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GRAPH THEORY AND BOVINE EPIDEMIOLOGY

MATHEMATICS OF LANGUAGE AND GRAMMAR

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Image courtesy of Matt Warden.
See article on page 20.

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Feature content should be submitted to the editor-in-chief at iain.moffatt@rhul.ac.uk.

News items should be sent to newsletter@lms.ac.uk.

Notices of events should be prepared using the template at lms.ac.uk/publications/lms-newsletter and sent to calendar@lms.ac.uk.

For advertising rates and guidelines see lms.ac.uk/publications/advertise-in-the-lms-newsletter.

CONTENTS

Forthcoming LMS Events

The following will take place in the next two months:

Noether Celebration: 11 Sept, London
tinyurl.com/LMS-IMANoether2018

Popular Lectures: 19 Sept, Birmingham
tinyurl.com/hu58wjk

Good Practice Scheme Workshop: 5 Oct, London
tinyurl.com/y73n9ze9

Fisher Trust Joint Meeting: 9 Oct, Edinburgh
tinyurl.com/y9vwuhws

A full listing of forthcoming LMS events can be found on page 54.

LMS President-Designate

The LMS is pleased to announce Jonathan Keating FRS as President-Designate. Professor Keating has been the Henry Overton Wills Professor of Mathematics at the University of Bristol since 2012 and Chair of the Heilbronn Institute for Mathematical Research since 2015. He will take over from the current President, Professor Caroline Series FRS, in November 2019. He is known for his contributions to random matrix theory and quantum chaos, and their connections with number theory.

He obtained his PhD from the University of Bristol in 1989 under Professor Sir Michael Berry FRS. He began his career at Bristol as a Royal Society Research Fellow, before taking up a lectureship at the University of Manchester in 1991 and returning to Bristol as Reader in Applied Mathematics in 1995.

His research mainly concerns the quantum mechanics of chaotic and complex systems, random matrix theory, and applications of random matrix theory to the theory of the Riemann zeta function and other L-functions. He was elected a Fellow of the Royal Society in 2009 and received the LMS Fröhlich Prize in 2010. He currently holds a Royal Society Wolfson Research Merit Award and an ERC Advanced Grant.

Professor Keating has served the wider mathematical community as a member of the editorial boards of several scientific journals, including *Nonlinearity*, for which he was an editorial board member (1997–2004) and joint Editor-in-Chief (2004–2012). Under his leadership, the Heilbronn Institute of Mathematical Research has continued to support LMS Research Schools and sponsor the Bronze Medal for Mathematical Sciences at the annual STEM for Britain competition in the House of Commons. He has also been involved directly with the work of the Society as a member of the LMS Prizes Committee (2015–2018).

Fields Medallists

The 2018 Medals were awarded at the Opening Ceremony of the International Congress of Mathematicians in Rio De Janeiro, Brazil.

Professor Caucher Birkar (University of Cambridge) received his award for his ‘proof of the boundedness of Fano varieties and for contributions to the minimal model program’.

Professor Peter Scholze (University of Bonn) received his award ‘for transforming arithmetic algebraic geometry over p-adic fields through his introduction of perfectoid spaces, with application to Galois representations and for the development of new cohomology theories’. He is an Honorary Member of the LMS.

Professor Alessio Figalli (ETH Zurich) received his award for his ‘contributions to the theory of optimal transport, and its application to partial differential equations, metric geometry, and probability’.

Professor Akshay Venkatesh (Stanford University) received his award for his ‘synthesis of analytic number theory, homogeneous dynamics, topology, and representation theory, which has resolved longstanding problems in areas such as the equidistribution of arithmetic objects’.

Professor Caroline Series, LMS President, said: “I would like to offer my heartiest congratulations to all the 2018 Fields Medallists, in particular Caucher Birkar of the University of Cambridge. I am delighted to have been present at the opening ceremony when the winners were announced. I will be presenting Peter with his LMS Honorary Membership certificate in Rio”.

LMS Elections 2018

Voting for the LMS Elections for Council and Nominating Committee will open on 4 October 2018. The slate of candidates can be found at lms.ac.uk/about/council/lms-elections and an online forum for discussion is available at discussions.lms.ac.uk/lmselections.

Instructions on how to vote will be sent to members by email or post before the ballot opens. Members are encouraged to check that their contact details are up to date at lms.ac.uk/user. The election results will be announced at the LMS AGM on 9 November.

LMS Honorary Members 2018

The LMS elected Peter Scholze (Bonn) and Maria J. Esteban (CNRS) as Honorary Members of the Society.

Peter Scholze has made revolutionary contributions to arithmetic geometry and the theory of automorphic forms, in particular to p-adic arithmetic geometry, and has made decisive contributions to several fundamental open problems in number theory.

Maria J. Esteban has made major contributions to the theory of nonlinear partial differential equations and the use of variational methods in relativistic quantum mechanics and quantum chemistry.

Bond Report: an Update

Members will be aware of Philip Bond's report *The Era of Mathematics* highlighting the importance of mathematics and its impact on society and the economy, which was highlighted by Caroline Series, LMS President in the July LMS Newsletter (page 4). The report makes a number of extremely ambitious and far-reaching proposals to improve communication between mathematics and policy makers and the training and support of mathematicians, especially those engaging with potential applications. Make sure you have looked at the 9 Key Recommendations to see the scale of it.

Personally, I believe that, if adopted, this represents an enormous opportunity to change the mathematical environment for the better, and that LMS members should do all they can to make sure that effort is directed towards the best possible outcome. By all means disagree with me, but do look at Bond's Key Recommendations, discuss them, and let us know what you think by posting comments on the blog at bond.lms.ac.uk

In the first instance, the Council for the Mathematical Sciences (CMS) has issued the statement of support below. The CMS was established in 2001 by the Institute of Mathematics and its Applications, LMS and Royal Statistical Society and joined by the Edinburgh Mathematical Society and Operational Research Society in 2008. It is an authoritative and objective body that exists to develop, influence and respond to UK policy issues that affect the Mathematical Sciences in higher education and research, the UK economy and society in general.

Again, post your thoughts on the blog mentioned above.

Statement from the CMS

The Mathematical Sciences community welcomes the Bond Review Report, which clearly demonstrates the importance of mathematics and its impact across an exceptionally wide range of disciplines, technologies and industries. The review has recognised the need to develop a sustainable people pipeline of top-level Mathematical Sciences talent within academia, nurturing individuals keen to engage with government, industry and wider research challenges.

The report highlights the current shortage of such individuals and makes a strong case for increased investment in the discipline. It also recognises the need to establish infrastructure to facilitate effective and timely communication and interaction between mathematicians and potential users.

The CMS has met with individuals from the Mathematical Sciences community for a high level discussion of the report and its recommendations and with regard to the various areas covered in the report it was agreed as follows:

(1) Governance: There is strong support for the establishment of an Academy for the Mathematical Sciences with careful consideration being given to the most appropriate governance model for, and remit of, such an institution and its relationship to existing learned societies.

(2) Skills: Knowledge Exchange in the Mathematical Sciences community should be encouraged and additional funding for academic Mathematical Sciences would be very welcome. A performance based incentive model for the Mathematical Sciences, similar to the Higher Education Innovation Funding as previously administered by HEFCE, would facilitate innovative engagement with the user community.

(3) National Resources and Infrastructure: These recommendations would be beneficial, and could largely be included under the framework of the proposed Academy for the Mathematical Sciences. The Heilbronn Institute is an excellent model on which to base at least one national centre but further consideration as to how to effectively replicate this would be needed.

(4) Regional Support: The establishment of regional Knowledge Exchange centres links strongly with the 'place' agenda stemming from the industrial strategy and there is strong support to explore this further.

(5) Government: Stronger links between the Mathematical Sciences community and government can only be beneficial and should be encouraged.

The Bond Review report on Knowledge Exchange in the Mathematical Sciences was published on 26 April 2018 and can be found at tinyurl.com/yc8ykx4m

John Greenlees
LMS Vice-President

The Gender Gap in Science Project

Members are encouraged to participate in the survey for the project *A Global Approach to the Gender Gap in Mathematical, Computing, and Natural Sciences: How to Measure It, How to Reduce It?* As reported in the last issue, it is a three-part project funded by the International Council for Science (ICSU) and supported by many international scientific bodies. The project is collecting data to develop a broader picture of the status of mathematicians and scientists across the world. The results will provide data about the situation of scientists and mathematicians worldwide, as well as focused information about women in these fields. The data will help inform interventions by ICSU and member unions to increase participation, especially for women. The survey is open until 31 October 2018 and can be accessed at gender-gap-in-science.org.

Christopher Zeeman Medal 2018

The 2018 Christopher Zeeman Medal is awarded to Dr Hannah Fry of University College London for her contributions to the public understanding of the mathematical sciences. Dr Fry has a sustained and distinguished record of communicating mathematics to the public, with a huge portfolio of public engagement activities including books, videos, radio, TV, and public talks, which between them reach vast audiences. A full citation can be found at tinyurl.com/yap8j9aq.

The Zeeman Medal is a joint prize awarded by the LMS and the Institute of Mathematics and its Applications. It was created and named in honour of Professor Sir Christopher Zeeman FRS (1925–2016). Further information on this award can be found at tinyurl.com/y8ncdvyn.

The medal will be awarded at a ceremony in spring 2019, where Dr Fry will deliver a lecture.

Louis Bachelier Prize 2018

The 2018 Louis Bachelier Prize is awarded to Pauline Barrieu (London School of Economics and Political Science). Pauline Barrieu's scientific contributions cover a wide range of topics spanning pricing, risk management and insurance. Her insightful papers are motivated by the needs of the practical world while being skillfully crafted with elegant analysis that illuminates the underlying structure of the problem in question.

The Louis Bachelier Prize is a biennial prize jointly awarded by the London Mathematical Society, the Natixis Foundation for Quantitative Research and the Société de Mathématiques Appliquées et Industrielles. The winners are prominent personalities, widely recognized in the academic and professional worlds of quantitative finance and/or risk management. Further information on the Louis Bachelier Prize can be found at tinyurl.com/ybdosyyy.

2018 Kavli Education Medal

Professor Alice Rogers OBE has received the Royal Society's Kavli Education Medal 2018, for her outstanding contributions to mathematics education.

Professor Rogers' distinguished career has included work with the LMS as Vice-President and Education Secretary, and also as an active member of the Women in Mathematics Committee, of which she was a founding member. Outside of the Society's work she has advocated for mathematics education as a member of the Advisory Committee on Mathematics Education, serving as Deputy Chair from 2009 to 2011. She was also a member of the Joint Council for Mathematics and made influential contributions to mathematics education through her long career at King's College London.

Professor Caroline Series, LMS President, said:

"Alice has made tremendous contributions to the LMS through her lengthy and wise service. It is a fitting tribute that her outstanding contributions to education have been recognised in this way."

The medal will be formally awarded at the Kavli Education Lecture 2018. The medal is awarded biennially to honour impact in the fields of science, mathematics or computing education.

2018 Sylvester Medal

Professor Dusa McDuff FRS, of Barnard College, Columbia University, has received the Royal Society's Sylvester Medal 2018.

Professor McDuff, who was made an Honorary Member of the LMS in 1996, receives the medal in recognition of her work leading developments in the field of symplectic geometry and topology.

Professor McDuff studied in Edinburgh and later Cambridge. She briefly held lectureships in York and Warwick before moving to the US in 1976, where she has held chairs at Stony Brook and later Columbia, while always maintaining her links with the UK. In 1994 she became only the second female mathematician (after Mary Cartwright) to be elected to the Royal Society.

Professor Caroline Series, LMS President, said:

"Dusa is a tremendously dynamic person with amazing energy, original ideas, and deep insights. She has been an inspiration to generations of female mathematicians, including myself. It was Dusa who first suggested the idea of a British Women in Maths Day, the forerunner of the present LMS Women in Mathematics Days. I am truly delighted that her outstanding mathematics and leadership have been recognised by the award of this prestigious medal."

The medal will be awarded at the Royal Society's Anniversary Day in November 2018.

Other News of Members

GWYN BELLAMY was awarded the Learned Society of Wales Dillwyn Medal for STEMM. The Dillwyn Medals are awarded in recognition of outstanding early career research.

PETER CAMERON was elected a Fellow of the Royal Society of Edinburgh.

EUROPEAN

Budget for Horizon Europe

The proposed EU budget for Horizon Europe (the successor framework to Horizon 2020) is smaller than anticipated by the EU science community, and puts Europe behind other advanced economies on scientific investment. The European scientific bodies Euroscience and the Initiative for Science in Europe have launched a petition to increase this budget from €100 billion to €160 billion for the seven-year period.

European Mathematical Society on Open Access

The European Mathematical Society has issued an updated (July 2018) position document on Open Access, in reaction to concurrent discussions at the European Science Open Forum (ESOF) in Toulouse: the document can be seen at tinyurl.com/yc9stjuk.

CIRM-Luminy

The research centre CIRM-Luminy near Marseilles has just launched two real estate developments with the aim of extending scientific and hotel reception capacities and upgrading the whole centre by 2020. This physical expansion is accompanied by academic growth, with new programmes being launched, one of them with industry, another one focusing on pluridisciplinary approaches. In 2018 CIRM-Luminy's flagship programme, the Jean-Morlet Chair, is led by Professor Genevieve Walsh (Tufts University) for a semester on *Geometry, Topology and Group Theory in Low Dimensions* and will also welcome Professor Kerrie Mengersen (QUT Brisbane) in the second semester, with a focus on *Bayesian Modelling and Analysis of Big Data*. Calls for proposals for events in the second semester of 2020 are now open (www.cirm-math.fr).

David Chillingworth
LMS/EMS Correspondent

OPPORTUNITIES

LMS Research Grant Schemes

The LMS offers a number of grant schemes to support mathematics researchers. An overview of these grant schemes can be found on page 14. These schemes offer a number of deadlines throughout the year.

The next deadline for Schemes 1–5 (Conference Grants, Visits to the UK, Joint Research Groups in the UK, Research in Pairs, Collaborations with Developing Countries) is 15 September 2018

The next deadline for Scheme 7 (Computer Science Small Grants) is 1 October 2018.

The next deadline for Schemes 8 & 9 (Postgraduate Research Conferences, Celebrating new appointments) is 15 October 2018

For full details and to download an application form, see lms.ac.uk/grants/research-grants.

Talks in Schools and Colleges: Holgate Session Leaders

Do you give mathematical talks in schools and colleges? The LMS currently has vacancies for Holgate Session Leaders, and invites applications from those interested in taking part in this scheme.

The Holgate Lectures and Workshops Sessions scheme provides session leaders who are willing to give talks or run workshops on mathematical subjects to groups of students or teachers. The sessions are intended to enrich and enhance mathematical education, looking both within and beyond the curriculum.

Holgate sessions are intended for school- and college-level students, from primary and secondary to A-Level or equivalent (including STEP/AEA). They may also cover adult education. Sessions may also be offered to other relevant groups who wish to enhance their professional development, e.g. mathematics teachers.

Holgate session leaders do not charge a fee for giving talks, but local organisers are expected to pay travel expenses and subsistence costs, together with any local costs of organising the session. The LMS will pay an annual honorarium to the session leaders, currently £450.

More information on the scheme and details of current Holgate session leaders can be found at tinyurl.com/holgate-workshops.

To apply, please send a short (maximum 2-page) CV and a letter outlining: a) what you could offer as a Holgate Session Leader, and b) what you believe the Holgate sessions could offer as an educational experience, to education@lms.ac.uk.

Although there is no strict person specification, applicants should have a track record in mathematics education, communicating with people and learning and/or teaching mathematics outside of HE. They may be research-active mathematicians in a university department or be someone mathematically or statistically qualified based outside of academia.

Appointments will be for an initial three-year term, renewable by agreement.

Women and Girls in Maths Events: Call for Expressions of Interest

The LMS is inviting expressions of interest from individuals or groups interested in hosting a Women in Mathematics or Girls in Mathematics event in 2018–19. The deadline for receipt of applications for either scheme is 5 pm Friday 14 September 2018.

Women in Mathematics events are aimed at academic mathematicians (from at least postgraduate level, but possibly including undergraduates), and are intended to help early career women mathematicians when considering the next stages in their careers. For details of previous events held organised by the Society, see tinyurl.com/ycw6nk5h. Funding is available for two events in the academic year 2018–19. £3,000 of funding is available for each event. To submit an expression of interest see tinyurl.com/yb4p6v8u.

Girls in Mathematics events should be aimed at schoolgirls, up to and including A-level or equivalent students, with mathematics as a main focus. Funding is available for two events in the academic year 2018–19. £500 of funding is available for each event. Details on how to submit an expression of interest can be found at tinyurl.com/y8b5hu93.

Spitalfields Day: Call for Proposals

The LMS offers funding of up to £1,000 towards the cost of a Spitalfields Day, a one-day event at which selected participants, often eminent experts from overseas, give survey lectures or talks accessible to a general mathematical audience. The Spitalfields Day is often associated with a long-term symposium and speakers will generally give lectures on topics of the symposium.

The funding is intended to cover actual supplementary costs for the event, e.g. subsidising a lunch for participants, and small travel grants of £50 to enable LMS members and research students to attend the event.

If you are interested in organising a Spitalfields Day, please write to lmsmeetings@lms.ac.uk. The format need not be precisely as described, but should be in a similar spirit. The next deadline for proposals is 15 September 2018; the subsequent deadline is 31 January 2019. Please note the Society cannot fund events retrospectively so applicants are advised to apply well in advance of the event.

BSHM Small Grants Scheme

The British Society for the History of Mathematics (BSHM) has introduced a small grants scheme to encourage the study of the history of mathematics. The aim of this grants scheme is to provide small grants (from £100 to £500) for specific research purposes. Funding will be allocated twice a year, with applications to be received by 30 September or 31 March each year. To apply see bshm.ac.uk.

Clay Research Fellowships

The Clay Mathematics Institute calls for nominations for the 2019 Clay Research Fellowships. Fellows are selected for their research achievements and their potential to become leaders in research mathematics. All are recent PhDs, and most are selected as they complete their thesis work. Terms range from one to five years, with most given in the upper range of this interval. Fellows are employed by the Clay Mathematics Institute, which is a US charitable foundation, but may hold their fellowships anywhere in the USA, Europe, or elsewhere in the world. Nominations should be received by 16 November. To nominate a candidate see tinyurl.com/y8ghv56w.

Emma Castelnuovo Award

The ICMI Emma Castelnuovo Award honours persons, groups, projects, institutions or organisations for exceptionally excellent and influential work in the practice of mathematics education. Nominations for the 2020 award should be sent by email to the Chair of the Committee by 31 March 2019. For further details see tinyurl.com/ybqb7ayy.

VISITS

Visit of Pablo Shmerkin

Dr Pablo Shmerkin (Universidad Torcuato Di Tella, Argentina) will visit the UK from 6 to 24 October. His research involves geometric measure theory and dynamical systems. He will give lectures at:

- University of Bristol, 11 October (contact Thomas Jordan: thomas.jordan@bristol.ac.uk)
- University of Manchester, 15 October (contact Tuomas Sahlsten: tuomas.sahlsten@manchester.ac.uk)
- University of St Andrews, 23 October (contact Jonathan Fraser: jmf32@st-andrews.ac.uk)

For further details contact Thomas Jordan. The visit is supported by an LMS Scheme 2 grant.

Visit of Vassilis Rothos

Professor Vassilis Rothos (AUTh) will visit the UK from 4 to 15 December. His research involves asymptotic expansions, applied analysis, numerics, and the calculus of variations. He will give lectures at:

- University of Essex, 4-6 December (contact Hadi Susanto: hsusanto@essex.ac.uk)
- University of Surrey, 7-9 and 13-15 December (contact Tom Bridges: T.Bridges@surrey.ac.uk)
- University of Edinburgh, 10-12 December (contact Noel Smyth: N.Smyth@ed.ac.uk)

For details contact Tom Bridges. Supported by an LMS Scheme 2 grant.

Visit of Yuri Yakubovich

Dr Yuri Yakubovich (St. Petersburg State University) will visit the School of Mathematics, Leeds, from 16 January to 9 February 2019. He works in probabilistic combinatorics. He will give the following seminars:

- Warwick University, 17 January (14.00), *Minimal Difference Partitions and Fractional Quantum Statistics* (contact D.Ueltschi@warwick.ac.uk)
- University of Oxford, 21 January (12.00), *Orderings of Gibbs Random Samples* (contact goldschm@stats.ox.ac.uk)
- Queen Mary University of London, 23 January (13.00), *Orderings of Gibbs Random Samples* (contact A.Gnedin@qmul.ac.uk)
- University of Leeds, 24 January (12.00), *Orderings of Gibbs Random Samples* (contact K.Colaneri@leeds.ac.uk)

For further details contact L.V.Bogachev@leeds.ac.uk.

Visit of Sotirios Konstantinou-Rizos

Dr Sotirios Konstantinou-Rizos (Demidov Yaroslavl State University) will visit the UK from 14 January to 5 February 2019. His expertise is in mathematical physics, integrable systems and Yang-Baxter maps. He will give lectures at:

- University of Essex, 17 January, *From continuous to discrete integrable systems via Darboux transformations* (contact grah@essex.ac.uk)
- University of Leeds, 25 January, *A noncommutative extension scheme for discrete integrable systems & related Yang-Baxter maps* (contact A.V.Mikhailov@leeds.ac.uk)
- Heriot-Watt University, 30 January, *Yang-Baxter maps related to NLS type equations* (contact A.Doikou@hw.ac.uk)

For further information contact Georgi Grahovski.

