patrick applied for a position there and was appointed in 1964. According to the Mathematics Genealogy Project he had two PhD students who graduated in 1979 and 1997. However, there appears to be another student, Chandrasekher Mukku, who studied for PhD 1977–80 and produced a thesis on *Aspects of Metric-Torsion Theories of Gravitation,* which involved reference to earlier work of Cartan, Sciama and Kibble.

In his professional career Patrick Dolan wrote a number of papers with collaborators, including A. Gerber and his (Dolan's) student, Bruno Muratori. In those papers the concept of the Weyl-Lanczos equations in four dimensions and of the Riemann–Lanczos equations and their integrability for general relativity is treated. The collaboration with Muratori was concerned with Ernst potentials in a vacuum, and separately gravitational potentials. Moreover, Patrick had the privilege of having a paper accepted by *Nature* in 1970, quite early in his career.

A notable initiative of Patrick at and for Imperial College London was the concept of a yearly Schrödinger Lecture, which arose from his suggestion, for which he will be particularly remembered as well as for his bright and cheerful personality.

Anthony Hill: 1930–2020

Anthony Hill, who was elected a member of the London Mathematical Society on 12 October 1979, died on 13 October 2020, aged 90.

Mark Thomson writes: The artist Anthony Hill was a singular figure in post-war British art who made work of lasting significance in three distinct but related areas. As an artist he developed a rational abstraction that remains at the peak of post-war modernism; as a writer he brought together luminaries of mathematics, architecture, physics and art in the seminal anthology *DATA — Directions in Art, Theory and Aesthetics* (1968); and among graph theorists he is known as the author of Hill's Conjecture, concerning the number of crossings in a complete graph.

Hill was born in Hampstead, London, in 1930. His father Adrian was an official First World War artist, whose convalescence from tuberculosis in 1938 led him to coin the term 'art therapy' and who became well known as the presenter of the BBC's 'Sketch Club' in the late 1950s. Anthony's grandfather Graham Hill was encouraged as a poet by Oscar Wilde and romantically linked with Lillie Langtry, who starred in several of his plays.

Anthony Hill burst onto the London art scene in the early 1950s and quickly became the principal theorist of the Constructionists, a group of established artists who had turned towards abstraction, led by Victor Pasmore. Hill was in his early 20s. By the mid-1950s he had abandoned painting and developed his mature form, the constructed relief. At first strictly orthogonal, Hill's relief constructions were made entirely from industrial materials such as copper, aluminium, perspex and PVC, and presented

a radical view of structure in abstract art. He was featured in the exhibition *This is Tomorrow* at the Whitechapel Gallery, London, in 1956 and the major international surveys *Konkrete Kunst* at Helmhaus in Zurich in 1960 and *Experiment in Constructie* at Amsterdam's Stedelijk Museum in 1962, placing him in the context of the pioneer abstract artists from the earlier twentieth century. From then until the mid-1980s he was a star of his field, exhibiting widely around the world, and being included in the foremost public and private collections.

Hill's interest in structure drew him to an engagement with graph theory and other aspects of topology. His work in this field led him in 1958 to what would become known as Hill's Conjecture, which was summarised in a paper co-authored with Frank Harary in 1963 *On the Number of Crossings in a Complete Graph*. His work in mathematics (he eventually wrote or contributed to 11 further papers) was recognised with the award of a Leverhulme Fellowship in 1970, and an Honorary Research Fellowship in the Department of Mathematics at University College, London.

Alongside this he was a prolific writer on art and its relations with science. Among the celebrated mathematicians, architects, scientists and artists included in *DATA* were the topologist L.E.J. Brouwer, with whom Hill had developed a correspondence, the quantum physicist David Bohm and the visionary urbanist Yona Friedman. Hill's own contribution featured a highly original analysis of the work of Piet Mondrian from a topological viewpoint.

In 1983 Hill had a celebrated mid-career retrospective at the Hayward Gallery in London. He was 53. In the later 1980s the art world became increasingly geared towards its market, and the idea of a rational aesthetic was overrun by one of its periodic returns to painting. Anthony countered this lurch towards the transactional with a redoubling of the work of his Dadaist alter ego Redo (pronounced ray-dough). Redo's acerbic collages and assemblages often included recycled materials found within walking distance of his flat in London's Charlotte Street: offcuts and disjecta from the nearby Saatchi & Saatchi advertising agency, and discarded X-rays from University College Hospital. The seemingly unlikely combination of the Dadaist Redo and the constructivist Anthony Hill (work he termed 'precisionist') was in fact a duality he shared with some of the leading artists from the earlier twentieth century such as Theo van Doesburg, Kurt Schwitters and, most importantly for Hill, Marcel Duchamp, with whom he also corresponded.

