LONDON MATHEMATICAL SOCIETY
EST. 1865

Whitehead Prize 2019
Citation for Professor David Conlon

Professor David Conlon of the University of Oxford is awarded a Whitehead Prize in recognition of his many contributions to Combinatorics. His particular expertise is Ramsey Theory, where he has made fundamental contributions to both the arithmetic and graph-theoretic sides of the subject.

Perhaps the most celebrated result of Conlon is his new upper bound for the diagonal Ramsey number $R(k,k)$, which he proved very early in his career. This is defined to be the least number N such that, if the edges of the complete graph K_N on N vertices are coloured red and blue, then there is guaranteed to be a copy of K_k, all of whose vertices are one colour.

A celebrated 1990 quote of Paul Erdős explains how hard the problem of computing the numbers $R(k,k)$ is thought to be. "Suppose aliens invade the earth and threaten to obliterate it in a year's time unless human beings can find $R(5,5)$. We could marshal the world's best minds and fastest computers, and within a year we could probably calculate the value. If the aliens demanded $R(6,6)$, however, we would have no choice but to launch a preemptive attack." Nearly thirty years later, the best bounds known are that $102 \leq R(6,6) \leq 165$.

A simple inductive argument due to Erdős and Szekeres, dating from 1935, gives $R(k,k) \leq 4^k$. Conlon's paper established, for the first time, a bound stronger than this one by an arbitrary power of k.

Another very well-regarded piece of work by Conlon is his joint work with Gowers on combinatorial theorems in random sets. Among the many results they establish is a version of Szemerédi's theorem in quite sparse random subsets of the integers. Their main result in this direction is that for every positive $\delta > 0$ and for every integer $k \geq 3$ there is a constant C such that a random subset S of $\{1, \ldots, N\}$ of size $CN^{(k-2)/(k-1)}$ will, almost surely as $N \to \infty$, have the *Szemerédi property* for sets of density δ: every subset $S' \subset S$ with $|S'| \geq \delta|S|$ contains k distinct elements in arithmetic progression. Szemerédi's theorem itself is the case when S is the (deterministic) set $\{1, \ldots, N\}$ itself. Given the notorious difficulty of that theorem, it is very remarkable that corresponding phenomena are understood in such sparse settings. Furthermore the result of Conlon and Gowers is completely sharp in that this result is false for sets of size $cN^{(k-2)/(k-1)}$ for suitably small c.

These two topics, while they give a flavour of Conlon's work, do not do justice to its breadth. Indeed, he has more than 50 publications on a wide spectrum of questions in the Ramsey theory of graphs, hypergraphs and arithmetic structures, as well as on extremal combinatorial questions.