New Standing Orders for the LMS?

Background

The LMS is a membership charity governed by a rather lengthy document called the standing orders. This consists of three parts, the Charter, Statues and By-laws, as shown in the table. The standing orders were last comprehensively reviewed in 1965. Since then, many changes have taken place both in the Society and in the wider world. The Society has grown considerably in size, the existing standing orders contain references to outdated procedures such as letter writing which are all but obsolete, and they are not written in gender-neutral language.

It is proposed that instead of a President-Designate, there is a President-Elect. He or she would be elected in year n and then become President (for a two-year term) in year $n+1$. The President-Elect would attend Council but not be a member of it.

Whenever our members are asked to vote on a motion at a General Meeting of the Society, they have the option to do so by proxy. At present, our detailed procedures for proxy voting have not been made explicit. It is proposed to describe them in the By-Laws.

	How many	For example
Charter	17 items	Aims, members, Council, General Meetings, committees, Statutes
Statutes	39 items	Types of members, admission, subscription, reciprocity agreements, General Meetings and Ordinary Meetings, quorum, voting, Council
By-laws	41 items	Officers and Members-at-Large, terms of office, nominations, elections, honorary members, President, Vice-Presidents, Treasurer, Secretaries

Thus it is time for a review. A small group of Council members together with the Chair of Nominating Committee has been working on this since 2014. Council has seen and discussed the various proposed changes, and the Departmental Representatives have also been consulted.

The essence of the proposed changes

Most of the changes are very minor. For example, the language has been made more inclusive, and references to posting letters have been changed to simply communicating. But there are proposals for a few more substantial changes, such as the following.

The Trustees of the charity are the twenty members of Council, consisting of eight Officers and twelve Members-at-Large. At present the Officers are elected for terms of office of one year, and can serve for up to ten consecutive terms. The Members-at-Large are elected for terms of office of two years, and can serve for up to three consecutive terms. These terms of office are being re-considered.

At present members can vote at the Annual General Meeting itself. It is proposed to change this so that the voting takes place prior to the AGM, with the results announced at the AGM as now.

At present, the number of signatures required to call a Special General Meeting is 20. This was fixed in 1965, when this number was about 2.5% of the membership. It is proposed to change the criterion to 2.5% of the membership, which would now mean about 60.

Proposed timetable

At the time of writing (early August), Council is still working on some details of the proposals. But there will be a long period of consultation with members of the Society during the winter of 2018. This will end in the new year, so that Council has time to consider the feedback and prepare a final version, to be presented to the General Meeting in June 2019.

Members will be notified when the consultation period begins, and will be invited to comment on a dedicated blog or to submit private comments by email. We would like to encourage all members to take part in this discussion.

Caroline Series, President
Stephen Huggett, General Secretary

LMS Council Diary – A Personal View

Council met at De Morgan House on Friday 29 June 2018 for a slightly shorter meeting due to the LMS General Meeting being held that same afternoon. As usual, the meeting began with an update on the President's activities since the last Council meeting, which included attending meetings of the Prizes committee, the Zeeman Medal Panel, and the Presidential Search Panel. She also attended the Abel Prize Ceremony in Oslo as well as the LMS Representatives Day and a Society meeting in Leicester.

Council then agreed that Jonathan Keating should be nominated as the Society's next President-Designate.

Chris Parker, the Chair of the Early Career Research Committee, had joined Council to introduce revised proposals for the Early Career Fellowship scheme, which had been under discussion for some time, and is planned to succeed the Society's Postdoctoral Mobility Grants. It is designed to provide support for researchers in the very early stages after their PhD, who had not already secured a postdoctoral position. Council agreed that the Early Career Fellowships be established, and that from September 2018, funding should be provided for eight six-month fellowships each year for an initial period of three years.

The President reported from the International Affairs Committee explaining that there were two bids, Paris and St Petersburg, to host the 2022 International Congress of Mathematics (ICM), and that a vote will be taken at the 2018 International Mathematical Union (IMU) General Assembly.

The President introduced an updated proposal for the celebration of the 21st Anniversary of the Society being located at De Morgan House. Council agreed the outline proposal. The celebration is to take place on 19 October 2019.

The Treasurer presented the Third-Quarter financial review 2017-18, and the budget allocation for 2018-19, as well as the planning figures 2019-21. In particular, it was noted that the grant budget had increased. It was also agreed that membership subscription rates for 2018-19 are to be increased by 5%, and that the Society is to offer half-price membership for a year to attendees of the induction course for new lecturers.

We also heard reports from the Education Committee, the Computer Science Committee, the Society Lectures and Meetings Committee, and the Publications Committee.

Finally, we agreed to note the slate for Council and Nominating Committee elections in 2018, and we approved the 30 applications for membership to be proposed to the Society meeting to be held that afternoon.

We then decamped to the BMA building down the road for the General Meeting and Hardy Lecture.

Brita Nucinkis

Annual LMS Subscription 2018–19

Payment for annual subscriptions for the period November 2018 – October 2019 is due on 1 November 2018, and payment should be received by 1 December 2018. Any payments received after this date may result in a delay in journal subscriptions being renewed.

LMS membership subscription rates 2018-19

Ordinary membership	£84.00	US$168.00
Reciprocity	£42.00	US$84.00
Career break or part-time working	£21.00	US$42.00
Associate membership	£21.00	US$42.00

Access to LMS Journals

LMS member benefits include free online access to the *Bulletin*, *Journal* and *Proceedings of the London Mathematical Society* and *Nonlinearity*, for personal use only. If you would like to receive free electronic access to these journals, please indicate your choices either on your online membership record under the 'Journal Subscription' tab or on the LMS subscription form.

Subscribing to the EMS and JEMS via the LMS

Members also have the option to pay their European Mathematical Society subscription via the LMS and subscribe to the Journal of the EMS. Please indicate your preference either on your online membership record under the 'Journal Subscription' tab or on the LMS subscription form.

Payment of membership fees for EWM

LMS members who are also members of European Women in Mathematics may pay for their EWM fees when renewing their LMS membership. You decide yourself your category of fees: high, normal, low. Please indicate your category of fee either on your online membership record under the 'Journal Subscription' tab or on the LMS subscription form. To join EWM please register first at tinyurl.com/y9ffpl73. It is not possible to join the EWM through the LMS.

Online renewal and payment

Members can log on to their LMS user account (lms.ac.uk/user) and make changes to their contact details and journal subscriptions under the 'My LMS Membership' tab. Members can also renew their subscription by completing the subscription form and including a cheque either in GBP or USD. We regret that we do not accept payment by cheques in Euros.

LMS member benefits

Members are entitled to the following benefits: voting in the LMS elections, free online access to selected journals, the bi-monthly Newsletter, use of the Verblunsky Members' Room at De Morgan House, and use of the Society's Library at UCL, among others. For a full list of member benefits, see lms.ac.uk/membership/member-benefits.

Membership of the London Mathematical Society

The standing and usefulness of the Society depends on the support of a strong membership, to provide the resources, expertise and participation in the running of the Society to support its many activities in publishing, grant-giving, conferences, public policy, influencing government, and mathematics education in schools. The Society's Council therefore hopes that all mathematicians on the staff of UK universities and other similar institutions will support mathematical research by joining the Society.

If current members know of friends or colleagues who would like to join the Society, please do encourage them to complete the online application form (lms.ac.uk/membership/online-application). Contact membership@lms.ac.uk for advice.

A full list of LMS member benefits can be found at lms.ac.uk/membership/member-benefits.

LMS Grant Schemes

The following is a list of the grant schemes offered by the LMS which are intended to support mathematics researchers. For full details and to download an application form, see lms.ac.uk/grants/research-grants.

RESEARCH GRANT SCHEMES 1–5

Deadline: 15 September 2018

Scheme 1 — Conferences Grants: Grants of up to £7,000 are available to provide partial support for conferences held in the United Kingdom. This includes a maximum of £4,000 for principal speakers, £2,000 to support the attendance of research students who are studying at universities in the UK, and £1,000 to support the attendance of participants from Scheme 5 eligible countries.

Scheme 2 — Visits to the UK: Grants of up to £1,500 are available to provide partial support for a visitor to the UK, who will give lectures in at least three separate institutions. Awards are made to the host towards the travel, accommodation and subsistence costs of the visitor. Potential applicants should note that it is expected the host institutions will contribute to the costs of the visitor.

Scheme 3 — Joint Research Groups in the UK: Grants of up to £4,000 are available to support joint research meetings held by mathematicians who have a common research interest and who wish to engage in collaborative activities, working in at least three different locations (of which at least two must be in the UK). Potential applicants should note that the grant award covers two years, and it is expected that a maximum of four meetings (or an equivalent level of activity) will be held per academic year.

Scheme 4 — Research in Pairs: For those mathematicians inviting a collaborator to the UK, grants of up to £1,200 are available to support a visit for collaborative research either by the grant holder to another institution abroad, or by a named mathematician from abroad to the home base of the grant holder. For those mathematicians collaborating with another UK-based mathematician, grants of up to £600 are available to support a visit for collaborative research.

Scheme 5 — Collaborations with Developing Countries: For those mathematicians inviting a collaborator to the UK, grants of up to £3,000 are available to support a visit for collaborative research, by a named mathematician from a country in which mathematics could be considered to be in a disadvantaged position, to the home base of the grant holder. For those mathematicians going to their collaborator's institution, grants of up to £2,000 are available to support a visit for collaborative research by the grant holder to a country in which mathematics could be considered to be in a disadvantaged position.

RESEARCH GRANT SCHEME 7

Deadline: 1 October 2018

Scheme 7 — Computer Science Small Grants: Funding is available to support a visit for collaborative research at the interface of Mathematics and Computer Science either by the grant holder to another institution within the UK or abroad, or by a named mathematician from within the UK or abroad to the home base of the grant holder.

RESEARCH GRANT SCHEMES 8 & 9

Deadline: 15 October 2018

Scheme 8 — Postgraduate Research Conferences: Grants of up to £4,000 are available to provide partial support for conferences held in the United Kingdom, which are organised by and are for postgraduate research students. The grant award will be used to cover the costs of participants.

Scheme 9 — Celebrating new appointments: Grants of up to £600 are available to provide partial support for meetings held in the United Kingdom to celebrate the new appointment of a lecturer at a UK university. Potential applicants should note that it is expected that the grant holder will be one of the speakers at the conference.

OTHER GRANT SCHEMES

The Society offers a number of other grant schemes to support mathematics, including education grants for teachers and other educators; caring supplementary grants to enable parents and carers to attend conferences and other research schools; travel grants for early career researchers; and support for mathematics overseas. See details at lms.ac.uk/grants.

REPORTS OF THE LMS

2018 LMS Popular Lectures

The 2018 LMS Popular Lectures were held on 4 July at King's College London. A diverse audience, including many students, enjoyed an evening of mathematics with a range of mathematical puzzles and a journey through the correct and incorrect use of statistics to assess risk. How statistically significant are the risks that we see? And what do the numbers mean?

Katie Steckles was first up with her presentation *Maths' Greatest Unsolved Puzzles*. The audience was asked to consider puzzles such as 'What is the smallest number that has all of the numbers 1 to 9 as a factor?', 'how many flips are required to turn a disordered stack of pancakes into an ordered stack', and the Collatz Conjecture. (The Collatz Conjecture is as follows. Consider a sequence starting with any positive integer n. Each term is obtained from the previous term as follows: if the previous term is even, the next term is one half the previous term. If the previous term is odd, the next term is 3 times the previous term plus 1. The conjecture is that no matter what value of n is, the sequence will always reach 1.)

After the break, Jennifer Rogers from the University of Oxford presented her lecture *Living is a Risky Business*. The audience was shown a number of media reports that highlighted the importance of the difference between relative and absolute increase. For example, headlines such as the apparent 20% increase risk of developing pancreatic cancer as a result of eating a bacon sandwich every day (from 5 in 400 to 6 in 400). She also highlighted some of the stranger examples of correlation not always meaning causation, e.g., the consumption of cheese and the number of people dying from becoming tangled in bedsheets. Dr Rogers went on to explain the significance of increased survival rates from medical trials where an increase in survival rate from 60% to 65% is not significant in a sample size of 100 but is in a sample of 1000. Analysis showed the probability of this difference in survival rates occurring naturally significantly dropped from 47% to 2.7% between sample sizes.

The audience engaged enthusiastically with the presentations and overall the lectures were very well received. If you weren't able to attend the London lectures, Katie and Jennifer will be presenting their lectures again in Birmingham on Wednesday 19 September. You can register online at lms.ac.uk/events/popular-lectures.

John Johnston
Society Communications Officer

Lincoln Maths And Physics Society

Jason Lotay using a lightsaber to help describe and explain the different dimensions

The newly founded University of Lincoln Maths And Physics Society (MAPS) hosted their first guest lecture on Monday 16 April 2018 with the support of an LMS grant for Undergraduate Society Meetings. The guest speaker was Professor Jason D. Lotay from University College London who gave an insightful talk titled *Adventures in the 7th Dimension*. Professor Lotay showed us how in 7 dimensions there exist special shapes that may help us unlock the mysteries of the universe. Looking for this unique geometry is challenging, but nature holds a possible solution (specifically, bubbles and thermodynamics). The lecture took us on a mathematical journey across multiple dimensions, exploring their role in art, science and popular culture.

Following the talk and Q&A session, refreshments were served which went down a treat. Considering that MAPS is such a new society and that this was the first ever guest lecture, a good turnout was witnessed of around 30–40 people; this included society members as well as staff and students of the University.

Lovedeep Bharaj
University of Lincoln Maths and Physics Society

The University of Lincoln Maths And Physics Society (MAPS) was supported by the Society with a grant of £410 under the Society's grant scheme to support meetings of Undergraduate Mathematical Societies at UK universities. Further details of this scheme can be found at lms.ac.uk/grants/LMS-Grants-Undergrad-Soc-Meetings.

LMS Midlands Regional Meeting

The LMS Midlands Regional Meeting 2018 took place at the University of Leicester on 4 June 2018. The invited speakers were Professor Minhyong Kim (University of Oxford), Professor Leila Schneps (Institut de Mathématiques de Jussieu Paris) and Professor Fabrizio Catanese (University of Bayreuth). The meeting started with official society business chaired by LMS President Professor Caroline Series. New members had the chance to sign the society book and thereby to be officially admitted to the LMS.

In the opening talk, *Diophantine Geometry and Principal Bundles*, Professor Minhyong Kim described fascinating new links between number theory, geometry and physics. He presented a new unifying approach on two seemingly separate lines of investigation in number theory and physics, namely Diophantine equations and the first least-action principle to explain the trajectory of light, both originating in research by Pierre de Fermat in the 17th century. Classically, the principle of least action was the key of the theoretical foundation of gauge theory, underlying quantum field theory and its applications to geometry and topology. In Kim's new approach towards Diophantine geometry, solutions of Diophantine equations are obtained via solutions of an arithmetic gauge theory which can be seen as an arithmetic manifestation of classical gauge theory, the study of principal bundles in geometry and mathematical physics.

In the second talk, Professor Schneps related Teichmüller theory to number theory in her inspirational talk *Pro-unipotent Grothendieck–Teichmüller Theory: Surprising Connections with Number Theory*. Grothendieck–Teichmüller theory was introduced by Alexander Grothendieck as a way to study the absolute Galois group of the rational numbers by considering its action on fundamental groups of varieties, in particular of moduli spaces of curves with marked points. Grothendieck suggested to study a larger profinite group, nowadays called the Grothendieck–Teichmüller group, which contains the absolute Galois group and respects the inertia generators and relations in the fundamental group. Studying generators and relations of this group reveals a pro-unipotent avatar, and an associated graded Lie algebra. In presenting this new Lie algebra approach, Schneps revealed many surprising and hidden links with number theory, for example Bernoulli numbers, cusp forms on $\mathrm{SL}_2(\mathbb{Z})$ and multiple zeta values arise very naturally within this Lie algebra framework.

In the final talk, Professor Fabrizio Catanese gave an inspiring overview, *Mathematical Mysteries behind the Interplay of Algebra and Topology in Moduli Theory*, on new connections between algebraic and topological aspects of the classification theory of algebraic varieties with symmetries. The famous Riemann existence theorem indicates the existence of certain polynomials, or algebraic functions satisfying certain very special properties. The topological approach, via Riemann's existence theorem, is particularly useful when analysing the moduli spaces of curves with symmetries. In turn, the study of curves with certain groups of automorphisms, is crucial for the construction of higher dimensional varieties, like for example algebraic surfaces and for the description of their moduli spaces. Catanese illustrated the two different approaches to the construction of moduli spaces, one via topology, namely Teichmüller theory, and the other through algebra, namely Geometric Invariant Theory. The latter approach also gives rise to an action of the absolute Galois Group on moduli spaces, which is particularly interesting in the case of projective classifying spaces, i.e., varieties with contractible universal cover. Finally, he addressed a famous open problem, pioneered by work of Kazhdan, namely whether a Galois conjugate of a projective classifying space is again one such by presenting many illuminating new examples and constructions.

The regional meeting was followed by an interdisciplinary workshop on *Galois covers, Grothendieck–Teichmüller theory and Dessins d'enfants* on 5–7 June 2018 featuring 16 invited international speakers, including the three regional meeting speakers. It was a very diverse workshop bringing together world leading experts as well as graduate students and early career researchers from geometry, topology, algebra and number theory. The workshop was very successful and initiated new links and future collaborations. Many talks provided an introduction or overview to the different themes of the workshop which was of great benefit especially for postgraduate students and early career researchers.

The Regional Meeting and Workshop was well attended including many PhD students and early career researchers. Additional financial support was provided from the University of Leicester, REPNET and EPSRC. A conference website with the complete information about the LMS Midlands Regional Meeting and the Workshop with abstracts and the slides of some of the talks is maintained at tinyurl.com/y974n9xv.

Frank Neumann and Sibylle Schroll
University of Leicester

LETTERS TO THE EDITOR

EPSRC's New Investigator Award

I would like to draw your attention to recent changes that the EPSRC has made to one of its grant schemes: their *New Investigator Award* has replaced the *First Grant* scheme and one of the resulting technical changes could adversely affect our community.

The funding cap on what can be applied for has now been removed, so a new investigator can apply for whatever is appropriate for their research. But money is, of course, limited and there is a real danger that people will apply for larger grants than they necessarily need, and, as a result, fewer grants will be awarded. This is what happened between 2003 and 2009 — the funding cap was removed on the old First Grant scheme and as a result the size of grants increased (typically to around £300k) and the number awarded decreased.

Obtaining research grants is now a key promotion criterion at many Universities (including my own). Larger grants will mean fewer grants which, in turn, will damage promotion prospects of junior staff. There is a real tension here between individual and collective behaviour.

As a member of the EPSRC Strategic Advisory Team for the Mathematical Sciences, I argued against this change, but the scheme cuts across all programme areas within EPSRC with a single set of criteria, irrespective of subject, leaving it up to reviewers and panels to make decisions on what size of grant is appropriate for a new investigator.

But what can the community do, collectively, to decide what is appropriate for a "New Investigator Award"? One possible action is through how we referee grant proposals, and how we act on grant panels. It could be as simple as referees paying closer attention to the costings of a grant, and criticising an application if the costings seem unreasonably large and cease to be value for money. Panels, in my experience, rarely notice costings.

I have written a blog entry for the EPSRC on this matter, see: tinyurl.com/y9gkumd5 and I, and the EPSRC, would welcome the community's views on this important issue. This can be done on the website.

Do we really, as a community, want fewer, larger grants? The title of the blog entry is *The bigger the better? Why less is more*, and this one line sums up the article.

Ian Strachan
University of Glasgow

Letters may be edited for style and space.

CORRECTIONS AND CLARIFICATIONS

From the July issue (477) of the *Newsletter*.

Mathematicians Honoured by the Royal Society (page 5): Kevin Costello was elected a Fellow of the Royal Society. Manuel del Pino was awarded a Royal Society Research Professorship to investigate how and when singularities occur in natural phenomena.

Records and Proceedings at LMS meetings
General Meeting: 29 June 2018

The meeting was held at the BMA House, Tavistock Square, London. Over 60 members and visitors were present for all or part of the meeting.

The meeting began at 3.30 pm with the President, Professor Caroline Series, FRS, in the Chair.

On a recommendation from Council it was agreed to elect Professor Charles Goldie and Professor Chris Lance as Scrutineers in the forthcoming Council elections. The President invited members to vote, by a show of hands, to ratify Council's recommendation. The recommendation was ratified unanimously.

The President, on Council's behalf, proposed that the following people be elected to Honorary Membership of the Society: Professor Maria J. Esteban, Director of Research at the CNRS and University Paris-Dauphine, and Professor Peter Scholze of the University of Bonn. This was approved by acclaim. The President read a short version of the citations, to be published in full in the *Bulletin of the London Mathematical Society*.