Parity Study No.2 First Version, 1970

Nicholas Serota, former director of Tate, summarised Hill's importance as an artist: "He played a remarkable part in the history of modernism in England after the Second World War. His work, his writing and his engagement with intellectual endeavour in Europe and America changed the way that art developed in London, moving the terms of reference beyond Paris without falling under the spell of New York." There are 17 works by Hill in the Tate collection, including several of his most significant constructions. In addition to this, Hill's Conjecture is still an open problem that is engaging mathematicians around the world.

In the mid-2000s Hill was hit by a lorry in the street, leaving him unable to walk without assistance and having a significant impact on his work. Despite this and the effects of depressive illness from which he had suffered throughout his adult life, Hill's tenacity and sheer grip on life sustained him, even beyond the premature death of his wife Yuriko in 2013. After this he had to give up the Charlotte Street flat where he had lived since 1956; his last few years were spent in a care home in St John's Wood. He died peacefully on 13 October 2020, a remarkable man with a unique legacy in art and mathematics.

Death Notices

We regret to announce the following deaths:

- Robin J. Chapman, formerly of Exeter University, who died on 18 October 2020.
- A.E.L. Davis, who died on 23 November 2020.

LMS Meeting

Society Meeting at JMM 2021

7 January 2021, 10–11am Mountain Standard Time/5–6pm Greenwich Mean Time

Website: tinyurl.com/y4he6vpn

This Society meeting forms part of the programme of the virtual Joint Mathematics Meeting (JMM) 2021, which will be co-hosted by the American Mathematical Society and the Mathematical Association of America.

The meeting will begin with Society business, followed by an LMS lecture by Sarah Zerbes (UCL) on *Special Values of L-functions.*

Abstract: L-functions are one of the central objects of study in number theory. There are many beautiful theorems and many more open conjectures linking their values to arithmetic problems. The most famous example is the conjecture of Birch and Swinnerton-Dyer, which is one of the Clay Millennium Prize Problems. I will discuss this conjecture and some related open problems, and I will describe some recent progress on these conjectures, using a tool called 'Euler systems'.

The lecture is aimed at a general mathematical audience. All interested, whether LMS members or not, are welcome to attend. For further details and to register, see tinyurl.com/b645x2n.

Burnside Rings for Profinite Groups

Location: Online
Date: 8 January 2021
Website: lancaster.ac.uk/maths/fcg

This meeting will focus on the generalisation of Burnside rings from finite to profinite groups and their applications in representation theory. The speakers are S. Bouc (Amiens), Z. Hall (Lancaster), V. Kelsey (St Andrews) and B. Nucinkis (RHUL). To register for the event, email the local organiser Nadia Mazza (n.mazza@lancaster.ac.uk). Supported by an LMS Joint Research Group grant.

BMC–BAMC 2021

Location: Online
Date: 6–9 April 2021
Website: tinyurl.com/y4xsgze9

This is the postponed 2020 event. There will be a full programme of plenary lectures, BMC sessions, BAMC mini-symposia, contributed talks, poster sessions, public lecture, and various social events. The software platform will be Sococo. Individual emails have been sent to all those who registered for the 2020 event. The Early Bird conference fee is a flat £15. Details and registration are available at the website.

Mathematics in Defence and Security

Location: Online
Date: 30–31 March 2021
Website: tinyurl.com/yynzgpv3

Science and technology play an increasingly important role in supporting the defence and security industries. Mathematics is fundamental to these two disciplines, providing a framework for understanding and solving the varied and complex problems faced, and to model systems and scenarios. These models are then used to estimate system performance, find weaknesses in real systems, and suggest improvements.

Mathematics of Operational Research

Location: Online
Date: 20–23 April 2021
Website: tinyurl.com/yb2v9fza

This IMA and OR Society conference will take a comprehensive view and showcase activity from across OR, and will welcome contributions which have a clear application focus as well as those which are theoretically driven. Contributions will be expected to showcase both significant new mathematics and OR relevance. The conference will host plenaries from leading international experts.

Marriages, Couples, and the Making of Mathematical Careers

Location: Online
Date: 29–30 April 2021
Website: mathmarriages.wordpress.com

This conference will use collaborative couples as a window on the ways in which a mathematical career is the product of many types of work: intellectual, clerical, and domestic. The call for papers is now open and proposed abstracts should be submitted by 12 February 2021. More information can be found on the website. Supported by the LMS and the British Society for the History of Mathematics.

Rigidity, Flexibility and Applications

Location: Lancaster University
Date: 19–23 July 2021
Website: tinyurl.com/y3gdqo6w

This LMS Research School will introduce talented young scientists to the increasingly active mathematical and inter-disciplinary research area of rigidity and flexibility of structures. Alongside the technical sessions, there will be plenary talks by experts in making an impact with mathematics. Application deadline: 31 January 2021. Registration for research students is only £50.