The President then announced the awards of prizes for 2018:

Pólya Prize: Professor Karen Vogtmann (University of Warwick)
Fröhlich Prize: Professor Francesco Mezzadri (University of Bristol)
Senior Berwick Prize: Professor Dr Marc Levine (University of Duisburg-Essen)
Whitehead Prizes: Professor Caucher Birkar (University of Cambridge)
Dr Ana Caraiani (Imperial College, London)
Dr Heather Harrington (University of Oxford)
Professor Valerio Lucarini (University of Reading)
Dr Filip Rindler (University of Warwick)
Dr Péter Várju (University of Cambridge)
Hirst Prize & Lectureship: Professor Jeremy Gray (The Open University)
Anne Bennett Prize: Dr Lotte Hollands (Heriot-Watt University)

The President announced Professor Jon Keating, FRS, as the President Designate for 2019–2021.

Nineteen people were elected to Ordinary Membership: Dr Timothy Adamo, Dr Hamid Alemi Ardakani, Dr Mathew Bullimore, Mr Kevin Comer, Professor Pierre Degond, Dr Laura Delaney, Mr Felix Goldstein, Dr Jonathan Hickman, Ms Fotini Karakatsani, Dr Oleg Kirillov, Dr Gordon Laing, Dr Peter Landrock, Miss Ems Lord, Dr Ciaran Mac an Bhaird, Dr Abu Bakar Maharani, Professor John Pryce, Mr Danny Sarpong, Dr Dumitru Trucu and Dr James Waldron.

Nine people were elected to Associate Membership: Mr Joseph Bennett. Dr Andrea Granelli, Mr Shaun Isherwood, Mr Joseph Macmillan, Mr Mehdi Mansourpour, Mr Ka Ching Ernest Ng, Mr Avishka Perera, Mr Alexander Round and Mr James Waterman.

Two people were elected to Reciprocity Membership: Dr Roseline Abah and Dr Zeinab Mansour.

Ten members signed the book and were admitted to the Society.

The President introduced a lecture given by Professor Konstanze Rietsch (King's College, London) on *Grassmannians and Polytopes*.

Following a break for tea, the President introduced the Hardy Lecture by Professor Lauren Williams (UC Berkeley/Harvard University) on *The Positive Grassmannian and Shallow Water Waves*.

At the end of the meeting, the President thanked both speakers for their brilliant lectures.

The President also thanked Chris Parker (University of Birmingham) and Anthony Byrne (LMS) for organising the Graduate Student Meeting in the morning and Anna Felikson (University of Durham) and Sibylle Schroll (University of Leicester), who gave talks at the Graduate Student Meeting. Rafael Prieto Curiel (University College London) was also congratulated on winning the prize for the best graduate student talk.

After the meeting, a reception was held at De Morgan House, followed by a dinner at the Montague on the Gardens Hotel.

Records and Proceedings at LMS meetings
Ordinary Meeting, 4 June 2018

The regional meeting was held at the Ken Edwards Lecture Theatre 3 (KE LT3), University of Leicester, as part of the Midlands Regional Meeting and Workshop on *Galois Covers, Grothendieck–Teichmüller Theory and Dessins d'enfants*. Over 40 members and guests were present for all or part of the meeting.

The meeting began at 1.30 pm with the President, Professor Caroline Series, FRS, in the Chair.

There were no members elected to Membership at this Society Meeting.

Six members signed the Members' Book and were admitted to the Society.

Professor Alexander Clark, Head of Mathematics at the University of Leicester, welcomed the Society's members and guests to the University and thanked the Society for its support of the Department.

Dr Frank Neumann, University of Leicester, introduced the first lecture given by Professor Minhyong Kim (University of Oxford) on *Diophantine geometry and principal bundles*.

After tea, Dr Sibylle Schroll, University of Leicester, introduced the second lecture by Professor Leila Schneps (Institut de Mathématiques de Jussieu Paris) on *Pro-unipotent Grothendieck–Teichmüller theory: surprising connections with number theory*.

Dr Neumann then introduced the final lecture given by Professor Fabrizio Catanese (University of Bayreuth) on *Mathematical Mysteries behind the Interplay of Algebra and Topology in Moduli Theory*.

Professor Series thanked the speakers for their excellent lectures and then expressed the thanks of the Society to the organisers, Dr Frank Neumann and Dr Sibylle Schroll, both of the University of Leicester, for a wonderful meeting and workshop.

Afterwards, a wine reception was held outside the Ken Edwards Lecture Theatre 2 (KE LT2). The Society dinner was held at the Kayal Restaurant in Leicester.

Records and Proceedings at LMS meetings
Special Ordinary Meeting at the British Mathematical Colloquium 2018

The meeting was held on 13 June 2018 at the School of Physics and Astronomy, St Andrews University, as part of the British Mathematical Colloquium 2018. Over 70 members and guests were present for all or part of the meeting.

The meeting began at 4.30 pm with the Vice-President, Professor Catherine Hobbs, in the Chair.

There were no members elected to Membership at this Society Meeting. Ten members signed the Members' Book and were admitted to the Society by the Vice-President.

There were no Records of Proceedings to approve at this meeting.

Professor Hobbs introduced Professor Laura DeMarco, of Northwestern University, who gave a lecture titled *Complex Dynamics and Arithmetic Equidistribution*.

Professor Hobbs warmly thanked the local organisers of the British Mathematical Colloquium, Professor Kenneth Falconer, Dr Colva Roney-Dougal and Dr Mike Todd amongst others, for organising a great colloquium.

The meeting closed at 5.40 pm, and was followed by a wine reception in the School of Physics and Astronomy, and a Colloquium Dinner at Lower College Hall in St Andrews.

How Graph Theory Can Help Control Cattle Diseases

JESS ENRIGHT

I think that everyone can benefit from a little graph theory — in this article I argue that this includes cows. I'll describe several ways that a British cattle dataset can be interpreted as a graph, and outline some algorithmic work on optimally modifying these graphs to limit worst-case disease outbreaks.

As anyone who has spent time on a rail journey through the British countryside will know, there are a lot of sheep and cattle in Britain. A large amount of information is recorded about the management of these animals, including careful recording of their movements between farms and markets. I first encountered cattle trading datasets in 2011, when I started a postdoctoral appointment at the University of Glasgow as part of a quantitative epidemiology group, and since then I have joined many other researchers in thinking about how considering British cattle movements as a network or graph can help us detect and control disease.

While cattle movement datasets in Britain may not be big in the "big data" sense, they're also not small. At the most recent count, there were about 9.8 million cattle in the United Kingdom[1], held on over 75,000 agricultural holdings. Movements of these cattle between agricultural holdings are recorded and reported to British government, and collected into a central database. This extensive dataset is invaluable for modelling infectious diseases of cattle, as well as monitoring the operation of the industry as a whole. Because of the established importance of these animal movements in major outbreaks of foot-and-mouth disease, and as a risk factor for bovine tuberculosis, there has been a very large amount of modelling, statistical, and simulation work using the cattle movement database, including work from a network science perspective.

In this article, I'll outline some of the work that has been done on cattle movements in Britain, with a focus on how graph theory can contribute. First, I talk about the cattle movement dataset and several different ways of representing it as a graph, and spend some time on how we can incorporate temporal information into this graph. Then I'll move on to a discussion of how the properties of these graphs have let us make progress on optimal graph modification problems for limiting worst-case disease.

From movements to graphs

The British Cattle Movement Service dataset contains movement records for individual animals, including the date of the movement, as well as births and deaths. It is one of my favourite datasets because it is large, detailed, has been collected for more than a decade, and is generally considered to be reliable.

When making a graph out of the cattle movement dataset, the first thing to decide is what entities will be vertices, and what contacts will be recorded as edges. The most common approach is to take the set of farms as the vertex set, with an edge between two farms if there has been animal trade between them over some time period of interest, but this is far from the only option. Sometimes a larger aggregation is more appropriate: for example, if we're looking at regional trade perhaps the counties of Britain should form the vertex set. Or, perhaps we need closer granularity, and take the set of individual bovines as the vertex set — of course, this may result in a larger graph than we are prepared to deal with, as at any given time there are approximately 9 million cattle living in Britain!

For the moment, let's say we take the set of agricultural holdings (including farms, markets, etc.) as our vertex set. What about edges? As I mentioned, a common choice is to link two agricultural holdings with an undirected edge if an animal has moved between them within some time window. This choice throws away a lot of potentially useful information, including the direction, weight, and timing of a contact. As methods and and available software have developed, these extra pieces of information are being more frequently used, and the cattle trading graph might be considered as a directed (as in the bottom left of Figure 1), weighted, or temporal graph. We'll return to the idea of a temporal graph later in this article.

[1] https://www.gov.uk/government/statistics/farming-statistics-livestock-populations-at-1-december-2016-uk

The majority of cattle trades in Britain are conducted through a relatively small number of auction markets. This means that in the graph in which both farms and markets occur in the vertex set, there are a relatively small number of vertices (the markets) with very high degree, and many vertices of comparatively low degree. More than that, the structure of the graph is relatively simple: the majority of edges form a hub-and-spoke-like graph. This sort of structure can make many computationally difficult problems much easier to solve, but may not be appropriate for every disease setting.

A slow-spreading disease that requires close contact to spread between animals is unlikely to spread at a market, and so sometimes it may be more appropriate to consider a *market-stripped* version of this graph in which we remove the markets and record a movement of an animal from farm u to farm v via a market as an edge from u to v (bottom right of Figure 1).

```
Animal1,2018/01/01,Farm_A,Market_1
Animal1,2018/01/01,Market_1,Farm_B
Animal2,2018/01/01,Farm_A,Market_1
Animal2,2018/01/01,Market_1,Farm_B
Animal1,2018/02/01,Farm_B,Market_1
Animal1,2018/02/01,Market_1,Farm_C
Animal3,2018/01/15,Farm_C,Market_2
Animal3,2018/01/15,Market_2,Farm_B
```

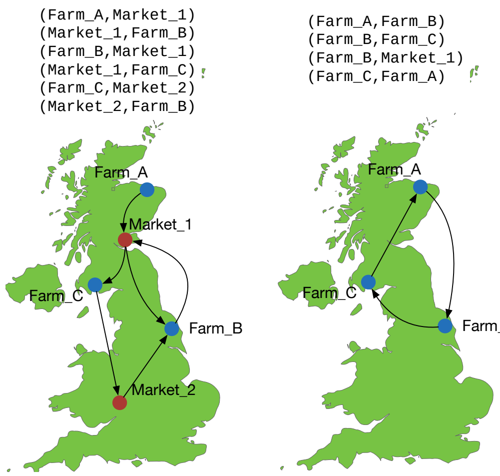


Figure 1. Examples of graphs derived from cattle movement records: at the top, a section of individual animal movements, showing the animal ID, date of movement, and source and destination locations. On the bottom left, a directed graph showing an edge from u to v if there is at least one animal movement from u to v in the set of movements. On the bottom right, a market-stripped graph, in which an animal's movement from farm u to market m to farm v on a single day is recorded as an edge from u to v. Edge sets for the two graphs are shown at the top of the maps.

Temporal graphs

Leaving out the temporal information when creating a graph from cattle movements is a serious omission that can give us a misleading picture of transmission pathways of disease. It is no surprise that the timing of cattle trades is important in modelling disease. Consider the simple case of a path of three vertices, with different orderings of the edges.

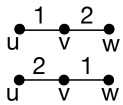


Figure 2. The same graph with two different assignments of times to edges. On the top, there is a temporally admissible path from u to w, on the bottom there is not.

If the edge from u to v occurs before the edge from v to w, then it's possible for an infection at u to spread to w, and we say the path from u to v to w is *temporally admissible*, whereas if the edges occur in the other order, then it is not. In a time-ignoring snapshot of more than a week of movements from the GB cattle trading dataset, approximately half of the static paths of three vertices and one fifth of the paths of four vertices are temporally feasible when timing is taken into account.

The detailed timing information in network datasets like the cattle movement dataset has partially motivated the increasing popularity of temporal or dynamic graphs both in algorithmic graph theory and in network science. While there are a variety of formalisms for temporal graphs, my favourite is one that fits well with the cattle movement data: a temporal graph is a pair $(G = (V, E), \Delta)$, where $G = (V, E)$ is a graph and Δ is a function from E to sets of timesteps, with $\Delta(e)$ denoting the timesteps at which $e \in E$ occurs. We can include directions or weights in our temporal graph by allowing G to be directed or weighted.

Because there are far more tools available for dealing with static graphs, there have been several attempts to capture the dynamic nature of trades in static graphs. Vernon and Keeling [9] describe a variety of static graphs derived from the cattle movements with the intention of capturing the dynamic nature of the graph. In common with many others, their primary tool is effectively a sequence of graphs, with one defined at each appropriate time step (in the case of the cattle movements, usually this timestep is a day).

Kim and Anderson [8] use a now-popular method that creates a static directed network in which the vertex set is formed of multiple copies of vertices from the temporal graph, with a vertex duplicated at each appropriate timestep, and edges going forward in time.

Heath et al. [7] use a line graph-like method that incorporates explicit information about an infectious period for a particular infectious disease of interest to produce a static graph that captures some of the key dynamic information: essentially, they create a graph in which the vertices are pairs of trades and days at which those trades occurred, and there is an edge from one trade/time pair to another if disease that spreads over the first could subsequently move over the second.

Formally, given a directed temporal graph $(G = (V, E), \Delta)$ and an infectious period δ_I, they produce a new directed graph $H = (P, F)$, in which the vertex set P is composed of (e, t) where $e \in E$ and $t \in \Delta(e)$, and the edge set F contains an edge $((e, t_e) \rightarrow (f, t_f))$ where $e = (u \rightarrow v)$ and $f = (v \rightarrow w)$ if and only if $t_e < \mathcal{T}(f)$ and $t_f - t_e \leq \delta_I$. This static representation captures the directionality both of the original graph, and of time.

Seasonal trends

Great Britain has (somewhat) distinct seasons, and so, as you might expect, there are seasonal patterns in the cattle movement network. We can use a limited, but easy-to-calculate measure of changes in the network over time: to compare two network snapshots at different times, calculate the proportion of edges in the graph that exist at both time-steps. If we consider month-long snapshots of the cattle trading graph, then we see a scallop-edged plot in Figure 3. For any given month, the most similar months are multiples of 12 months previous, and the least similar are those in opposite seasons. For example, April of this year is most similar to April in previous years, and most dissimilar to November. In addition to the seasonal effect, we see a gradual decrease in similarity with an increase in temporal distance — the farther in the past we look, the more dissimilar the graph is. As you might expect, we see higher similarity scores at higher levels of aggregation: when we consider only county-to-county level edges, we see much more similarity than at the farm-to-farm level of detail.

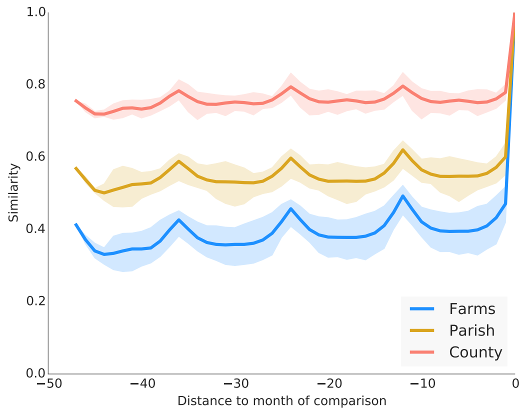


Figure 3. From [1]: A plot of the mean similarity (calculated using a method of set similarity counting equivalent to the Jacquard similarity) of cattle movements in Great Britain between each month of 2011 and months up to four years previous. The blue line shows similarity of unaggregated movements, the yellow movements to parishes, and the pink movements to counties. The background shaded envelopes are the minimal areas that include the similarity plots of each of the individual months in 2011 (the solid line is the mean over all 12 months).

Modification of livestock movement graphs

A significant volume of work has shown that the strategic removal of important vertices or edges (by a variety of measures) is far more effective in decreasing simulated disease outbreak size than random interventions — this is pleasingly logical. For example, it is not surprising that in a network with a relatively small number of high-degree vertices (often dealers or markets in the cattle network), removing those vertices will limit disease spread on the network more effectively than removing random farms. Gates and Woolhouse [6] provide evidence of this effect for edges, using a simple edge centrality measure, and use this observation to motivate a heuristic rewiring method to modify the cattle trading network with the aim of limiting disease spread. Roughly speaking, Gates and Woolhouse select edges with a high potential to spread disease across the network, and rewire them using a heuristic matching process that preserves in- and out-degree. Their approach gives networks with lower simulated endemic disease prevalence than the baseline real-data networks.

Given that removing edges and vertices strategically can make a large difference in expected disease prevalence, we might address the idea in a more formal algorithmic sense and ask: given a graph and a budget for edge or vertex removal, what is the best possible choice? As a first attempt at answering this question, Kitty Meeks and I [3] focussed on removing edges to limit the maximum connected component size in a graph, as maximum component size is an upper bound on the largest possible outbreak size.

In general, this problem is NP-complete, so we expect that it cannot be solved efficiently on general graphs. However, if we restrict the graphs we are interested in to a limited class, we can make progress toward an efficient algorithm (by which we mean one that takes computational time asymptotically bounded by a polynomial function of the size of the input).

We focussed our attention on graphs of limited treewidth, which are graphs admitting a certain type of decomposition (see "Treewidth") for two reasons: first, the tree decompositions of graphs with limited treewidth have a strong history of supporting efficient algorithms, particularly via several key meta-theorems that promise the existence of polynomial-time algorithms for problems that can be encoded in specified logical frameworks, and secondly (and possibly more remarkably), a static aggregation of the cattle trading graph in Scotland has relatively low treewidth.

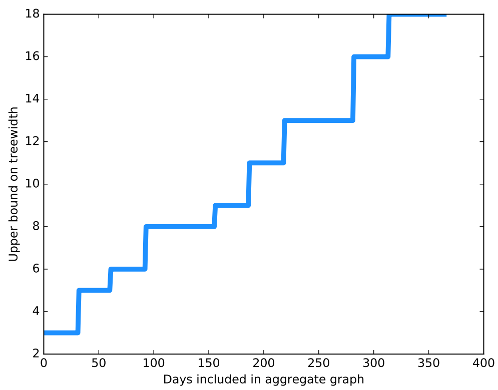


Figure 4. From [3]: a plot of treewidth of the static market-stripped graph derived from cattle trades in Scotland over increasing portions of 2013.

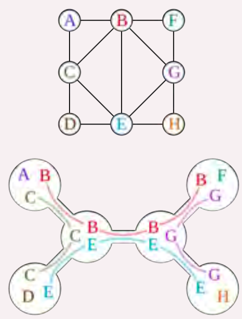
We created a series of aggregations of the Scottish cattle movement graph with markets stripped over the year 2013. For each day in 2013, we create a graph of all movements (ignoring direction and timing) from the beginning of the year to that day, and calculated and plotted an upper bound on the treewidth of that graph. Because the graph grows over the year, we expect the treewidth to be non-decreasing, and this is what we see in Figure 4. The entire year of movements has a treewidth of at most 18, and the first six months a treewidth under 10. The running time of a tree-decomposition-based algorithm is typically exponential in the treewidth of the graph — a running time that is exponential in a treewidth of 10 is perhaps not ideal, but is generally feasible, as we discovered in a number of related computational experiments [3].

Treewidth

A *tree decomposition* is an assignment of all vertices of a graph $G = (V, E)$ to bags at nodes of a tree that must follow two rules:

- for every vertex $v \in V$, the set of nodes of the tree that are assigned v must induce a connected subtree of the tree, and
- every pair of vertices u, v that are adjacent in G must co-occur in at least one bag on the tree

The width of such a decomposition is one less than the largest number of vertices assigned to a single bag. The *treewidth* of a graph is the smallest width of a tree decomposition over all possible tree decompositions of that graph. Here is an example of a valid tree decomposition, reproduced from the wikimedia commons [5].



Note that each subtree corresponding to nodes containing a single vertex is connected, and every pair of vertices that are adjacent occur together at a node at least once. This decomposition has width 2.

Incorporating temporal information — Reachability sets

Where the maximum component size is an upper bound on the size of the largest outbreak on a static and undirected graph, the maximum reachability set size serves the same role in a temporal graph, either directed or undirected.

If $(G = (V, E), \Delta)$ is a temporal graph, then we say vertex $v \in V$ is *reachable* from vertex $u \in V$ if there is a path of edges from u to v such that every edge occurs temporally after the one before it in the path: that is, the path goes forward in time. The *reachability set* of a vertex v is the set of vertices that are reachable from v, including v itself.

Recently, we have been investigating several approaches to modifying temporal graphs to limit the size of the maximum reachability set, including edge deletion (as we did to limit component size in static graphs), and assigning or re-ordering the times of edges (with George Mertzios and Viktor Zamaraev). This is very much work in progress, with current results reported in [2, 4].

Concluding thoughts

I have only talked about cattle movements here, but there are a wide variety of agricultural datasets that I've enjoyed viewing as graphs. I believe that graph theory has a lot to contribute to the analysis and modelling of the processes and systems that produce these datasets.

When we were trying to find small edge deletions to limit component sizes in the cattle trading graphs in Scotland, we were lucky that the graphs had low treewidth, which we were able to exploit to devise and implement an algorithm. In the future, several of my collaborators and I are planning to investigate why these graphs have low treewidth, and what other properties and classes arise in data-derived graphs that we can exploit for the design of efficient algorithms. There are some obvious candidates — for example, graphs derived from geographical proximity are often planar, or may be well-described with random geometric models.

There is a lot of work to be done on optimisation problems on these agriculturally-derived graphs, particularly on graphs that incorporate direction or temporal information. While there has been a recent boom of activity in the algorithmics of temporal graphs, there is still an enormous gulf between the large collection of algorithmic machinery available for static graphs, and that available for temporal graphs, even when the temporal graph is fully specified. Data-driven problems like those I've encountered working with cattle movements can benefit significantly from these recent and coming advances, as well as jump-starting work on theoretically appealing algorithmic questions — I'm looking forward to being inspired by cows for years to come!

FURTHER READING

[1] J. Enright and R.R. Kao, Epidemics on dynamic networks, Epidemics, in press.
[2] J. Enright and K. Meeks, Changing times to optimise reachability in temporal graphs, CoRR abs/1802.05905 (2018).
[3] J. Enright and K. Meeks, Deleting edges to restrict the size of an epidemic: A new application for treewidth, Algorithmica 80 (2018) 1857–1889.
[4] J. Enright, et al. Deleting edges to restrict the size of an epidemic in temporal networks, CoRR abs/1805.06836 (2018).
[5] D. Epstein, A graph and its tree decomposition, 2007.
[6] M.C. Gates and M.E.J. Woolhouse, Controlling infectious disease through the targeted manipulation of contact network structure, Epidemics 12 (2015), 11–19, Papers arising from Epidemics 4.
[7] M.F. Heath, M. Vernon, and C. Webb, Construction of networks with intrinsic temporal structure from UK cattle movement data, BMC Veterinary Research 4 (2008) 11.
[8] H. Kim and R. Anderson, Temporal node centrality in complex networks, Physical Review E 85 (2012) 026107+.
[9] M.C. Vernon and M.J. Keeling, Representing the UK's cattle herd as static and dynamic networks, Proceedings of the Royal Society B: Biological Sciences 276 (2009) 469–476.

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Unifying the Mathematics of Natural Language Grammar and Data

MEHRNOOSH SADRZADEH

Mathematical linguistics reasons about natural language using formal tools. Different approaches model different aspects of language. Some encode the grammar in axiomatic structures; others build vector representations for words using contextual data. Is there a mathematical structure that unifies the two?

Natural language has different characteristic features and mathematical linguists look for mathematical structures that can formalise and reason about them. Two main features herein are rules of grammar and patterns of meaning. Rules of grammar enable us to put the words of a language together and form phrases and sentences. Words have various meanings attached to them; these get composed to form meanings for phrases and sentences to which they belong. Two main challenges in the field are how to model meanings of words and how to formalise the grammar so that it can be applied to meanings of words.

On the grammatical side, mathematical structures with noncommutative notions of multiplication and division have proven useful. Words are assigned grammatical types that get encoded into elements of the structure. Some words have atomic types, such as "men", which is a noun phrase; some have complex types, such as "sleep", which is an intransitive verb: it takes noun phrases and produces sentences, such as "men sleep". These types get encoded using the division operator. Multiplication is used to model the juxtaposition of the types. If we denote the noun phrase type by n and the sentence type by s, an intransitive verb will get the type $n \to s$ and "men sleep" gets the type $n \cdot n \to s$, from which s is derivable.

On the side of meaning, recently, and in the presence of large quantities of data, a model called "distributional semantics" has gained attention and success. This model conjectures that words that often occur in the same contexts have similar meanings. Examples are "beer" and "wine", which often occur in the context of "drink, alcoholic, pub", and "cat" and "dog", which often occur in the context of "pet, furry, stroke". The mathematical model used here employs vectors to store co-occurrence counts over their contexts. The predictions of the model have been verified on a variety of word similarity datasets and applied to tasks such as indexing and automatic extraction of thesauri.

To obtain meaning representations for sentences, we need to combine the distributional semantics of words with a model of grammar. It turns out that if word meanings are modelled by higher order vectors, known as *tensors*, and their juxtaposition by composition of tensors, known as *tensor contraction*, one obtains vectors for sentences. For instance, build a one dimensional vector for "men", a two dimensional vector, i.e. a matrix, for "kill", then use their composition, in this case matrix multiplication, to obtain a vector for the sentence "men kill". This idea has led to the development of a unified model of grammar and data; the model has been implemented on large corpora of data and applied to sentence similarity tasks such as verb disambiguation and paraphrasing.

Formalising the grammar

Consider the basic fragment of a language consisting of noun phrases, adjectives, intransitive and transitive verbs, e.g., as given in the following vocabulary:

$$\Sigma = \{\text{men, dogs, cute, kill}\},$$

where "men" and "dogs" are noun phrases, "cute" is an adjective and "kill" can be an intransitive verb, e.g., as in the sentence "men kill", or a transitive verb, e.g., as in the sentence "men kill dogs". Suppose we want to encode the following rules of English grammar:

- an adjectival noun phrase is formed by the occurrence of an adjective to the left of a noun phrase, e.g., as in "cute dogs";
- an intransitive sentence is formed by the occurrence of an intransitive verb to the right of a noun phrase, e.g., as in "men kill";

- a transitive sentence is formed by the occurrence of a transitive verb to the right and left of two noun phrases, e.g., “men kill cute dogs”.

We need a mathematical structure with a non-commutative operator to encode the occurrence of one grammatical type after or before another. We also need to have two notions of implication to encode the fact that certain grammatical types input others, either from the left or the right, and produce new grammatical types. For example, an adjective inputs a noun phrase from its right and produces a noun phrase, and an intransitive verb inputs a noun phrase from its left and produces a sentence. An example of such a structure is a *residuated monoid* (see “Residuated Monoids”).

In the 1950s, J. Lambek, then a ring theoretician, used residuated monoids to model the grammatical structure of sentences of natural language. His seminal paper [6] founded a new field in mathematical linguistics, referred to by *categorial*, *type categorial*, or *type logical* grammars; for advances and example of application to different fragments of English see [8].

Given a vocabulary Σ for the language $\mathscr{L}$ and a set of its basic grammatical types $\mathscr{B}$, we define the type dictionary of that language to be the relation

$$\mathscr{D} \subseteq \Sigma \times \mathscr{T}(\mathscr{B}),$$

for $\mathscr{T}(\mathscr{B})$ the free monoid generated over $\mathscr{B}$. The type dictionary is used to assign grammatical types to the words of language. For instance, for $\mathscr{B} = \{n, s\}$, the type dictionary of our exemplar preceding language is as follows:

$$\begin{aligned}&\mathscr{D} =\\ &\{(\text{men}, n), (\text{dogs}, n), (\text{cute}, n \leftarrow n),\\ &(\text{kill}, n \to s), (\text{kill}, (n \to s) \leftarrow n)\}.\end{aligned}$$

This says the following:

- “men” and “dogs” are noun phrases.
- Adjective “cute” takes a noun phrase on its right and produces a (adjectival) noun phrase.
- Verb “kill” can be an intransitive verb, in which case it takes a noun phrase on its left to produce a sentence, or it can be a transitive verb, where it takes a noun phrase on its right to produce a verb phrase, which then takes a noun phrase on its left to produces a sentence.

Residuated Monoids

Recall that a monoid $(L, \cdot, 1)$ is a set endowed with a unital operator called multiplication. A monoid without a unit is a semigroup. A partially ordered monoid $(L, \leq, \cdot, 1)$ is a monoid with a partial ordering preserved by its multiplication.

Monoid multiplication can have inverse-like operators, like division, known as *residuation*. These are used to formalise implicative notions. For instance, the implication of classical logic is the right residual to its conjunction. Since conjunction is commutative, it only has one implication, for which we have:

$$c \wedge a \leq b \iff a \leq c \to b\,.$$

In this example, $\leq$ denotes logical inference. A corollary of residuation formalises the logical rule of *Modus Ponens*:

$$c \wedge c \to b \leq b\,.$$

In monoids, since the multiplication is not necessarily commutative, we have two implications, one for each argument of multiplication.

A residuated monoid $(L, \leq, \cdot, 1, \to, \leftarrow)$ is a partially ordered monoid with two families of order preserving maps $c \to -$ and $- \leftarrow c$, as the right residuals to $c \cdot -$ and $- \cdot c$, for any $c \in L$. This means that for $a, b \in L$ we have:

$$\begin{aligned} c \cdot a \leq b &\iff a \leq c \to b,\\ a \cdot c \leq b &\iff a \leq b \leftarrow c\,.\end{aligned}$$

Two useful corollaries of the above are:

$$c \cdot c \to b \leq b, \qquad b \leftarrow c \cdot c \leq b\,.$$

An example of a residuated monoid is the set of functions on natural numbers with composition and the identity function as multiplication and its unit, and the natural number ordering extended to functions as the ordering. Here, the residuals are defined as follows:

$$\begin{aligned} c \to b &:= \max\{a \in \mathbb{N} \mid c \cdot a \leq b\},\\ b \leftarrow c &:= \max\{a \in \mathbb{N} \mid a \cdot c \leq b\}.\end{aligned}$$

Multiplying the above types in the right order then allows us to derive sentences, for instance we have:

$$
\begin{array}{ll}
\text{men kill} & n \cdot (n \to s) \leq s\,. \\
\text{men kill dogs} & n \cdot ((n \to s) \leftarrow n) \cdot n \\
 & \leq n \cdot (n \to s) \\
 & \leq s\,. \\
\text{men kill cute dogs} & n \cdot ((n \to s) \leftarrow n) \cdot (n \leftarrow n) \cdot n \\
 & \leq n \cdot ((n \to s) \leftarrow n) \cdot n \\
 & \leq n \cdot (n \to s) \\
 & \leq s\,.
\end{array}
$$

To wrap up, we define a monoid grammar to be the tuple $(\Sigma, \mathcal{B}, \mathcal{D}, \{s\})$, for s a designated element of $\mathcal{B}$. Herein, a sequence of words $w_1 w_2 \cdots w_n$ forms a grammatical sentence when for $(w_i, t_i) \in \mathcal{D}$ we have:

$$t_1 \cdot t_2 \cdot \cdots \cdot t_n \leq s\,.$$

This partial ordering is referred to as the *grammatical reduction* of $w_1 w_2 \cdots w_n$.

Representing meanings of words

Vectors are often used to model phenomena that have a magnitude and a direction, such as force, velocity, and acceleration. At first sight, word meaning does not seem to share anything with these examples and it might be hard to imagine words having a magnitude and a direction. In the presence of large quantities of data, however, one can use the context of a word to model its meaning. The meaning of a word can then be modelled by a vector, whose magnitude is the frequency of its occurrence in context and whose direction denotes the degree of its contextual similarity. The idea behind this approach was put forward in the 1950s by linguists such as Harris and Firth, see for example [5]. Later, it was referred to as the *distributional hypothesis*, got implemented on large corpora of data using vectors, and is now routinely taught in textbooks such as [3].

According to the distributional hypothesis, words that often occur in the same context have similar meanings. In order to build a model of word meaning based on the distributional hypothesis, one forms a *co-occurrence matrix*. The columns of this matrix are called *context words*; the rows of this matrix are called *target words*. The value of a cell c_{ij} at the crossing of the context word c_i and the target word t_j is a function of the number of times the two words co-occurred. Co-occurrence can be proximity in a window of k, e.g., 5 or 10 words, being part of the same sentence, sharing a grammatical relation with each other and such like. The function is often a logarithmic function on conditional probabilities.

As an example, consider the following sample sentences from the British National Corpus.

I'm a keen pet owner and I can't recommend a **dog** enough for its constant company and affection.

Growling like a mother **cat** may make the owner feel ridiculous.

Every pet lover knows the moment when a **dog** approaches a new dish of food.

Ninety-five per cent of our **cats** are fed a canned food.

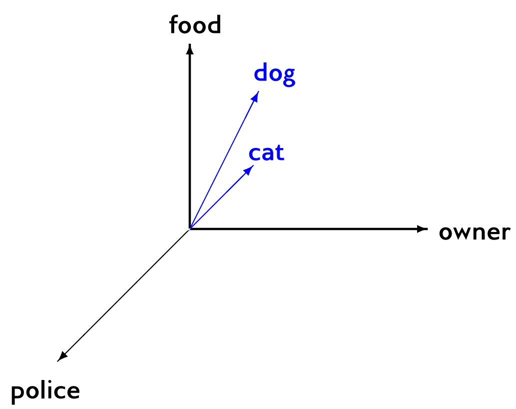
Dog food is subject to stricter controls than human food.

When you look after a police **dog** it becomes your pet as well.

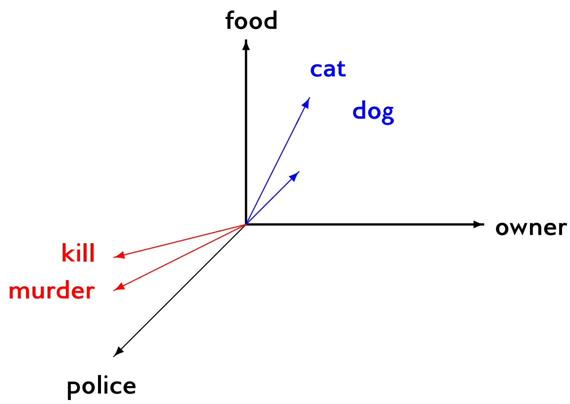
From this one builds a co-occurrence matrix with "occurring in the same sentence" as the proximity window, a set of context words containing {food, owner, police} and the target words, lemmatised versions of {dog, cat}.

	food	owner	pet	police	mother	...
dog	2	1	2	1	0	...
cat	1	1	0	0	1	...

Co-occurrence matrices can be embedded in vector spaces, in which the rows corresponding to each word are represented by a vector. We refer to these as *co-occurrence vector spaces*. For example, the 3d projection of the vector space version of the above co-occurrence matrix is the following.



The cosine of the angle between word vectors is the most common similarity metric in distributional semantics. Clearly, in the above vector space, cat and dog have a high degree of similarity. In order to see the contrast, consider a version of the above vector space, built from a piece of text that also contains the target words: "kill" and "murder".



Here, "dog" and "cat" have a high degree of similarity to each other, and so do "kill" and "murder", whereas "cat" and "kill" have a low degree of similarity, as do pairs (cat, murder), (dog, kill), and (dog, murder).

Co-occurrence matrices and the notion of cosine similarity have been tested on different datasets, such as WordSim-353, consisting of 353 pairs of nouns such as *(plane, car)*, SimLex-999, which consists of noun, verb, as well as adjective pairs, such as *(bad, awful)* and *(kill, destroy)*, and the synonymy part of the TOEFL test. Through similarity, vector models of word meanings have found applications in a wide range of natural language tasks, such as entity recognition, parsing, and semantic role labelling, as well as in question answering, summarization, and automatic essay grading.

Representing meanings of sentences

A mapping can be defined between a monoid grammar $(\Sigma, \mathcal{B}, \mathcal{D}, \{s\})$ and tensor products of a co-occurrence vector space W with itself. We denote this mapping by $\mathcal{F}$. The basic grammatical types in $\mathcal{B}$ are assigned the atomic space W:

$$\mathcal{F}(t) = W \qquad t \in \mathcal{B}.$$

A shorthand for linear expansion

The sum notation for the linear expansions of tensors is often shorthanded using a tradition set by Einstein:

$$T_{i_1 i_2 \cdots i_n} \rightsquigarrow \sum_{i_1 i_2 \cdots i_n} C_{i_1 i_2 \cdots i_n} \overrightarrow{b_{i_1}} \otimes \overrightarrow{b_{i_2}} \otimes \cdots \otimes \overrightarrow{b_{i_n}},$$

for $\overrightarrow{b_i}$ a basis vector of W. The simplest case of the above shorthands vectors:

$$T_i \rightsquigarrow \sum_i C_i \overrightarrow{b_i}.$$

Words with type a are assigned vectors in $\mathcal{F}(a)$:

$$\mathcal{F}(w) = T_i \in W \qquad (w, t) \in \mathcal{D}.$$

The complex types in $\mathcal{T}(\mathcal{B}) \setminus \mathcal{B}$ are assigned tensors of W by structural induction as follows.

$$\begin{aligned} \mathcal{F}(t_1 \cdot t_2) &= \mathcal{F}(t_1) \otimes \mathcal{F}(t_2), \\ \mathcal{F}(t_1 \to t_2) &= \mathcal{F}(t_1) \otimes \mathcal{F}(t_2), \\ \mathcal{F}(t_1 \leftarrow t_2) &= \mathcal{F}(t_1) \otimes \mathcal{F}(t_2). \end{aligned}$$

So the three operations of the residuated monoid, its multiplication $(\cdot)$ and its two residuals ($\to$ and $\leftarrow$), get mapped to the tensor product. This uses the fact that linear maps from $U \to U'$ are in correspondence with elements of the space $U^* \otimes U'$. So a word with type $t_1 \to t_2$ will get assigned a matrix, that is a linear map from U to U'. If U is finite dimensional then choosing an orthonormal basis in it identifies U^* and U and this map becomes an element of $U \otimes U'$. In the above, $U = U' = W$, and W is a co-occurrence vector space and these are by definition finite.

Similar to the atomic case, words with complex types are assigned elements of tensor spaces, the rank of which is the same as the number of operations in their type. Formally (for notation see "A shorthand for linear expansion"), we have

$$\mathcal{F}(w) = T_{i_1 i_2 \cdots i_n} \in \underbrace{W \otimes \cdots \otimes W}_{n}$$

when the type of w is

$$(w, t_1 \bigcirc \cdots \bigcirc t_n) \in \mathcal{D}$$

for $\bigcirc$ either of $\cdot, \to, \leftarrow$.

Using the above definitions and the grammatical structure provided by our monoid grammar, we can now assign a tensor to a grammatical sentence $w_1 w_2 \cdots w_n$. This is via tensor contraction (see "Tensor Contraction") on $\mathcal{F}$ of each of the words w_i of the sentence. This idea is used in [1, 2] to develop similar mappings for three different categorial grammars.

As an example, the mapping on the monoid grammar of the previous section assigns the co-occurrence vector space W to the basic types n and s. This means that "men" and "dogs" get assigned vectors in W; they are the rows corresponding to the two words in the co-occurrence matrix underlying W. For the sake of coherence with the tensor notation, we denote them by T_k^{dogs} and T_j^{men}. To $n \leftarrow n$, this mapping assigns the tensor space $W \otimes W$ and thus to the adjective "cute" the matrix T_{lk}^{cute}. An intransitive verb gets assigned a similar type, e.g., T_{ij}^{kill}. The type $(n \to s) \leftarrow n$ gets assigned the space $W \otimes W \otimes W$, and the transitive verb "kill" the cube T_{ijk}^{kill}.

Sentences "men kill", "men kill dogs", and "men kill cute dogs" get assigned tensors in W via the following contractions.

men kill	$T_{ij}^{\text{kill}} T_j^{\text{men}}$
men kill dogs	$(T_{ijk}^{\text{kill}} T_k^{\text{dogs}}) T_j^{\text{men}}$
men kill cute dogs	$(T_{ijl}^{\text{kill}} (T_{lk}^{\text{cute}} T_k^{\text{dog}})) T_j^{\text{men}}$

Tensor Contraction

For $T_{i_1 i_2 \cdots i_n}$ an element of $\underbrace{W \otimes \cdots \otimes W}_{n}$ and $T'_{i'_n i'_{n+1} \cdots i'_{n+k}}$ an element of $\underbrace{W \otimes \cdots \otimes W}_{k+1}$, the contraction is formed as follows

$$T_{i_1 i_2 \cdots i_n} T'_{i'_n i'_{n+1} \cdots i'_{n+k}}$$

and is a tensor in $\underbrace{W \otimes \cdots \otimes W}_{n+k-1}$.

Concrete Constructions and Experiments

Amongst the various ways tensors are built from data, two models stand out. One is a model that employs machine learning and the other a model that uses analytical methods.

The machine learning model approximates a matrix for words with one argument, e.g., adjectives and intransitive verbs, as follows. We first build co-occurrence vectors $T_j^{x_i}$ for the word w and its arguments x_i, that is, for each of the wx_i phrases. Then, a matrix T_{ij}^{w} is learned for w in such a way that it provides a good approximation for the wx_i's, through tensor contraction, that is:

$$\text{from} \quad \left.\begin{array}{c} \overrightarrow{wx_1} \\ \overrightarrow{wx_2} \\ \cdots \\ \overrightarrow{wx_n} \end{array}\right\} \overset{\text{we learn}}{\Longrightarrow} T_{ij}^{w} \text{ s.t. } \left\{\begin{array}{c} T_{1j}^{w} T_j^{x_1} \sim \overrightarrow{wx_1} \\ T_{2j}^{w} T_j^{x_2} \sim \overrightarrow{wx_2} \\ \cdots \\ T_{nj}^{w} T_j^{x_n} \sim \overrightarrow{wx_n}. \end{array}\right.$$

For instance, suppose we want to build a matrix for the adjective "red". We observe that it has modified nouns "car", "carpet", "flower", in the corpus. We build the following co-occurrence vectors using the method described in the section on vectors:

$$\overrightarrow{\text{red car}}, \overrightarrow{\text{red carpet}}, \overrightarrow{\text{red flower}}.$$

From the above, we then learn a matrix T_{ij}^{red} for "red" such that after it contracts with each of its modified nouns, it provides a good approximation of the above co-occurrence vectors:

$$\begin{array}{rcl} T_{ij}^{red} T_j^{car} & \sim & \overrightarrow{\text{red car}}, \\ T_{ij}^{red} T_j^{carpet} & \sim & \overrightarrow{\text{red carpet}}, \\ T_{ij}^{red} T_j^{flower} & \sim & \overrightarrow{\text{red flower}}. \end{array}$$

For words with more than one argument, such as those that are transitive (2 arguments) and ditransitive (3 arguments), one follows a similar procedure to build cubes and hypercubes, etc., but using the multi step version of the approximation algorithm. This model only works if the words and their arguments have occurred together a reasonable amount of time and other wise suffers from data sparsity.