Nonlinearity and Coherent Structures

Location: Loughborough University
Date: 7–9 July 2021
Website: tinyurl.com/y2qc2jwr

The aim of this IMA conference is to bring together researchers working on aspects of nonlinear phenomena and to encourage interaction between experts from different areas such as Applied Mathematics, Mathematical Analysis, Fluid Dynamics, Engineering and Physics. Recent theoretical developments, new computational methods and experimental findings will be presented and discussed.

Young Researchers in Algebraic Number Theory III (Y-RANT)

Location: University of Bristol
Date: 18–20 August 2021
Website: tinyurl.com/y4m3jotj

Y-RANT is a (relatively) new conference aimed at postgraduate students and early career researchers in algebraic number theory, promoting discussion and sharing of ideas between members of the community. Supported by an LMS Scheme 8 Postgraduate Research Conference grant. A limited amount of funding will be available to support UK students.

LMS Invited Lectures Series 2022: Call for Proposals

The annual LMS Invited Lectures Series consists of meetings held in the UK at which a single speaker gives a course of about ten expository lectures, examining some subject in depth, over a five day period (Monday to Friday) during a University vacation. The meetings are residential and open to all interested.

A £1,250 honorarium is offered directly from the LMS to the Invited Lecturer and £6,000 in funding is given to the host department to cover both the Lecturer's expenses (travel, accommodation and subsistence) and support attendance at the lectures.

Proposals for the Invited Lectures 2022

Any member who would like to suggest a topic and lecturer and be prepared to organise the meeting at their own institution or a suitable conference centre can submit a proposal. For further details, please visit the Society's website: tinyurl.com/y98espkj. The deadline for proposals is 1 February 2021.

LMS Invited Lecturer 2021

The LMS Invited Lecture Series 2021 on equations in groups and complexity will be given by Professor Olga Kharlampovich (CUNY Graduate Center and Hunter College) at the University of Newcastle (dates to be determined).

Recent previous Invited Lecturers:

- 2020: Professor Yulia Mishura (University of Kyiv) *Fractional Calculus and Fractional Stochastic Calculus, including Rough-Paths, with Applications*, Zoom via Brunel University, 15-19 June.
- 2019: Professor Søren Asmussen (Aarhus University) *Advanced Topics in Life Insurance Mathematics*, ICMS in Edinburgh, 20-24 May.
- 2018: Professor Art Owen (Stanford University) *From the Foundations of Simulation to Quasi Monte Carlo*, Warwick University, 9-13 July.

Enquiries about the Invited Lectures may be addressed to Professor Brita Nucinkis, the Chair of the Society Lectures and Meetings Committee: lmsmeetings@lms.ac.uk.

Covid-19: Owing to the coronavirus pandemic, many events may be cancelled, postponed or moved online. Members are advised to check event details with organisers.

Society Meetings and Events

January 2021

7 Society Meeting at the Joint AMS & MAA Meeting (492)

April 2021

8 Society Meeting at the BMC–BAMC 2021, online

May 2021

14 LMS–UCL Meeting on the Educational Times, London

June 2021

2-4 Midlands Regional Meeting and Workshop, Lincoln

22 Society Meeting at the 8ECM, Portorož, Slovenia

July 2021

2 General Meeting of the Society, London

September 2021

6-10 Northern Regional Meeting, Conference in Celebration of the 60th Birthday of Bill Crawley-Boevey, University of Manchester

January 2022

4-6 South West & South Wales Regional Meeting, Swansea

Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website (www.lms.ac.uk/content/calendar). Please send updates and corrections to calendar@lms.ac.uk.

January 2021

8 Burnside Rings for Profinite Groups, Lancaster (online) (492)

20-22 Fry Inaugural Series: Challenges and Recent Advances in Mathematical Physics, Heilbronn Institute, Bristol (490)

March 2021

14 International Day of Mathematics (491)

30-31 Mathematics in Defence and Security IMA Conference (online) (492)

April 2021

6-9 British Mathematical Colloquium and British Applied Mathematics Colloquium 2021 (online) (492)

20-23 Mathematics of Operational Research (online) (492)

29-30 Marriages, Couples, and the Making of Mathematical Careers (online) (492)

June 2021

20-26 8th European Congress of Mathematics, Portorož, Slovenia (492)

July 2021

7-9 Nonlinearity and Coherent Structures, Loughborough University (492)
12-16 New Challenges in Operator Semigroups, St John's College, Oxford (490)
19-23 Rigidity, Flexibility and Applications, Lancaster University (492)

August 2021

16-20 IWOTA, Lancaster University (481)

September 2021

1-3 Scaling Limits: From Statistical Mechanics to Manifolds, Cambridge (489)
19-24 8th Heidelberg Laureate Forum, Heidelberg, Germany
21-23 Conference in Honour of Sir Michael Atiyah, Isaac Newton Institute, Cambridge (487)

October 2022

18-20 Young Researchers in Algebraic Number Theory III, University of Bristol (492)

July 2022

24-26 7th IMA Conference on Numerical Linear Algebra and Optimization, Birmingham (487)