The analytic method argues that a tensor has to be populated by the properties of its arguments. Words with one argument are sums of the co-occurrence vectors of their arguments. Words with two arguments are sums of the tensor products of the co-occurrence vectors of their arguments, and so on:

$$T_i^{w} = \sum \overrightarrow{x_i}, T_{ij}^{w} = \sum_i \overrightarrow{x_i} \otimes \overrightarrow{y_i}, T_{ijk}^{w} = \sum_i \overrightarrow{x_i} \otimes \overrightarrow{y_i} \otimes \overrightarrow{z_i}, \cdots .$$

This method produces tensors whose rank is one lower than the rank of the tensor assigned to the word via our $\mathcal{F}$ mapping. One can elevate these ranks by embedding the acquired tensors into higher order ones by padding the extra dimensions either by zeros, or by a plausibility measure, which tells us how likely it is that the tensor has related any set of its arguments to each other.

Similar to the word vector case, the tensor representations have been tested on various tasks. These tasks measure the degree of similarity of pairs of sentences. For example, the disambiguation dataset of [7] measures degrees of similarity of intransitive sentences to disambiguate between different meanings of intransitive verbs. The following sample shows the entries for disambiguating "strayed" between its two meanings of "roamed" and "digressed".

```
(child strayed, child roamed)
(child strayed, child digressed)
(discussion strayed, discussion roamed)
(discussion strayed, discussion digressed)
```

This task is extended in [4] to transitive sentences to disambiguate between different meanings of transitive verbs. The following sample shows the pairs used to disambiguate "drew" between its two meanings of "depicted" and "attracted".

```
(old man drew sword, old man depicted sword)
(old man drew sword, old man attracted
  sword)
(annual report drew attention, annual report
  depicted attention)
(annual report drew attention, annual report
  attracted attention)
```

Examples of other tasks where this approach is used are those of measuring degrees of similarity of sentences for paraphrasing and of measuring degrees of similarities of words and their dictionary definitions for term-definition classification tasks.

Acknowledgements

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FURTHER READING

[1] M. Baroni, R. Bernardi, R. Zamparelli, Frege in Space: A Program of Compositional Distributional Semantics, Linguistic Issues in Language Technology 9 (2014) 241–346.
[2] B. Coecke, E. Grefenstette, M.Sadrzadeh, Lambek vs. Lambek: Functorial vector space semantics and string diagrams for Lambek calculus, Annals of Pure and Applied Logic 164 (2013) 1079–1100.
[3] D. Jurafsky and J.H. Martin, Speech and Language Processing (3rd ed. draft), 2017, tinyurl.com/ya6lko7f.
[4] E. Grefenstette and M. Sadrzadeh, Concrete models and empirical evaluations for a categorical compositional distributional model of meaning, Comput. Linguist. 41 (2015) 71–118.
[5] Z. Harris, Methods in structural linguistics, University of Chicago Press, 1951.
[6] J. Lambek, The mathematics of sentence structure, Amer. Math. Monthly 65 (1958) 154–170.
[7] J. Mitchell, M. Lapata, Composition in Distributional Models of Semantics, Cognitive Science 34 (2010) 1388–1429.
[8] G. Morrill, Categorial Grammar: Logical Syntax, Semantics, and Processing, Oxford University Press, 2015.

Mehrnoosh Sadrzadeh

Mehrnoosh Sadrzadeh is a Senior Lecturer in Computer Science at Queen Mary University of London. Her current research interests are in the use of logic and algebra to model and reason about natural language structures and data. She also has an interest in encouraging women to study Computer Science. Mehrnoosh was born by the Caspian sea and is a keen wild water swimmer.

Apollonius Circle Counting

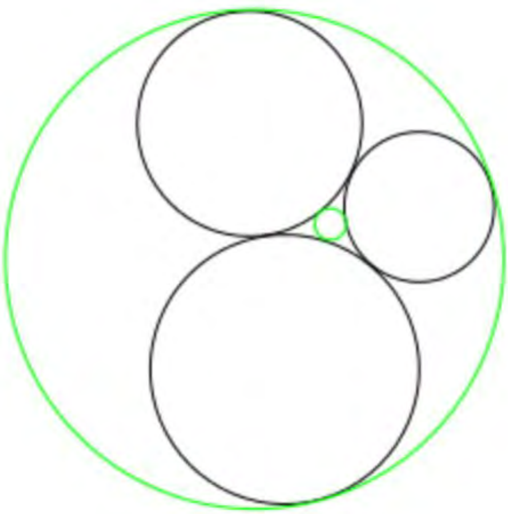
MARK POLLICOTT

Circle packings in the plane date back over 2000 years and have attracted the attention of princesses and Nobel prize winners alike. The most famous are the Apollonian circle packings, consisting of a family of infinitely many circles with disjoint interiors, constructed iteratively by starting from a finite family of mutually tangent circles and successively inscribing circles into the spaces between them. We give a brief history of them and describe some recent developments.

Apollonius and his circles

The famous geometer Apollonius of Perga (circa 240–190 BC) made many important contributions to mathematics, perhaps the best known nowadays being the coining of the modern terms *ellipse*, *hyperbola* and *parabola*. Apollonius also established the following basic result.

Theorem 1 (Apollonius). *Given three tangent circles in the plane there are precisely two more circles each of which is mutually tangent to the original three.*



The three initial circles (of radii r_1, r_2, r_3, say) and the new Apollonian circles (of radii r_0, r_4, say)

There is a simple modern proof of this theorem using Möbius maps [7]. By transforming to infinity one of the points where two of the original circles touch the image circles are straight lines. The new circles are then simply translates of the third circle.

Descartes and Princess Elizabeth of Bohemia

Princess Elizabeth (1618–1680) was the daughter of King Frederick of Bohemia, whose ill-fated reign lasted a mere 1 year and 4 days. Her superior education included instruction from the great French mathematician and philosopher René Descartes (1596–1650). Included in their correspondence was a formula relating the radii r_0, r_4 of the two additional circles given by Apollonius' theorem to the radii r_1, r_2 and r_3 of the original three circles.

Theorem 2 (Descartes–Princess Elizabeth). *The two possible solutions ξ to the quadratic equation*

$$\frac{1}{\xi^2}+\frac{1}{r_1^2}+\frac{1}{r_2^2}+\frac{1}{r_3^2}=\left(\frac{1}{\xi}+\frac{1}{r_1}+\frac{1}{r_2}+\frac{1}{r_3}\right)^2$$

correspond to r_0 and $-r_4$.

Sadly, when the princess petitioned Queen Christina of Sweden for support in recovering her lands the only consequence was that Descartes went to Stockholm, dying there of pneumonia due to having 5 am audiences in a draughty palace. Thwarted in her attempts to recover her family's lands in Bohemia, the princess retreated to a nunnery.

Frederick Soddy

The formula was rediscovered two centuries later by Frederick Soddy (1877–1956), and for this reason the circles are sometimes called *Soddy circles*. Soddy is probably more famous for having won the Nobel Prize for Chemistry in 1921, and having introduced the terms *isotopes* and *chain reaction*.

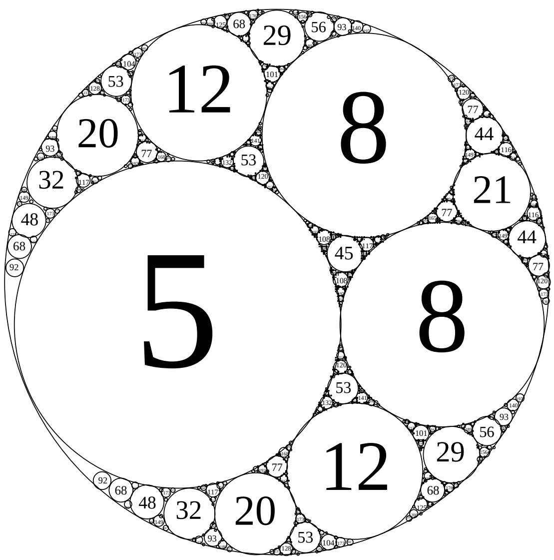
In 1936 he published the same theorem in the form of a poem in the journal *Nature* entitled *The Kiss Precise* [8]. (See "Part of *The kiss precise*".)

Part of *The kiss precise*

Four circles to the kissing come.
The smaller are the benter.
The bend is just the inverse of
The distance from the center.
Though their intrigue left Euclid dumb
There's now no need for rule of thumb.
Since zero bend's a dead straight line
And concave bends have minus sign,
The sum of the squares of all four bends
Is half the square of their sum.

Counting infinitely many circles

We can continue to add circles ad infinitum by iteratively applying Theorem 1 to every triple of mutually tangent circles we see. Let $(r_n)_{n=0}^{\infty}$ enumerate their radii.



A fractal set $\mathscr{A}$ in the plane

Example. If we start with $\frac{1}{r_0} = 3$, $\frac{1}{r_1} = 5$, $\frac{1}{r_2} = \frac{1}{r_3} = 8$ then we can order the reciprocals of all the new radii. In the example above this becomes

$$(1/r_n) = 5, 8, 8, 12, 12, 20, 20, 21, 29, 29, 32, 32, \cdots$$

The sequence of radii tends to infinity (or equivalently the sequence of radii (r_n) tends to zero) since the total area of the disjoint disks enclosed by the circles is $\pi \sum r_n^2 < +\infty$ which is bounded by the area inside the outer circle.

Why do we get natural numbers?

It is an interesting property of the identity that if r_0, r_1, r_2, r_3 are reciprocals of natural numbers then so is r_4 since

$$\frac{1}{r_0} + \frac{1}{r_4} = 2\left(\frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_0} + \frac{1}{r_3}\right)$$

by the formula of Descartes–Princess Elizabeth. Proceeding iteratively, we see that r_n is the reciprocal of a natural number for all $n \geq 0$.

It is a natural question to ask how quickly such the sequence $(1/r_n)$ grows (see "The sequence looks a bit like. . .".)

The sequence looks a bit like. . .

One might compare this with a similar looking result for prime numbers

$$2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, \ldots$$

For any $x > 0$ let $\pi(x)$ denote the finite number of primes numbers less than x. Since there are infinitely many primes we see that $\pi(x)$ tends to infinity as x tends to infinity. A more refined estimate comes from the following famous theorem proved in 1896.

Theorem 3 (Prime Number Theorem). *The function $\pi(x)$ is asymptotic to $\frac{x}{\log x}$ as $x \longrightarrow +\infty$, i.e., $\lim_{x\to+\infty} \frac{\pi(x)}{x/\log x} = 1$.*

One might (rightly) imagine that these two results might be proved in a similar way.

Kontorovich-Oh

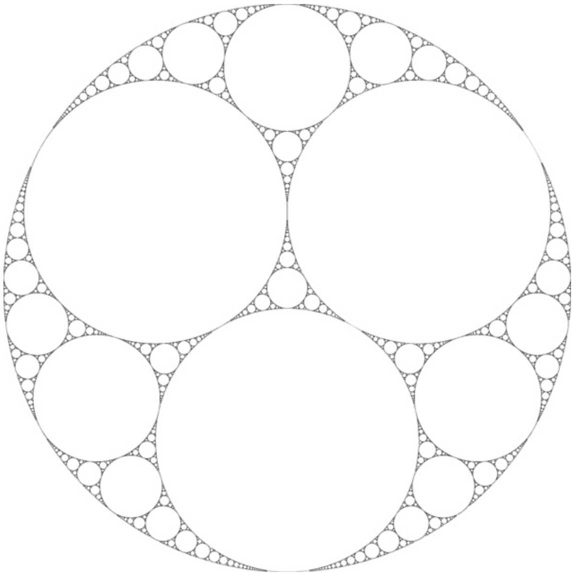
We denote by $N(\epsilon) = \#\{n : r_n > \epsilon\}$ the finite number of circles with radii greater than $\epsilon > 0$. We have already observed that $N(\epsilon) \to +\infty$ as $\epsilon \to 0$. This was considerably strengthened by A. Kontorovich and H. Oh in the following theorem.

Theorem 4 (Kontorovich–Oh). *There exist $C > 0$ and $\delta > 0$ such that the number $N(\epsilon) \sim \epsilon^{-\delta}$ as $\epsilon \to 0$, i.e.,*

$$\lim_{\epsilon\to 0} \frac{N(\epsilon)}{\epsilon^{-\delta}} = C.$$

The value δ

We can consider the compact set $\overline{\mathscr{A}}$ in the plane given by the closure of the countable union of circles $\mathscr{A}$. The value of δ is equal to the Hausdorff dimension of this set $\mathscr{A}$ (i.e., the natural notion of "size" for fractal sets). Although no explicit expression is known for δ it has been numerically estimated to be $\delta = 1 \cdot 30568\ldots$ [4].



A fractal set $\mathscr{A}$ in the plane

Any initial configuration of circles in the plane can be mapped onto any other by a Möbius map, which will also map the corresponding circles in each packing. Since the Hausdorff Dimension is preserved by any Lipschitz map we deduce that the same value of δ occurs for any initial choice of r_1, r_2 and r_3.

A pinch of complex analysis

Rather than describe the original proof in [5] it is possible to give a proof of the counting theorem for radii of circles in the Apollonian circle packing which is analogous to that of Hadamard's famous proof of the Prime Number Theorem [6]. (See "Comparison with the Prime Number Theorem".) We can consider the complex function

$$\eta(s) = \sum_n r_n^s$$

where the summation is over all the radii of circles in the Apollonian circle packing $\mathscr{A}$. This converges to a non-zero analytic function for $Re(s) > \delta$ and an analytic function with the following properties:

(1) $\eta(s)$ has a simple pole at $s = \delta$; and

(2) $\eta(s)$ has no zeros or poles on the line $Re(s) = \delta$.

In particular, we can write

$$\eta(s) = \frac{C}{s-\delta} + \psi(s)$$

where $\psi(s)$ is analytic in a neighbourhood of $Re(s) \geq \delta$. We can also write

$$\eta(s) = \int_0^\infty r^s \, dN(r)$$

and then the properties (1) and (2) above are enough to ensure that by the classical Ikehara–Wiener tauberian theorem we have that $N(r) \sim Cr^{-\delta}$ as $r \to 0$ [3].

Comparison with the Prime Number Theorem

The original proof of the Prime Number Theorem used the Riemann Zeta function

$$\zeta(s) = \sum_{n=1}^\infty \frac{1}{n^s}$$

which converges to a non-zero analytic function for $Re(s) > 1$ and the analytic extension has the following properties:

(1) $\zeta(s)$ has a simple pole at $s = 1$; and

(2) $\zeta(s)$ has no zeros or poles on the line $Re(s) = 1$.

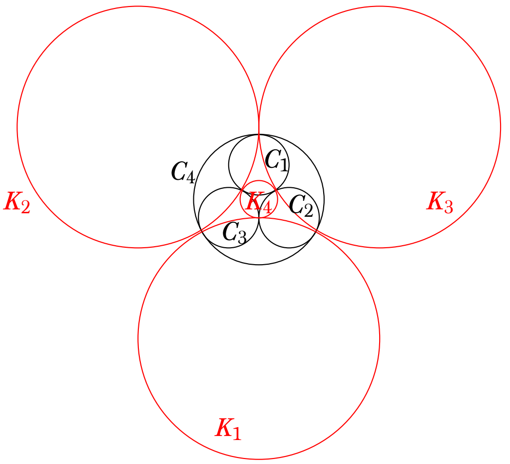
The asymptotic formula for $\pi(x)$ then follows from an application of the classical Ikehara-Wiener tauberian theorem.

Phillip Beecroft

There remains the non-minor detail of proving the properties (1) and (2) for $\eta(s)$.

A useful dynamical approach to generating circle packings was discovered by Philip Beecroft (1818–1862), a school teacher from Hyde, in Greater Manchester, who was the son of a miller and lived quietly with his two sisters. His approach was based on inversions in four complementary circles, a result he published in the pleasantly named *Lady's and Gentleman's diary* from 1842 [1, 2].

Theorem 5 (Beecroft). *Given 4 mutually tangent circles C_1, C_2, C_3, C_4 we can associate 4 new mutually tangent "dual" circles K_1, K_2, K_3, K_4 passing through the points $C_i \cap C_j$ $(1 \leq i < j \leq 4)$.*



The 4 original circles C_1, C_2, C_3, C_4 and their 4 dual circles K_1, k_2, K_3, K_4

In the course of his work, Beecroft too rediscovered the formula of Descartes and Princess Elizabeth.

Apollonian groups

Considering the circles to lie in the complex plane, for each of the dual circles K_i, with center c_i and radius r_i, say, we can define a Möbius map $T_i(z) = \frac{r^2(z-c)}{|z-c|}$ $(i = 1, 2, 3, 4)$. These maps generate a group $\mathcal{G}$ called the *Apollonian group*.

The group $\mathcal{G}$ gives a systematic way to study the circles in $\mathcal{A}$. The images gC_i of the four initial circles C_i under elements $g \in \mathcal{G}$ correspond to the circles in $\mathcal{A}$, with a few trivial exceptions. In particular, we can write

$$\eta(s) = \sum_{i=1}^{4} \sum_{g \in \Gamma} r(gC_i).$$

where $r(\cdot)$ just denotes the radius of a circle.

Of course, one might ask what we have gained through writing the function $\eta(s)$ in this way. In fact the group is closely related to an example of a Kleinian group and the function $\eta(s)$ is essentially what is called a Poincaré series. There are dynamical methods available to establish (1) and (2).

FURTHER READING

[1] P. Beecroft, The Concordent Circles, The Lady's and Gentleman's Diary, The Company of Stationers, London (1843)
[2] H. Coxeter An Absolute Property of Four Mutually Tangent Circles. In: Prékopa A., Molnár E. (eds) Non-Euclidean Geometries. Mathematics and Its Applications, vol 581. Springer, Boston, MA, 2006.
[3] F. Ellison and W. Ellison, *Prime Numbers*, Wiley, New York, 1985.
[4] C. McMullen, Hausdorff dimension and conformal dynamics III: Computation of dimension, Amer. J. Math. 120 (1998) 691–721.
[5] A. Kontorovich and H. Oh, Apollonian circle packings and closed horospheres on hyperbolic 3-manifolds, J. Amer. Math. Soc. 24 (2011) 603–648.
[6] M. Pollicott and M. Urbański, Asymptotic Counting in Conformal Dynamical Systems, Memoir of the Amer. Math. Soc., in press.
[7] P. Sarnak, Integral Apollonian Packings, Amer. Math. Monthly 118 (2011) 291–306.
[8] F. Soddy, The Kiss Precise, Nature, 137 (1936) 1021.

Mark Pollicott

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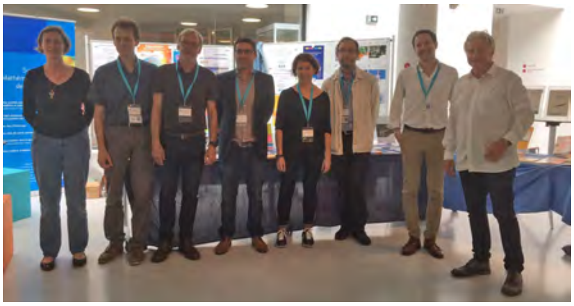
Reciprocal Societies: Société Mathématique de France



The SMF team

The Société Mathématique de France (SMF) is a learned society founded in 1872, as part of a particular effort made at that time to enhance and reinforce the French scientific communities. The SMF was created to develop and promote Mathematics and Mathematics research in France. Nowadays, the SMF is proud to count more than 1,800 members, and our publishing house distributes our journals and books to hundreds of institutions and individuals around the world. The SMF is regularly consulted by politicians and journalists and seeks to convey the voice of the mathematical community to the government and public. To reach our objectives, we are active in many areas:

Organisation of conferences

The SMF holds regular regular 3-day workshops on the latest developments in mathematics, and many other events. In addition, in 2016 the SMF held its first national congress, gathering the French mathematical community and presenting various aspects of mathematics, including mathematics education, the history of mathematics, and mathematics and industry. A second national congress was held in June 2018, and one is planned for every two years thereafter.

Shaping the future of mathematics teaching

We are involved in the key questions related to mathematics teaching at all levels. For instance, we contributed to the recent Villani–Torossian report on the history and development of mathematics teaching and alternative pedagogies that could be employed to improve the mathematics knowledge of French students.

Diffusion of information to our members

We exploit many channels (twitter, website, newsletter) to keep our members updated with the latest news. Our quarterly information letter, *la Gazette*, received a complete revamp three years ago to keep it fresh and responsive to members' needs.

Contributing to the research community

Conference cycles aimed at secondary school students are frequently organised. Inspired by the Miklos Schweitzer contest in Hungary, in 2017 we initiated a national mathematics competition for masters' students: the "concours SMF junior". This competition offers teams of at most three participants the opportunity to work for 10 days on 10 problems. More than 200 students participated in the 2017 competition.

Diffusion of knowledge: SMF publishing

One very important part of the SMF's work is its non-profit publishing house. We publish four journals (including *Annales Scientifiques de l'ENS* and *Bulletin de la SMF*) and many collections of books (*Astérisque, Documents Mathématiques*), for which we oversee the whole editorial process, from the editorial secretary to the distribution to our readers. Special attention is given to the reviewing process as well as to the quality of our produced documents and books, and we invest a lot of energy to keep them very high quality.

CIRM

The SMF, together with the CNRS and Aix-Marseille University, runs the Centre International of Rencontres Mathématiques (CIRM), located in Luminy (close to Marseille). The CIRM welcomes 3,500 participants a year, and is currently expanding: starting in 2020, we will be able to welcome two conferences simultaneously per week, with a brand new building and a renovated restaurant. Don't hesitate to participate in upcoming calls for proposals!

Further information about the SMF can be found on its website smf.emath.fr/.

Stéphane Seuret
President of the SMF

Editor's note: the LMS and the SMF have a reciprocity agreement meaning members of either society may benefit from discounted membership of the other.

Oh, the Places You'll Apply!

CAROLYN CHUN AND DAVID PHILLIPS

To be successful in academia requires mastering a variety of skills. A great deal of planning ahead is required to obtain a permanent job in mathematical academia. In this article, we focus on the academic market in the USA.

Caveat emptor

In this article, we ignore the dire statistics concerning the number of scientists with doctoral degrees that are unable to stay in academia.

Carolyn is a pure mathematician working in matroid theory, and David specializes in operations research[1]. As we discuss below, this difference in specialty results in different experiences. However, we note that our viewpoints are only a small sample and anyone applying for jobs is encouraged to talk to individuals in their own specialty. For the record, Carolyn and David combined have applied for over 500 jobs and have been offered fewer than 10 of them.

We focus here on the search for a job in a mathematics department in the United States. Recipients of degrees in the math sciences, e.g., mathematics, statistics, or operations research, should also consider departments outside of mathematics. We stress that our advice and information focuses on the academic situation in the United States.

It's a good thing mathematicians have a strong work ethic

The skill set required to learn graduate-level mathematics, do original research, write and defend a dissertation[2], and obtain a PhD takes years to cultivate. In the US, depending on specialty, this typically takes between four and seven years, as most PhD candidates do not begin their program with a master's degree level of mathematical training. The skill set required to obtain a job as an academic can seem completely disjoint, but the application process is no less important than obtaining the degree itself. The work that it takes is often minimized, and applying to academic jobs may seem almost an afterthought, but this work will either launch or sink an academic's career, and it varies from place to place.

In the US, the typical academic ranks are assistant professor, associate professor, and professor. The latter is also called full professor. "Lecturer" in the US refers to a teaching position that is not necessarily permanent or research-active. Another title in US job ads is "visiting professor," which is usually a one- to three-year postdoc position that is either research-intensive or teaching-intensive. Some universities have named professorships. Among Carolyn's collaborators are Boyd Professor James Oxley, and the Herbert Butts Memorial LSU Alumni Association Departmental Professor in Mathematics, Guoli Ding. Roughly speaking, assistant professor in the US corresponds to lecturer in the UK and associate professor corresponds to senior lecturer. It gets a bit harder to directly compare the UK ranks of reader and professor with the US ranks of full professor or named professor, as the details vary between institutions.

In addition to these ranks, there is also the process for obtaining permanence at a US academic job, called tenure. An assistant professor is considered "tenure-track," and the other ranks (excluding lecturer and visiting professor) are considered "tenured." In what follows, we discuss tenure-track assistant professorships. Obtaining tenure is a process that usually takes about six years and is itself a complicated endeavor that depends on the institution. Departments hire assistant professors with the expectation that they will obtain tenure, although tenure is never a given. Describing the associated challenges are beyond the scope of this article and we merely note that the joys of completing applications do not end with obtaining the tenure-track

[1] also known as "operational research" in the UK.

[2] also known as a "doctoral thesis" in the UK. (In the US, the term thesis is typically associated with masters and bachelors degrees.)

assistant professorship. There are also schools[3] that do not offer tenure.

Charming nuances to observe in American schools

According to the Washington Post[4], there are approximately 5,300 schools in the US that offer degrees past the American high school level. There is considerable heterogeneity in these different schools and we limit our attention to describing the schools that would hire tenure-track assistant professors in a mathematics department. Such schools are typically called colleges and universities, although college is a term often used to describe any school of higher learning[5]. For our purposes, the important feature that distinguishes schools are the types of degrees offered. Some schools have as their main focus, or even exclusively offer, undergraduate degrees (associates and bachelors), while others offer graduate degrees (masters and doctorates) as well as undergraduate degrees. The latter tend to be considered universities although all possible permutations of school names and degree offerings exist. For example, The College of William and Mary and Dartmouth College are both universities with mathematics departments that offer graduate degrees and the Naval Postgraduate School exclusively grants graduate degrees. The degree focus of the school determines the proportion of students who are at the undergraduate, masters, or doctoral level. This proportion, in turn, determines many important factors for faculty such as the skill level of student research collaborators, the types of grant funding that may be needed, and the teaching requirements.

In the US, almost all academic jobs in mathematics departments are not full-year jobs. The faculty is paid for nine to ten months and is responsible for obtaining summer salary during the period of time between the end of the academic year in May and the beginning of the new school year in August. Most US schools use a semester system, where classes are offered during the autumn term and during the spring term, with limited class offerings during the summer term. Other schools use a quarter system, offering classes during three periods of time in addition to summer classes.

Playing the numbers game

The winning strategy when it comes to the job market is brute force method.

(1) Apply early.

(2) Apply often.

(3) Apply everywhere!

Anyone seeking an academic job in mathematics in the US will need to begin applying to jobs around August of the year before they would like to start the job, assuming that they intend to start at the beginning of the fall semester. To be clear: over one year of lead time may be required to get a job in the US. Lead time for applying to jobs in Europe and the UK market is usually much less. The steps for putting a job application packet together are detailed in the next section.

A mathematician's first job after graduate school is often not in the place she will be long term. She may do several post-docs or visiting assistant professorships, although this is dependent on specialty. Anyone who likes shopping is welcome to think about the job hunt as shopping for a next venue. Whether you intend a brief stay of a year, or you intend to remain indefinitely, we suggest that quality of life is best ensured by assessing the location. Cost of living, public transportation and proximity to an airport, healthcare options, language barriers, cultural factors, and climate conditions are all worth taking into account.

A major factor in these decisions is the availability of jobs that start at a convenient time. The absurd truth of the matter is that the majority of math[6] professors working in the US did not have a particular dream school in mind to work at, but instead applied to the schools that were hiring the year they graduated or completed a postdoc, accepted the job that they were offered, gained tenure, and intend to stay until retirement.

[3]In the US, school is used as a generic term to refer to any institution of learning. We understand this usage can be disconcerting for those in the UK. However, in the interest of acclimating the UK reader to our (charming?) US vernacular, we liberally use school, operations research, other American academic slang, and American pop references. In a seeming tangent, we insist that watching *The Godfather I* and *II* is an essential part of any complete mathematical education.

[4]https://www.washingtonpost.com/news/grade-point/wp/2015/07/20/how-many-colleges-and-universities-do-we-really-need/

[5]For example, see https://www.usnews.com/best-colleges

[6]also known as "maths" in the UK.

Another hard truth that does not receive enough attention when studying for a PhD is that the quality of your research may be secondary to your field of research in the job search. When a school is ready to hire an assistant professor in mathematics, it is often interested in particular fields of study. Since the job search greatly impacts an academic's career, this means that an academic is well advised to understand how her specialty relates to other areas.

Carolyn, the matroid theorist, obtained her PhD in 2009 and worked as a research postdoc from 2009 until 2015, so she sampled the job market every year in between and has this to report: the general category of "pure mathematics" is rarely the direct target of a job ad, whereas statistics and the general category of "applied math" are often direct targets for job ads. Further specifications are popular, too. Mathematical biology, computational mathematics, the fields of analysis, algebra, and geometry are popular fields for assistant professor openings as of the writing of this article. At one job that she applied to, there were about 400 applications for an assistant professorship of unspecified field, whereas an assistant professorship in math education at the same school received about 40 applications. As a pure mathematician, she would not advise anyone who loves studying matroids to give up their passion, but it can be worthwhile to find other sides to your field. For example, matroid theory is related to graph theory and coding theory, which can be considered applied math, and can justify a matroid theorist to consider applied math positions.

David obtained his PhD in operations research in 2007, the first year of his first tenure-track job. There are few post-doctoral opportunities in operations research. Most graduates either take a tenure-track job or work in industry after completing their doctorate. For a variety of reasons, including the "two-body problem,"[7] he was on the market from 2006 until 2011 when he accepted his current job. Echoing Carolyn's advice, at every job he applied for, he found considerably more success when he found connections between optimization and other mathematical specialties such as real analysis, graph theory, and even matroid theory. In addition to finding potential research collaborations, he was able to assess and demonstrate his fit with potential colleagues.

Coffee can be converted into paperwork (as well as theorems)

There are a lot of reasons to become a mathematics professor. The beauty of mathematics, the satisfaction of being an educator, and the joy of paperwork, to name just three. Anyone who could not find joy in paperwork would never want to write a dissertation, a curriculum vitae (CV), a teaching statement, a research statement, and a grant application, right? Maybe it's not fair to categorize these items as "paperwork," but maybe it is fair. History will judge us.

There is no way that we know of to avoid paperwork and become a successful academic. We can suggest many procrastination techniques that have worked well for us in postponing paperwork for a finite amount of time, but the most gratifying way to do paperwork that we have found is to complete it as soon as possible. The pitying condescension with which one who has completed her paperwork approaches her colleagues who have not completed theirs is sufficient reward.

For those of us who do not love paperwork, however, luckily there are lots of ways to get a job. Except in academia. Excelling at plumbing is a good way to get a plumbing job. Excelling at mathematics, excelling at communicating in written and oral forms, excelling at teaching and caring about students, excelling at mentoring students of all abilities and dispositions, and excelling at meeting deadlines and completing committee work are required to get an academic job. It's a lot to take on board. The written communication skills are particularly relevant during the job application process. All of the following documents are usually required to complete a job application:

(1) a CV,
(2) a teaching statement,
(3) a research statement,
(4) three letters of recommendation, one of which specifically addresses teaching, and
(5) a cover letter.

There are excellent resources online for putting each of the first three items together. For Item (4), the applicant will need to identify prospective letter writers well in advance of applying to jobs. For Item (5),

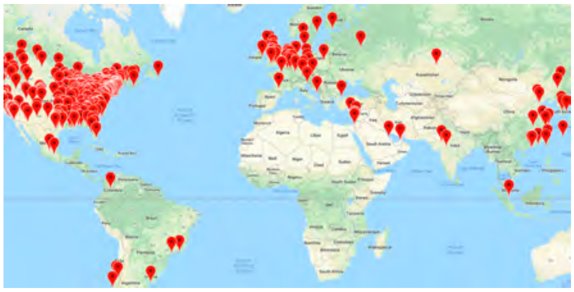
[7]In the two-body problem, one has a romantic partner that is also an academic but is at a school distant enough to preclude reasonable co-habitation.

each letter should be specific to the school and position. A cover letter to a research-heavy position will be different from a cover letter to a teaching-heavy position. It is considered good form to know something relevant about the school, gleaned from the website or by word of mouth, and to include the fact that the applicant is a good fit for the position due to that information. The applicant should indicate whether or not she will attend the Joint Mathematical Meetings (JMM) that January in the cover letter, since many US institutions have representatives at the JMM and conduct interviews there. The JMM is the largest mathematical gathering in the US every year, and its location varies. It is worth reviewing the topics of the sessions offered at the JMM. The applicant should endeavor to attend the meetings and to give a talk, either by asking an organizer for an invitation from an appropriate session, or by giving a contributed talk. This advice is also true for operations research and statistics albeit with their main conferences, i.e., the INFORMS Annual Meeting (INFORMS) for operations research and the Joint Statistical Meetings (JSM) for statistics, although the applicant in those fields is still encouraged to attend JMM as well.

Current job openings listed on www.mathjobs.org for math PhD’s. Red dots on the map indicate job openings. Image used with permission from the AMS.

Optimizing your journey

The American Mathematical Society (AMS) Employment Services for PhD Mathematicians (tinyurl.com/y9dw96mo) provides lots of support for job applicants. It is considered by many to be the canonical website for PhD mathematician job seekers in the US. Here is a quote from their website: “The AMS provides the automated MathJobs.Org job application system plus job ads on the web and on paper, and helps arrange interviews at the January meetings.” The January meetings referred to here are the JMM. This webpage includes links to four services for those seeking jobs, including MathJobs.Org, which is the job listing site that the authors of this article used most heavily.



Current job openings listed on www.mathjobs.org for math PhD’s. Red dots indicate job openings. Image used with permission from the AMS.

It is free to create a profile on MathJobs.Org, where you can store all of the documents necessary to apply to jobs, and then apply to those jobs with just a few clicks. Here is some information from the website about the website: “MathJobs.Org was officially launched in August, 2000. In the last 12 months, 1008 employers from 202 states in 44 countries have used the service for 2279 job ads (with 1168 ads-only); 36956 unique users have logged in; 2361 applicants are currently on the ’Job Wanted’ list; 8091 applicants have uploaded 197487 new documents; 7785 applicants have submitted 147447 new applications, with an average of 18.94 positions applied per applicant; 21742 reference writers have used the system, with 16205 reference writers uploaded 28718 new letters.” This data on the site highlights the amount of traffic it receives. This is good news, aside from the ratio of job seekers in the US to jobs being about nineteen to one: MathJobs.Org can be considered a one-stop shop.

We note that the ease of applying to jobs on MathJobs.Org contributes to the high number of applications for jobs that can be applied to through the site. Some schools list information about open positions on the site but only accept applications through other avenues, and those positions are likely to receive fewer applications since they are slightly more onerous to apply for. You may want to apply to jobs that only list job openings on their websites or where applications are only accepted via snail mail to ensure that the number of co-applicants is relatively low.

Take a deep breath and send it all in

In this beginning stage of the process, you submit your lovely paperwork as described previously, typically via MathJobs.Org. Once these materials are in, faculty at schools begin their decision processes to identify the most promising applicants for further screening. Whether this decision is made by a single individual, a subset of the department, or the whole department is school-dependent. At this stage, given the likely large number of applicants, the department must reduce the number of applicants being considered to a manageable number. It is critical you do not create an easy way for your paperwork to go the way of the dodo. In particular, the you are strongly advised to do the following:

- Make every effort to have materials in on time. For example, a common deadline statement is, "For full consideration, please apply by 8/1/2018.[8]"
- Understand what materials are requested and if any variations on the basic template described above are requested. For example, a current job listing for a teaching focused assistant professorship states, "To apply, please upload a single PDF file which includes a cover letter, a detailed resume and evidence of teaching excellence. "
- Email the contact person to ensure submitted materials have arrived. This both ensures no mishaps with the system have occurred and also signals interest in the job.

Submitting applications is an enormous amount of work — 100 is a normal number of applications to submit when seeking your first job — and applicants really deserve a lot of chocolates, pats on the back, and free drinks for reaching this point in the process. The process is not over, however, so you should keep your game-face on.

What to expect when you're expecting (a response to your applications)

After you have submitted your materials, the next step of the process is the screening interviews. This can be an in-person interview at one of the annual meetings mentioned above (JMM, INFORMS, or JSM) or a phone interview. The degree of competitiveness varies across both school and type of interview, although typically the phone interview signifies you have made it past one level of screening. At this point, the department is trying to decide which applicants to invite for an on-campus interview. Again, the numbers do not favor you. We therefore advise you to follow these steps:

- Read the job listing again with care and be sure to understand what the department is looking for.
- Find out as much as possible about the faculty at the department including their research specialties and courses taught.
- Be prepared to answer questions about teaching, research, and grant experiences. Think carefully about challenges faced and lessons learned in these areas.
- Prepare a list of questions about the institution. You may want to inquire about typical teaching loads, research collaborations, grant expectations, and student research opportunities. It is also a good opportunity to ask about the culture of the department, the cost of living nearby, and the job satisfaction that the interviewer has experienced in the department.

Planning ahead to apply to the US academic market

- The overwhelming majority of positions begin in the autumn.
- Application deadlines may be as early as the autumn of the year before the autumn that the position is for.
- If the applicant would like to begin working in January, then she should apply for autumn, and ask to delay by one semester after receiving a job offer.
- The applicant should plan to attend the Joint Mathematical Meetings in January.[a]

[a]For those in operations research, the applicant should additionally plan to attend INFORMS in late October or early November, and for statisticians, the applicant should plan to attend the JSM in August.

[8]Note that this means August 1, 2018. In the US, we (how charming!) switch the month and day from the convention used by nearly all of the rest of the world. We also find the metric system *terrifying*.

The big show

After you have gone through the screening interview process, the next step is the on-campus interview. This typically consists of one to two days of interviews with individual or groups of faculty, meeting students, and giving one or two job talks. Depending on the school, one job talk will focus on your research and the second on teaching. In the latter case, applicants usually give a talk to students with one or more faculty present. The importance of the job talks cannot be overerestimated: this is often a major part of the decision process for most schools. We offer the following advice.

- Suss out the knowledge level of the audience in their particular research area and be sensitive to ensure enough introductory material is present. Depth at the end of a talk is encouraged but be sure to start from a point which any member of the audience can understand.
- If there is a teaching talk, be sure to be organized and understand fully what material should be covered. Extra material such as slides and handouts can be helpful and demonstrate preparation as long as they are used effectively.
- Be aware of your rights under the law: For example, it is illegal to use gender, race, romantic partners, parenting, or disabilities to make employment decisions.
- At this stage, you are also interviewing the school: ask questions about the school that indicate a sincere vision of you being a faculty member at the school. Where would the faculty advise you to live? What opportunities for collaborations and summer funding exist? What courses would you be expected to teach? What, if any, travel funds are available to faculty?

They make you an offer that you can't refuse

Finally, you have received a job offer! Congratulations are in order from being converted from an applicant to a prospective employee. Typically, the chair of the department makes the offer and conducts subsequent discussions. At this stage in the process, you have to negotiate such things as salary, start date, and any additional remuneration, e.g., moving costs or start-up funds. We advise you as follows:

- Ask for time to consider the offer.
- Consult with others, e.g., your advisor.
- Consider the relative merits of other offers including non-academic offers.
- Consider your constraints, e.g., is there a two-body problem or some current job-related start date?

After these questions are answered, always negotiate. Respectfully ask for higher salary, more start-up money, a computer, a lighter initial teaching load, and travel funds. Patience is the best strategy here within the time constraints the department indicates. The chair will indicate what flexibility the department and school has, if any.

And then make a decision. Notify your letter writers of the outcome and thank them for their help. Whether or not you are successful, you are entitled to consume your weight in chocolate! Those are the rules! Also, keep in mind that any applicant who undergoes all of this bother and ends up leaving academia is likely to become exceedingly wealthy and happy. Her work ethic and big strong brains are a credit to her training, her family, and her country.

Carolyn Chun

Carolyn Chun is an assistant professor in mathematics at the United States Naval Academy. Her main research interests are graphs, matroids, and delta-matroids, but she procrastinates by writing experimental fiction and making comics of herself, her husband, her twin sister, and her tiny, hilarious daughter.

David Phillips

David Phillips is an associate professor in mathematics at the United States Naval Academy. His research interests are in convex and combinatorial optimization and their applications in data sciences. He never procrastinates under the logic system where "never" means "always".

Sublime Symmetry

Guildhall Art Gallery, City of London
11 May – 28 October 2018, free entry

Review by David Singerman

This is a really beautiful exhibition centred around the work of William de Morgan. It also has mathematical significance in more than one way. For a start, William was the son of Augustus De Morgan, the first President of the London Mathematical Society, and this exhibition also has a special room devoted to Augustus and the LMS, of which more later.

William de Morgan (1839–1917) was a ceramic designer of the late Victorian period. He was a friend of the designer William Morris and is a central figure in the arts and crafts movement. He began his work designing stained glass windows, went on to pottery in 1862 and then, in 1872, shifted his interest to ceramics.

From the gallery's website:

> He was undoubtedly the most intriguing and inventive ceramic designer of the late Victorian period. His conjuring of fantastical beasts to wrap themselves around the contours of ceramic hollowware and his manipulation of fanciful flora and fauna to meander across tile panels fascinated his contemporaries and still captivates today.
>
> Over his career William De Morgan revolutionised the field of ceramic design with his reinvention of lusterware, dedication to studying and perfecting Middle Eastern designs, invention and use of his own kilns and his wonderful patterns.

Sublime Symmetry builds on these earlier observations to reassess — through a rigorous examination of the mathematical devices used in his designs — De Morgan as a natural mathematician and talented draughtsman. The exhibition showcases magnificent ceramics from the De Morgan Collection and designs on paper on loan from the V&A. The pieces have all been chosen to demonstrate the mathematical concepts which are the basis for De Morgan's beautiful and colourful ceramic designs.

The LMS room

This exhibition has a special room devoted to the LMS and in particular to Augustus De Morgan, William's father and the first LMS President. There are posters about work associated with the work of Augustus, for example on set theory there is a display illustrating Venn diagrams. On a rather deeper level there is an interesting poster about the four-colour theorem. On October 23 1852, Augustus wrote to his friend William Rowan Hamilton, "A student of mine asked me today to give him a reason for a fact which I do not know as a fact — and do not yet. He said that if a figure be anyhow divided into compartments differently coloured so that figures with any portion of common boundary are differently coloured, four colours may be wanted but not more." Although the four-colour problem was proposed in 1852, it was not solved until 1976, through the work of Appel and Haken.

There is also a poster about Ada Lovelace, a gifted, perceptive and knowledgeable mathematician and expositor of the computing machines devised by Charles Babbage. In 1843, she wrote a remarkable paper which explained the principles of Babbage's most ambitious invention his unbuilt 'Analytical Engine'. This paper contains far-sighted reflections on computing "beyond number" — even computer music and artificial intelligence. A large table of mathematical formulae, sometimes inaccurately called the "first computer programme" was used to calculate Bernoulli numbers as an example. In her early years, Ada Lovelace had an intense two-year correspondence course with Augustus De Morgan, who rated her highly as a mathematician and capable of much more than her 1843 paper.

The Elements of Euclid by Oliver Byrne

Also on display is Oliver Byrne's *The Elements of Euclid* (left), where coloured diagrams and symbols are used "for the greater ease of learners". De Morgan considered Byrne as an odd character among "minor mathematical writers". Both he and Charles Dodgson wondered if Byrne's work could be used in the teaching of Euclid. However, Byrne's work is visually stunning and makes a nice link between William's creations and the mathematics of the LMS room.

Another interesting mathematical display is of the models of geometric surfaces, including Plücker's quartic surfaces, which were donated to the society in 1866 by the geometer Thomas Archer Hirst.

To sum up, I would urge all mathematicians to visit this exhibition. It closes on October 28. It contains many very beautiful creations and there is more mathematical significance here than in the mathematics exhibition at the Winton Gallery of the Science Museum, which was reviewed in the *Newsletter* in April 2017.

David Singerman

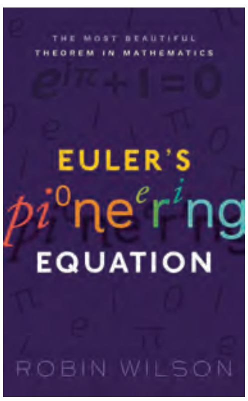
David Singerman is an emeritus professor at the University of Southampton. His main interests have been on Fuchsian groups and Riemann surfaces, in particular the theory of maps (or "dessin d'enfants") on Riemann surfaces.

Editor's note: Sublime Symmetry is supported by the LMS. One of the Newsletter's editors (J. B.-G.) is involved in the exhibition, but was not involved in the preparation of this review.

Euler's Pioneering Equation

by Robin Wilson, Oxford University Press, 2018, £14.99, US$ 19.95, ISBN: 978-0-19-879492-9

Review by Andrew N.W. Hone



Euler's identity

$$e^{ix} = \cos x + i \sin x \quad (1)$$

is one of the most fundamental relations in mathematics, providing a bridge between real and complex numbers, and between exponential and trigonometric functions. I can still remember feeling a sense of wonder when one of my A-Level teachers first explained this identity, and I'm not the first person to have been impressed by it: the special case $x = \pi$ is equivalent to the equation

$$e^{i\pi} + 1 = 0, \quad (2)$$

which was awarded the top 'score for beauty' in a 1988 poll conducted by *The Mathematical Intelligencer*, and in 2004 was ranked second in a list of *The greatest equations ever* by the readers of *Physics World* magazine. In his new book, Robin Wilson reveals the wonder and beauty of Euler's equation to the uninitiated reader.

The beauty of (2) was measured with MRI scans by neuroscientists at UCL in 2014: when mathematicians looked at Euler's equation, it consistently evoked responses from the same emotional centres in the brain as are used to appreciate art. The great aesthetic appeal of the equation is surely due to the fact that, via the operations of addition, multiplication and exponentiation, it combines five of the most important numbers in mathematics, namely 1, 0, π, e and $\sqrt{-1}$. Wilson devotes an eponymous chapter to each number in turn, leading to a synthesis with (2) in the final chapter. The order of material more or less follows the historical development of the relevant concepts, allowing a gradual increase in the level of mathematical sophistication required of the reader.

The first chapter deals with the counting numbers, and the many different ways of writing them and naming them throughout history. At some stage, and in more than one part of the world, a symbol for zero was introduced as a place-holder in a place-value system for writing numbers; in Mesopotamia, then later in Mayan mathematics, and independently in India. However, it was a highly non-trivial step to go from a place-holder to the notion of zero as a number in its own right. In the 7th century, Brahmagupta almost laid out all the correct rules for integer arithmetic, but stumbled when it came to dealing with division by zero. Ultimately, the decisive step of treating zero as a number led to an even more inclusive notion of arithmetic, with both fractions and irrational numbers involved, culminating in Dedekind's 19th century construction of the real number line. The second chapter describes the developments in arithmetic that came out of the concept of the number zero, and ends by explaining the distinction between algebraic numbers, like $\sqrt{2}$, and transcendental numbers, like π and e — the subjects of the next two chapters.

A popular method for calculating π, originating with the sophists Antiphon and Bryson, and developed by Archimedes, consists of approximating a circle of unit radius by a regular polygon: the area of each inscribed (circumscribed) polygon gives a lower (upper) bound. Around 500 AD, Zu Chongzhi and his son Zu Gengzhi obtained the bounds $3.1415926 < \pi < 3.1415927$ by using polygons with 24576 sides. In 1579, François Viète took 393216 sides to get an approximation accurate to 9 decimal places, and found an exact infinite product expression:

$$\frac{2}{\pi} = \prod_{n=0}^{\infty} C_n, \qquad C_0 = \frac{1}{\sqrt{2}}, \qquad C_{n+1} = \sqrt{\frac{1+C_n}{2}}.$$

Wallis found another sort of infinite product, which inspired Brouncker to derive a whole series of continued fractions, like

$$\frac{4}{\pi} = 1 + \cfrac{1^2}{2 + \cfrac{3^2}{2 + \cfrac{5^2}{2 + \cfrac{7^2}{2 + \cdots}}}}$$

and other special cases of a gamma function formula due to Euler, Stieltjes and Ramanujan [1].

However, such expressions produce very slowly convergent approximations to π: much better results come from arctan addition formulae, or from more exotic series found by Ramanujan using modular forms, which with current computing technology have allowed π to be calculated to over 20 trillion places. As Wilson remarks, such a ridiculous number of digits is completely irrelevant for all practical purposes, but is used to test the speed and accuracy of new computers. Unsurprisingly, some non-mathematicians would prefer it if π had a more convenient value, like 3.2: the State of Indiana passed a bill to this effect in 1897 — fortunately never ratified!

There is the fascinating story of the development of logarithms by Napier and Briggs, who succeeded in taming exponential growth by replacing multiplication with addition through the rule $\log ab = \log a + \log b$. This soon reappeared in integral calculus: de Sarasa observed that the same rule arose from Saint-Vincent's calculation of the area under the graph of a hyperbola. Exponentials appeared in compound interest, when Jakob Bernoulli calculated the interest at successively shorter time intervals to find the limit $\lim_{n\to\infty}(1 + x/n)^n = \mathrm{e}^x$; and Newton discovered the power series for e^x. However, it had to wait for Euler in the 18th century to fully explain the properties of the exponential function, and the fact that it is the inverse of the logarithm. The chapter on exponentials ends with various applications: catenaries, derangements, and differential equations for population growth, the cooling of a cup of tea, and radioactive decay.

In the next chapter, the arithmetic and geometry of complex numbers are introduced from scratch, including the solution of polynomial equations, the fundamental theorem of algebra, De Moivre's theorem and the roots of unity. An entertaining account of Hamilton's discovery of quaternions is provided, and even a brief description of octonions. Finally, in the last chapter all the different threads are drawn together, as the basic properties of exponentials and logarithms of complex numbers are explained, together with Euler's identity (1). The history of the equation (2) turns out to be rather convoluted: in 1702 Johann Bernoulli found a formula for the area of a sector of a circle which gives $\log(-1) = \mathrm{i}\pi$, while in 1714 Roger Cotes obtained the identity corresponding to taking the logarithm of both sides of (1); and Euler published the proof of (1) in 1748, yet never explicitly wrote down the particular case $x = \pi$.

Visually the book is very appealing: 88 figures serve to illustrate the main text, which is interspersed with 11 boxes presenting detailed proofs of technical results for readers with above average mathematical knowledge. The style throughout is very engaging, with numerous anecdotes, amusing quotes, rhymes and mnemonics for the digits of π and e. The book would be good for anyone who is interested in the history of mathematics, but it is particularly suitable as an inspiration for young mathematicians. I have tried it out on an 11-year-old, who was instantly attracted by the pictures of different number systems in the first chapter, and on a 15-year-old, who has found it enjoyable and easy to read so far.

Robin Wilson has produced a wonderful introduction to some of the most fundamental ideas in mathematics. I would highly recommend that you give a copy to any inquisitive young person you know.

FURTHER READING

[1] Thomas J. Osler, Lord Brouncker's forgotten sequence of continued fractions for pi, Int. J. Sci. Math. Educ. 41 (2010) 105–110.

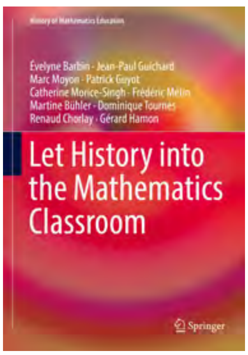
Andrew Hone

Andy Hone is Professor of Mathematics at the University of Kent, but he wrote this review in Sydney, where he is currently a Visiting Professorial Fellow at the University of New South Wales. He likes doing yoga, going walking, and singing (mostly in the shower these days, if his family will let him).

Let History into the Mathematics Classroom

by E Baerbin et al, Springer International Publishing, 2018, hardback, pp146. £99.99, US$ 139.99, ISBN 978-3-319-57149 2.

Review by Sean Springer



This interesting and engaging book consists of a series of papers, translated from French, which make the case for the history of mathematics to be an integral part of the teaching of the subject in schools today. The authors' reasoning being that such an approach emphasises the practical origins of topics as tools for problem solving; topics which today can sometimes be presented divorced from any practical purposes — let alone the original ones. As a blunt quotation from a Medieval author given on page 17 of the book puts it:

> 'It would be shameful for someone to praise whatever skill it might be and not know what it actually is, its genre, what it is about and all the other things that have gone before it.'

Each of the book's ten chapters outlines the use of a historical perspective on a mathematical topic in teaching a group of students. Each follows the same broad structure: an introduction to the area of maths with its origins, an outline of the particular tasks carried out in class and finally a reflection on the activity.

The inclusion of anecdotal pupil responses and survey results are particularly welcome as is the fact that the authors take time to present mistakes made by the students, mistakes which are appreciated as being as enlightening to the reader as the pupils' successes. In a few places, the authors anticipate pupil apathy and express doubts about the ambition of some of the activities; this disarming and relatable honesty could well be as effective as the clear success of the activities in convincing any skeptical teachers to embrace their ideas.

While the book stands as an interesting read on many topics to a mathematician, it is certainly pitched as a call to reflect on and modify the way mathematics is taught and so its practicality requires examination. While schools are not yet the tightly plotted pursuit of exam grades that they are sometimes accused of being, it is still necessary to consider how these activities can be accommodated within existing syllabuses. Some chapters are more of a digression from the standard curriculum and so would prove more of a challenge than those chapters which offer history and problem solving linked to a topic already explicitly covered.

The breadth of topics means that a teacher wishing to try one of these activities can do so with secondary pupils of any age: it spans from basic angle work (Chapter 1) to Euler's Method (Chapter 7). The experiments of the book were carried out with a diverse range of school groups; most strikingly a class on inscribing squares in triangles (based on a Marolois text from 1616) was run with a group following a fashion course with "most having some difficulty in written and spoken French". It seems throughout that the French students have, as a matter of course, a greater confidence with geometry than is perhaps usual in most schools and this fashion class can recall and use Thales' theorem. Even allowing for the fact that students here would call this similar triangles (rather than attribute it to Thales), it is a level of prior knowledge which would need to be accounted for before undertaking these particular tasks.

An interesting theme running through the book is the importance that languages play in the history of mathematics; be it the challenges of Spanish calligraphy, Euler's multilingual correspondence or the fascinating structure of Sanskrit writings (as concise metred verses intended to be memorised). The authors are honest about the problems presented by the accessibility of the older texts' language to students but, on balance, the 'hook' that the original material provides seems to outweigh the risk of obscuring the meaning or disengaging students.

A particularly interesting chapter concerns Nomography (the study of graphical tables for performing calculations). For the vast majority of teachers today, not even log tables have been a practical tool, so for the current cohort of students the use of tables of any kind is a curious historical fact. The ability of parabolas to multiply numbers comes up naturally when studying Conic Sections but, after reading this book, I found myself asking why I was not using it at GCSE when students study coordinate geometry. Many of the tables could be shown naturally within the Key Stage 3 curriculum and while using them in this way does not perhaps fully embrace the philosophy of this book, it is at least a way of incorporating elements which then naturally lead to giving the historical context and an outline of the broader ideas (and in this instance with much younger students than the 16+ target of this chapter).

This book has many strengths: the broad and engaging range of topics, the clear structure of the lessons and the detailed presentation of the history with a good use of images. To teachers, in particular, this book is certainly a compelling call to value the importance of history in the maths classroom but there is much of interest to any mathematician here.

Sean Springer

Sean Springer has long liked sums and books. He has a PhD and PGCE from Queen's University, Belfast and since graduating in 2010 has taught Mathematics at the Belfast Royal Academy. In his spare time he enjoys improving his French and spending too much time and money building Lego.

Obituaries of Members

Frederick Bernard de Neumann: 1943 – 2018

Bernard de Neumann, who was elected a member of the London Mathematical Society on 16 January 1981, died on 18 April 2018 aged 75.

Bev Littlewood writes: Frederick Bernard de Neumann — known to all as Bernard — attended the Royal Hospital School, Holbrook, whose teachers kindled his life-long interest in mathematics. He studied at Birmingham University, where he obtained a BSc and an MSc in mathematics. After university he went in 1963 to work in Jozef Skwirzynski's mathematics group at Marconi Research Centre at Great Baddow. He worked there with, among many others, Sir Eric Eastwood FRS, following the latter's retirement as Director of Research of GEC. During his time at MRC the company was doing cutting-edge engineering, and Bernard played a significant role as what he called a "working mathematician" collaborating with people throughout the GEC empire. A colleague of his at the time described him, appreciatively, as a "pure" applied mathematician. Much of his work was defence-related, and details remain under wraps even now. However, some of his mathematical work helped to make it possible to receive images from deep space missions, such as the first colour pictures from the Mars surface sent by the Viking Landers 1 and 2. He also invented and patented a self-configuring multi-processor computer, from which emerged ideas used in contactless smart cards and RFI tagging chips.

Bernard spent 25 years at Baddow, and told me he enormously enjoyed his work there, interacting widely throughout the company on very varied problems. Eventually, however, because of management changes, the company culture of a "collegial family" of talented scientists and engineers, under which he had thrived, gradually disappeared. He left in 1988.

Bernard and I first met in 1981 when we both helped Jozef Skwirzynski organize a NATO Advanced Study Institute on Electronic System Effectiveness and Life Cycle Costing, which was held at UEA, Norwich, in July 1982. This ASI was influential in the creation of a community of researchers and industrial practitioners working in systems "dependability" (roughly: reliability, safety, security). Most notably, it highlighted the emerging importance of software in complex systems. Bernard and I, along with a handful of other ASI participants, set up the Centre for Software Reliability shortly afterwards. Eventually two CSR research centres were established, one at City University (with me as Director) and one at Newcastle University (with Tom Anderson as director).

Following his departure from Baddow in 1988, Bernard joined City University, first as an Honorary Visiting Professor of Mathematics and shortly afterwards, in 1989, as Research Professor of Software Engineering in the City CSR research centre, where he worked on stochastic modeling of the reliability of software-based systems. During his time at City he published several books, including "Software Certification" and "Mathematical Structures for Software Engineering". He retired from City University in 1996, but continued to use his mathematical talents in consulting roles in a wide variety of organisations, including NASA, ESA, NATO, MoD.

In retirement he had several pro bono roles, including: member of the Court of Essex University; member of the Council of the Institute of Mathematics and Its Applications; non-executive director of the Essex Health Authority; archivist of the Royal Hospital School.

For many years he had an interest in genealogy, particularly his own ancestors, who were middle European nobility serving in the retinues of Dukes, Grand Dukes, Kings and Emperors. His ancestor Carl Friedrich Bernhard von Neumann came to London from Munich in 1833 and stayed. Bernard styled himself von Neumann in his publications in German. He suspected — but was not able to establish — a distant relationship with John von Neumann. If mathematical talent has some genetic basis, this may not be an unreasonable supposition.

At the time of his death Bernard was writing two books: one about the history of the Royal Hospital School, and another about the exploits during World War 2 of his father. Captain Peter de Neumann, "The man from Timbuktu", was awarded the George Medal for bravery; he was unjustly convicted of piracy by the wartime Vichy French government and served as a prisoner of war in Timbuktu.

In 2005 a portrait of Bernard by the painter John Wonnacott CBE won the Ondaatje Prize of the Royal Society of Portrait Painters (tinyurl.com/y85pbrta). This superb painting captures very well the Bernard that his friends knew — empty wine glass and all. It is also not without mathematical interest. Bernard told me about his discussions with Wonnacott concerning the latter's interest in ultra-wide-angle landscapes, and Bernard's reconciling this with projective geometry. I can imagine these conversations: Bernard's interests and enthusiasms were wide-ranging. I have fond memories of dinners with him in various parts of the country at conferences and research project meetings over almost forty years.

In the last year of Bernard's life he faced a succession of hospital crises as a result of his cancers. Sadly, his wife Jill died in November 2017 and he missed her terribly. But the last time I met him, when he knew he had only a few weeks left, he cheerfully curtailed my reminiscences of times past, saying he still wanted to think of the future. We ended up discussing his current reading: Daniel Ellsberg's recent book, *The Doomsday Machine: Confessions of a Nuclear War Planner*. He is going to be greatly missed by his many friends.

John Roe: 1959 – 2018

John Roe, who was elected a member of the London Mathematical Society on 16 May 1986, died on 9 March 2018 after a four-year battle with cancer.

Nigel Higson (The Pennsylvania State University) writes: John was born on 6 October 1959 in Shropshire. He was fascinated by mathematics from an early age. He wrote of "a vivid memory of startling an elementary school teacher by using logarithms (which my father had explained to me) to solve the multiplication homework that she had set." John was educated at Rugby School, Cambridge and Oxford, where he was a doctoral student of Michael Atiyah. His thesis concerned new extensions of the Atiyah–Singer index theorem to complete Riemannian manifolds.

Throughout his career, John's research centered on coarse geometry, which is the study of the large-scale features of metric spaces, ignoring small-scale structure. More specifically, John was a pioneer in combining coarse geometry with index theory. From the outset, operator algebras played an important role in John's work, and the key mathematical concept that ties index theory and coarse geometry together is now named the *Roe algebra* in honor of its discoverer. The Roe algebra, along with other concepts introduced into coarse geometry by John, have become so indispensable to coarse geometry as to seem to be, by now, defining features of the subject.

John worked at Oxford University from 1986 to 1998 as a lecturer and tutor. But prior to taking up those positions he visited Berkeley, California for a year, and it was there that he met his future wife, Liane. John and Liane's two children were born in Oxford. In 1998 the family moved to the United States, and John became professor of mathematics at the Pennsylvania State University.

John was passionate about teaching. He invariably prepared carefully written lecture notes for his classes, and he always made these available to his students. He wrote over half a dozen undergraduate- or graduate-level books, and in addition he published many sets of lecture notes online, several of them through the American Mathematical Society's *Open Math Notes* project. John taught his last class during the Fall 2017 semester, while he was critically ill. His last book, *Mathematics for Sustainability,* was published by Springer only weeks before his death.

John was an extraordinarily responsible and unboundedly energetic faculty colleague, and he served as head of the mathematics department at Penn State from 2006 to 2012. Whenever and wherever he saw a need for his department to evolve, he made change happen. The lasting improvements at Penn State for which John is responsible include an extensive training program for graduate student teaching assistants, the reorganization of nearly every administrative aspect of the department's undergraduate program, and the planning of extensive renovations to the mathematics building.

John's interests and passions ranged widely. Most notably he was deeply religious, and a committed and active member of the congregations to which he belonged. John and Liane endured tragedy in 2016 when their transgender son, Eli, died following a struggle with depression. John's online writings about religion, about Eli, and about many other things, are preserved on his website at Penn State.

In the forward to John's book *Mathematics for Sustainability,* Francis Su wrote that "this project grew out of a conviction and prayerful reflection that [John's] knowledge as a mathematician and an educator could be channelled into wise action on matters that will impact us all." Su's words about John apply more broadly; they are a fitting epitaph for a man who affected and improved the lives of those around him, and those afar, in a multitude of ways.

Finance, Insurance, Probability & Statistics

Location: King's College London
Date: 10–11 September 2018
Website: tinyurl.com/yd7trowf

This workshop will bring together a rich mix of academics and practitioners focusing on the mathematical challenges of regulatory requirements and of the ever increasing amount of data. Supported by an LMS Conference grant.

British Science Festival 2018

Location: University of Hull
Date: 11–14 September 2018
Website: tinyurl.com/zuybjoc

Mathematics-related events include lectures by Jennifer Rogers (Oxford), Colin Wright (Denbridge Marine), Laura Bonnett (Liverpool, Rosalind Franklin Award Lecture), Stuart Davidson (NPL), Sandra Vaiciulyte (Greenwich) and MathsJam (organiser Katie Steckles).

LMS–IMA Joint Meeting: Noether Celebration

11 September 2018, De Morgan House, London

Website: tinyurl.com/LMS-IMANoether2018

Speakers: K. Brading (Duke), E. Mansfield (Kent), C. Praeger (W. Australia), N. Schappacher (IRMA/UFR de Mathématique et d'Informatique), R. Siegmund-Schultze (Agder).

The meeting, which includes lunch and a reception, is free to attend. Please register for your place online. A dinner at £30 per person will be held after the reception. To attend, email lmsmeetings@lms.ac.uk.

Das Kontinuum - 100 Years Later

Location: University of Leeds
Date: 11–15 September 2018
Website: tinyurl.com/y7og2738

Weyl proposed in *Das Kontinuum* (1918) a radically new 'predicative' foundation of analysis, showing how large portions of the subject could be developed assuming only the natural numbers. This conference brings together mathematicians and philosophers working in areas related to Weyl's legacy, on this centennial anniversary of *Das Kontinuum*. Supported by an LMS Conference grant.

LMS Popular Lectures

Location: Birmingham
Date: 19 September 2018
Website: lms.ac.uk/events/popular-lectures

These are free annual events, open to all, which present exciting topics in mathematics and its applications to a wide audience. Speakers: Katie Steckles (*Maths's Greatest Unsolved Puzzles*) and Jennifer Rogers (University of Oxford; *Risky Business*).

Representations of Groups and Hecke Algebras

Location: University of Birmingham
Date: 18–19 September 2018
Website: tinyurl.com/yc9c2ho7

This two-day conference in memory of Anton Evseev will celebrate his mathematical achievements. Speakers include Joe Chuang, Ryan Davies, Radha Kessar, Sinead Lyle, Gunter Malle and Michael Livesey, among others. All are welcome to attend. Registration is available via the conference website. The meeting is supported by an LMS Conference grant.

LMS Good Practice Scheme Workshop

Location: De Morgan House, London
Date: 5 October 2018
Website: tinyurl.com/y73n9ze9

The workshop will focus on the Society's updated Benchmarking Survey on UK women in mathematics and good practice. To register email womeninmaths@lms.ac.uk.

Fisher Centenary

Tuesday 9 October 2018, 8.30 am - 7.00 pm, Royal College of Surgeons, Edinburgh

Website: tinyurl.com/yd2khjok

This joint meeting celebrates the centennial of R.A. Fisher's famous 1918 paper on the theory of quantitative trait inheritance. Speakers: N. Barton (Kloster Neuburg), J. Hadfield (Edinburgh), S. Browning (Seattle), H. Cordell (Newcastle), E. Buckler (Ithaca), R. Mott (London), J. Pemberton (Edinburgh). The Fisher Memorial Lecture will be given by Michael Goddard (Melbourne) at 5 pm, followed by a reception. Register online by 21 September. The meeting is sponsored by the Fisher Memorial Trust, the Genetics Society, the Galton Institute, the LMS and the Royal Statistical Society.

Random Processes and Heavy-tailed Phenomena

Location: Durham University
Date: 17 October 2018
Website: tinyurl.com/y7mjwlyd

This event focuses on probabilistic models and processes that exhibit heavy-tailed phenomena. Registration is free but required; see details online. Supported by an LMS Celebrating New Appointments Scheme 9 grant.

Intersections of Geometric Analysis and Mathematical Relativity

Location: Queen Mary University of London
Date: 30–31 October 2018
Website: tinyurl.com/y7q6anrg

This event unites researchers in Geometric Analysis and Mathematical Relativity, featuring talks by experts from the UK, USA and Europe. Supported by an LMS Celebrating New Appointments Scheme 9 grant.

LMS Computer Science Colloquium

Location: De Morgan House, London
Date: 8 November 2018
Website: tinyurl.com/cscolloquium18

Aimed at PhD students and post-docs, the theme of this event will be 'Quantum Computing: Unique Mathematical Perspectives'. See the website for details of speakers and to register. Funds are available to help with travel costs. Free attendance for students; £5 for all others (payable on the day).

Baker Memorial Conference

Location: Isaac Newton Institute, Cambridge
Date: 18 October 2018
Website: tinyurl.com/yd4hnvrw

This conference is being held in honour of Alan Baker and his mathematics. Speakers include Enrico Bombieri, Kálmán Győry, David Maser and Gisbert Wüstholz. Organised by Béla Bollobás and Imre Leader.

BCS-FACS Evening Seminar 2018

Location: De Morgan House, London
Date: 1 November 2018
Website: tinyurl.com/bcsfacs

This event is organised by the LMS and the BCS Specialist Group for practioners in Formal Aspects of Computing Science. Bill Roscoe (Oxford) will talk on *Verifying CSP and its offspring*. To register interest, email lmscomputerscience@lms.ac.uk.

Young Researchers in Algebraic Number Theory

Location: University of Sheffield
Date: 8–9 November 2018
Website: tinyurl.com/yaoqz99j

This event is for early career researchers in algebraic number theory and arithmetic geometry to present their research to a network of their peers. Funding is available for UK-based participants thanks to support from an LMS Scheme 8 Postgraduate Conference grant.

LMS Meeting

LMS Graduate Student Meeting

9 November 2018, 10.00 am – 3.00 pm, BMA House, Tavistock Square, London

Website: tinyurl.com/y8hafqjx

This is a free event aimed at a general mathematical audience. The meeting will include student presentations of their current work, with a prize awarded for the best student talk. It will be followed by the LMS AGM, and a wine reception at De Morgan House. Travel grants of up to £50 are available for students who attend both the Graduate Student Meeting and the AGM; email lmsmeetings@lms.ac.uk for details.

LMS Meeting

LMS Annual General Meeting

9 November 2018; 3.00 – 6.00 pm, BMA House, Tavistock Square, London

Website: tinyurl.com/y7q5dsgk

This year's AGM will include a Naylor Lecture given by John R. King (Nottingham). It is free to attend and the lecture will be aimed at a general mathematical audience. The meeting will be followed by a wine reception at De Morgan House. The Society's Annual Dinner will be held after the reception at The Montague on the Gardens, at a cost of £58.00 per head, including drinks. To reserve a place at the dinner, email AnnualDinner_RSVP@lms.ac.uk.

Trends in Persistent Homology

Location: Queen's University Belfast
Date: 16 November 2018
Website: tinyurl.com/y9tfwznl

The aim of this meeting is to bring together researchers from pure and applied fields to discuss different applications of persistent homology. Supported by an LMS Celebrating New Appointments Scheme 9 grant.

Blackett Memorial Lecture

Location: The Royal Society, London
Date: 28 November 2018
Website: tinyurl.com/yd2rdmt9

The lecture will be given by Sir Alan Wilson, Chief Executive of The Alan Turing Institute and Professor of Urban and Regional Systems in the Centre for Advanced Spatial Analysis at University College London. Entry free. Online registration compulsory.

Careers Open Day

Location: The Repertory Theatre, Birmingham
Date: 27 November 2018
Website: tinyurl.com/y9w9tv5x

This is an opportunity for employers working in operational research, analytics, data science and other related fields to connect with upcoming or recent graduates. The programme of talks will offer first-hand experience, career advice and case studies.

Mathematical Challenges of Big Data

Location: Double Tree West End, London
Date: 10–11 December 2018
Website: tinyurl.com/yd325pb8

This IMA conference is an opportunity for researchers and practitioners to meet, network and showcase the very latest research in a broad range of topics. Please submit abstracts of 100–200 words by 28 September 2018.

Advances in Applied Algebraic Geometry

Location: University of Bristol
Date: 11–14 December 2018
Website: tinyurl.com/y9rnokqz

This conference will cover the rapidly-growing field of applied algebraic geometry. The talks will be accessible to graduate students. The conference is supported by an LMS Conference grant, Heilbronn Institute, IMA and the Bristol Statistics Group. Register at the conference website.

Finite Dimensional Algebras, Homotopy Theory, and Geometry

Location: University of Glasgow
Date: 13–14 December 2018
Website: tinyurl.com/y9hsmzab

This conference is in connection with the appointments of Sira Gratz and Greg Stevenson. It will be loosely based around their research areas. Limited financial support available for graduate students and early career researchers.

LMS Meeting

LMS South West & South Wales Regional Meeting

17 December 2018; 2.00 pm Exeter University

Website: tinyurl.com/ydymqdeq

The meeting forms part of a workshop on *Number Fields and Function Fields: Two Parallel Worlds*, 18–20 December 2018. Funding for travel expenses is available for Society members and research students. See the website for further details, to register and to reserve a place at the dinner. The cost of the dinner will be approximately £35, including drinks.

XIV Brunel-Bielefeld Workshop on Random Matrix Theory and Applications

Location: Brunel University London
Date: 14–15 December 2018
Website: tinyurl.com/ycs4kjhf

This event brings together leading scientists working in Random Matrix Theory and its applications. Participation deadline: 5 November 2018. Some funding available for early-career researchers. Supported by an LMS Conference grant and CRC1283 (Germany).

Operators, Operator Families and Asymptotics II

Location: University of Bath
Date: 14–17 January 2019
Website: tinyurl.com/OOFA19-info

The workshop will address applications of the techniques of operator theory to the asymptotic analysis of parameter-dependent differential equations and boundary-value problems. Register by 9 December.

Variational Approaches to Problems in Solid Mechanics

Location: University of Warwick
Date: 7–8 January 2019
Website: tinyurl.com/y9tt8u7f

Solid Mechanics and the calculus of variations have been intertwined branches of study since the time of Euler. This workshop will bring together junior faculty members UK-wide.

British Postgraduate Model Theory Conference 2019

Location: University of Manchester
Date: 16–18 January 2019
Website: tinyurl.com/y7uctdcc

This conference is an opportunity for postgraduate students and postdocs working in model theory to present their research. Supported by an LMS Postgraduate Conference grant.

Society Meetings and Events

September 2018

11 Joint Society Meeting with IMA: Noether Celebration, London
19 LMS Popular Lecture, Birmingham

October 2018

5 LMS Good Practice Scheme Workshop, London
9 Joint Society Meeting with the Fisher Trust, Galton Institute, Genetics Society and RSS; Royal College of Surgeons, Edinburgh

November 2018

1 BCS-FACS Evening Seminar, London
9 LMS Graduate Student Meeting, London
9 Society Meeting and AGM, London

December 2018

17 LMS South West & South Wales Regional Meeting, Exeter

May 2019

20-24 LMS Invited Lecture Series 2019, Professor Søren Asmussen (Aarhus University), ICMS, Edinburgh

Calendar of Events

This calendar lists Society meetings and other mathematical events. Further information may be obtained from the appropriate LMS Newsletter whose number is given in brackets. A fuller list is given on the Society's website (www.lms.ac.uk/content/calendar). Please send updates and corrections to calendar@lms.ac.uk.

September 2018

2–4 Modern Mathematical Methods in Science and Technology, Kalamata, Greece (475)
3–7 Dynamics Days Europe 2018, Loughborough University (476)
3–7 Model Sets and Aperiodic Order, Durham University (476)
3–7 Renormalisation in Quantum Field Theory and in Stochastic Partial Differential Equations, INI, Cambridge (476)
4–6 33rd British Topology Meeting, The Open University (477)
7–8 LMS Prospects in Mathematics Meeting, University of Warwick (477)
10–11 IMS–FIPS 2018 Workshop, King's College London (478)
10–12 BSDEs, Information and McKean-Vlasov Equations, University of Leeds (477)
10–13 Arithmetic Ramsey Theory, University of Manchester (477)
11 Joint Society Meeting with IMA: Noether Celebration, London (478)
11–14 Dragon Applied Topology, Swansea University, Singleton Campus (477)
11–14 British Science Festival 2018, University of Hull (478)
11–15 Das Kontinuum — 100 Years Later, University of Leeds (478)
14–15 Theoretical and Computational Discrete Mathematics, University of Derby (476)
17 Combinatorial Algebraic Geometry, University of Bristol (477)
17–19 Probability and Nonlocal PDEs: Interplay and Cross-Impact, Swansea University (477)
18–19 Representations of Groups and Hecke Algebras, University of Birmingham (478)
19 LMS Popular Lectures, University of Birmingham (478)
21 Functor Categories for Groups, Senate House, London (477)
24–26 Clay Research Conference 2018, Mathematical Institute, Oxford (476)
24–28 Derived Algebraic Geometry and Chromatic Homotopy Theory, INI, Cambridge (476)

October 2018

- 5 LMS Good Practice Scheme Workshop, De Morgan House, London (478)
- 8 Baker Memorial Conference, Isaac Newton Institute, Cambridge
- 9 Joint Society Meeting with the Fisher Trust, Galton Institute, Genetics Society and RSS; Royal College of Surgeons, Edinburgh (478)
- 17 Random Processes and Heavy-tailed Phenomena, Durham University (478)
- 22–26 Quantum Field Theory, Renormalisation and Stochastic Partial Differential Equations, INI, Cambridge (476)
- 30–31 Intersections of Geometric Analysis and Mathematical Relativity, Queen Mary University of London (478)

November 2018

- 1 BCS-FACS Evening Seminar, London (478)
- 6–7 Women in Mathematics: Opportunities for the Future 2018, University of Bristol
- 8 LMS Computer Science Colloquium, London (478)
- 8–9 Young Researchers in Algebraic Number Theory, University of Sheffield (478)
- 9 Graduate Student Meeting, London (478)
- 9 Society Meeting and AGM, London (478)
- 16 Trends in Persistent Homology, Queen's University Belfast (478)
- 27 Careers Open Day, The Repertory Theatre, Birmingham (478)
- 28 Blackett Memorial Lecture, The Royal Society, London (478)

December 2018

- 3–7 Manifolds Workshop, INI, Cambridge (477)
- 5–11 Topology and Applications, Cochin, India (477)
- 10–11 Mathematical Challenges of Big Data IMA Conference, London (478)
- 10–12 Mathematical Sciences and Technology 2018, Hotel Equatorial Penang, Malaysia
- 10–14 Conclusions and Future Directions Workshop, INI, Cambridge (477)
- 11–14 Advances in Applied Algebraic Geometry, University of Bristol (478)
- 13–14 Finite Dimensional Algebras, Homotopy Theory, and Geometry, University of Glasgow (478)
- 14–15 XIV Brunel-Bielefeld Workshop on Random Matrix Theory and Applications, Brunel University London (478)
- 17 LMS South West & South Wales Regional Meeting, Exeter (478)

January 2019

- 7–10 Variational Problems in Geometry and Mathematical Physics, University of Leeds (478)
- 14–17 Operators, Operator Families and Asymptotics II, University of Bath (478)
- 16–18 British Postgraduate Model Theory Conference 2019, University of Manchester (478)

April 2019

- 25–26 Mathematics of Operational Research, Aston University, Birmingham

May 2019

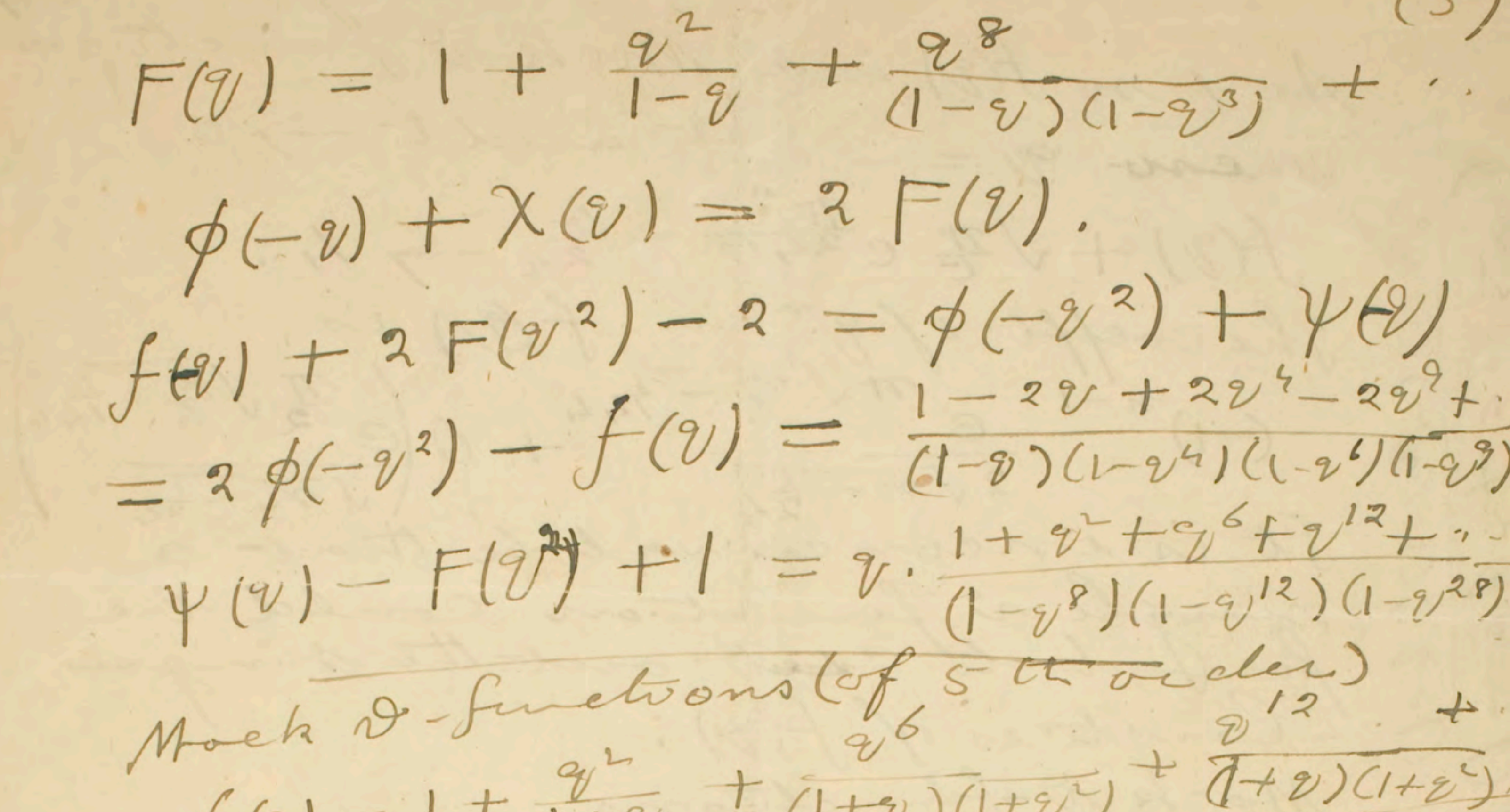
- 20–24 LMS Invited Lecture Series, Søren Asmussen (Aarhus University), ICMS, Edinburgh (477)

July 2019

- 29–2 Aug British Combinatorial Conference 2019, University of Birmingham

August 2019

- 4–9 Theory and Practice: an Interface or a Great Divide? Maynooth University (476)


$$F(q) = 1 + \frac{q^2}{1-q} + \frac{q^8}{(1-q)(1-q^3)} + \cdots$$
$$\phi(-q) + \chi(q) = 2F(q).$$
$$f(q) + 2F(q^2) - 2 = \phi(-q^2) + \psi(q)$$
$$= 2\phi(-q^2) - f(q) = \frac{1 - 2q + 2q^4 - 2q^9 + \cdots}{(1-q)(1-q^4)(1-q^6)(1-q^9)\cdots}$$
$$\psi(q) - F(q^2) + 1 = q \cdot \frac{1 + q^2 + q^6 + q^{12} + \cdots}{(1-q^8)(1-q^{12})(1-q^{28})\cdots}$$
Mock ϑ-functions (of 5th order)

Srinivasa Ramanujan:
in celebration of the centenary of his election as a Fellow of the Royal Society

Scientific discussion meeting
Part of the Royal Society scientific programme

Organised by Professor Ken Ono, Professor George E Andrews, Professor Manjul Bhargava and Professor Robert C Vaughan FRS.

15 – 16 October 2018

The Royal Society
6 – 9 Carlton House Terrace, London, SW1Y 5AG

Find out more at **royalsociety.org/events/for-scientists**

THE ROYAL SOCIETY

Image: Extract from Srinivasa Ramanujan's letter on *Mock Theta Functions*.